C H A P T E R 33

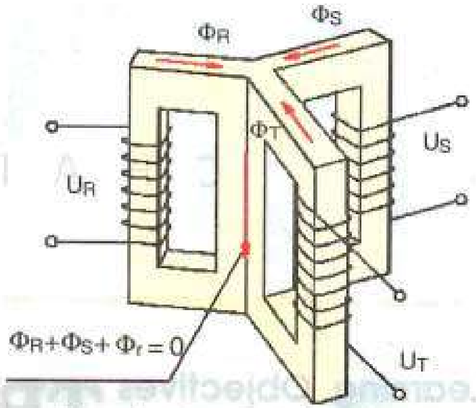
TRANSFORMER: THREE PHASE

Learning Objectives

- Three-phase Transformers
- Three-phase Transformer Connections
- Star/Star or Y/Y Connection
- Delta-Delta or Connection
- Wye/Delta or Y/ Connection
- Delta/Wye or /Y Connection
- Open-Delta or V-V Connection
- Power Supplied by V-V Bank
- Scott Connection or T-T Connection
- Three-phase to Two-Phase Conversion and vice-versa
- Parallel Operation of 3-phase Transformers
- Instrument Transformers
- Current Transformers
- Potential Transformers

Three phase transformers are used throughout industry to change values of three phase voltage and current. Three phase power is the most common way in which power is produced.

33.1. Three-Phase Transformer



Three phase transformer inner circuits

Large scale generation of electric power is usually 3-phase at generated voltages of 13.2 kV or somewhat higher. Transmission is generally accomplished at higher voltages of 110, 132, 275, 400 and 750 kV for which purpose 3-phase transformers are necessary to step up the generated voltage to that of the transmission line. Next, at load centres, the transmission voltages are reduced to distribution voltages of 6,600, 4,600 and 2,300 volts. Further, at most of the consumers, the distribution voltages are still reduced to utilization voltages of 440, 220 or 110 volts. Years ago, it was a common practice to use suitably interconnected three single-phase transformers instead of a single 3-phase transformer. But these days, the latter is gaining popularity because of improvement in design and manufacture but principally because of better acquaintance of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a 3 phase transformer are that it occupies less floor space for equal rating, weighs less, costs about 15% less and further, that only one unit is to be handled and connected.

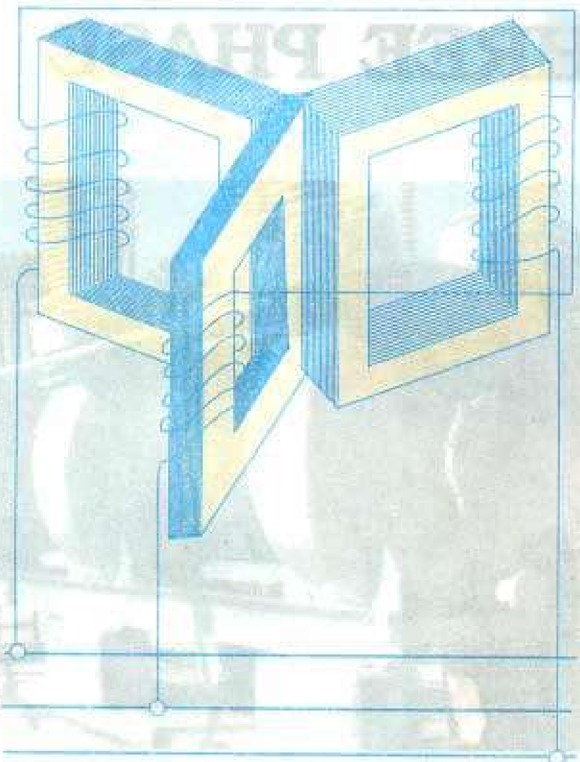


Fig. 33.1

Like single-phase transformers, the three-phase transformers are also of the core type or shell type. The basic principle of a 3-phase transformer is illustrated in Fig. 33.1 in which only primary windings have been shown interconnected in star and put across 3-phase supply. The three cores are 120° apart and their empty legs are shown in contact with each other. The centre leg, formed by these three, carries the flux produced by the three-phase currents I_R, I_Y and I_B. As at any instant $I_R + I_Y + I_B = 0$, hence the sum of three fluxes is also zero. Therefore, it will make no difference if the common leg is removed. In that case any two legs will act as the return for the third just as in a 3-phase system any two conductors act as the return for the current

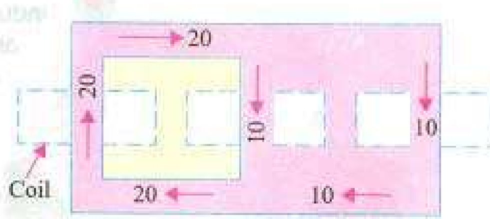


Fig. 33.2 (*a*)

in the third conductor. This improved design is shown in Fig. 33.2 (*a*) where dotted rectangles indicate the three windings and numbers in the cores and yokes represent the directions and magnitudes of fluxes at a particular instant. It will be seen that at any instant, the amount of 'up' flux in any leg is equal to the sum of 'down' fluxes in the other two legs. The core type transformers are usually wound with circular cylindrical coils.

In a similar way, three single-phase shell type transformers can be combined together to form a 3-phase shell type unit as shown in Fig. 33.2(*b*). But some saving in iron can be achieved in constructing a single 3-phase transformer as shown in Fig. 33.3. It does not differ from three single-phase transformers put side by side. Saving in iron is due to the joint use of the magnetic paths between the coils. The three phases, in this case, are more independent than they are in the core type transformers, because each phase has a magnetic circuit independent of the other.

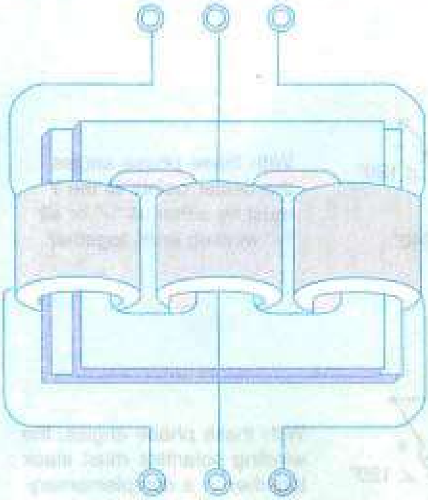


Fig. 33.2 (*b*)

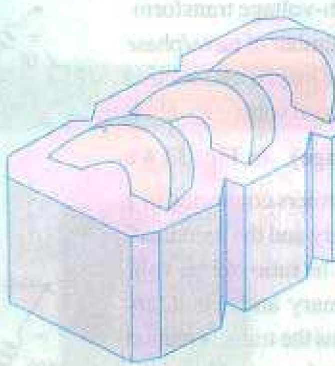
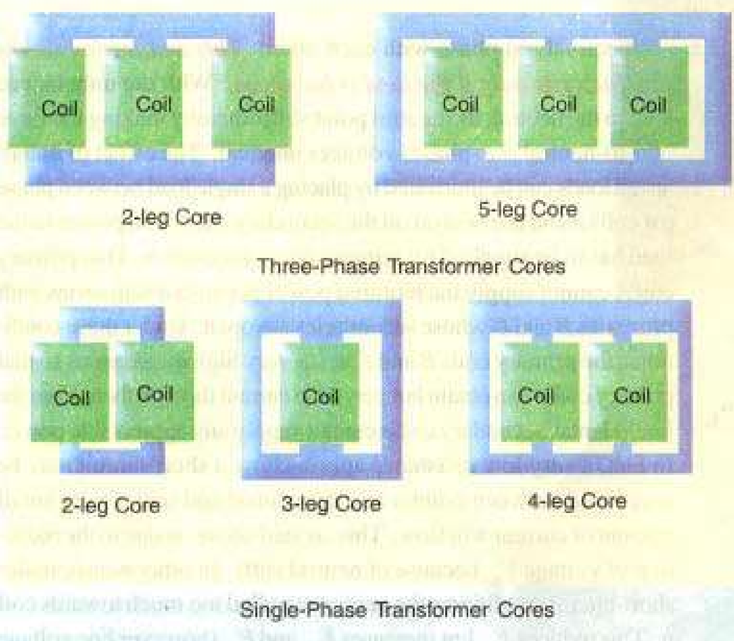


Fig. 33.3



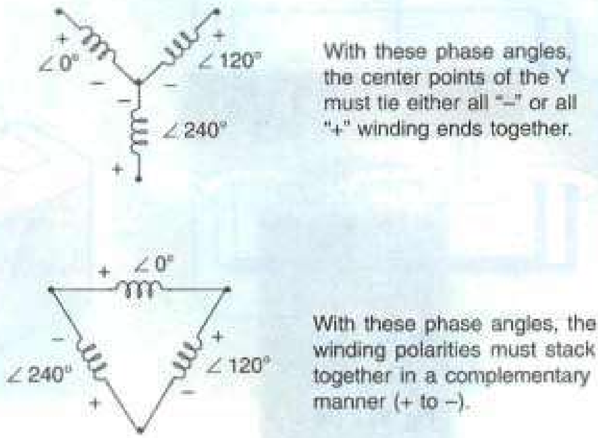
One main drawback in a 3-phase transformer is that if any one phase becomes disabled, then the whole transformer has to be ordinarily removed from service for repairs (the shell type may be operated open

Δ or Vee but this is not always feasible). However, in the case of a 3-phase bank of single-phase transformers, if one transformer goes out of order, the system can still be run open-Δ at reduced capacity or the faulty transformer can be readily replaced by a single spare.

33.2. Three-phase Transformer Connections

There are various methods available for transforming 3-phase voltages to higher or lower 3-phase voltages *i.e.* for handling a considerable amount of power. The most common connections are (*i*) $Y-Y$ (*ii*) $\Delta-\Delta$ (*iii*) $Y-\Delta$ (*iv*) $\Delta-Y$ (*v*) open-delta or $V-V$ (*vi*) Scott connection or $T-T$ connection.

33.3. Star/Star or *Y/Y* Connection



This connection is most economical for small, high-voltage transformers because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only $1/\sqrt{3}$ of line voltage). In Fig. 33.4 a bank of 3 transformers connected in Y on both the primary and the secondary sides is shown. The ratio of line voltages on the primary and secondary sides is the same as the transformation ratio of each transformer. However, there is a phase shift of 30° between the phase voltages and line voltages both on the primary and secondary sides. Of course, line voltages on both sides as well as primary voltages are respectively in phase with each other. ***This connection works satisfactorily only if the load is balanced.*** With the unbalanced load to the neutral, the neutral point shifts thereby making the three line-to-neutral (*i.e.* phase) voltages unequal. The effect of unbalanced loads can be illustrated by placing a single load between phase (or coil) a and the neutral on the secondary side. The power to the load has to be supplied by primary phase (or coil) A. This primary coil A cannot supply the required power because it is in series with primaries B and C whose secondaries are open. Under these conditions, the primary coils B and C act as very high impedances so that primary coil A can obtain but very little current through them from the line. Hence, secondary coil a cannot supply any appreciable power. In fact, a very low resistance approaching a short-circuit may be connected between point a and the neutral and only a very small amount of current will flow. This, as said above, is due to the reduction of voltage E_{an} because of neutral shift. In other words, under short-circuit conditions, the neutral is pulled too much towards coil a. This reduces E_{an} but increases E_{bn} and E_{cn} (however line voltage E_{AB}, E_{BC} and E_{CA} are unaffected). On the primary side, E_{AN} will be

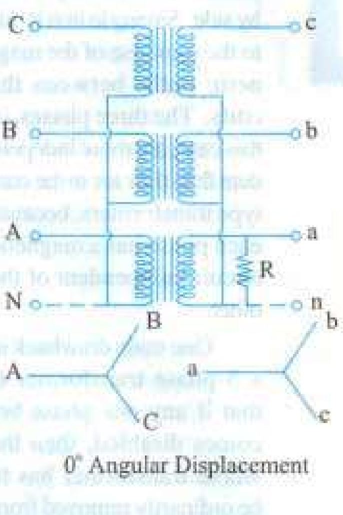


Fig. 33.4

practically reduced to zero whereas E_{BN} and E_{CN} will rise to nearly full primary line voltage. This difficulty of shifting (or floating) neutral can be obviated by connecting the primary neutral (shown dotted in the figure) back to the generator so that primary coil A can take its required power from between its line and the neutral. It should be noted that if a single phase load is connected between the lines a and b, there will be a similar but less pronounced neutral shift which results in an overvoltage on one or more transformers.

Another advantage of stabilizing the primary neutral by connecting it to neutral of the generator is that it eliminates distortion in the secondary phase voltages. This is explained as follows. For delivering a sine wave of voltage, it is necessary to have a sine wave of flux in the core, but on account of the characteristics of iron, a sine wave of flux requires a third harmonic component in the exciting current. As the frequency of this component is thrice the frequency of the circuit, at any given instant, it tends to flow either towards or away from the neutral point in all the three transformers. If the primary neutral is isolated, the triple frequency current cannot flow. Hence, the flux in the core cannot be a sine wave and so the voltages are distorted. But if the primary neutral is earthed *i.e.* joined to the generator neutral, then this provides a path for the triple-frequency currents and e.m.fs. and the difficulty is overcome. Another way of avoiding this trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in Δ and provides a circuit in which the triple-frequency component of the magnetising current can flow (with an isolated neutral, it could not). In that case, a sine wave of voltage applied to the primary will result in a sine wave of phase voltage in the secondary. As said above, the advantage of this connection is that insulation is stressed only to the extent of line to neutral voltage *i.e.* 58% of the line voltage.

33.4. Delta-Delta or $\Delta - \Delta$ Connection

This connection is economical for large, low-voltage transformers in which insulation problem is not so urgent, because it increases the number of turns/phase. The transformer connections and voltage triangles are shown in Fig. 33.5. The ratio of transformation between primary and secondary line voltage is exactly the same as that of each transformer. Further, the secondary voltage triangle *abc* occupies the same relative position as the primary voltage triangle *ABC i.e.* there is no angular displacement between the two. Moreover, there is no internal phase shift between phase and line voltages on either side as was the case in $Y - Y$ connection. This connection has the following advantages :

1. As explained above, in order that the output voltage be sinusoidal, it is necessary that the magnetising current of the transformer must contain a third harmonic component. In this case, the third harmonic component of the magnetising current can flow in the Δ-connected transformer primaries without flowing in the line wires. The three phases are 120° apart which is $3 \times 120 = 360°$ with respect to the third harmonic, hence it merely circulates in the Δ. Therefore, the flux is sinusoidal which results in sinusoidal voltages.

2. No difficulty is experienced from unbalanced loading as was the case in $Y - Y$ connection. The three-phase voltages remain practically constant regardless of load imbalance.

3. An added advantage of this connection is that if one transformer becomes disabled, the system can continue to operate in open-delta or in $V - V$ although with reduced available capacity. The reduced capacity is 58% and not 66.7% of the normal value, as explained in Art. 33.7.

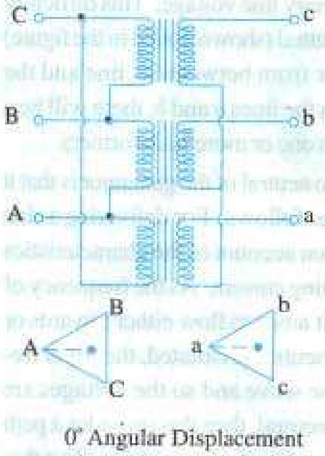


Fig. 33.5

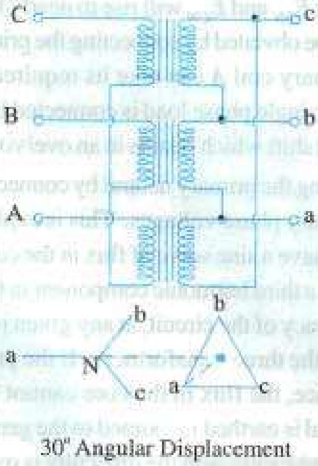


Fig. 33.6

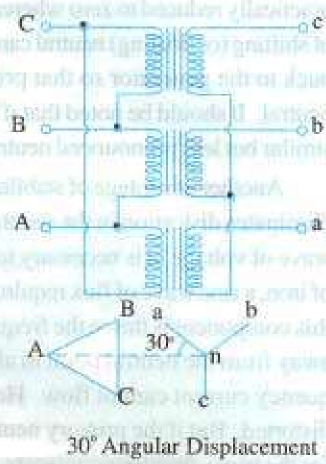


Fig. 33.7

33.5. Wye/Delta or Y/Δ Connection

The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is Y-connected with grounded neutral as shown in Fig. 33.6. The ratio between the secondary and primary line voltage is $1/\sqrt{3}$ times the transformation ratio of each transformer. There is a 30° shift between the primary and secondary line voltages which means that a $Y-\Delta$ transformer bank cannot be paralleled with either a $Y-Y$ or a $\Delta-\Delta$ bank. Also, third harmonic currents flows in the Δ to provide a sinusoidal flux.

33.6. Delta/Wye or Δ/Y Connection

This connection is generally employed where it is necessary to step up the voltage as for example, at the beginning of high tension transmission system. The connection is shown in Fig. 33.7. The neutral of the secondary is grounded for providing 3-phase 4-wire service. In recent years, this connection has gained considerable popularity because it can be used to serve both the 3-phase power equipment and single-phase lighting circuits.

This connection is not open to the objection of a floating neutral and voltage distortion because the existence of a Δ-connection allows a path for the third-harmonic currents. It would be observed that the primary and secondary line voltages and line currents are out of phase with each other by 30°. Because of this 30° shift, it is impossible to parallel such a bank with a $\Delta-\Delta$ or $Y-Y$ bank of transformers even though the voltage ratios are correctly adjusted. The ratio of secondary to primary voltage is $\sqrt{3}$ times the transformation ratio of each transformer.

Example 33.1. *A 3-phase, 50-Hz transformer has a delta-connected primary and star-connected secondary, the line voltages being 22,000 V and 400 V respectively. The secondary has a star-connected balanced load at 0.8 power factor lagging. The line current on the primary side is 5 A. Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in kW ?*

Solution. It should be noted that in three-phase transformers, the *phase* transformation ratio is equal to the turn ratio but the terminal or line voltages depend upon the method of connection employed. The Δ/Y connection is shown in Fig. 33.8.

Fig. 33.8

Phase voltage on primary side $= 22{,}000$ V

Phase voltage on secondary side $= 400/\sqrt{3}$

$\therefore \quad K = 400/22{,}000 \times \sqrt{3} = 1/55\sqrt{3}$

Primary phase current $= 5/\sqrt{3}$ A

Secondary phase current $= \dfrac{5}{\sqrt{3}} \div \dfrac{1}{55\sqrt{3}} = 275$ A

Secondary line current $=$ **275 A**

$\therefore$ Output $= \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 400 \times 275 \times 0.8 =$ **15.24 kW.**

Example 33.2. *A 500-kVA, 3-phase, 50-Hz transformer has a voltage ratio (line voltages) of 33/11-kV and is delta/star connected. The resistances per phase are : high voltage 35 Ω, low voltage 0.876 Ω and the iron loss is 3050 W. Calculate the value of efficiency at full-load and one-half of full-load respectively (a) at unity p.f. and (b) 0.8 p.f.* **(Electrical Machinery, Madras Univ. 1985)**

Solution. Transformation ratio $K = \dfrac{11{,}000}{\sqrt{3} \times 33{,}000} = \dfrac{1}{3\sqrt{3}}$

Per phase $\quad R_{02} = 0.876 + (1/3\sqrt{3})^2 \times 35 = 2.172\,\Omega$

Secondary phase current $= \dfrac{500{,}000}{\sqrt{3} \times 11{,}000} = \dfrac{500}{11\sqrt{3}}$ A

Full-load condition

Full load total Cu loss $= 3 \times (500/11\sqrt{3})^2 \times 2.172 = 4{,}490$ W; Iron loss $= 3{,}050$ W

Total full-load losses $= 4{,}490 + 3{,}050 = 7{,}540$ W; Output at unity p.f. $= 500$ kW

$\therefore$ F.L. efficiency $= 500{,}000/507{,}540 = 0.9854$ or **98.54%** ; Output at 0.8 p.f. $= 400$ kW

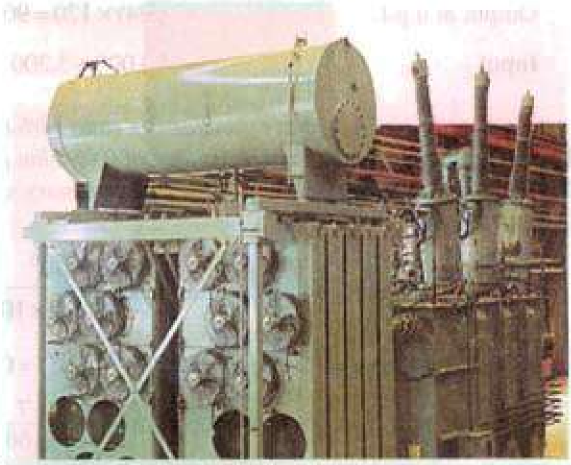
$\therefore$ Efficiency $= 400{,}000/407.540$
$= 0.982$ or **98.2%**

Half-load condition

Output at unity p.f. $= 250$ kW

Cu losses $= (1/2)^2 \times 4{,}490$
$=$ **1,222 W**

Total losses
$= 3{,}050 + 1{,}122 = 4{,}172$ W



Three-phase transformer

$\therefore \quad \eta = 250{,}000/254{,}172 = 0.9835 = \mathbf{98.35\ \%}$

Output at 0.8 p.f. $= 200$ kW $\therefore \quad \eta = \mathbf{200{,}000/204{,}172 = 0.98 \quad or \quad 98\%}$

Example 33.3. *A 3-phase, 6,600/415-V, 2,000-kVA transformer has a per unit resistance of 0.02 and a per unit leakage reactance of 0.1. Calculate the Cu loss and regulation at full-load 0.8 p.f. lag.* **(Electrical Machines-I, Bombay Univ. 1987)**

Solution. As seen from Art. 27 – 16, $\% R = \%$ Cu loss $= \dfrac{\text{Cu loss}}{VA} \times 100$

Now, $\% R = 0.02 \times 100 = 2\ \%$ $\quad \therefore \quad 2 = \dfrac{\text{Cu loss}}{2{,}000} \times 100 \quad \therefore$ Cu loss $=$ **40 kW**

Now, percentage leakage reactance $= 0.1 \times 100 = 10\%$

$$\text{Regn.} = v_r \cos\phi + v_x \sin\phi = 2 \times 0.8 + 10 \times 0.6 = \mathbf{7.6\%}$$

Example 33.4. *A 120-kVA, 6,000/400-V, Y/Y 3-ph, 50-Hz transformer has an iron loss of 1,600 W. The maximum efficiency occurs at 3/4 full load.*

Find the efficiencies of the transformer at

(i) full-load and 0.8 power factor *(ii) half-load and unity power factor*

(iii) the maximum efficiency. **(Elect. Technology Utkal Univ. 1987)**

Solution. Since maximum efficiency occurs at 3/4 full-load, Cu loss at 3/4 full-load equals iron loss of 1,600 W.

Cu loss at 3/4 F.L. $= 1{,}600$ W; Cu loss at F.L. $= 1{,}600 \times (4/3)^2 = 2{,}845$ W

(i) F.L. output at 0.8 p.f. $= 120 \times 0.8 = 96$ kW $= 96{,}000$ W

Total loss $= 1{,}600 + 2{,}845 = 4{,}445$ W

$$\therefore \quad \eta = \frac{96{,}000}{100{,}445} \times 100 = \mathbf{95.57\ \%}$$

(ii) Cu loss at 1/2 full-load $= (1/2)^2 \times 2{,}845 = 710$ W

Total loss $= 710 + 1{,}600 = 2310$ W

Output at 1/2 F.L. and u.p.f. is $= 60$ kW $= 60{,}000$ W; $\eta = \dfrac{60{,}000}{62{,}310} \times 100 = \mathbf{96.57\%}$

(iii) Maximum efficiency occurs at 3/4 full-load when iron loss equals Cu loss.

Total loss $= 2 \times 1{,}600 = 3{,}200$ W

Output at u.p.f. $= (3/4) \times 120 = 90$ kW $= 90{,}000$ W

Input $= 90{,}000 + 3{,}200 = 93{,}200$ W $\therefore \eta = \dfrac{90{,}000}{93{,}200} \times 100 = \mathbf{96.57\%}$

Example 33.5. *A 3-phase transformer, ratio 33/6.6-kV, Δ/Y, 2-MVA has a primary resistance of 8 Ω per phase and a secondary resistance of 0.08 ohm per phase. The percentage impedance is 7%. Calculate the secondary voltage with rated primary voltage and hence the regulation for full-load 0.75 p.f. lagging conditions.* **(Elect. Machine-I, Nagpur, Univ. 1993)**

Solution. F.L. secondary current $= \dfrac{2 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 175$ A

$K = 6.6/\sqrt{3} \times 33 = 1/8.65$; $R_{02} = 0.08 + 8/8.65^2 = 0.1867\ \Omega$ per phase

Now, secondary impedance drop per phase $= \dfrac{7}{100} \times \dfrac{6{,}600}{\sqrt{3}} = 266.7$ V

$$\therefore \quad Z_{02} = 266.7/175 = 1.523\ \Omega \text{ per phase}$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.523^2 - 0.1867^2} = 1.51\ \Omega/\text{phase}$$

Drop per phase $= I_2 (R_{02} \cos\phi + X_{02} \sin\phi) = 175\ (0.1867 \times 0.75 + 1.51 \times 0.66) = 200$ V

Secondary voltage/phase $= 6,600/\sqrt{3} = 3,810$ V $\quad\therefore\quad V_2 = 3,810 - 200 = 3,610$ V

$\therefore$ Secondary *line* voltage $= 3,610 \times \sqrt{3} =$ **6,250 V**

%regn. $= 200 \times 100/3,810 =$ **5.23 %**

Example 33.6. *A 100-kVA, 3-phase, 50-Hz 3,300/400 V transformer is Δ-connected on the h.v. side and Y-connected on the l.v. side. The resistance of the h.v. winding is 3.5 Ω per phase and that of the l.v. winding 0.02 Ω per phase. Calculate the iron losses of the transformer at normal voltage and frequency if its full-load efficiency be 95.8% at 0.8 p.f. (lag).*

(A.C. Machines-I, Jadavpur Univ. 1989)

Solution. F.L. output $= 100 \times 0.8 = 80$ kW; Input $= 80/0.958 = 83.5$ kW

Total loss $=$ Input $-$ Output $= 83.5 - 80 = 3.5$ kW

Let us find full-load Cu losses for which purpose, we would first calculate R_{02}.

$$K = \frac{\text{secondary voltage/phase}}{\text{primary voltage/phase}} = \frac{400/\sqrt{3}}{3,300} = \frac{4}{33\sqrt{3}}$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (4/\sqrt{3} \times 33)^2 \times 3.5 = 0.037\ \Omega$$

Full-load secondary phase current is $I_2 = 100,000/\sqrt{3} \times 400 = 144.1$ A

Total Cu loss $= 3 I_2^2 R_{02} = 3 \times 144.1^2 \times 0.037 = 2,305$ W

Iron loss $=$ Total loss $-$ F.L. Cu loss $= 3,500 - 2,305 =$ **1,195 W**

Example 33.7. *A 5,000-kVA, 3-phase transformer, 6.6/33-kV, Δ/Y, has a no-load loss of 15 kW and a full-load loss of 50 kW. The impedance drop at full-load is 7%. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV.*

Solution. Full-load $\quad I_2 = 5 \times 10^6/\sqrt{3} \times 33,000 = 87.5$ A

Impedance drop/phase $= 7\%$ of $(33/\sqrt{3}) = 7\%$ of 19 kV $= 1,330$ V

$\therefore\quad Z_{02} = 1,330/87.5 = 15.3\ \Omega$/phase; F.L. Cu loss $= 50 - 15 = 35$ kW

$\therefore\quad 3 I_2 R_{02} = 35,000; \quad R_{02} = 35,000/3 \times 8.75^2 = 1.53\ \Omega$/phase

$\therefore\quad X_{02} = \sqrt{15.3^2 - 1.53^2} =$ **15.23 Ω**

When load is 3,200 kW at 0.8 p.f.

$I_2 = 3,200/\sqrt{3} \times 33 \times 0.8 = 70$ A; drop $= 70\,(1.53 \times 0.8 + 15.23 \times 0.6) = 725$ V/phase

$$\therefore\ \%\ \text{regn.} = \frac{725 \times 100}{19,000} = 3.8\ \%$$

Primary voltage will have to be increased by 3.8%.

$\therefore$ Primary voltage $= 6.6 + 3.8\%$ of $6.6 = 6.85$ kV $=$ **6,850 V**

Example 33.8. *A 3-phase transformer has its primary connected in Δ and its secondary in Y. It has an equivalent resistance of 1% and an equivalent reactance of 6%. The primary applied voltage is 6,600 V. What must be the ratio of transformation in order that it will deliver 4,800 V at full-load current and 0.8 power factor (lag) ?* **(Elect. Technology-II, Magadh Univ. 1991)**

Solution. Percentage regulation

$$= v_r \cos\phi + v_x \sin\phi$$

$= 1 \times 0.8 + 6 \times 0.6 = 4.4\%$

Induced secondary e.m.f. (line value)

$= 4{,}800 + 4.4\%$ of $4{,}800 = 5{,}010$ V, as in Fig. 33.9.

Secondary phase voltage

$= 5{,}010/\sqrt{3} = 2{,}890$ V

Transformation ratio

$K = 2{,}890/6{,}600 =$ **0.437.**

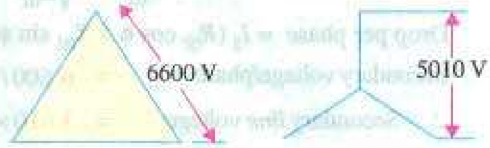


Fig. 33.9

Example 33.9. *A 2000-kVA, 6,600/400-V, 3-phase transformer is delta-connected on the high voltage side and star-connected on the low-voltage side. Determine its % resistance and % reactance drops, % efficiency and % regulation on full load 0.8 p.f. leading given the following data :*

S.C. test ; H.V. data : 400 V, 175 A and 17 kW

O.C. test; L.V. data : 400 V, 150 A and 15 kW **(Basic Elect., Machines Nagpur Univ. 1993)**

Solution. From S.C. test data, we have

Primary voltage/phase $= 400$ V; Primary current/phase $= 175/\sqrt{3} = 100$ A

$$\therefore \quad Z_{01} = \frac{400}{101} = 3.96\ \Omega$$

$$I_1^2 R_{01} = \frac{17000}{3} \text{ or } R_{01} = 0.555\ \Omega;\ X_{01} = \sqrt{3.96^2 - 0.555^2} = 3.92\ \Omega$$

$$\% R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{101 \times 0.555}{6{,}600} \times 100 = \mathbf{0.849}$$

$$\% X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{101 \times 3.92}{6{,}600} \times 100 = \mathbf{6}$$

$$\% \text{ regn} = v_r \cos\phi - v_x \sin\phi = 0.49 \times 0.8 - 6 \times 0.6 = -2.92\%$$

Full-load primary line current can be found from

$$\sqrt{3} \times 6{,}600 \times I_1 = 2000 \times 1{,}000;\ I_1 = 175 \text{ A}$$

It shows that S.C. test has been carried out under full-load conditions.

Total losses $= 17 + 15 = 32$ kW; F.L. output $= 2{,}000 \times 0.8 = 1600$ kW

$$\eta = 1{,}600/1{,}632 = 0.98 \text{ or } \mathbf{98\%}$$

Example 33.10. *A 3-ph, delta/star connected 11,000/440 V, 50 Hz transformer takes a line current of 5 amp, when secondary Load of 0.8 Lagging p.f. is connected. Determine each coil-current and output of transformer.* **(Amravati Univ. 1999)**

Solution. Due to delta/star connections the voltage ratings of the two sides on per phase basis are :

Primary coil rating $= 11{,}000$ V, Secondary coil rating $= \dfrac{440}{\sqrt{3}} = 254$ volts

Primary coil-current $= 5/\sqrt{3} = 2.887$ amp

Each coil is delivering equal volt. amps.

Since three phase volt – amps $= 3 \times 11{,}000 \times 5/\sqrt{3}$

$= 95266$

Volts amps/phase $= 31755$

This corresponds to the secondary coil-current of I_2, given by

$$I_2 = \frac{31755}{254} = 125 \text{ amp. This is shown in Fig. 33.10.}$$

Total Output of transformer, in kVA = 31,755

Since, the p.f. given is 0.8 lagging.

The total output power in kW $= 31.755 \times 0.80 = $ **25.4 kW**

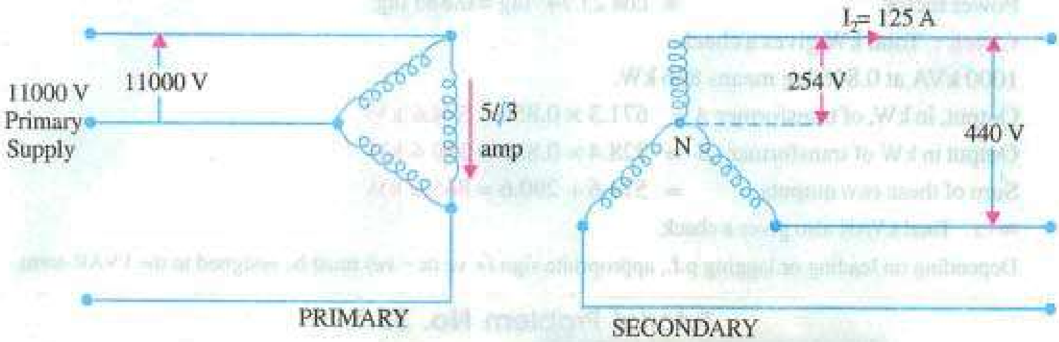


Fig. 33.10. Transformer coil currents

Example 33.11. *A load of 1000 kVA at 0.866 p.f. lagging is supplied by two 3 phase transformers of 800 kVA capacity operating in parallel. Ratio of transformation is same : 6600/400 V, delta/star. If the equivalent impedances referred to secondary are (0.005 + j.015) ohm and (0.012 + j 0.030) ohm per phase respectively. Calculate load and power factor of each transformer.*

(Amravati Univ. 1999)

Solution. Total load $= 1000$ kVA

$\cos\phi = 0.866$ Lag, $\phi = 30°$ lag

Total power output $= 866$ kW

Secondary current with star connection,

$$I_2 = \frac{1000}{\sqrt{3}\times 440}\times 1000 = \textbf{1312.2 amp}$$

If the two transformers are identified as *A* and *B*, with their parameters with subscripts of *a* and *b*, we have :

$$\dot{Z}_a = 0.005 + j\,0.015 = 0.0158 \angle 71.56° \text{ ohm}$$

$$\dot{Z}_b = 0.012 + j\,0.030 = 0.0323 \angle 68.2° \text{ ohm}$$

$$\dot{Z}_a + \dot{Z}_b = 0.017 + j0.045 = 0.0481 \angle 69.3° \text{ ohm}$$

$\dot{I}_{2a}$ = secondary current of transformer *A*

$\dot{I}_{2b}$ = secondary current of transformer *B*

$$\dot{I}_{2a} = \frac{\dot{Z}_b}{\dot{Z}_b + \dot{Z}_a}\times \mathbf{I}_2 = \frac{0.0323 \angle 68.2°}{0.0481 \angle 69.3°}\times 1312 \angle -30°$$

$$= 88.1 \angle -31.1°$$

$$\dot{I}_{2b} = \frac{\dot{Z}_a}{\dot{Z}_a + \dot{Z}_b}\times \mathbf{I}_2 = \frac{0.0158 \angle 71.56°}{0.0481 \angle 69.3°}\times 1312 \angle -30°$$

$$= 43.1 \angle -27.74°$$

For Transformer A

Load $= 3 \times 254 \times 881 \times 10^{-3} = 671.3$ kVA

Power factor $= \cos 31.1° \text{ lag} = 0.856 \text{ lag}$

For Transformer *B*

Load $= 3 \times 254 \times 431 \times 10^{-3} = 328.4 \text{ kVA}$

Power factor $= \cos 23.74° \text{ lag} = 0.885 \text{ lag}$

Check : Total kW gives a check.

1000 kVA at 0.866 lag means 866 kW.

Output, in kW, of transformer A = $671.3 \times 0.856 =$ **574.6 kW**

Output in kW of transformer B = $328.4 \times 0.885 =$ **290.6 kW**

Sum of these two outputs = $574.6 + 290.6 =$ **865.2 kW**

Note. Total kVAR also gives a check.

Depending on leading or lagging p.f., appropriate sign (+ *ve* or − *ve*) must be assigned to the kVAR-term.

Tutorial Problem No. 33.1

1. A 3-phase, star-connected alternator generates 6,360 V per phase and supplies 500 kW at a p.f. 0.9 lagging to a load through a step-down transformer of turns 40 : 1. The transformer is delta connected on the primary side and star-connected on the secondary side. Calculate the value of the line volts at the load. Calculate also the currents in (*a*) alternator windings (*b*) transformer primary windings (*c*) transformen secondary windings. **[476 V (*a*) 29.1 A (*b*) 16.8 A (*c*) 672 A]**

2. A 11,000/6,600 V, 3-ϕ, transformer has a star-connected primary and a delta-connected secondary. It supplies a 6.6 kV motor having a star-connected stator, developing 969.8 kW at a power factor of 0.9 lagging and an efficiency of 92 per cent. Calculate (*i*) motor line and phase currents (*ii*) transformer secondary current and (*iii*) transformer primary current.

[(*a*) Motor ; $I_L = I_{ph} = 126.3$ A (*b*) phase current 73 A (*c*) 75.8 A]

33.7. Open-Delta or $V - V$ connection

If one of the transformers of a $\Delta - \Delta$ is removed and 3-phase supply is connected to the primaries as shown in Fig. 33.11, then three equal 3-phase voltages will be available at the secondary terminals on no-load. This method of transforming 3-phase power by means of only two transformers is called the open $- \Delta$ or $V - V$ connection.

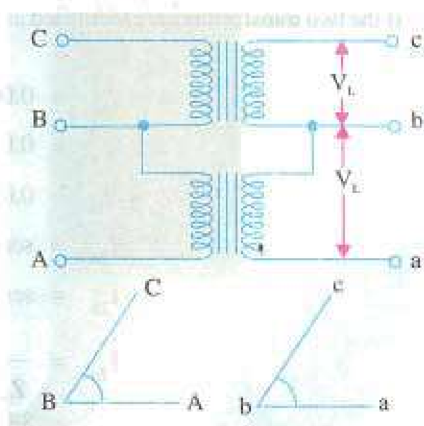


Fig. 33.11

It is employed :

1. when the three-phase load is too small to warrant the installation of full three-phase transformer bank.
2. when one of the transformers in a $\Delta - \Delta$ bank is disabled, so that service is continued although at reduced capacity, till the faulty transformer is repaired or a new one is substituted.
3. when it is anticipated that in future the load will increase necessitating the closing of open delta.

One important point to note is that the total load that can be carried by a $V - V$ bank is *not* two-third of the capacity of a $\Delta - \Delta$ bank but it is only 57.7% of it. That is a reduction of 15% (strictly, 15.5%) from its normal rating. Suppose there is $\Delta - \Delta$ bank of three 10-kVA transformers. When one transformer is removed, then it runs in $V - V$. The total rating of the two transformers is 20 kVA. But the capacity of the $V - V$ bank is not the sum of the transformer kVA ratings but only 0.866 of it *i.e.* $20 \times 0.866 = 17.32$ (or $30 \times 0.57 = 17.3$ kVA). The fact that the ratio of V-capacity to Δ-capacity is $1/\sqrt{3} = 57.7\%$ (or nearly 58%) instead of $66\frac{2}{3}$ per cent can be proved as follows :

As seen from Fig. 33.12 (*a*)

$$\Delta - \Delta \text{ capacity} = \sqrt{3} \,.\, V_L \,.\, I_L = \sqrt{3} \,.\, V_L\,(\sqrt{3}\,.\,I_S) = 3V_L I_S$$

In Fig. 33.12 (*b*), it is obvious that when $\Delta - \Delta$ bank becomes $V - V$ bank, the secondary line current I_L becomes equal to the secondary phase current I_S.

$$\therefore \qquad V - V \text{ capacity} = \sqrt{3}\,.\,V_L I_L = \sqrt{3}\,V_L\,.\,I_S$$

$$\therefore \qquad \frac{V - V \text{ capacity}}{\Delta - \Delta \text{ capacity}} = \frac{\sqrt{3}\,.\,V_L I_S}{3V_L\; I_S} = \frac{1}{\sqrt{3}} = 0.577 \text{ or } 58 \text{ per cent}$$

It means that the 3-phase load which can be carried ***without exceeding the ratings*** of the transformers is 57.7 per cent of the original load rather than the expected 66.7%.

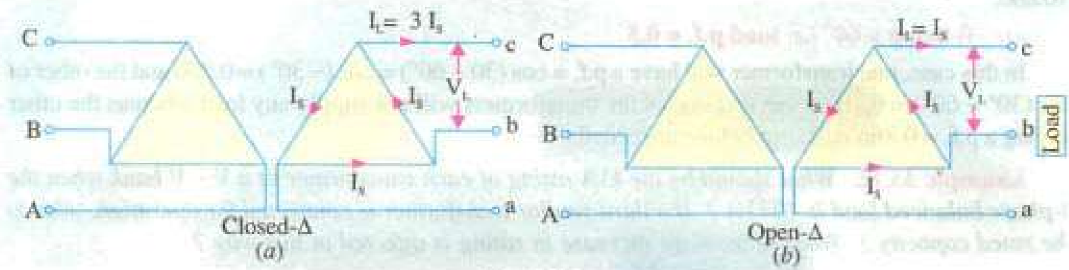


Fig. 33.12

It is obvious from above that when one transformer is removed from a $\Delta - \Delta$ bank.

1. the bank capacity is reduced from 30 kVA to $30 \times 0.577 = 17.3$ kVA and not to 20 kVA as might be thought off-hand.
2. only 86.6% of the rated capacity of the two remaining transformers is available (*i.e.* $20 \times 0.866 = 17.3$ kVA). In other words, ratio of *operating* capacity to *available* capacity of an open-Δ is 0.866. This factor of 0.866 is sometimes called the ***utility factor***.
3. each transformer will supply 57.7% of load and not 50% when operating in $V - V$ (Ex. 33.13).

However, it is worth noting that if three transformers in a $\Delta - \Delta$ bank are delivering their *rated load** and one transformer is removed, the overload on *each* of the two remaining transformers is 73.2% because

$$\frac{\text{total load in } V - V}{VA/\text{transformer}} = \frac{\sqrt{3}\,.\,V_L I_S}{V_L I_S} = \sqrt{3} = 1.732$$

This over-load may be carried temporarily but some provision must be made to reduce the load if overheating and consequent breakdown of the remaining two transformers is to be avoided.

The disadvantages of this connection are :

1. The average power factor at which the *V*-bank operates is less than that of the load. ***This power factor is actually 86.6% of the balanced load power factor.*** Another significant point to note is that, except for a balanced unity power factor load, the two transformers in the $V - V$ bank operate at different power factors (Art. 33.8).

2. Secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

It may, however, be noted that if two transformers are operating in $V - V$ and loaded to rated capacity (in the above example, to 17.3 kVA), the addition of a third transformer increases the total capacity by $\sqrt{3}$ or 173.2% (*i.e.* to 30 kVA). It means that for an increase in cost of 50% for the third transformer, the increase in capacity is 73.2% when converting from a $V - V$ system to a $\Delta - \Delta$ system.

* In Ex. 33.13, the three transformers are not supplying their rated load of $20 \times 3 = 60$ kVA but only 40 kVA.

33.8. Power Supplied by $V - V$ Bank

When a $V - V$ bank of two transformers supplies a balanced 3-phase load of power factor $\cos \phi$, then one transformer operates at a p.f. of $\cos (30° - \phi)$ and the other at $\cos (30° + \phi)$. Consequently, the two transformers will not have the same voltage regulation.

$\therefore \quad P_1 = \text{kVA} \cos (30° - \phi)$ and $P_2 = \text{kVA} \cos (30° + \phi)$

(i) When $\phi = 0$ *i.e.* load p.f. = 1

Each transformer will have a p.f. $= \cos 30° = 0.866$

(ii) When $\phi = 30°$ *i.e.* load p.f. = 0.866.

In this case, one transformer has a p.f. of $\cos (30° - 30°) = 1$ and the other of $\cos (30° + 30°) = 0.866$.

(iii) When $\phi = 60°$ *i.e.* load p.f. = 0.5

In this case, one transformer will have a p.f. $= \cos (30 - 60°) = \cos (-30°) = 0.866$ and the other of $\cos (30° + 60°) = 0$. It means that one of the transformers will not supply any load whereas the other having a p.f. = 0.866 will supply the entire load.

Example 33.12. *What should be the kVA rating of each transformer in a $V - V$ bank when the 3-phase balanced load is 40 kVA ? If a third similar transformer is connected for operation, what is the rated capacity ? What percentage increase in rating is affected in this way ?*

Solution. As pointed out earlier, the kVA rating of each transformer has to be 15% greater.

$\therefore$ kVA/trasformer $= (40/2) \times 1.15 = \mathbf{23}$

$\Delta - \Delta$ bank rating $= 23 \times 3 = 69$; Increase $= [(69 - 40)/ 40] \times 100 = \mathbf{72.5\%}$

Example 33.13. *A $\Delta - \Delta$ bank consisting of three 20-kVA, 2300/230-V transformers supplies a load of 40 kVA. If one transformer is removed, find for the resulting $V - V$ connection*

(i) *kVA load carried by each transformer*

(ii) *per cent of rated load carried by each transformer*

(iii) *total kVA rating of the V-V bank*

(iv) *ratio of the V-V bank to $\Delta - \Delta$ bank transformer ratings.*

(v) *per cent increase in load on each transformer when bank is converted into V-V bank.*

Solution. (i) As explained earlier in Art. 33.7, $\dfrac{\text{total kVA load in } V - V \text{ bank}}{\text{VA/transformer}} = \sqrt{3}$

$\therefore$ kVA load supplied by *each* of the two transformers $= 40/\sqrt{3} = \mathbf{23.1\ kVA}$

Obviously, each transformer in V - V bank does not carry 50% of the original load but 57.7%.

(ii) per cent of rated load $= \dfrac{\text{kVA load/transformer}}{\text{kVA rating/transformer}} = \dfrac{23.1}{20} = \mathbf{115.5\ \%}$

carried by each transformer.

Obviously, in this case, each transformer is overloaded to the extent of 15.5 per cent.*

(iii) kVA *rating* of the V - V bank $= (2 \times 20) \times 0.866 = \mathbf{34.64\ kVA}$

(iv) $\dfrac{V - V \text{ rating}}{\Delta - \Delta \text{ rating}} = \dfrac{34.64}{60} = 0.577$ or **57.7%**

---

* Overloading becomes 73.2% only when full rated load is supplied by the $\Delta - \Delta$ bank (*i.e.* $3 \times 20 = 60$ kVA in this case) before it becomes $V - V$ bank.

As seen, the rating is reduced to 57.7% of the original rating.

(*v*) Load supplied by each transformer in $\Delta - \Delta$ bank = 40/3 = 13.33 kVA

$\therefore$ Percentage increase in load supplied by each transformer

$$= \frac{\text{kVA load/transformer in } V - V \text{ bank}}{\text{kVA load/transformer in } \Delta - \Delta \text{ bank}} = \frac{23.1}{13.3} = 1.732 = \mathbf{173.2\ \%}$$

It is obvious that each transformer in the $\Delta - \Delta$ bank supplying 40 kVA was running underloaded (13.33 *vs* 20 kVA) but runs overloaded (23.1 *vs* 20 kVA) in $V - V$ connection.

Example 33.14. *A balanced 3-phase load of 150 kW at 1000 V, 0.866 lagging power factor is supplied from 2000 V, 3-phase mains through single-phase transformers (assumed to be ideal) connected in (i) delta-delta (ii) Vee-Vee. Find the current in the windings of each transformer and the power factor at which they operate in each case. Explain your calculations with circuit and vector diagrams.*

Solution. (*i*) Delta-Delta Connection

$$\sqrt{3}\ V_L I_L \cos\phi = 150{,}000$$

$$\sqrt{3} \times 1000 \times I_L \times 0.866 = 150{,}000 \therefore I_L = 100 \text{ A}$$

$\therefore$ Secondary line current = 100 A; secondary phase current = $100/\sqrt{3}$ = **57.7 A**

Transformation ratio = 1000/2000 = 1/2

$\therefore$ Primary phase current = 57.7/2 = **28.85 A**

(*ii*) Vee-Vee Connection

Let I be the secondary line current which is also the phase current in $V - V$ connection. Then

$$\sqrt{3} \times 1000 \times I \times 0.866 = 150{,}000 \quad \therefore \quad I = 100 \text{ A}$$

$\therefore$ Secondary phase current = **100 A**; primary phase current = 100 × 1/2 = **50 A**

Transformer power factor = 86.6 per cent of 0.866 = **0.75 (lag).**

Example 33.15. (*a*) *Two identical 1-phase transformers are connected in open-delta across 3-phase mains and deliver a balanced load of 3000 kW at 11 kV and 0.8 p.f. lagging. Calculate the line and phase currents and the power factors at which the two transformers are working.*

(*b*) *If one more identical unit is added and the open delta is converted to closed delta, calculate the additional load of the same power factor that can now be supplied for the same temperature rise. Also calculate the phase and line currents.* **(Elect. Machinery-I, Madras Univ. 1987)**

Solution. (*a*) If I is the line current, then

$$\sqrt{3} \times 11{,}000 \times I \times 0.8 = 3{,}000{,}000 \qquad \mathbf{I = 197\ A}$$

Since, this also represents the phase current,

$\therefore$ Secondary phase current = **197 A**; Transformer p.f. = 86.6 per cent of 0.8 = **0.693**

(*b*) Additional load = 72.5 per cent of 3000 = **2175 kW**

Total load = 3000 + 2175 = 5175 kW

Now, $\sqrt{3} \times V_L I_L \cos\phi$ = 5,175,000 or $\sqrt{3} \times 11{,}000 \times I_L \times 0.8$ = 5,175,000

$\therefore$ I_L = **340 A**; phase current = $340/\sqrt{3}$ = **196 A**

Example 33.16. *Two transformers connected in open delta supply a 400-kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V. What is the (a) kVA supplied by each transformer ? (b) kW supplied by each transformer ?* **(Elect. Machines-I, Gwalior Univ. 1991)**

Solution. As stated in Art 33.7, the ratio of operating capacity to available capacity in an open-Δ is 0.866. Hence, kVA of each transformer is one-half of the total kVA load divided by 0.866.

(*a*) kVA of each transformer $= \dfrac{(400/2)}{0.866} = $ **231 kVA**

(*b*) As stated in Art 33.8, the two transformers have power factors of cos (30° − φ) and cos (30° + φ).

$\therefore \quad P_1 = \text{kVA} \cos(30° - \phi)$ and $P_2 = \text{kVA}\cos(30 + \phi)$

Now, load p.f. $= \cos\phi = 0.866; \quad \phi = \cos^{-1}(0.866) = 30°$

$\therefore \quad P_1 = 231 \times \cos 0° = $ **231 kW**; $P_2 = 231 \times \cos 60° = $ **115.5 kW**

Obviously, $P_1 + P_2$ must equal $400 \times 0.86 = 346.5$ kW

Tutorial Problem No. 33.2

1. Three 1100/110-V transformers connected delta-delta supply a lighting load of 100 kW. One of the transformers is damaged and removed for repairs. Find
   (*a*) What currents were flowing in each transformer when the three transformers were in service ?
   (*b*) What current flows in each transformer when the third is removed ? and
   (*c*) The output kVA of each transformer if the transformers connected in open Δ supply the full-load with normal heating ?

[(*a*) primary = 30.3 A; secondary 303 A (*b*) primary = 30.3 $\sqrt{3}$A ; secondary = 303$\sqrt{3}$A (*c*) 33.33 kVA] ***(Elect. Machines-I, Gwalior Univ. Apr. 1977)***

33.9. Scott Connection or *T - T* Connection

This is a connection by which 3-phase to 3-phase transformation is accomplished with the help of two transformers as shown in Fig. 33.13. Since it was first proposed by Charles *F*. Scott, it is frequently referred to as Scott connection. This connection can also be used for 3-phase to 2-phase transformation as explained in Art. 33.10.

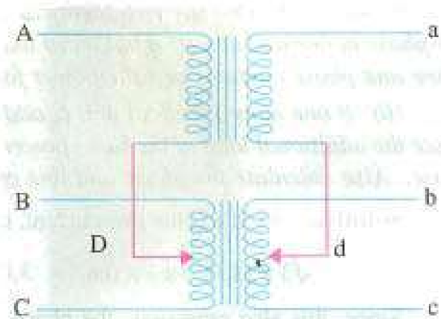


Fig. 33.13

One of the transformers has centre taps both on the primary and secondary windings (Fig. 33.13) and is known as the ***main*** transformer. It forms the horizontal member of the connection (Fig. 33.14).

The other transformer has a 0.866 tap and is known as ***teaser*** transformer. One end of both the primary and secondary of the teaser transformer is joined to the centre taps on both primary and secondary of the main transformer respectively as shown in Fig. 33.14 (*a*). The other end *A* of the teaser primary and the two ends *B* and *C* of the main transformer primary are connected to the 3-phase supply.

The voltage diagram is shown in Fig. 33.14 (*a*) where the 3-phase supply line voltage is assumed to be 100 V and a transformation ratio of unity. For understanding as to how 3-phase transformation results from this arrangement, it is desirable to think of the primary and secondary vector voltages as forming geometrical T_S' (from which this connection gets its name).

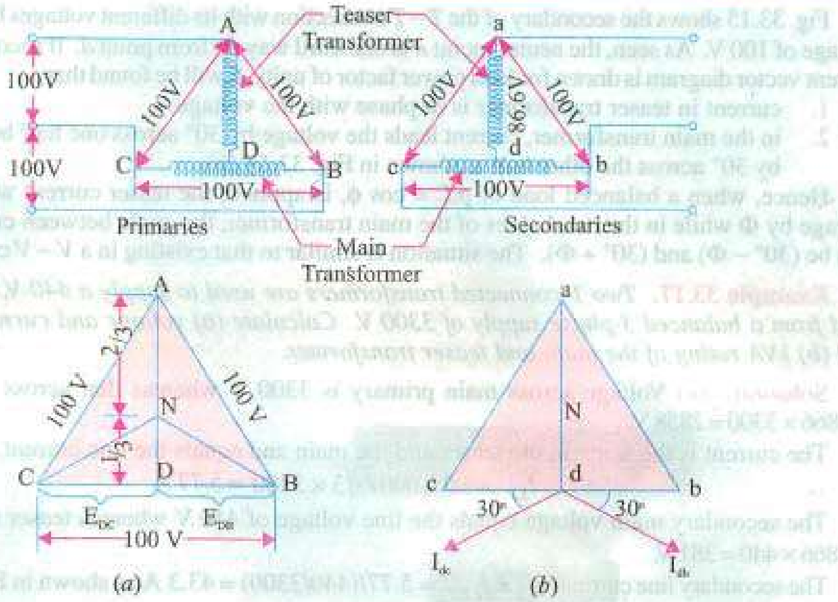


Fig. 33.14

In the primary voltage T of Fig. 33.14 (a), E_{DC} and E_{DB} are each 50 V and differ in phase by 180°, because both coils DB and DC are on the same magnetic circuit and are connected in opposition. Each side of the equilateral triangle represents 100 V. The voltage E_{DA} being the altitude of the equilateral triangle is equal to $(\sqrt{3}/2) \times 100 = 86.6$ V and lags behind the voltage across the main by 90°. The same relation holds good in the secondary winding so that abc is a symmetrical 3-phase system.

With reference to the secondary voltage triangle of Fig. 33.14 (b), it should be noted that for a load of unity power factor, current I_{db} lags behind voltage E_{db} by 30° and I_{dc} leads E_{dc} by 30°. In other words, the teaser transformer and each half of the main transformer, all operate at different power factors.

Obviously, the full rating of the transformers is not being utilized. The teaser transformer operates at only 0.866 of its rated voltage and the main transformer coils operate at cos 30° = 0.866 power factor, which is equivalent to the main transformer's coils working at 86.6 per cent of their kVA rating. Hence the capacity to rating ratio in a T–T. connection is 86.6% — the same as in $V - V$ connection if two identical units are used, although heating in the two cases is not the same.

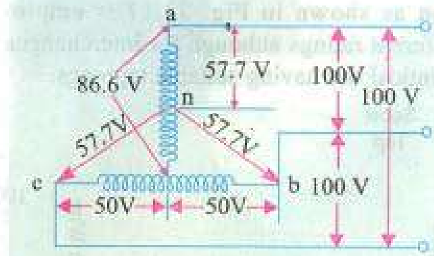


Fig. 33.15

If, however, both the teaser primary and secondary windings are designed for 86.6 volts only, then they will be operating at full rating, hence the combined rating of the arrangement would become (86.6 + 86.6)/(100 + 86.6) = 0.928 of its total rating.* In other words, ratio of kVA utilized to that available would be 0.928 which makes this connection more economical than open-Δ with its ratio of 0.866.

* Alternatively, VA capacity available is $= V_L I_L + (0.866\ V_L) I_L = 1.866\ V_L I_L$ where I_L is the primary line current. Since 3-phase power is supplied, volt-amperes actually utilized $= 1.732\ V_L I_L$. Hence, ratio of kVA actually utilized to those available is $= 1.732\ V_L I_L / 1.866\ V_L I_L = 0.928$.

Fig. 33.15 shows the secondary of the $T-T$ connection with its different voltages based on a nominal voltage of 100 V. As seen, the neutral point n is one third way up from point d. If secondary voltage and current vector diagram is drawn for load power factor of unity, it will be found that

1. current in teaser transformer is in phase with the voltage.
2. in the main transformer, current leads the voltage by 30° across one half but lags the voltage by 30° across the other half as shown in Fig. 33.14 (*b*).

Hence, when a balanced load of p.f. = cos ϕ, is applied, the teaser current will lag or lead the voltage by Φ while in the two halves of the main transformer, the angle between current and voltage will be $(30° - \Phi)$ and $(30° + \Phi)$. The situation is similar to that existing in a $V-V$ connection.

Example 33.17. *Two T-connected transformers are used to supply a 440-V, 33-kVA balanced load from a balanced 3-phase supply of 3300 V. Calculate (a) voltage and current rating of each coil (b) kVA rating of the main and teaser transformer.*

Solution. (*a*) Voltage across main primary is 3300 V whereas that across teaser primary is $= 0.866 \times 3300 = 2858$ V.

The current is the same in the teaser and the main and equals the line current.

$$\therefore \quad I_{LP} = 33,000/\sqrt{3} \times 3300 = 5.77 \text{ A} \qquad \text{—Fig. 33.16}$$

The secondary main voltage equals the line voltage of 440 V whereas teaser secondary voltage $= 0.866 \times 440 = 381$ V.

The secondary line current, $I_{LS} = I_{LP}/k = 5.77/(440/3300) = 43.3$ A as shown in Fig. 33.16.

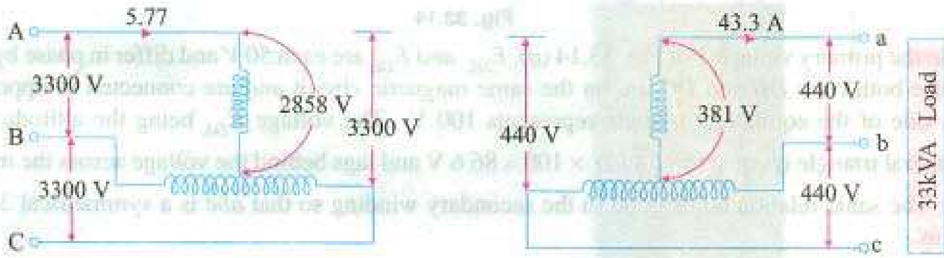


Fig. 33.16

(*b*) Main kVA $= 3300 \times 5.77 \times 10^{-3} =$ **19 kVA**

Teaser kVA $= 0.866 \times \text{main kVA} = 0.866 \times 19 =$ **16.4 kVA**

33.10. Three-phase to Two-phase Conversion and *vice-versa*

This conversion is required to supply two-phase furnaces, to link two-phase circuit with 3-phase system and also to supply a 3-phase apparatus from a 2-phase supply source. For this purpose, Scott connection as shown in Fig. 33.17 is employed. This connection requires two transformers of different ratings although for interchangeability and provision of spares, both transformers may be identical but having suitable tappings.

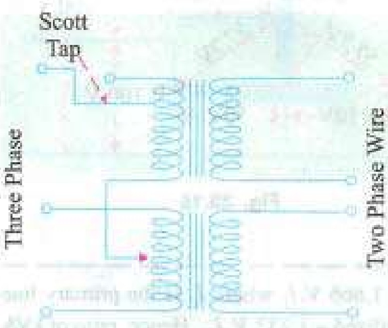
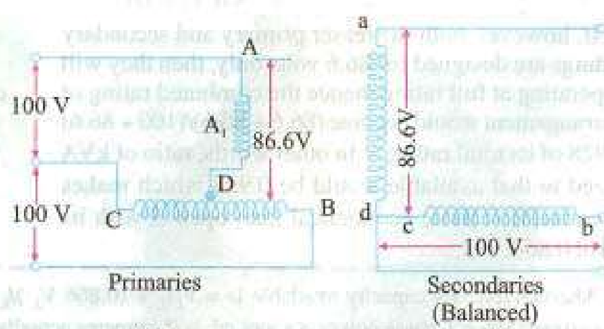


Fig. 33.17



(*a*) (*b*)

Fig. 33.18

If, in the secondaries of Fig. 33.14 (*b*), points *c* and *d* are connected as shown in Fig. 33.18 (*b*), then a 2-phase, 3-wire system is obtained. The voltage E_{dc} *is 86.6 V but* E_{Cb} = 100 V, hence the

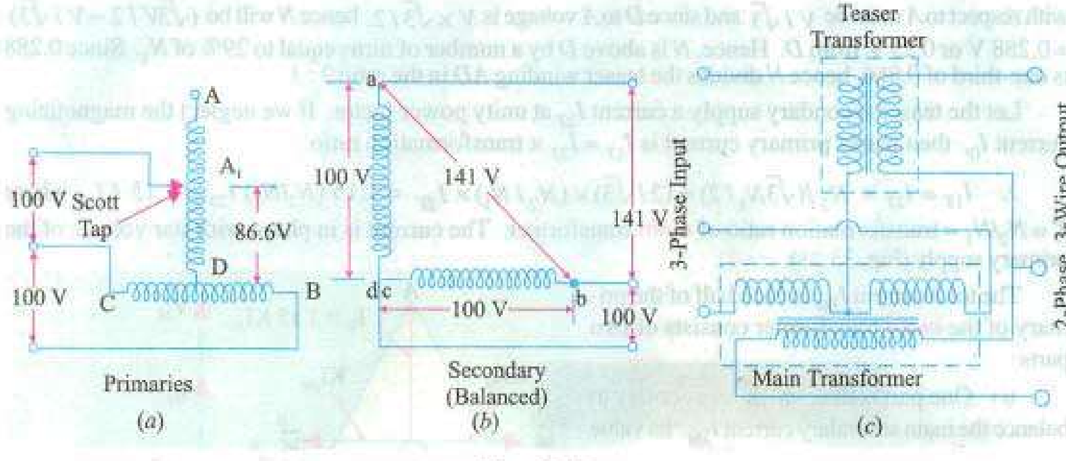


Fig. 33.19

resulting 2-phase voltages will be unequal. However, as shown in Fig. 33.19 (*a*) if the 3-phase line is connected to point A_1, such that DA_1 represents 86.6% of the teaser primary turns (which are the same as that of main primary), then this will increase the volts/turn in the ratio of 100 : 86.6, because now 86.6 volts are applied across 86.6 per cent of turns and not 100% turns. In other words, this will make volts/turn the same both in primary of the teaser and that of the main transformer. If the secondaries of both the transformers have the same number of turns, then secondary voltage will be equal in magnitude as shown, thus resulting in a symmetrical 2-phase, 3-wire system.

Consider the same connection drawn slightly differently as in Fig. 33.20. The primary of the main transformer having N_1 turns is connected between terminals *CB* of a 3-phase supply. If supply line voltage is *V*, then obviously $V_{AB} = V_{BC} = V_{CA} = V$ but voltage between *A* and *D* is $V \times \sqrt{3}/2$. As said

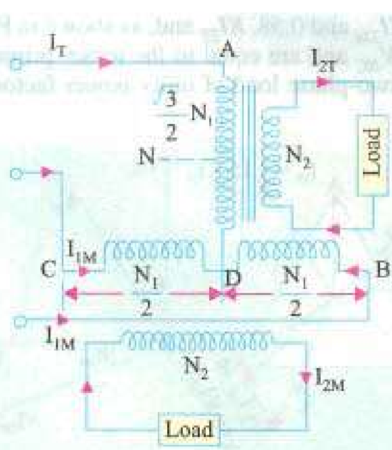


Fig. 33.20

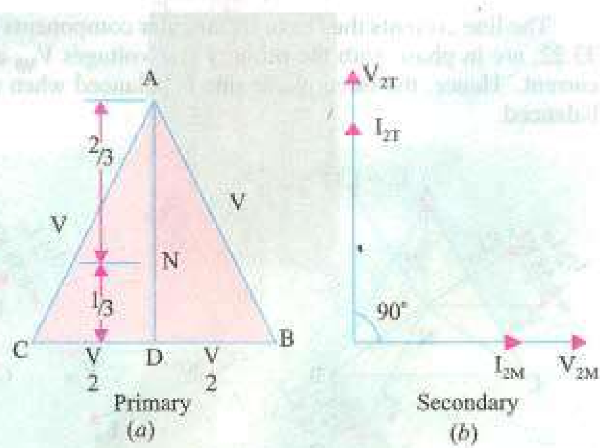


Fig. 33.21

above, the number of turns between *A* and *D* should be also $(\sqrt{3}/2)\ N_1$ for making volt/turn the same in both primaries. If so, then for secondaries having equal turns, the secondary terminal voltages will be equal in magnitude although in phase quadrature.

It is to be noted that point D is not the neutral point of the primary supply because its voltage with respect to any line is not $V/\sqrt{3}$. Let N be the neutral point. Its position can be determined as follows. Voltage of N with respect to A must be $V/\sqrt{3}$ and since D to A voltage is $V \times \sqrt{3}/2$, hence N will be $(\sqrt{3}V/2 - V/\sqrt{3})$ $= 0.288$ V or 0.29 V from D. Hence, N is above D by a number of turns equal to 29% of N_1. Since 0.288 is one-third of 0.866, hence N divides the teaser winding AD in the ratio 2 : 1.

Let the teaser secondary supply a current I_{2T} at unity power factor. If we neglect the magnetizing current I_0, then teaser primary current is $I_{1T} = I_{2T} \times$ transformation ratio.

$\therefore \quad I_{1T} = I_{2T} \times N_2/(\sqrt{3}N_1/2) = (2/\sqrt{3}) \times (N_2/N_1) \times I_{2T} = 1.15\,(N_2/N_1)\,I_{2T} = 1.15\,KI_{2T}$ where $K = N_2/N_1$ = transformation ratio of ***main*** transformer. The current is in phase with star voltage of the primary supply (Fig. 33.21).

The total current I_{1M} in each half of the primary of the ***main*** transformer consists of two parts :

(*i*) One part is that which is necessary to balance the main secondary current I_{2M}. Its value is

$$= I_{2M} \times \frac{N_2}{N_1} = KI_{2M}$$

(*ii*) The second part is equal to one-half of the teaser primary current *i.e.* $\frac{1}{2}I_{1T}$. This is so because the main transformer primary forms a return path for the teaser primary current which divides itself into two halves at mid-point D in either direction. The value of each half is $= I_{1T}/2 = 1.15\,KI_{2T}/2 = 0.58\,KI_{2T}$.

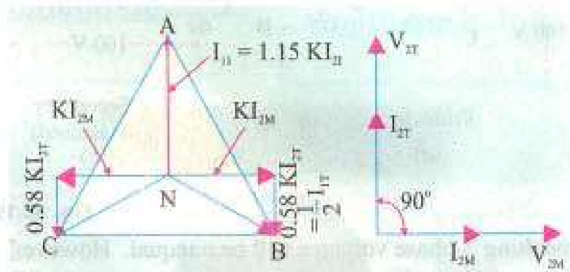


Fig. 33.22

Hence, the currents in the lines B and C are obtained vectorially as shown in Fig. 33.22. It should be noted that as the two halves of the teaser primary current flow in opposite directions from point D, they have no magnetic effect on the core and play no part at all in balancing the secondary ampere-turns of the main transformer.

The line currents thus have rectangular components of KI_{2M} and 0.58, KI_{2T} and, as shown in Fig. 33.22, are in phase with the primary star voltages V_{NB} and V_{NC} and are equal to the teaser primary current. Hence, the three-phase side is balanced when the two-phase load of unity power factor is balanced.

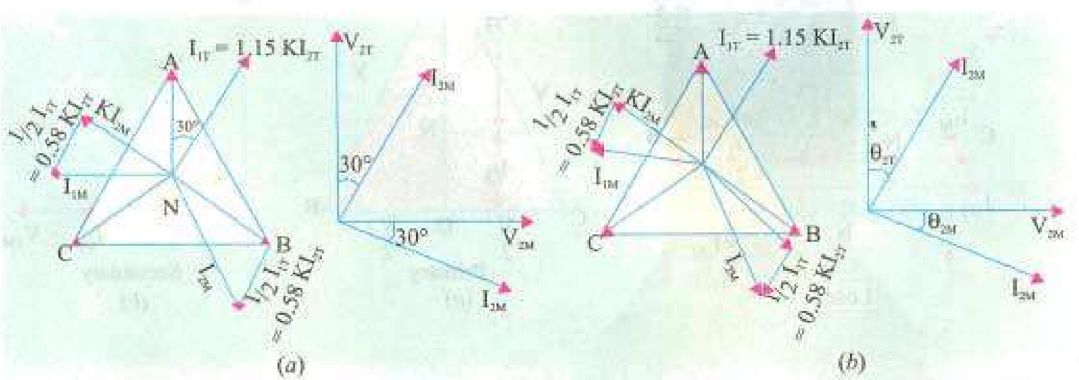


Fig. 33.23

Fig. 33.23 (*a*) illustrates the condition corresponding to a balanced two-phase load at a lagging power factor of 0.866. The construction is the same as in Fig. 33.22. It will be seen that the 3-phase side is again balanced. But under these conditions, the main transformer rating is 15% greater than that of the teaser, because its voltage is 15% greater although its current is the same.

Hence, ***we conclude that if the load is balanced on one side, it would always be balanced on the other side.***

The conditions corresponding to an unbalanced two-phase load having different currents and power factors are shown in Fig. 33.23 (*b*). The geometrical construction is similar to those explained in Fig. 33.22 and 33.23 (*a*).

Summarizing the above we have :

1. Teaser transformer primary has $\sqrt{3}/2$ times the turns of main primary. But volt/turn is the same. Their secondaries have the same turns which results in equal secondary terminal voltages.

2. If main primary has N_1 turns and main secondary has N_2 turns, then main transformation ratio is N_2/N_1. However, the transformation ratio of teaser is

$$N_2/(\sqrt{3}N_1/2) = 1.15\ N_2/N_1 = 1.15\ K$$

3. If the load is balanced on one side, it is balanced on the other side as well.

4. Under balanced load conditions, main transformer rating is 15% greater than that of the teaser.

5. The currents in either of the two halves of main primary are the vector sum of KI_{2M} and 0.58 KI_{2T} (or $\frac{1}{2}I_{1T}$).

Example 33.18. *Two transformers are required for a Scott connection operating from a 440-V, 3-phase supply for supplying two single-phase furnaces at 200 V on the two-phase side. If the total output is 150 kVA, calculate the secondary to primary turn ratio and the winding currents of each transformer.*

Solution. Main Transformer

Primary volts $= 440$ V ; secondary volts $= 200$ V $\therefore \frac{N_2}{N_1} = \frac{200}{440} = \frac{1}{2.2}$

Secondary current $= 150{,}000/2 \times 200 =$ **375 A**

$\therefore$ Primary current $= 375 \times 1/_{2.2} =$ **197 A**

Teaser Transformer

Primary volts $= (\sqrt{3}/2 \times 440) = 381$ V ; Secondary volts $= 200$ V

$$\frac{\text{secondary turns}}{\text{primary turns}} = \frac{200}{381} = \frac{1}{1.905} \text{ (also teaser ratio} = 1.15 \times 1/2.2 = 1/1.905)$$

Example 33.19. *Two single-phase furnaces working at 100 V are connected to 3300-V, 3-phase mains through Scott-connected transformers. Calculate the current in each line of the 3-phase mains when the power taken by each furnace is 400-kW at a power factor of 0.8 lagging. Neglect losses in the transformers.* **(Elect. Machines-III, South Gujarat Univ. 1988)**

Solution. Here $K = 100/3{,}300 = 1/33$ (main transformer)

$$I_2 = \frac{400{,}000}{0.8 \times 100} = 5{,}000\text{A (Fig. 33.24); Here } I_{2T} = I_{2M} = I_2 = 5{,}000 \text{ A}$$

As the two-phase load is balanced, the 3-phase side is also balanced.

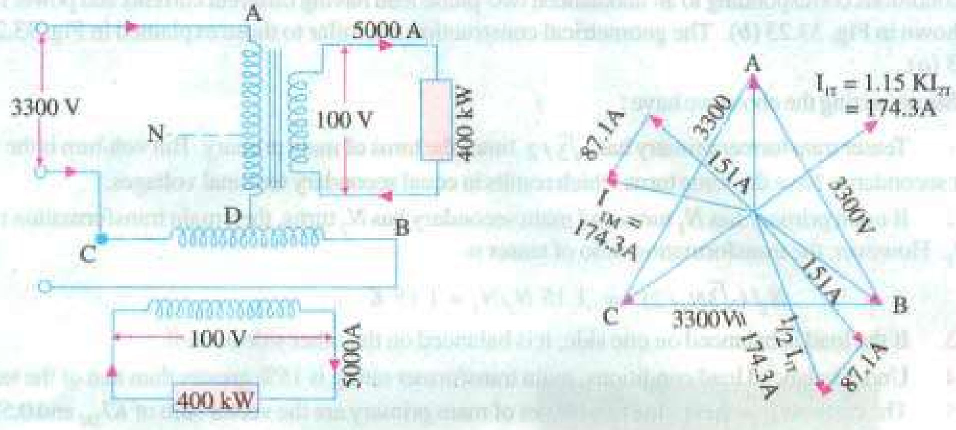
Primary phase currents are $= 1.15\ KI_2 = 1.15 \times (1/33) \times 5{,}000 = 174.3$ A

Since for a star-connection, phase current is equal to line current,

$\therefore$ Line current $=$ **174.3 A**

Note. We have made use of the fact that since secondary load is balanced, primary load is also balanced. If necessary, I_{1M} can also be found.

I_{1M} is the vector sum of (*i*) KI_{2M} and (*ii*) $\frac{1}{2}\ I_{1T}$ or 0.58 KI_{2T}.



(*a*) (*b*)

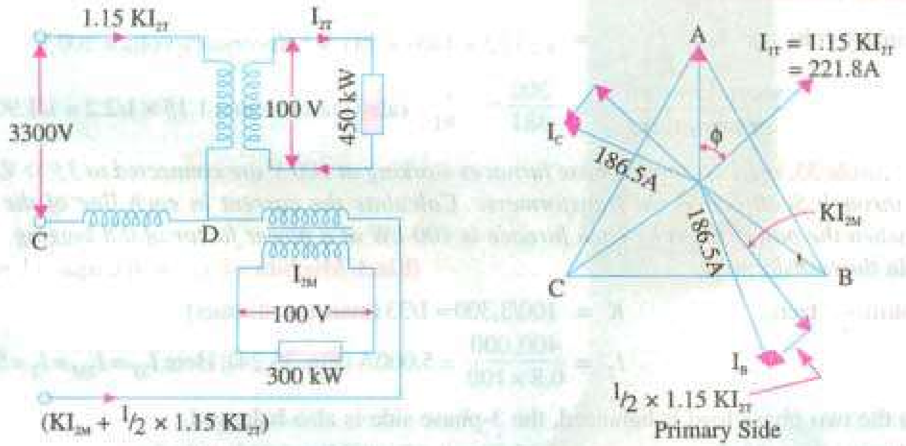
Fig. 33.24

Now, KI_{2M} = (1/33) × 5,000 = 151 A and 0.58 $KI_2 = \frac{1}{2}\ I_{1T}$ = 174.3/2 = 87.1 A

$\therefore$ $I_{1M} = \sqrt{151^2 + 87.1^2}$ = **174.3 A**

Example 33.20. *In a Scott-connection, calculate the values of line currents on the 3-phase side if the loads on the 2-phase side are 300 kW and 450 kW both at 100 V and 0.707 p.f. (lag) and the 3-phase line voltage is 3,300 V. The 300-kW load is on the leading phase on the 2-phase side. Neglect transformer losses.* **(Elect. Technology, Allahabad Univ. 1991)**

Solution. Connections are shown in Fig. 33.25 (*a*) and phasor diagram in Fig. 33.25 (*b*).



(*a*) (*b*)

Fig. 33.25

Here, K = 100/3,300 = 1/33

Teaser secondary current is I_{2T} = 450,000/100 × 0.707 = 6360 A

Teaser primary current is I_{1T} = 1.15 KI_{2T} = 1.5 × (1/33) × 6360 = 221.8 A

As shown in Fig. 33.25 (*b*), main primary current I_{1M} has two rectangular components.

(*i*) KI_{2M} where I_{2M} is the secondary current of the main transformer and

(*ii*) Half of the teaser primary current $\frac{1}{2} I_{1T} = \frac{1}{2} \times 1.15\ KI_{2T} = 0.577\ KI_{2T}$

Now $KI_{2M} = \frac{1}{33} \times \frac{300,000}{100 \times 0.707} = 128.58$ A ; Also $\frac{1}{2} I_{1T} = \frac{1}{2} \times 221.8 = 110.9$A

Main Primary current $= \sqrt{128.58^2 + 110.9^2} = 169.79$A

Hence, the 3-phase line currents are **221.8 A** in one line and **169.79 A** in each of the other two.

Example 33.21. *Two electric furnaces are supplied with 1-phase current at 80 V from a 3-phase, 11,000 V system by means of two single-phase Scott-connected transformers with similar secondary windings. When the load on one furnace is 500 kW (teaser secondary) and on the other 800 kW (secondary of main transformer) what current will flow in each of the 3-phase lines (a) at unity power factor and (b) at 0.5 power factor ? Neglect phase displacement in and efficiency of, the transformers.* **(Electrical Engineering, Madras Univ. 1987)**

Solution. The connections are shown in Fig. 33.26 and the phasor diagrams for unity and 0.5 p.f. are shown in Fig. 33.27 (*a*) and (*b*) respectively.

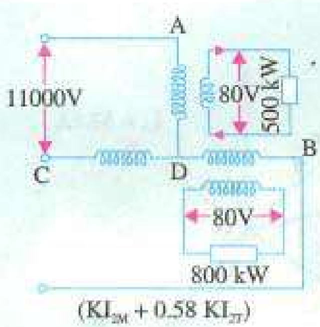


Fig. 33.26

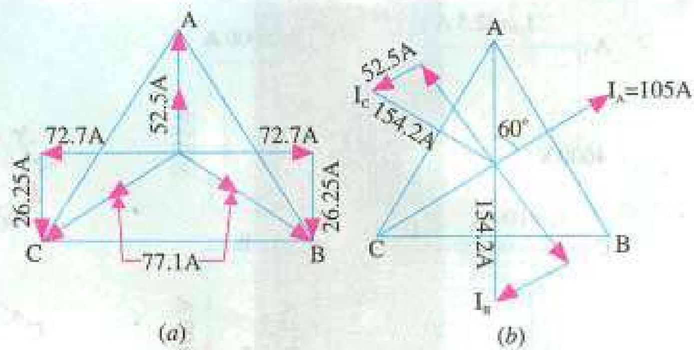


Fig. 33.27

Here, $K = 80/11,000 = 2/275$

(*a*) Unity p.f.

With reference to Fig. 33.27 (*a*), we have $I_{2T} = 500,000/80 \times 1 = 6,250$ A

Teaser primary current $I_{1T} = 1.15\ KI_{2T} = 1.15 \times (2/275) \times 6,250 = 52.5$ A

For the main transformer primary

(*i*) $KI_{2M} = \frac{2}{275} \times \frac{800,000}{80 \times 1} = 72.7$ A (*ii*) $\frac{1}{2} \times I_{1T} = 52.5/2 = 26.25$ A

Current in the primary of the main transformer is $= \sqrt{72.7^2 + 26.25^2} = 77.1$ A

Hence, one 3-phase line carries **52.5 A** whereas the other 2 carry **77.1 A** each [Fig. 33.27 (*a*)].

(*b*) 0.5 p.f.

With reference to Fig. 33.27 (*b*) we have $I_{2T} = 500,000/80 \times 0.5 = 12,500$ A

Teaser primary current $I_{1T} = 1.15 \times (2/275) \times 12,500 = 105$ A

For the main transformer primary

(i) $$KI_{2M} = \frac{2}{275} \times \frac{800{,}000}{80 \times 0.5} = 145.4 \text{ A} \quad (ii) \ \frac{1}{2} I_{1T} = 105/2 = 52.5 \text{ A}.$$

Current in the primary of the main transformer is $= \sqrt{145.4^2 + 52.5^2} = 154.2$ A.

Hence, one 3-phase line carries **105 A** and the other two carry **154.2 A** each.

Note : Part (*b*) need not be worked out in full because at 0.5 p.f., each component current and hence the resultant are doubled. Hence, in the second case, answers can be found by multiplying by a factor of 2 the line currents found in (*a*).

Example 33.22. *Two furnaces are supplied with 1-phase current at 50 V from a 3-phase, 4.6 kV system by means of two 1-phase, Scott-connected transformers with similar secondary windings. When the load on the main transformer is 350 kW and that on the other transformer is 200 kW at 0.8 p.f. lagging, what will be the current in each 3-phase line ? Neglect phase displacement and losses in transformers.* **(Electrical Machinery-II, Bangalore Univ. 1991)**

Solution. Connections and vector diagrams are shown in Fig. 33.28.

$$K = 50/4{,}600 = 1/92; I_{2T} = 200{,}000/50 \times 0.8 = 5{,}000 \text{ A}$$
$$I_{1T} = 1.15\, KI_{2T} = 1.1 \times (1/92) \times 5{,}000 = 62.5 \text{ A}$$

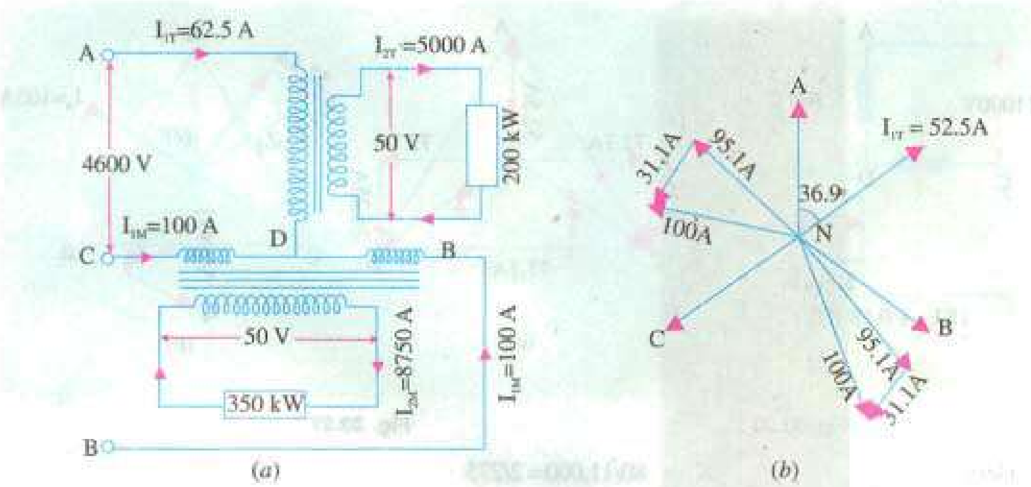


Fig. 33.28

As shown in Fig. 33.28 (*b*), main primary current I_{1M} has two rectangular components.

(i) KI_{2M} where $I_{2M} = 350{,}000/50 \times 0.8 = 8{,}750$ A $\therefore KI_{2M} = 8{,}750/92 = 95.1$ A

(ii) $(1/2)\, I_{1T} = 62.5/2 = 31.3$ A $\quad \therefore \quad I_{1M} = \sqrt{95.1^2 + 31.3^2} = 100$ A

$\therefore$ Current in line A = **62.5 A**; Current in line B = **100 A**; Current in line C = **100 A.**

Example 33.23. *Two single-phase Scott-connected transformers supply a 3-phase four-wire distribution system with 231 volts between lines and the neutral. The h.v. windings are connected to a two-phase system with a phase voltage of 6,600 V. Determine the number of turns in each section of the h.v. and l.v. winding and the position of the neutral point if the induced voltage per turn is 8 volts.*

Solution. As the volt/turn is 8 and the h.v. side voltage is 6,600 V, the h.v. side turns are = 6,600/8 = **825** on both transformers.

Now, voltage across points B and C of main winding

$= \text{line voltage} = 231 \times \sqrt{3} = 400\ \text{V}$

No. of turns on the l.v. side of the main transformer = 400/8 = **50**

No. of turns on the l.v. side of teaser transformer =

$\sqrt{(3/2)} \times$ mains turns

$= \sqrt{3} \times 50/2 =$ **43** (whole number)

The neutral point on the 3-phase side divides teaser turns in the ratio 1 : 2.

$\therefore$ Number of turns between A and $N = (2/3) \times AD = (2/3) \times 43 =$ **29**

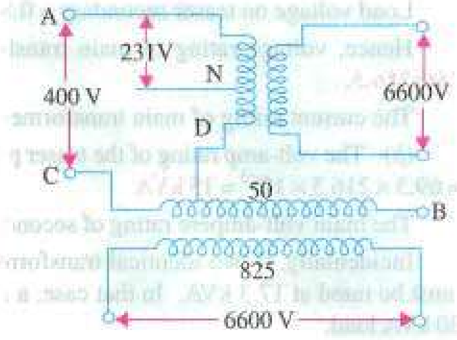


Fig. 33.29

Hence, neutral point is located on the 29th turn from A downwards (Fig. 33.29).

Example 33.24. *A Scott-connected (2 to 3-phase) transformer links a 6,000 V, 2-phase system with a 440 V; 3-phase system. The frequency is 50 Hz, the gross core area is 300 cm^2, while the maximum flux density is to be about 1.2 Wb/m^2. Find the number of turns on each winding and the point to be tapped for the neutral wire on the 3-phase side. If the load is balanced on the one side of such a transformer, find whether it will also be balanced on the other side.* **(London Univ.)**

Solution. Use the transformer voltage equation 1,

$E = 4.44\, fN\, \Phi_x$ volt

Gross core area = 300 cm^2

Assuming net iron = 0.9 of gross area, and considering the h.v. side, we have

$6000 = 4.44 \times 50 \times N_1 \times 1.2\,(300 \times 0.9 \times 10^{-4})$

$N_1 =$ **834**

Hence, h.v. sides of both transformers have 834 turns each.

Now $K = 440/6000 = 11/150$

$\therefore$ Turns on the l.v. side of main transformer

$N_2 = 834 \times 11/150 =$ **61**

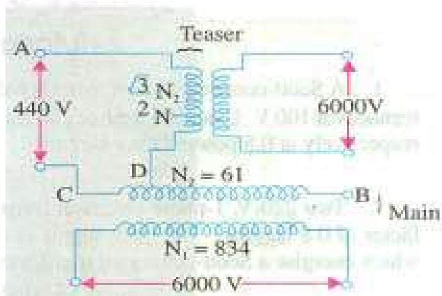


Fig. 33.30

Turns on the l.v. side of teaser = $(\sqrt{3}/2) \times 61 =$ **53**

With reference to Fig. 33.30, number of turns in $AN = 53 \times 2/3 =$ **35**

Example 33.25. *A 2-phase, 4-wire, 250 V system is supplied to a plant which has a 3-phase motor load of 30 kVA. Two Scott-connected transformers supply the 250 V motors. Calculate (a) voltage (b) kVA rating of each transformer. Draw the wiring connection diagram.*

Solution. (*a*) Both the main and the teaser have the same voltage rating as the supply voltage *i.e.* 250 V. The current in the main and the teaser coils is the same as the supply current and is

$$= \frac{\text{Total kVA}}{2 \times \text{Line voltage}} = \frac{30{,}000}{2 \times 250} = 60\ \text{A arcsin}\ \theta$$

On the three-phase side, current is the same in all coils and is equal to the load line current $= 30{,}000/\sqrt{3} \times 250 = 69.3\ \text{A}$

Load voltage on main secondary = line voltage = 250 V

Load voltage on teaser secondary = 0.866 × 250 = 216.5 V

Hence, voltage rating of main transformer is **250/250** whereas that of teaser transformer is **250/216.5.**

The current rating of main transformer is **60/69.3** and it is the same for the teaser transformer.

(*b*) The volt-amp rating of the teaser primary as well as secondary is the same *i.e.* $60 \times 250 \times 10^{-3}$ $= 69.3 \times 216.5 \times 10^{-3}$ = **15 kVA**

The main volt-ampere rating of secondary is = $250 \times 69.3 \times 10^{-3}$ = **17.3 kVA**

Incidentally, if two identical transformers are used for providing inter-changeability, then both must be rated at 17.3 kVA. In that case, a total capacity of 34.6 kVA would be required to provide a 30 kVA load.

The wiring connections are shown in Fig. 33.31.

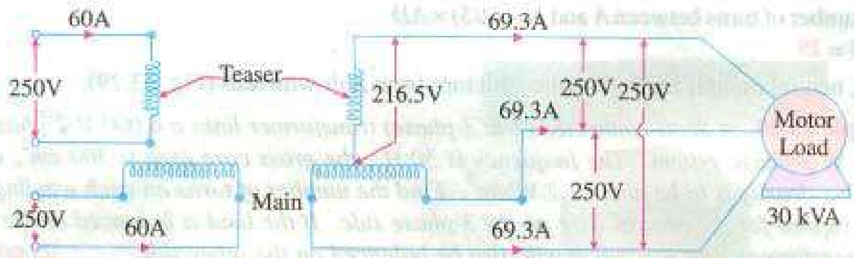


Fig. 33.31

Tutorial Problem No. 33.3

1. A Scott-connected transformer is fed from a 6,600-V, 3-phase network and supplies two single-phase furnaces at 100 V. Calculate the line currents on the 3-phase side when the furnaces take 400 kW and 700 kW respectively at 0.8 power factor lagging. **(Elect. Machines II, Indore Univ. 1977)**

[With 400 kW on teaser, line currents are 87. 2 A; 139 A; 139 A]

2. Two 220-V, 1-phase electrical furnaces take loads of 350 kW and 500 kW respectively at a power factor of 0.8 lagging. The main supply is at 11-kV, 3-phase, 50 Hz. Calculate current in the 3-phase lines which energise a Scott-connected transformer combination. **(Elect. Machines, Madras Univ. 1978)**

[With 350 kW on teaser line currents are : 45.7 A ; 61.2 A; 61.2 A]

3. Two electric furnaces are supplied with 1-phase current at 80 V from 3-phase, 11,000-V supply mains by means of two Scott-connected transformers with similar secondary windings. Calculate the current flowing kW respectively in each of the 3-phase lines at U.P.P. when the loads on the two transformers are 550 kW of 800 kW.

[With 550 kW on teaser, line currents are : 57.5 A; 78.2; 78.2 A] (Electrical Machines-I, Madras University, 1977)

33.11. Parallel Operation of 3-phase Transformers

All the conditions which apply to the parallel operation of single-phase transformers also apply to the parallel running of 3-phase transformers but with the following additions :

1. The voltage ratio must refer to the terminal *voltage of primary and secondary*. It is obvious that this ratio may not be equal to the ratio of the number of turns per phase. For example, if V_1, V_2 are the primary and secondary terminal voltages, then for Y/Δ connection, the turn ratio is $V_2/(V_1/\sqrt{3})$ $= \sqrt{3}V_2/V_1$.

2. The phase displacement between primary and secondary voltages must be the same for all transformers which are to be connected for parallel operation.

3. The phase sequence must be the same.

4. All three transformers in the 3-phase transformer bank will be of the same construction either core or shell.

Note. *(i)* In dealing with 3-phase transformers, calculations are made for one phase only. The value of equivalent impedance used is the equivalent impedance per phase referred to secondary.

(ii) In case the impedances of primary and secondary windings are given separately, then primary impedance must be referred to secondary by multiplying it with (transformation ratio)^2.

(iii) For Y/Δ or Δ/Y transformers, it should be remembered that the voltage ratios as given in the questions, refer to terminal voltages and are quite different from turn ratio.

Example 33.26. *A load of 500 kVA at 0.8 power factor lagging is to be shared by two three-phase transformers A and B of equal ratings. If the equivalent delta impedances as referred to secondary are (2 + j6) Ω for A and (2 + j5) Ω for B, calculate the load supplied by each transformer.*

Solution. $$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A}+\mathbf{Z_B}} = \mathbf{S}\frac{1}{1+(\mathbf{Z_A}/\mathbf{Z_B})}$$

Now $$S = 500\,(0.8 - j0.6) = (400 - j300)$$

$$\mathbf{Z_A/Z_B} = (2+j6)/(2+j5) = 1.17 + j\,0.07;\ \mathbf{Z_B/Z_A} = (2+j5)/(2+j6) = 0.85 - j\,0.05$$

$$\mathbf{S_A} = (400 - j300)/(2.17 + j0.07) = 180 - j\,144.2 = 230.7\angle -38.7^\circ$$

$$\cos\phi_A = 0.78 \text{ lagging}$$

$$\mathbf{S_B} = (400 - j\,300)/(1.85 - j0.05) = 220.1 - j\,156 = 270\angle -40^\circ 28' \;\therefore\; \cos\Phi_B = \mathbf{0.76\ lagging.}$$

Example 33.27. *State (i) the essential and (ii) the desirable conditions to be satisfied so that two 3-phase transformers may operate successfully in parallel.*

A 2,000-kVA transformer (A) is connected in parallel with a 4,000 kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage voltage drops in the windings at their rated loads are as follows :

Transformer A	*resistance 2% ;*	*reactance 8 %*
Transformer B	*resistance 1.6 % ;*	*reactance 3 %*

(Elect. Engineering-II, Bombay Univ. 1987)

Solution. On the basis of 4,000 kVA

$$\%\,\mathbf{Z_A} = (4{,}000/2{,}000)\,(2+j8) = (4+j16) = 16.5\angle 76^\circ$$

$$\%\,\mathbf{Z_B} = (1.6+j3);\ \%\,Z_A + \%\,Z_B = (5.6+j16) = 19.8\angle 73.6^\circ$$

$$\mathbf{S} = 5{,}000\angle -36.9^\circ = (4{,}000 - j\,3{,}000)$$

Now $$\mathbf{S_B} = \mathbf{S}.\frac{\mathbf{Z_A}}{\mathbf{Z_A}+\mathbf{Z_B}} = 5{,}000\angle -36.9^\circ \times \frac{16.5\angle 76^\circ}{19.8\angle 73.6^\circ}$$

$$= 5{,}000\angle -36.9^\circ \times 0.832\angle 2.4^\circ = 4{,}160\angle -34.5^\circ = (3{,}425 - j\,2{,}355)$$

$$\mathbf{S_A} = \mathbf{S} - \mathbf{S_B} = (4{,}000 - j\,3{,}000) - (3{,}425 - j\,2355)$$

$$= (575 - j\,645) = 864\angle -48.3^\circ$$

$$\cos\phi_B = \cos 34.5^\circ = 0.824 \text{ (lag)};\ \cos\phi_A = \cos 48.3^\circ = \mathbf{0.665\ (lag).}$$

Example 33.28. *A load of 1,400 kVA at 0.866 p.f. lagging is supplied by two 3-phase transformers of 1,000 kVA and 500 kVA capacity operating in parallel. The ratio of transformation is the same in both : 6,600/400 delta-star. If the equivalent secondary impedances are (0.001 + j 0.003) ohm and (0.0028 + j0.005) ohm per phase respectively, calculate the load and power factor of each transformer.* **(Elect. Engg-I, Nagpur Univ. 1993)**

Solution. On the basis of 1000 kVA, $\mathbf{Z}_A = (0.001 + j\,0.003)\ \Omega$

$$\mathbf{Z}_B = (1000/500)\ (0.0028 + j\,0.005) = (0.0056 + j\,0.01)\ \Omega$$

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{(0.001 + j\,0.003)}{(0.0066 + j\,0.013)} = \frac{3.162 \times 10^{-3}\ \angle 71.6^\circ}{14.57 \times 10^{-3}\ \angle 63.1^\circ} = 0.2032\ \angle 8.5^\circ$$

$$\mathbf{S} = 1400\ \angle \cos^{-1}(0.866) = 1400\ \angle -30^\circ = (1212 - j\,700)$$

$$\mathbf{S}_B = \mathbf{S}\,.\,\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = 1400\ \angle -30^\circ \times 0.2032\ \angle 8.5^\circ$$

$$= 284.5\ \angle -21.5^\circ = 265 - j\,104$$

$$\mathbf{S}_A = \mathbf{S} - \mathbf{S}_B = (1212 - j\,700) - (265 - j\,104) = (947 - j\,596) = 1145\ \angle -32.2^\circ$$

$$\cos \phi_A = \cos 32.3^\circ = 0.846 \text{ (lag)};\ \cos \phi_B = \cos 21.5^\circ = \mathbf{0.93\ (lag).}$$

Example 33.29. *Two 3-phase transformers A and B having the same no-load line voltage ratio 3,300/400-V supply a load of 750 kVA at 0.707 lagging when operating in parallel. The rating of A is 500 kVA, its resistance is 2% and reactance 3%. The corresponding values for B are 250 kVA; 1.5% and 4% respectively. Assuming that both transformers have star-connected secondary windings, calculate*

(a) the load supplied by each transformer,

(b) the power factor at which each transformer is working,

(c) the secondary line voltage of the parallel circuit.

Solution. On the basis of 500 kVA,

$$\%\ \mathbf{Z}_A = 2 + j3,\ \%\ \mathbf{Z}_B = (500/200)\,(1.5 + j4) = (3 + j8)$$

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{2 + j3}{5 + j1} = 0.3\angle -9.3^\circ;\ \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{3 + j8}{5 + j11} = 0.711\ \angle 3.8^\circ$$

Now, $\mathbf{S} = 750\angle -45^\circ$

(a) $$\mathbf{S}_A = \mathbf{S}\,\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = 750\angle -45^\circ \times 0.711\ \angle 3.8^\circ$$

$$= 533\angle -41.2^\circ = (400 - j351)$$

$$\mathbf{S}_B = 750\ \angle -45^\circ \times 0.3\ \angle 09.3^\circ = 225\ \angle -54.3^\circ$$

(b) $$\cos \phi_A = \cos 41.2^\circ = 0.752 \text{ (lag)};\ \cos \phi_B = \cos 54.3^\circ = \mathbf{0.5835} \text{ (lag)}$$

(c) Since voltage drop of each transformer is the same, its value in the case of transformer *A* would only be calculated. Now, for transformer *A*, kW = 400 for the active component of the current and kVAR = 351 for the reactive component.

$\therefore$ % resistive drop = $2 \times 400/500$

= 1.6%; % reactive drop = $3 \times 351/500 = 2.1\%$

Total percentage drop = 1.6 + 2.1 = 3.7

Secondary line voltage = $400 - (3.7 \times 400/100) =$ **385.2 V.**

33.12. Instrument Transformers

In d.c. circuit when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with a high resistance connected in series with them. But it is not convenient to use this methods with alternating current and voltage instruments. For this purpose, specially constructed accurate ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. These instrument transformers are of two kinds : (*i*) current transformers for measuring large alternating currents and (*ii*) potential transformers for measuring high alternating voltages.

Current transformers

33.13. Current Transformers

These transformers are used with low-range ammeters to measure currents in high-voltage alternating-current circuits where it is not practicable to connect instruments and meters directly to the lines. In addition to insulating the instrument from the high voltage line, they step down the current in a known ratio. The current (or series) transformer has a primary coil of one or more turns of thick wire connected in *series* with the line whose current is to be measured as shown in Fig. 33.32. The secondary consists of a large number of turns of fine wire and is connected across the ammeter terminals (usually of 5-ampere bracket should be removed or 1-ampere range).

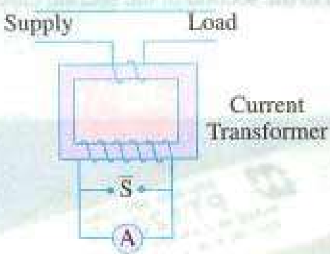


Fig. 33.32

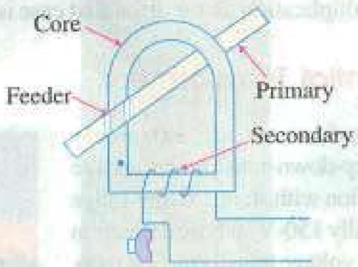


Fig. 33.33

As regards voltage, the transformers is of step-up variety but it is obvious that current will be stepped down. Thus, if the current transformer has *primary to secondary* current ratio of 100 : 5, then it steps up the voltage 20 times whereas it steps down the current to 1/20th of its actual value. Hence, if we know current ratio (I_1/I_2) of the transformer and the reading of the a.c. ammeter, the line current can be calculated. In fact, line current is given by the current transformation ratio times the reading on the ammeter. One of the most commonly used current transformer is the one known as *clamp-on or clip-on type*. It has a laminated core which is so arranged that it can be opened out at hinged section by merely pressing a triggr-like projection (Fig. 33.33). When the core is thus opened, it permits the admission of very heavy current-carrying bus bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current carrying conductor or feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle.

Small current transformer

It should be noted that, since the ammeter resistance is very low, the current transformer normally works short circuited. If for any reason, the ammeter is taken out of the secondary winding, then this winding must be short-circuited with the help of short-circulating switch S. If this is not done, then due to the absence of counter amp-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating and a high voltage across the secondary terminals. This is not the case with ordinary constant-potential transformers, because their primary current is determined by the load in their secondary whereas in a current transformer, the primay current is determined entirely by the load on the system and not by the load on its own secondary.

Hence, *the secondary of a current transformer should never be left open under any circumstances.*

Example 33.30. *A 100 : 5 transformer is used in conjunction with a 5-amp ammeter. If the latter reads 3.5 A, find the line current.*

Solution. Here, the ratio 100 : 5 stands for the ratio of primary-to-secondary currents *i.e.* $I_1/I_2 = 100/5$

$\therefore$ Primary (or line) current $= 3.5 \times (100/5) =$ **70 A**

Example 33.31. *It is desired to measure a line current of the order of 2,000 A to 2,500 A. If a standard 5-amp ammeter is to be used along with a current transformer, what should be the turn ratio of the latter ? By what factor should the ammeter reading be multiplied to get the line current in each case ?*

Solution. $I_1/I_2 = 2000/5 = 400$ or $2500/5 = 500$. Since $I_1/I_2 = N_2/N_1$ hence $N_2/N_1 = 400$ in the first case and 500 in the second case. It means that $N_1 : N_2 :: 1 : 400$ or $1 : 500$.

Ratio or multiplication factor in the first case is 400 and in the second in the second case 500.

33.14. Potential Transformers

These transformers are extremely accurate-ratio step-down transformers and are used in conjunction with standard low-range voltmeters (usually 150-V) whose deflection when divided by voltage transformation ratio, gives the true voltage on the high voltage side. In general, they are of the shell-type and do not differ much from the ordinary two-winding transformers discussed so far, except that their power rating is extremely small. Upto voltages of 5,000, potential transformers are usually of the dry type, between 5,000 and 13,800 volts, they may be either dry type or oil immersed type, although for voltages above 13,800 they are always oil immersed type. Since their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100 W. For safety, the secondary should be completely insulated from the high-voltage primary and should be, in addition, grounded for affording protection to the operator. Fig. 33.34 shows the connections of such a transformer.

Small Potential transformer

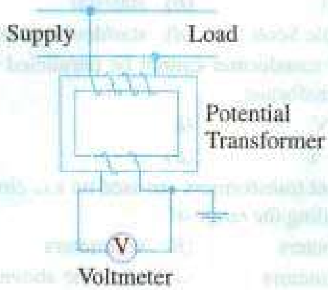


Fig. 33.34

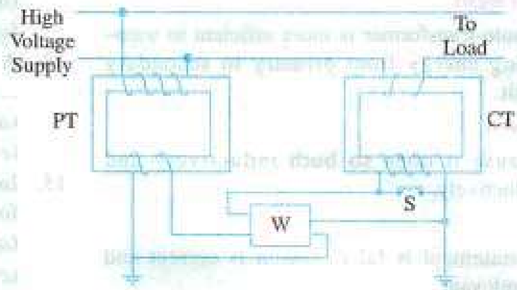


Fig. 33.35

Fig. 33.35 shows the connections of instrument transformers to a wattmeter. While connecting the wattmeter, the relative polarities of the secondary terminals of the transformers with respect to their primary terminals must be known for connections of the instruments.

OBJECTIVE TEST – 33

1. Which of the following connections is best suited for 3-phase, 4-wire service ?
   (*a*) Δ – Δ (*b*) Y – Y
   (*c*) Δ – Y (*d*) Y – Δ
2. In a three-phase Y-Y transformer connection, neutral is fundamental to the
   (*a*) suppression of harmonics
   (*b*) passage of unbalanced currents due to unbalanced loads
   (*c*) provision of dual electric service
   (*d*) balancing of phase voltages with respect to line voltages.
3. As compared to Δ – Δ bank, the capacity of the *V* – *V* bank of transformers is .... percent.
   (*a*) 57.7 (*b*) 66.7
   (*c*) 50 (*d*) 86.6
4. If three transformers in a Δ – Δ are delivering their rated load and one transformer is removed, then overload on *each* of the remaining transformers is .... percent.
   (*a*) 66.7 (*b*) 173.2
   (*c*) 73.2 (*d*) 58
5. When a V – V system is converted into a Δ – Δ system, increase in capacity of the system is .... percent.
   (*a*) 86.6 (*b*) 66.7
   (*c*) 73.2 (*d*) 50
6. For supplying a balanced 3 – φ load of 40-kVA, rating of each transformer in V – V bank should be nearly .... kVA.
   (*a*) 20 (*b*) 23
   (*c*) 34.6 (*d*) 25
7. When a closed – Δ bank is converted into an open – Δ bank, each of the two remaining transformers supplies .... percent of the original load.
   (*a*) 66.7 (*b*) 57.7
   (*c*) 50 (*d*) 73.2
8. If the load p.f. is 0.866, then the average p.f. of the V – V bank is
   (*a*) 0.886 (*b*) 0.75
   (*c*) 0.51 (*d*) 0.65
9. A T – T connection has higher ratio of utilization that a V – V connection only when
   (*a*) identical transformers are used
   (*b*) load power factor is leading
   (*c*) load power factor is unity
   (*d*) non-identical transformers are used.
10. The biggest advantage of T - T connection over the V - V connection for 3-phase power transformation is that it provides
    (*a*) a set of balanced voltages under load
    (*b*) a true 3-phase, 4-wire system
    (*c*) a higher ratio of utilization
    (*d*) more voltages.
11. Of the following statements concerning parallel operation of transformers, the one which is **not** correct is
    (*a*) transformers must have equal voltage ratings
    (*b*) transformers must have same ratio of transformation
    (*c*) transformers must be operated at the same frequency
    (*d*) transformers must have equal kVA ratings.

12. **Statement**

An auto-transformer is more efficient in transferring energy from primary to secondary circuit.

Reason

Because it does so both inductively and conductively.

Key

(*a*) statement is false, reason is correct and relevant

(*b*) statement is correct, reason is correct but irrelevant

(*c*) both statement and reason are correct and are connected to each other as cause and effect

(*d*) both statement and reason are false.

13. Out of the following given choices for poly phase transformer connections which one will you select for three-to-two phase conversion ?

(*a*) Scott (*b*) star/star

(*c*) double Scott (*d*) star/double-delta

14. A T – T transformer cannot be paralleled with .... transformer.

(*a*) V – V (*b*) Y – Δ

(*c*) Y – Y (*d*) Δ – Δ

15. Instrument transformers are used on a.c. circuits for extending the range of

(*a*) ammeters (*b*) voltmeters

(*c*) wattmeters (*d*) all of the above.

16. Before removing the ammeter from a current transformer, its secondary must be short-circuited in order to avoid

(*a*) excessive heating of the core

(*b*) high secondary e.m.f.

(*c*) increase in iron losses

(*d*) all of the above.

ANSWERS

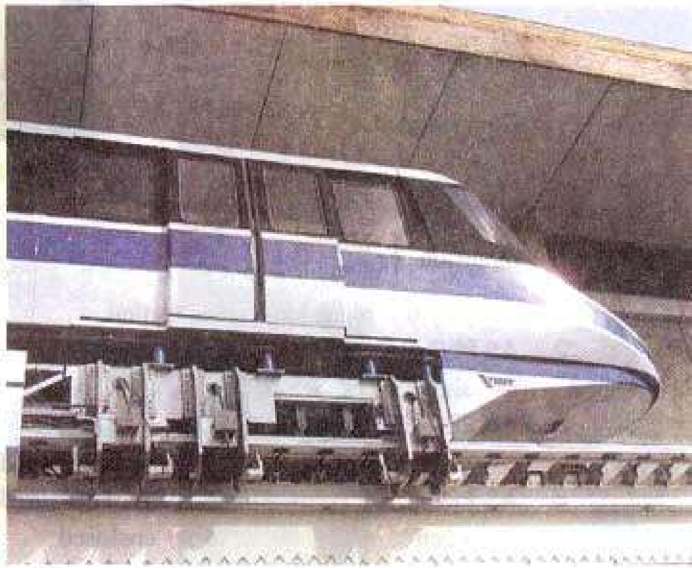
1. *c* 2. *a* 3. *a* 4. *c* 5. *c* 6. *b* 7. *b* 8. *b* 9. *d* 10. *b* 11. *d* 12. *c* 13. *a* 14. *b* 15. *d* 16. *d*

CHAPTER 34

INDUCTION MOTOR

Learning Objectives

- Classification of AC Motors
- Induction Motor: General Principal
- Construction
- Phase-wound Rotor
- Mathematical Proof
- Relation between Torque and Rotor Power Factor
- Condition for Maximum Starting Torque
- Rotor E.M.F and Reactance under Running Conditions
- Condition for Maximum Torque Under Running Conditions
- Rotor Torque and Breakdown Torque
- Relation between Torque and Slip
- Full-load Torque and Maximum Torque
- Starting Torque and Maximum Torque
- Torque/Speed Characteristic Under Load
- Complete Torque/Speed Curve of a Three-phase Machine
- Power Stages in an Induction Motor
- Torque Developed by an Induction Motor
- Induction Motor Torque Equation
- Variation in Rotor Current
- Sector Induction Motor
- Magnetic Levitation
- Induction Motor as a Generalized Transformer
- Power Balance Equation
- Maximum Power Output



The high-speed magnetic levitation trains employ the principle of linear induction motor

34.1. Classification of A.C. Motors

With the almost universal adoption of a.c. system of distribution of electric energy for light and power, the field of application of a.c. motors has widened considerably during recent years. As a result, motor manufactures have tried, over the last few decades, to perfect various types of a.c. motors suitable for all classes of industrial drives and for both single and three-phase a.c. supply. This has given rise to bewildering multiplicity of types whose proper classification often offers considerable difficulty. Different a.c. motors may, however, be classified and divided into various groups from the following different points of view :

1. **AS REGARDS THEIR PRINCIPLE OF OPERATION**

(A) Synchronous motors

(*i*) plain and (*ii*) super—

(B) Asynchronous motors

(*a*) **Induction motors**

(*i*) Squirrel cage { single, double

(*ii*) Slip-ring (external resistance)

(*b*) **Commutator motors**

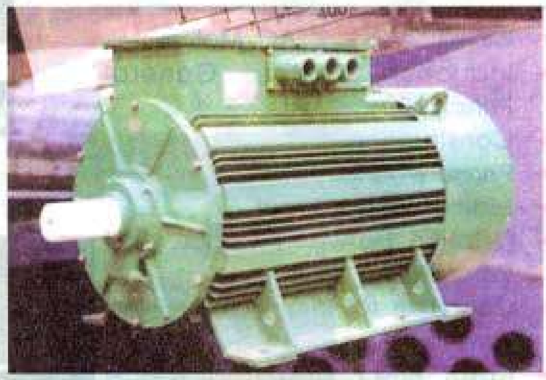
(*i*) Series { single phase, universal

(*ii*) Compensated { conductively, inductively

(*iii*) shunt { simple, compensated

(*iv*) repulsion { straight, compensated

(*v*) repulsion-start induction (*vi*) repulsion induction



Three phase high voltage asynchronous motors

2. **AS REGARDS THE TYPE OF CURRENT**

(*i*) single phase (*ii*) three phase

3. **AS REGARDS THEIR SPEED**

(*i*) constant speed (*ii*) variable speed (*iii*) adjustable speed

4. **AS REGARDS THEIR STRUCTURAL FEATURES**

(*i*) open (*ii*) enclosed (*iii*) semi-enclosed
(*iv*) ventilated (*v*) pipe-ventilated (*vi*) riverted frame eye etc.

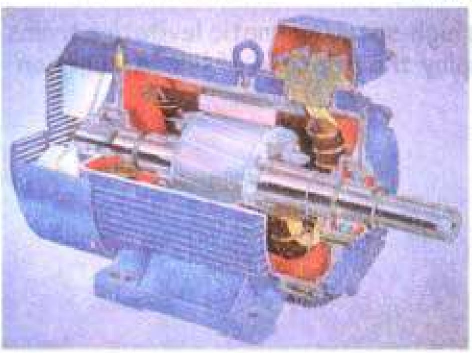


Fig. 34.1 Squirrel cage AC induction motor opened to show the stator and rotor construction, the shaft with bearings, and the cooling fan.

34.2. Induction Motor : General Principle

As a general rule, conversion of electrical power into mechanical power takes place in the *rotating* part of an electric motor. In d.c. motors, the electric power is *conducted* directly to the armature (*i.e.* rotating part) through brushes and commutator (Art. 29.1). Hence, in this sense, a d.c. motor can be called a *conduction* motor. However, in a.c. motors, the rotor does not receive electric power by conduction but by *induction* in exactly the same way as the secondary of a 2-winding transformer receives its power from the primary. That is why such motors are known as ***induction*** motors. In fact, an induction motor can be treated as a ***rotating transformer*** *i.e.* one in which primary winding is stationary but the secondary is free to rotate (Art. 34.47).

Of all the a.c. motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages and also some dis-advantages:

Advantages:

1. It has very simple and extremely rugged, almost unbreakable construction (especially squirrel-cage type).
2. Its cost is low and it is very reliable.
3. It has sufficiently high efficiency. In normal running condition, no brushes are needed, hence frictional losses are reduced. It has a reasonably good power factor.
4. It requires minimum of maintenance.
5. It starts up from rest and needs no extra starting motor and has not to be synchronised. Its starting arrangement is simple especially for squirrel-cage type motor.

Disadvantages:

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of a d.c. shunt motor.

34.3. Construction

An induction motor consists essentially of two main parts :

(*a*) a stator and (*b*) a rotor.

(*a*) Stator

The stator of an induction motor is, in principle, the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings [Fig.34.2 (*a*)]. The stator carries a 3-phase winding [Fig.34.2 (*b*)] and is fed from a 3-phase supply. It is wound for a definite number of poles*, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and ***vice versa***. It will be shown in Art. 34.6 that the stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by $N_s = 120f/P$). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.

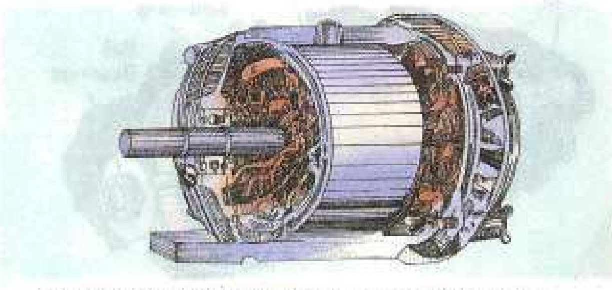


Fig. 34.2 (*a*) Unwound stator with semi-closed slots. Laminations are of high-quality low-loss silicon steel. (*Courtesy : Gautam Electric Motors*)

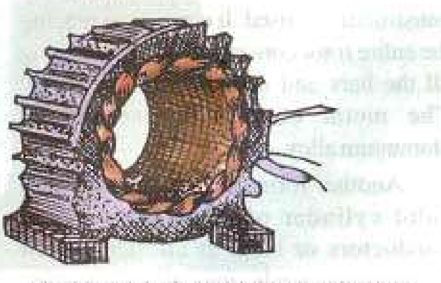


Fig. 34.2 (*b*) Completely wound stator for an induction motor. (*Courtesy : Gautam Electric Motors*)

* The number of poles P, produced in the rotating field is $P = 2n$ where n is the number of stator slots/pole/phase.

(*b*) **Rotor**

(*i*) ***Squirrel-cage rotor*** **:** Motors employing this type of rotor are known as squirrel-cage induction motors.

(*ii*) ***Phase-wound or wound rotor*** **:** Motors employing this type of rotor are variously known as 'phase-wound' motors or 'wound' motors or as 'slip-ring' motors.

34.4. Squirrel-cage Rotor

Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, it should be

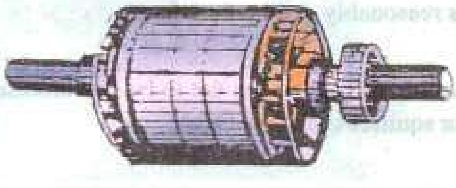


Fig. 34.3 (*a*) Squirrel-cage rotor with copper bars and alloy brazed end-rings (*Courtesy : Gautam Electric Motors*)

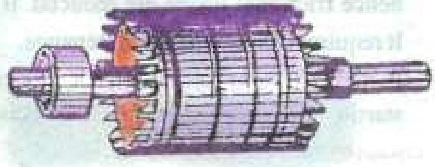


Fig. 34.3 (*b*) Rotor with shaft and brings (*Courtesy : Gautam Electric Motors*)

noted clearly, are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded or bolted to two heavy and stout short-circuiting end-rings, thus giving us, what is so picturesquely called, a squirrel-case construction (Fig. 34.3).

It should be noted that the ***rotor bars are permanently short-circuited on themselves*****,** hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes.

The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (Fig. 34.4). This is useful in two ways :

(*i*) it helps to make the motor run quietly by reducing the magnetic hum and

(*ii*) it helps in reducing the locking tendency of the rotor *i.e.* the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction between the two.*

In small motors, another method of construction is used. It consists of placing the entire rotor core in a mould and casting all the bars and end-rings in one piece. The metal commonly used is an aluminium alloy.

Another form of rotor consists of a solid cylinder of steel without any conductors or slots at all. The motor operation depends upon the production of eddy currents in the steel rotor.

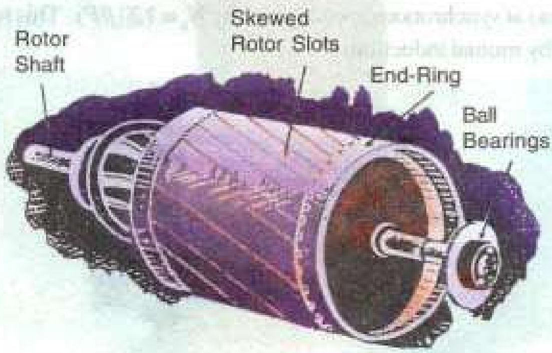


Fig. 34.4

* Other results of skew which may or may not be desirable are (*i*) increase in the effective ratio of transformation between stator and rotor (*ii*) increased rotor resistance due to increased length of rotor bars (*iii*) increased impedance of the machine at a given slip and (*iv*) increased slip for a given torque.

34.5. Phase-wound Rotor

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase even ***when the stator is wound two-phase***

The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them [Fig. 34.5 (*b*)]. These three brushes are further externally connected to a 3-phase star-connected rheostat [Fig. 34.5 (*c*)]. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor, as shown in Fig. 34.6 (*a*) (Ex.34.7 and 34.10) and for changing its speed-torque/current characteristics. When running under normal conditions, the ***slip-rings are automatically short-circuited*** by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-case rotor.

Fig. 34.5 (*a*)

Fig. 34.6 (*b*) shows the longitudinal section of a slip-ring motor, whose structural details are as under :

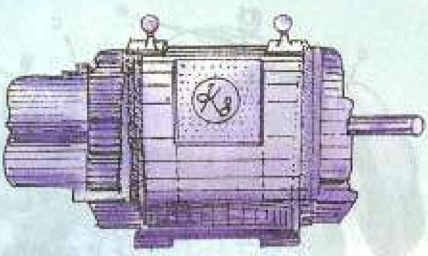


Fig. 34.5 (*b*) Slip-ring motor with slip-rings brushes and short-circuiting devices (*Courtesy : Kirloskar Electric Company*)

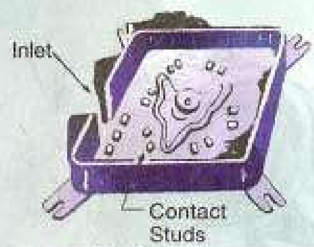


Fig. 34.5 (*c*)

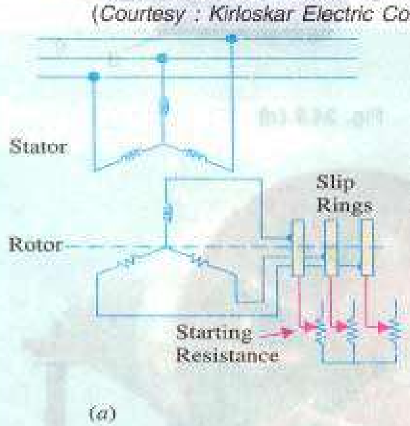


Fig. 34.6 (a)

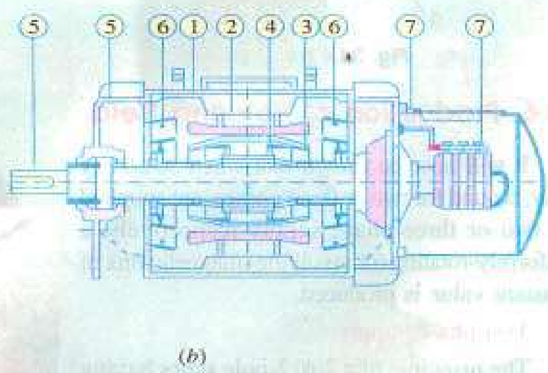


Fig. 34.6 (*b*) Longitudinal section of a Jyoti splash-proof slip-ring motor (*Courtesy : Jyoti Colour-Emag Ltd.*)

1. **Frame.** Made of close-grained alloy cast iron.
2. **Stator and Rotor Core.** Built from high-quality low-loss silicon steel laminations and flash-enamelled on both sides.
3. **Stator and Rotor Windings.** Have moisture proof tropical insulation embodying mica and high quality varnishes. Are carefully spaced for most effective air circulation and are rigidly braced to withstand centrifugal forces and any short-circuit stresses.
4. **Air-gap.** The stator rabbets and bore are machined carefully to ensure uniformity of air-gap.
5. **Shafts and Bearings.** Ball and roller bearings are used to suit heavy duty, toruble-free running and for enhanced service life.
6. **Fans.** Light aluminium fans are used for adequate circulation of cooling air and are securely keyed onto the rotor shaft.
7. **Slip-rings and Slip-ring Enclosures.** Slip-rings are made of high quality phosphor-bronze and are of moulded construction.

Fig. 34.6 (*c*) shows the disassembled view of an induction motor with squirrel-cage rotor. According to the labelled notation (*a*) represents stator (*b*) rotor (*c*) bearing shields (*d*) fan (*e*) ventilation grill and (*f*) terminal box.

Similarly, Fig. 34.6 (*d*) shows the disassembled view of a slip-ring motor where (*a*) represents stator (*b*) rotor (*c*) bearing shields (*d*) fan (*e*) ventilation grill (*f*) terminal box (*g*) slip-rings (*h*) brushes and brush holders.

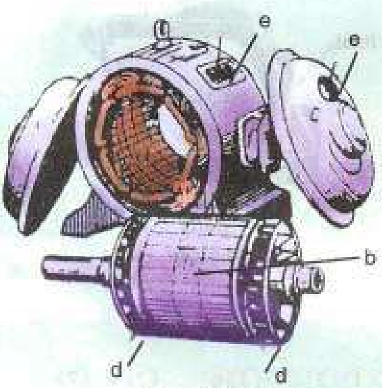


Fig. 34.6 (*c*)

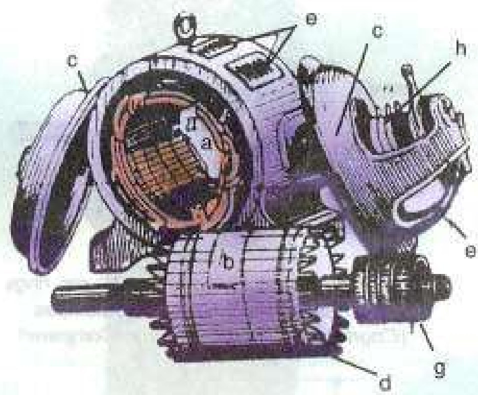


Fig. 34.6 (*d*)

34.6. Production of Rotating Field

It will now be shown that when stationary coils, wound for two or three phases, are supplied by two or three-phase supply respectively, a uniformly-rotating (or revolving) magnetic flux of constant value is produced.

Two-phase Supply

The principle of a 2-ϕ, 2-pole stator having two identical windings, 90 space degrees apart, is illustrated in Fig. 34.7.

Induction motor

The flux due to the current flowing in each phase winding is assumed sinusoidal and is represented in Fig. 34.9. The assumed positive directions of fluxes are those shown in Fig. 34.8.

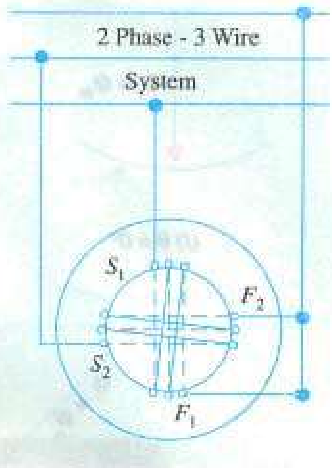


Fig. 34.7

Let Φ_1 and Φ_2 be the instantaneous values of the fluxes set up by the two windings. The resultant flux Φ_r at any time is the vector sum of these two fluxes (Φ_1 and Φ_2) at that time. We will consider conditions at intervals of 1/8th of a time period *i.e.* at intervals corresponding to angles of 0°, 45°, 90°, 135° and 180°. It will be shown that resultant flux Φ_r is constant in magnitude *i.e.* equal to Φ_m-the maximum flux due to either phase and is making one revolution/cycle. In other words, it means that the resultant flux rotates synchronously.

(*a*) When $\theta = 0°$ *i.e.* corresponding to point 0 in Fig. 34.9, $\Phi_1 = 0$, but Φ_2 is maximum *i.e.* equal to Φ_m and negative. Hence, resultant flux $\Phi_r = \Phi_m$ and, being negative, is shown by a vector pointing downwards [Fig. 34.10 (*i*)].

(*b*) When $\theta = 45°$ *i.e.* corresponding to point 1 in Fig. 34.9. At this instant, $\Phi_1 = \Phi_m / \sqrt{2}$ and is positive; $\Phi_2 = \Phi_m / \sqrt{2}$ but is still negative. Their resultant, as shown in Fig. 34.10 (*ii*), is $\Phi_r = \sqrt{[(\phi_m / \sqrt{2})^2 + (\phi_m / \sqrt{2})^2]} = \Phi_m$ although this resultant has shifted 45° clockwise.

(*c*) When $\theta = 90°$ *i.e.* corresponding to point 2 in Fig. 34.9. Here $\Phi_2 = 0$, but $\Phi_1 = \Phi_m$ and is positive. Hence, $\Phi_r = \Phi_m$ and has further shifted by an angle of 45° from its position in (*b*) or by 90° from its original position in (*a*).

(*d*) When $\theta = 135°$ *i.e.* corresponding to point 3 in Fig. 34.9. Here, $\Phi_1 = \Phi_m / \sqrt{2}$ and is positive, $\Phi_2 = \Phi_m / \sqrt{2}$ and is also positive. The resultant $\Phi_r = \Phi_m$ and has further shifted clockwise by another 45°, as shown in Fig. 34.10 (*iv*).

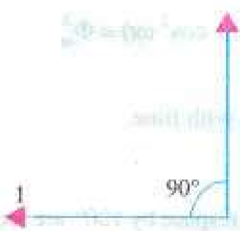


Fig. 34.8

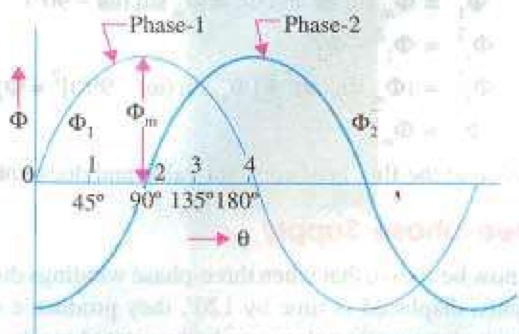


Fig. 34.9

(*e*) When $\theta = 180°$ *i.e.* corresponding to point 4 in Fig. 34.9. Here, $\Phi_1 = 0$, $\Phi_2 = \Phi_m$ and is positive. Hence, $\Phi_r = \Phi_m$ and has shifted clockwise by another 45° or has rotated through an angle of 180° from its position at the beginning. This is shown in Fig. 34.10(*v*).

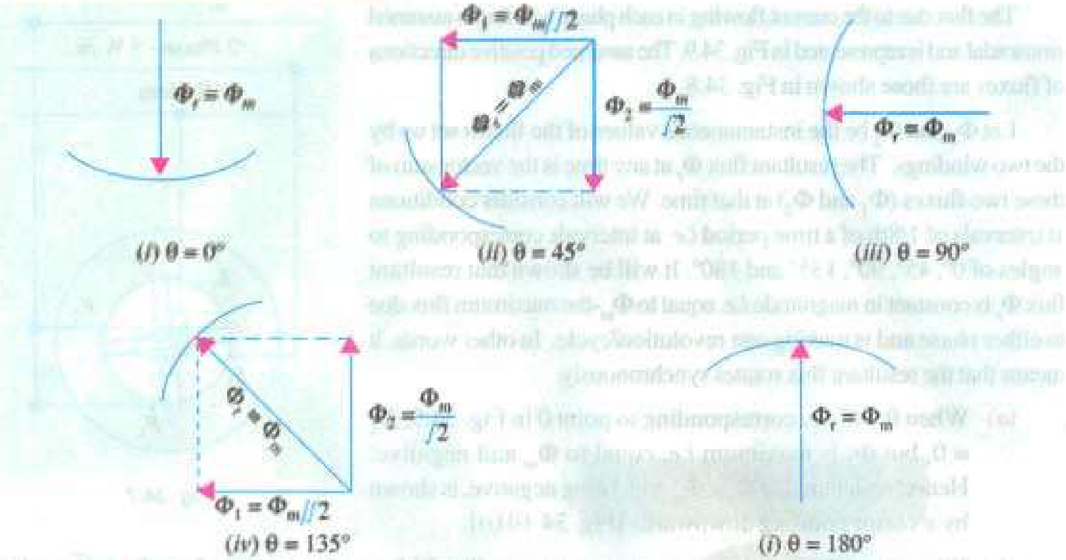


Fig. 34.10

Hence, we conclude

1. *that the magnitude of the resultant flux is constant and is equal to Φ_m — the maximum flux due to either phase.*
2. *that the resultant flux rotates at synchronous speed given by $N_s = 120\, f/P$ rpm.*

However, it should be clearly understood that in this revolving field, there is no actual revolution of the flux. The flux due to each phase changes periodically, according to the changes in the phase current, but the magnetic flux itself does not move around the stator. It is only the *seat* of the resultant flux which keeps on shifting synchronously around the stator.

Mathematical Proof

Let $\Phi_1 = \Phi_m \sin \omega t$ and $\Phi_2 = \Phi_m \sin(\omega t - 90°)$

$\therefore \quad \Phi_r^2 = \Phi_1^2 + \Phi_2^2$

$\Phi_r^2 = (\Phi_m \sin \omega t)^2 + [\Phi_m \sin(\omega t - 90°)]^2 = \Phi_m^2(\sin^2 \omega t + \cos^2 \omega t) = \Phi_m^2$

$\therefore \quad \Phi_r = \Phi_m$

It shows that the flux is of constant value and does not change with time.

34.7. Three-phase Supply

It will now be shown that when three-phase windings displaced in space by 120°, are fed by three-phase currents, displaced in time by 120°, they produce a resultant magnetic flux, which rotates in space as if actual magnetic poles were being rotated mechanically.

The principle of a 3-phase, two-pole stator having three identical windings placed 120 space degrees apart is shown in Fig. 34.11. The flux (assumed sinusoidal) due to three-phase windings is shown in Fig 34.12.

The assumed positive directions of the fluxes are shown in Fig 34.13. Let the maximum value of flux due to any one of the three phases be Φ_m. The resultant flux Φ_r, at any instant, is given by the vector sum of the individual fluxes, Φ_1, Φ_2 and Φ_3 due to three phases. We will consider values of Φ_r at four instants 1/6th time-period apart corresponding to points marked 0, 1, 2 and 3 in Fig. 34.12.

(*i*) When $\theta = 0°$ *i.e.* corresponding to point 0 in Fig. 34.12.

Here $\Phi_1 = 0$, $\Phi_2 = -\frac{\sqrt{3}}{2}\Phi_m$, $\Phi_3 = \frac{\sqrt{3}}{2}\Phi_m$. The vector for Φ_2 in Fig. 34.14 (*i*) is drawn in a direction opposite to the direction assumed positive in Fig. 34.13.

$$\therefore \quad \Phi_r = 2 \times \frac{\sqrt{3}}{2}\Phi_m \cos\frac{60°}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2}\Phi_m = \frac{3}{2}\Phi_m$$

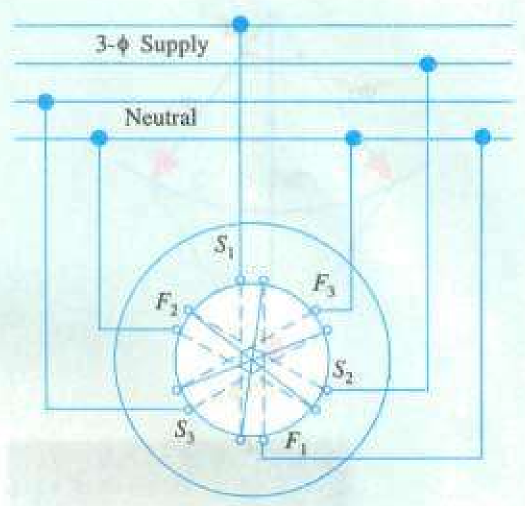


Fig. 34.11

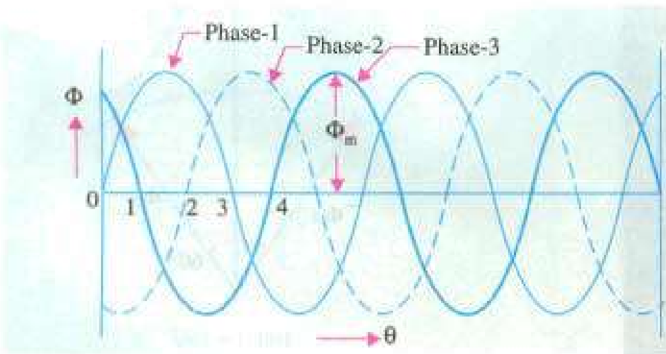


Fig. 34.12

Fig. 34.13

(*ii*) when $\theta = 60°$ *i.e.* corresponding to point 1 in Fig. 34.12.

Here $\Phi_1 = \frac{\sqrt{3}}{2}\Phi_m$ ...drawn parallel to *OI* of Fig. 34.13 as shown in Fig. 34.14 (*ii*)

$\Phi_2 = -\frac{\sqrt{3}}{2}\Phi_m$ ...drawn in opposition to *OII* of Fig. 34.13.

$\Phi_3 = 0$

$$\therefore \quad \Phi_r = 2 \times \frac{\sqrt{3}}{2}\Phi_m \times \cos 30° = \frac{3}{2}\Phi_m \qquad \text{[Fig. 34.14 (}ii\text{)]}$$

It is found that the resultant flux is again $\frac{3}{2}\Phi_m$ but has rotated clockwise through an angle of 60°.

(*iii*) When $\theta = 120°$ *i.e.* corresponding to point 2 in Fig. 34.12.

Here, $\Phi_1 = \frac{\sqrt{3}}{2}\Phi_m$, $\Phi_2 = 0$, $\Phi_3 = -\frac{\sqrt{3}}{2}\Phi_m$

It can be again proved that $\Phi_r = \frac{3}{2}\Phi_m$.

So, the resultant is again of the same value, but has further rotated clockwise through an angle of 60° [Fig. 34.14 (*iii*)].

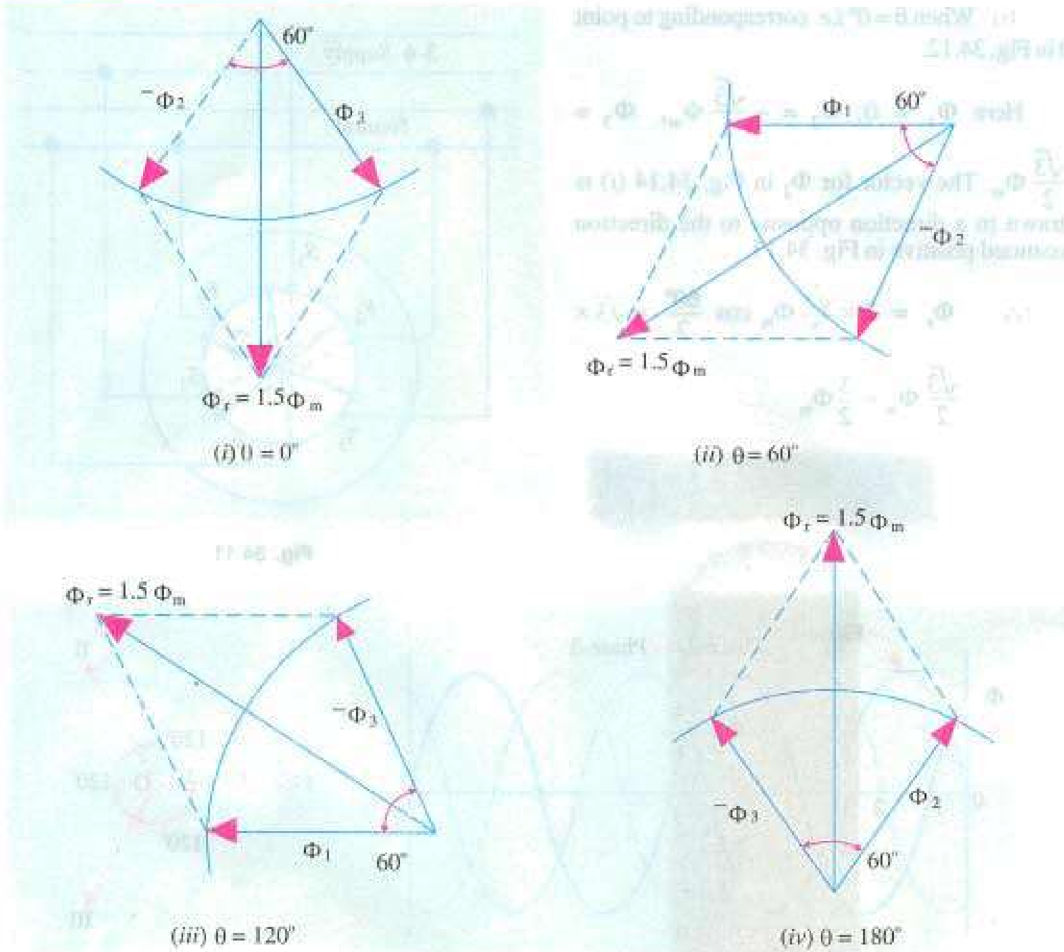


Fig. 34.14

(*iv*) When $\theta = 180°$ *i.e.* corresponding to point 3 in Fig. 34.12.

$$\Phi_1 = 0,\ \Phi_2 = \frac{\sqrt{3}}{2}\Phi_m,\ \Phi_3 = -\frac{\sqrt{3}}{2}\Phi_m$$

The resultant is $\frac{3}{2}\Phi_m$ and has rotated clockwise through an additional angle 60° or through an angle of 180° from the start.

Hence, we conclude that

1. the resultant flux is of constant value = $\frac{3}{2}\Phi_m$ *i.e. 1.5 times the maximum value of the flux due to any phase.*
2. *the resultant flux rotates around the stator at synchronous speed given by $N_s = 120\ f/P$.*

Fig. 34.15 (*a*) shows the graph of the rotating flux in a simple way. As before, the positive directions of the flux phasors have been shown separately in Fig. 34.15 (*b*). Arrows on these flux phasors are reversed when each phase passes through zero and becomes negative.

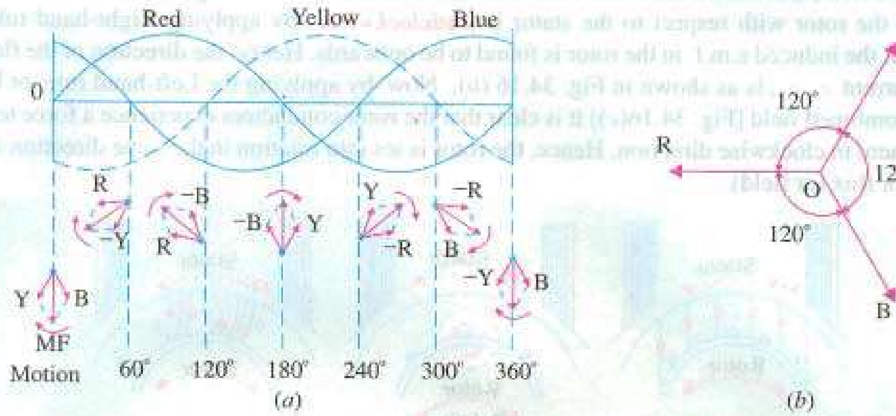


Fig. 34.15

As seen, positions of the resultant flux phasor have been shown at intervals of 60° only. The resultant flux produces a field rotating in the clockwise direction.

34.8. Mathematical Proof

Taking the direction of flux due to phase 1 as reference direction, we have

$$\Phi_1 = \Phi_m (\cos 0° + j \sin 0°) \sin \omega t$$
$$\Phi_2 = \Phi_m (\cos 240° + j \sin 240°) \sin (\omega t - 120°)$$
$$\Phi_3 = \Phi_m (\cos 120° + j \sin 120°) \sin (\omega t - 240°)$$

Expanding and adding the above equations, we get

$$\Phi_r = \frac{3}{2} \Phi_m (\sin \omega t + j \cos \omega t) = \frac{3}{2} \Phi_m \angle 90° - \omega t$$

The resultant flux is of constant magnitude and does not change with time 't'.

34.9. Why Does the Rotor Rotate ?

The reason why the rotor of an induction motor is set into rotation is as follow:

When the 3-phase stator windings, are fed by a 3-phase supply then, as seen from above, a magnetic flux of constant magnitude, but rotating at synchronous speed, is set up. The flux passes through the air-gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter, according to Faraday's laws of electro-magnetic induction. ***The frequency of the induced e.m.f. is the same as the supply frequency.*** Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is given by Fleming's Right-hand rule. Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the ***same*** direction as that of the flux and tries to catch up with the rotating flux.

Windings of induction electric motor

The setting up of the torque for rotating the rotor is explained below :

In Fig 34.16 (*a*) is shown the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is ***anticlockwise***. By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current *alone*, is as shown in Fig. 34.16 (*b*). Now, by applying the Left-hand rule, or by the effect of combined field [Fig. 34.16(*c*)] it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).

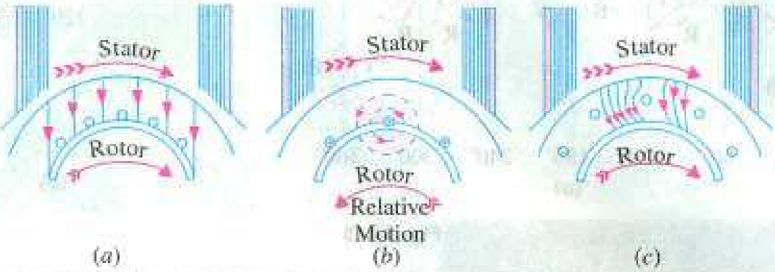


Fig. 34.16

34.10. Slip

In practice, the rotor never succeeds in 'catching up' with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor.*

The difference between the synchronous speed N_s and the actual speed N of the rotor is known as *slip*. Though it may be expressed in so many revolutions/second, yet it is usual to express it as a percentage of the synchronous speed. Actually, the term '*slip*' is descriptive of the way in which the rotor 'slips back' from synchronism.

$$\%\text{ slip} \quad s = \frac{N_s - N}{N_s} \times 100$$

Sometimes, $N_s - N$ is called the ***slip speed.***

Obviously, rotor (or motor) speed is $N = N_s(1 - s)$.

It may be kept in mind that revolving flux is rotating synchronously, relative to the stator (*i.e.* stationary space) but at slip speed relative to the rotor.

34.11. Frequency of Rotor Current

When the rotor is stationary, the frequency of rotor current is *the same as the supply frequency.* But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip-speed. Let at any slip-speed, the frequency of the rotor current be f'. Then

$$N_s - N = \frac{120 f'}{P} \qquad \text{Also,} \quad N_s = \frac{120 f}{P}$$

Dividing one by the other, we get, $\dfrac{f'}{f} = \dfrac{N_s - N}{N_s} = s$ $\qquad \therefore \quad f' = sf$

As seen, rotor currents have a frequency of $f' = sf$ and when flowing through the individual

* It may be noted that as the load is applied, the natural effect of the load or braking torque is to slow down the motor. Hence, slip increases and with it increases the current and torque, till the driving torque of the motor balances the retarding torque of the load. This fact determines the speed at which the motor runs on load.

phases of rotor winding, give rise to rotor magnetic fields. These individual rotor magnetic fields produce a combined rotating magnetic field, whose speed relative to rotor is

$$= \frac{120\,f'}{P} = \frac{120\,sf}{P} = sN_S$$

However, the rotor itself is running at speed N with respect to space. Hence,

speed of rotor field in *space* = speed of rotor magnetic field relative to rotor + speed of rotor relative to space

$$= sN_s + N = sN_s + N_s(1 - s) = N_s$$

It means that no matter what the value of slip, rotor currents and stator currents each produce a sinusoidally distributed magnetic field of constant magnitude and constant space speed of N_s. In other words, both the rotor and stator fields rotate synchronously, which means that they are stationary with respect to each other. These two synchronously rotating magnetic fields, in fact, superimpose on each other and give rise to the actually existing rotating field, which corresponds to the magnetising current of the stator winding.

Example 34.1. *A slip-ring induction motor runs at 290 r.p.m. at full load, when connected to 50-Hz supply. Determine the number of poles and slip.*

(Utilisation of Electric Power AMIE Sec. B 1991)

Solution. Since N is 290 rpm; N_s has to be somewhere near it, say 300 rpm. If N_s is assumed as 300 rpm, then $300 = 120 \times 50/P$. Hence, $P = 20$. $\therefore$ $s = (300 - 290)/300 = 3.33\%$

Example 34.2. *The stator of a 3-φ induction motor has 3 slots per pole per phase. If supply frequency is 50 Hz, calculate*

(*i*) *number of stator poles produced and total number of slots on the stator*

(*ii*) *speed of the rotating stator flux (or magnetic field).*

Solution. (*i*) $P = 2n = 2 \times 3 =$ **6 poles**

Total No. of slots = 3 slots/pole/phase × 6 poles × 3 phases = **54**

(*ii*) $N_s = 120 f/P = 120 \times 50/6 =$ **1000 r.p.m.**

Example 34.3. *A 4-pole, 3-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate :*

(*i*) *the speed at which the magnetic field of the stator is rotating.*

(*ii*) *the speed of the rotor when the slip is 0.04.*

(*iii*) *the frequency of the rotor currents when the slip is 0.03.*

(*iv*) *the frequency of the rotor currents at standstill.*

(Electrical Machinery II, Banglore Univ. 1991)

Solution. (*i*) Stator field revolves at synchronous speed, given by

$N_s = 120 f/P = 120 \times 50/4 =$ **1500 r.p.m.**

(*ii*) rotor (or motor) speed, $N = N_s(1 - s) = 1500(1 - 0.04) =$ **1440 r.p.m.**

(*iii*) frequency of rotor current, $f' = sf = 0.03 \times 50 = 1.5$ r.p.s = **90 r.p.m**

(*iv*) Since at standstill, $s = 1$, $f' = sf = 1 \times f = f =$ **50Hz**

Example 34.4. *A 3-φ induction motor is wound for 4 poles and is supplied from 50-Hz system. Calculate* (*i*) *the synchronous speed* (*ii*) *the rotor speed, when slip is 4% and* (*iii*) *rotor frequency when rotor runs at 600 rpm.*

(Electrical Engineering-I, Pune Univ. 1991)

Solution. (*i*) $N_s = 120 f/P = 120 \times 50/4 = 1500$ rpm

(*ii*) rotor speed, $N = N_s(1 - s) = 1500(1 - 0.04) =$ **1440 rpm**

(iii) when rotor speed is 600 rpm, slip is

$$s = (Ns - N)/N_s = (1500 - 600)/1500 = 0.6$$

rotor current frequency, $f' = sf = 0.6 \times 50 =$ **30 Hz**

Example 34.5. *A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor, at full-load is 3%, calculate the full-load speed of the motor.*

Solution. Let N = actual motor speed; Supply frequency, $f = 12 \times 500/120 = 50$ Hz. Synchronous speed $N_s = 120 \times 50/8 = 750$ r.p.m.

$$\% \text{ slip} \quad s = \frac{N_s - N}{N} \times 100\,; \quad 3 = \frac{750 - N}{750} \times 100 \qquad \therefore N = \mathbf{727.5 \text{ r.p.m.}}$$

Note. Since slip is 3%, actual speed N is less than N_s by 3% of N_s *i.e.* by $3 \times 750/100 = 22.5$ r.p.m.

34.12. Relation Between Torque and Rotor Power Factor

In Art. 29.7, it has been shown that in the case of a d.c. motor, the torque T_a is proportional to the product of armature current and flux per pole *i.e.* $T_a \propto \phi I_a$. Similarly, in the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current. However, there is one more factor that has to be taken into account *i.e.* the power factor of the rotor.

$\therefore \quad T \propto \phi I_2 \cos \phi_2 \quad$ or $\quad T = k \phi I_2 \cos \phi_2$

where I_2 = rotor current at standstill

ϕ_2 = angle between rotor e.m.f. and rotor current

k = a constant

Denoting rotor e.m.f. at ***standstill*** by E_2, we have that $E_2 \propto \phi$

$\therefore \quad T \propto E_2 I_2 \cos \phi_2$

or $\quad T = k_1 E_2 I_2 \cos \phi_2$

where k_1 is another constant.

The effect of rotor power factor on rotor torque is illustrated in Fig. 34.17 and Fig. 34.18 for various values of ϕ_2. From the above expression for torque, it is clear that as ϕ_2 increases (and hence, $\cos \phi_2$ decreases) the torque decreases and ***vice versa***.

In the discussion to follow, the stator flux distribution is assumed sinusoidal. This revolving flux induces in each rotor conductor or bar an *e.m.f.* whose value depends on the flux density, in which the conductor is lying at the instant considered ($e = Blv$ volt). Hence, the induced e.m.f. in the rotor is also sinusoidal.

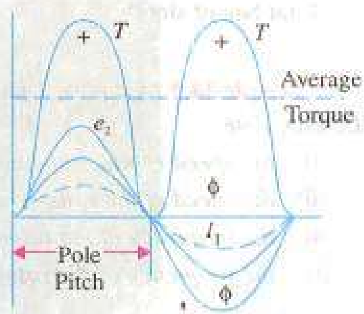


Fig. 34.17

(i) Rotor Assumed Non-inductive (or $\phi_2 = 0$)

In this case, the rotor current I_2 is in phase with the e.m.f. E_2 induced in the rotor (Fig. 34.17). The instantaneous value of the torque acting on each rotor conductor is given by the product of instantaneous value of the flux and the rotor current ($F \propto BI_2l$). Hence, torque curve is obtained by plotting the products of flux ϕ (or flux density B) and I_2. It is seen that the torque is always positive *i.e.* unidirectional.

(ii) Rotor Assumed Inductive

This case is shown in Fig. 34.18. Here, I_2 lags behind E_2 by an angle $\phi_2 = \tan^{-1} X_2/R_2$ where R_2 = rotor resistance/phase; X_2 = rotor reactance/phase at ***standstill***

It is seen that for a portion '*ab*' of the pole pitch, the torque is negative *i.e.* reversed. Hence, the total torque which is the difference of the forward and the backward torques, is considerably reduced. If $\phi_2 = 90°$, then the total torque is zero because in that case the backward and the forward torques become equal and opposite.

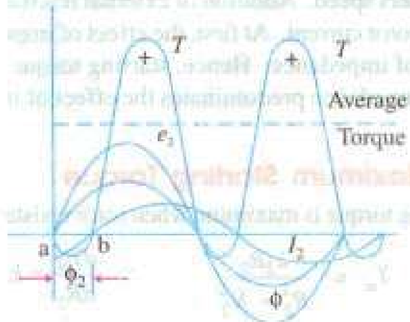


Fig. 34.18

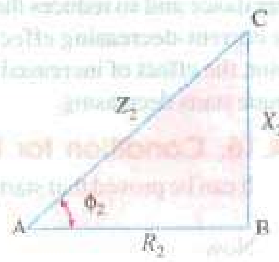


Fig. 34.19

34.13. Starting Torque

The torque developed by the motor at the instant of starting is called starting torque. In some cases, it is greater than the normal running torque, whereas in some other cases it is somewhat less.

Let E_2 = rotor *e.m.f.* per phase at ***standstill***;

R_2 = rotor resistance/phase

X_2 = rotor reactance/phase at ***standstill***

$\therefore \quad Z_2 = \sqrt{(R_2^2 + X_2^2)}$ = rotor impedance/phase at ***standstill*** ...Fig. 34.19

Then, $$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}}; \quad \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

Standstill or starting torque $T_{st} = k_1 E_2 I_2 \cos\phi_2$ ...Art. 34.12

or $$T_{st} = k_1 E_2 \cdot \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \quad ...(i)$$

If supply voltage V is constant, then the flux Φ and hence, E_2 both are constant.

$$\therefore \quad T_{st} = k_2 \frac{R_2}{R_2^2 + X_2^2} = k_2 \frac{R_2}{Z_2^2}$$ where k_2 is some other constant.

Now, $$k_1 = \frac{3}{2\pi N_s}, \quad \therefore \quad T_{st} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Where Ns → synchronous speed in rps.

34.14. Starting Torque of a Squirrel-cage Motor

The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor currents equals the supply frequency. Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2, with the result that the starting torque per ampere is very poor. It is roughly 1.5 times the full-load torque, although the starting current is 5 to 7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.

34.15. Starting Torque of a Slip-ring Motor

The starting torque of such a motor is increased by improving its power factor by adding external resistance in the rotor circuit from the star-connected rheostat, the rheostat resistance being progressively cut out as the motor gathers speed. Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

34.16. Condition for Maximum Starting Torque

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now $$T_{st} = \frac{k_2 R_2}{R_2^2 + X_2^2} \qquad \therefore \quad \frac{dT_{st}}{dR_2} = k_2 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2\,(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

or $$R_2^{\,2} + X_2^{\,2} = 2R_2^{\,2} \qquad \therefore \quad R_2 = X_2.$$

34.17. Effect of Change in Supply Voltage on Starting Torque

We have seen in Art. 34.13 that $T_{st} = \dfrac{k_1 E_2^2\, R_2}{R_2^2 + X_2^2}$. Now $E_2 \propto$ supply voltage V

$$\therefore \quad T_{st} = \frac{k_3 V^2 R_2}{R_2^2 + X_2^2} = \frac{k_3 V^2 R_2}{Z_2^2}$$ where k_3 is yet another constant. Hence $T_{st} \propto V^2$.

Clearly, the torque is very sensitive to any changes in the supply voltage. A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque. This fact is of importance in star-delta and auto transformer starters (Art. 33-11).

Example 34.6. *A 3-6 induction motor having a star-connected rotor has an induced e.m.f. of 80 volts between slip-rings at standstill on open-circuit. The rotor has a resistance and reactance per phase of 1 Ω and 4 Ω respectively. Calculate current/phase and power factor when (a) slip-rings are short-circuited (b) slip-rings are connected to a star-connected rheostat of 3 Ω per phase.*

(Electrical Technology, Bombay Univ. 1987, and similar example: Rajiv Gandhi Techn. Univ. Bhopal, Dec. 2000)

Solution. Standstill e.m.f./rotor phase = $80/\sqrt{3} = 46.2$ V

(a) Rotor impedance/phase $= \sqrt{(1^2 + 4^2)} = 4.12\,\Omega$

Rotor current/phase $= 46.2/4.12 =$ **11.2 A**

Power factor $= \cos\phi = 1/4.12 =$ **0.243**

As p.f. is low, the starting torque is also low.

(b) Rotor resistance/phase $= 3 + 1 = 4\,\Omega$

Rotor impedance/phase $= \sqrt{(4^2 + 4^2)} = 5.66\,\Omega$

∴ Rotor current/phase $= 46.2/5.66 =$ **8.16 A**; $\cos\phi = 4/5.66 =$ **0.707**.

Hence, the starting torque is increased due to the improvement in the power factor. It will also be noted that improvement in p.f. is much more than the decrease in current due to increased impedance.

Example 34.7. *A 3-phase, 400-V, star-connected induction motor has a star-connected rotor with a stator to rotor turn ratio of 6.5. The rotor resistance and standstill reactance per phase are 0.05 Ω and 0.25 Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to obtain maximum torque at starting and what will be rotor starting current with this resistance?*

Solution. Here $K = \dfrac{1}{6.5}$ because transformation ratio K is defined as

$$= \frac{\text{rotor turns/phase}}{\text{stator turns/phase}}$$

Standstill rotor e.m.f./phase, $E_2 = \frac{400}{\sqrt{3}} \times \frac{1}{6.5} = 35.5$ volt

It has been shown in Art. 34.16 that starting torque is maximum when $R_2 = X_2$ *i.e.* when $R_2 = 0.25\ \Omega$ in the present case

$\therefore$ External resistance/phase required $= 0.25 - 0.05 =$ **0.2 Ω**

Rotor impedance/phase $= \sqrt{(0.25^2 + 0.25^2)} = 0.3535\ \Omega$

Rotor current/phase, $I_2 = 35.5/0.3535 =$ **100 A (approx)**

Example 34.8. *A 1100-V, 50-Hz delta-connected induction motor has a star-connected slip-ring rotor with a phase transformation ratio of 3.8. The rotor resistance and standstill leakage reactance are 0.012 ohm and 0.25 ohm per phase respectively. Neglecting stator impedance and magnetising current determine.*

(i) the rotor current at start with slip-rings shorted

(ii) the rotor power factor at start with slip-rings shorted

(iii) the rotor current at 4% slip with slip-rings shorted

(iv) the rotor power factor at 4% slip with slip-rings shorted

(v) the external rotor resistance per phase required to obtain a starting current of 100 A in the stator supply lines. **(Elect. Machines AMIE Sec. B 1992)**

Solution. It should be noted that in a Δ/Y connection, primary phase voltage is the same as the line voltage. The rotor phase voltage can be found by using the phase transformation ratio of 3.8 *i.e.* $K = 1/3.8$.

Rotor phase voltage at standstill $= 1100 \times 1/3.8 = 289.5$V

(i) Rotor impedence/phase $= \sqrt{0.012^2 + 0.25^2} = 0.2503\ \Omega$

Rotor phase current at start $= 289.5/0.2503 =$ **1157 A**

(ii) $p.f. = R_2/Z_2 = 0.012/0.2503 =$ **0.048 lag**

(iii) at 4% slip, $X_r = sX_2 = 0.04 \times 0.25 = 0.01\ \Omega$

$\therefore$ $Z_r = \sqrt{0.012^2 + 0.01^2} =$ **0.0156 Ω**

$E_r = sE_2 = 0.04 \times 289.5 = 11.58$ V; $I_2 = 11.58/0.0156 =$ **742.3 A**

(iv) $p.f. = 0.012/0.0156 =$ **0.77**

(v) $I_2 = I_1/K = 100 \times 3.8 = 380$ A; E_2 at standstill $= 289.5$ V

$Z_2 = 289.5/380 = 0.7618\ \Omega$; $R_2 = \sqrt{Z_2^2 - X_2^2} = \sqrt{0.7618^2 - 0.25^2} = 0.7196\ \Omega$

$\therefore$ External resistance reqd./phase $= 0.7196 - 0.012 =$ **0.707 Ω**

Example 34.9. *A 150-kw, 3000-V, 50-Hz, 6-pole star-connected induction motor has a star-connected slip-ring rotor with a transformation ratio of 3.6 (stator/rotor). The rotor resistance is 0.1 Ω/phase and its per phase leakage reactance is 3.61 mH. The stator impedance may be neglected. Find the starting current and starting torque on rated voltage with short-circuited slip rings.*

(Elect. Machines, A.M.I.E. Sec. B, 1989)

Solution. $X_2 = 2\pi \times 50 \times 5.61 \times 10^{-3} = 1.13\ \Omega$

$K = 1/3.6$, $R_2' = R_2/K^2 = (3.6)^2 \times 0.1 = 1.3\ \Omega$

$X_2 = 2\pi \times 50 \times 3.61 \times 10^{-3} = 1.13\ \Omega$; $X_2' = (3.6)^2 \times 1.13 = 14.7\Omega$

$$I_{st} = \frac{V}{(R_2')^2 + (X_2')^2} \cdot \frac{3000/\sqrt{3}}{\sqrt{(1.3)^2 + (14.7)^2}} = \mathbf{117.4\ A}$$

Now, $N_s = 120 \times 50/6 = 1000$ rpm $= (50/3)$ rps

$$T_{st} = \frac{3}{2\pi N_s} \cdot \frac{V^2 R_2'}{(R_2')^2 + (X_2')^2} = \frac{3}{2\pi\,(50/3)} \times \frac{(3000/\sqrt{3})^2 \times 1.3}{(1.3^2 + 14.7^2)} = \mathbf{513\ N\text{-}m}$$

Tutorial Problem No. 34.1

1. In the case of an 8-pole induction motor, the supply frequency was 50-Hz and the shaft speed was 735 r.p.m. What were the magnitudes of the following: *(Nagpur Univ., Summer 2000)*
   (*i*) synchronous speed (*ii*) speed of slip
   (*iii*) per unit slip (*iv*) percentage slip
   [750 r.p.m. ; 15 r.p.m.; 0.02 ; 2%]
2. A 6-pole, 50-Hz squirrel-cage induction motor runs on load at a shaft speed of 970 r.p.m. Calculate:-
   (*i*) the percentage slip
   (*ii*) the frequency of induced current in the rotor. **[3% ; 1.5 Hz]**
3. An 8-pole alternator runs at 750. r.p.m. and supplies power to a 6-pole induction motor which has at full-load a slip of 3%. Find the full-load speed of the induction motor and the frequency of its rotor e.m.f. **[970 r.p.m. ; 1.5 Hz]**
4. A 3-phase, 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between the slip-rings on open-circuit. Calculate the current and power factor at standstill when the rotor winding is joined to a star-connected external circuit, each phase of which has a resistance of 10 Ω and an inductance of 0.04 H. The resistance per phase of the rotor winding is 0.2 Ω and its inductance is 0.04 H.

   Also, calculate the current and power factor when the slip-rings are short-circuited and the motor is running with a slip of 5 per cent. Assume the flux to remain constant.
   [10.67 A; 0.376; 21.95 A; 0.303]
5. Obtain an expression for the condition of maximum torque of an induction motor. Sketch the torque-slip curves for several values of rotor circuit resistance and indicate the condition for maximum torque to be obtained at starting.

   If the motor has a rotor resistance of 0.02 Ω and a standstill reactance of 0.1 Ω, what must be the value of the total resistance of a starter for the rotor circuit for maximum torque to be exerted at starting ? **[0.08 Ω]** *(City and Guilds, London)*
6. The rotor of a 6-pole, 50-Hz induction motor is rotated by some means at 1000 r.p.m. Compute (*i*) rotor voltage (*ii*) rotor frequency (*iii*) rotor slip and (iv) torque developed. Can the rotor rotate at this speed by itself ? **[(*i*) 0 (*ii*) 0 (*iii*) 0 (*iv*) 0; No]** *(Elect. Engg. Grad I.E.T.E. June 1985)*
7. The rotor resistances per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.024 ohm and 0.12 ohm respectively. Find the speed at maximum torque. Also find the value of the additional rotor resistance per phase required to develop 80% of maximum torque at starting.
   [1200 r.p.m. 0.036 Ω] *(Elect. Machines, A.M.I.E. Sec. B, 1990)*
8. The resistance and reactance per phase of the rotor of a 3-phase induction motor are 0.6 ohm and 5 ohms respectively. The induction motor has a star-connected rotor and when the stator is connected to a supply of normal voltage, the induced e.m.f. between the slip rings at standstill is 80 V. Calculate the current in each phase and the power factor at starting when (*i*) the slip-rings are shorted, (*ii*) slip-rings are connected to a star-connected resistance of 4 ohm per phase.

[(*i*) 9.17 amp, 0.1194 lag (*ii*) 6.8 amp, 0.6765 lag] *[Rajiv Gandhi Technical University, Bhopal, 2000]*

34.18. Rotor E.M.F. and Reactance Under Running Conditions

Let E_2 = *standstill* rotor induced e.m.f./phase

X_2 = *standstill* rotor reactance/phase, f_2 = rotor current frequency at *standstill*

When rotor is stationary *i.e.* $s = 1$, the frequency of rotor e.m.f. is the same as that of the stator supply frequency. The value of e.m.f. induced in the rotor at standstill is maximum because the relative speed between the rotor and the revolving stator flux is maximum. In fact, the motor is equivalent to a 3-phase transformer with a short-circuited rotating secondary.

When rotor starts running, the relative speed between it and the rotating stator flux is decreased. Hence, the rotor induced e.m.f. which is directly proportional to this relative speed, is also decreased (and may disappear altogether if rotor speed were to become equal to the speed of stator flux). Hence, for a slip s, the rotor induced e.m.f. will be s times the induced e.m.f. at standstill.

Therefore, under *running conditions* $E_r = sE_2$

The frequency of the induced e.m.f. will likewise become $f_r = sf_2$

Due to decrease in frequency of the rotor e.m.f., the rotor reactance will also decrease.

$$\therefore \qquad X_r = sX_2$$

where E_r and X_r are rotor e.m.f. and reactance under *running* conditions.

34.19. Torque Under Running Conditions

$$T \propto E_r I_r \cos\phi_2 \text{ or } T \propto \phi\, I_r \cos\phi_2 \qquad (\because E_r \propto \phi)$$

where E_r = rotor e.m.f./phase under *running conditions*

I_r = rotor current/phase under *running conditions*

Now $E_r = sE_2$

$$\therefore \qquad I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos\phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \qquad \text{—Fig. 34.20}$$

$$\therefore \qquad T \propto \frac{s\,\Phi\, E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{k\Phi \,.\, s\,.\, E_2 R_2}{R_2^2 + (sX_2)^2}$$

$$\text{Also } T = \frac{k_1 \,.\, sE_2^2\, R_2}{R_2^2 + (sX_2)^2} \qquad (\because E_2 \propto \phi)$$

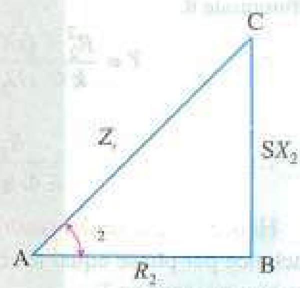


Fig. 34.20

where k_1 is another constant. Its value can be proved to be equal to $3/2\,\pi N_s$ (Art. 34.38). Hence, in that case, expression for torque becomes

$$T = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\, R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\, R_2}{Zr^2}$$

At standstill when $s = 1$, obviously

$$T_{st} = \frac{k_1 E_2^2\, R_2}{R_2^2 + X_2^2} \left(\text{or} = \frac{3}{2\pi N_S} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \right) \quad \text{the same as in Art. 34.13.}$$

Example 34.10. *The star connected rotor of an induction motor has a standstill impedance of (0.4 + j4) ohm per phase and the rheostat impedance per phase is (6 + J2) ohm.*

The motor has an induced emf of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find (*i*) *rotor current at standstill with the rheostat is in the circuit.*

(*ii*) *when the slip-rings are short-circuited and motor is running with a slip of 3%.*

(Elect.Engg. I, Nagpur Univ. 1993)

Solution. (1) Standstill Conditions

Voltage/rotor phase $= 80/\sqrt{3} = 46.2$. V; rotor and

starter impedance/phase $= (6.4 + j6) = 8.77 \angle 43.15°$

Rotor current/phase $= 46.2/8.77 =$ **5.27 A** (p.f. = cos 43.15° = 0.729)

(2) Running Conditions. Here, starter impedance is cut out.

Rotor voltage/phase, $E_r = sE_2 = 0.03 \times 46.2 = 1.386$ V

Rotor reactance/phase, $X_r = 0.03 \times 4 = 0.12\ \Omega$

Rotor impedance/phase, $Z_r = 0.4 + j0.12 = 0.4176 \angle 16.7°$

Rotor current/phase $= 1.386/0.4176 =$ **3.32 A** (p.f. = cos 16.7° = 0.96)

Note. It has been assumed that flux across the air -gap remains constant

34.20. Condition for Maximum Torque Under Running Conditions

The torque of a rotor under *running* conditions is

$$T = \frac{k\Phi sE_2R_2}{R_2^2 + (sX_2)^2} = k_1 \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \quad ...(i)$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then putting it equal to zero. However, it is simpler to put $Y = \frac{1}{T}$ and then differentiate it.

$$\therefore \quad Y = \frac{R_2^2 + (sX_2)^2}{k\,\Phi\, sE_2R_2} = \frac{R_2}{k\,\Phi\, sE_2} + \frac{sX_2^2}{k\Phi\, E_2R_2};\ \frac{dY}{ds} = \frac{-R_2}{k\Phi\, s^2E_2} + \frac{X_2^2}{k\Phi E_2R_2} = 0$$

$$\therefore \quad \frac{R_2}{k\,\Phi\, s^2E_2} = \frac{X_2^2}{k\,\Phi\, E_2R_2} \quad \text{or } R_2^{\,2} = s^2 X_2^{\,2} \text{ or } R_2 = sX_2$$

Hence, torque under *running condition* is maximum at that value of the slip s which makes rotor reactance per phase equal to rotor resistance per phase. This slip is sometimes written as s_b and the maximum torque as T_b.

Slip corresponding to maximum torque is $s = R_2/X_2$

Putting $R_2 = sX_2$ in the above equation for the torque, we get

$$T_{max} = \frac{k\,\Phi\, s^2\, E_2X_2}{2\, s^2\, X_2^2}\left(\text{or } \frac{k\,\Phi\, sE_2R_2}{2R_2^2}\right) \text{or } T_{max} = \frac{k\,\Phi\, E_2}{2\, X_2}\left(\text{or } \frac{k\,\Phi\, sE_2}{2\, R_2}\right) \quad ...(ii)$$

Substituting value of $s = R_2/X_2$ in the other equation given in (*i*) above, we get

$$T_{max} = k_1 \frac{(R_2/X_2)\,.\,E_2^2\,.\,R_2}{R_2^2 + (R_2/X_2)^2\,.\,X_2^2} = k_1 \frac{E_2^2}{2\,X_2}$$

Since, $k_1 = 3/2\pi N_s$, we have $T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2X_2}$ N-m

From the above, it is found

1. *that the maximum torque is independent of rotor resistance as such.*
2. *however, the speed or slip at which maximum torque occurs is determined by the rotor*

resistance. As seen from above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).

3. *maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.*
4. *maximum torque varies directly as the square of the applied voltage.*
5. *for obtaining maximum torque at starting (s =1), rotor resistance must be equal to rotor reactance.*

Example 34.11. *A 3-phase, slip-ring, induction motor with star-connected rotor has an induced e.m.f. of 120 volts between slip-rings at standstill with normal voltage applied to the stator. The rotor winding has a resistance per phase of 0.3 ohm and standstill leakage reactance per phase of 1.5 ohm.*

Calculate (i) rotor current/phase when running short-circuited with 4 percent slip and (ii) the slip and rotor current per phase when the rotor is developing maximum torque.

(Elect. Engg.-II, Pune Univ. 1989)

Solution. (*i*) induced e.m.f./rotor phase, $E_r = sE_2 = 0.04 \times (120/\sqrt{3}) = 2.77\text{V}$

rotor reactance/phase, $X_r = sX_2 = 0.04 \times 1.5 = 0.06\ \Omega$

rotor impedance/phase $= \sqrt{0.3^2 + 0.06^2} = 0.306\ \Omega$

rotor current/phase $= 2.77/0.306 = 9\text{A}$

(*ii*) For developing maximum torque,

$$R_2 = sX_2 \quad \text{or} \quad s = R_2/X_2 = 0.3/1.5 = 0.2$$

$$X_r = 0.2 \times 1.5 = 0.3\ \Omega, Z_r = \sqrt{0.3^2 + 0.3^2} = 0.42\ \Omega$$

$$E_r = sE_2 = 0.2 \times (120/\sqrt{3}) = 13.86\text{ V}$$

$\therefore$ Rotor current/phase $= 13.86/0.42 = 33\text{ A}$

34.21. Rotor Torque and Breakdown Torque

The rotor torque at any slip s can be expressed in terms of the maximum (or breakdown) torque T_b by the following equation

$$T = T_b \left[\frac{2}{(s_b/s) + (s/s_b)} \right]$$ where s_b is the breakdown or pull-out slip.

Example 34.12. *Calculate the torque exerted by an 8-pole, 50-Hz, 3-phase induction motor operating with a 4 per cent slip which develops a maximum torque of 150 kg-m at a speed of 660 r.p.m. The resistance per phase of the rotor is 0.5 Ω.* **(Elect. Machines, A.M.I.E. Sec. B, 1989)**

Solution. $N_s = 120 \times 50/8 = 750$ r.p.m.

Speed at maximum torque $= 660$ r.p.m. Corresponding slip $s_b = \dfrac{750 - 660}{750} = 0.12$

For maximum torque, $R_2 = s_b X_2$

$\therefore$ $X_2 = R_2/s_b = 0.5/0.12 = 4.167\ \Omega$

As seen from Eq. (*ii*) of Art. 34.20,

$$T_{max} = k\,\Phi\,E_2 \cdot \frac{s_b}{2R_2} = k\,\Phi\,E_2 \cdot \frac{0.12}{2 \times 0.5} = 0.12\,k\,\Phi\,E_2 \qquad \ldots(i)$$

When slip is 4 per cent

As seen from Eq. (*i*) of Art. 34.20

$$T = k\,\Phi\,E_2 \frac{sR_2}{R_2^2 + (sX_2)^2} = k\Phi E_2 \frac{0.04 \times 0.5}{0.5^2 + (0.04 \times 4.167)^2} = \frac{0.02\,k\,\Phi\,E_2}{0.2778}$$

$$\therefore \quad \frac{T}{T_{max}} = \frac{T}{150} = \frac{0.02}{0.2778 \times 0.12} \quad \therefore \quad T = \textbf{90 kg-m}$$

Alternative Solution

$$T_b = 150 \text{ kg.m}; \; s_b = 0.12, \; s = 4\% = 0.04, \; T = ?$$

$$T = T_b \left(\frac{2}{(s_b / s) + (s / s_b)} \right) \qquad \text{...Art 34.21}$$

$$= 150 \left(\frac{2}{(0.12/0.04) + (0.04/0.12)} \right) = \textbf{90 kg-m}$$

34.22. Relation Between Torque and Slip

A family of torque/slip curves is shown in Fig. 34.21 for a range of $s = 0$ to $s = 1$ with R_2 as the parameter. We have seen above in Art. 34.19 that

$$T = \frac{k\,\Phi\,sE_2R_2}{R_2^2 + (sX_2)^2}$$

It is clear that when $s = 0$, $T = 0$, hence the curve starts from point O.

At normal speeds, close to synchronism, the term $(s\,X_2)$ is small and hence negligible *w.r.t.* R_2.

$$\therefore \quad T \propto \frac{s}{R_2}$$

or $\quad T \propto s$ if R_2 is constant.

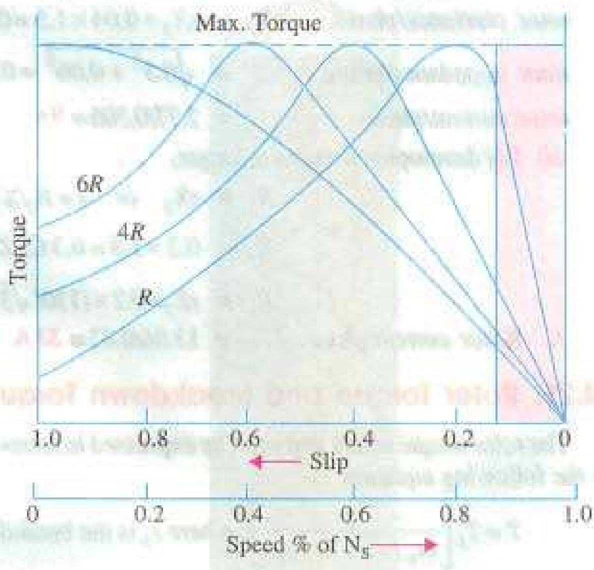


Fig. 34.21

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load on the motor), the torque also increases and becomes maximum when $s = R_2/X_2$. This torque is known as '*pull-out*' or '*breakdown*' torque T_b or stalling torque. As the slip further increases (*i.e.* motor speed falls) with further increase in motor load, then R_2 becomes negligible as compared to (sX_2). Therefore, for large values of slip

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of $s = 0$ and that corresponding to maximum torque. The operating range is shown shaded in Fig. 34.21.

It is seen that although maximum torque does not depend on R_2, yet the *exact location* of T_{max} is dependent on it. Greater the R_2, greater is the value of slip at which the maximum torque occurs.

34.23. Effect of Change in Supply Voltage on Torque and Speed

As seen from Art. 34.19, $T = \dfrac{k\Phi\, sE_2R_2}{R_2^2 + (sX_2)^2}$

As $\quad E_2 \propto \phi \propto V$ where V is supply voltage $\quad \therefore T \propto sV^2$

Obviously, torque at any speed is proportional to the square of the applied voltage. If stator voltage decreases by 10%, the torque decreases by 20%. Changes in supply voltage not only affect the starting torque T_{st} but torque under running conditions also. If V decreases, then T also decreases. Hence, for maintaining the same torque, slip increases *i.e.* speed falls.

Let V change to V', s to s' and T to T'; then $\dfrac{T}{T'} = \dfrac{sV^2}{s'V'^2}$

34.24. Effect of Changes in supply Frequency on Torque and Speed

Hardly any important changes in frequency take place on a large distribution system except during a major disturbance. However, large frequency changes often take place on isolated, low-power systems in which electric energy is generated by means of diesel engines or gas turbines. Examples of such systems are : emergency supply in a hospital and the electrical system on a ship etc.

The major effect of change in supply frequency is on motor ***speed***. If frequency drops by 10%, then motor speed also drops by 10%. Machine tools and other motor-driven equipment meant for 50 Hz causes problem when connected to 60-Hz supply. Everything runs $(60 - 50) \times 100/50 = 20\%$ faster than normal and this may not be acceptable in all applications. In that case, we have to use either gears to reduce motor speed or an expensive 50-Hz source.

A 50-Hz motor operates well on a 60-Hz line provided its terminal voltage is raised to $60/50 = 6/5$ (*i.e.* 120%) of the name-plate rating. In that case, the new breakdown torque becomes equal to the original breakdown torque and the starting torque is only slightly reduced. However, power factor, efficiency and temperature rise remain satisfactory.

Similarly, a 60-Hz motor can operate satisfactorily on 50-Hz supply provided its terminal voltage is reduced to 5/6 (*i.e.* 80%) of its name-plate rating.

34.25. Full-load Torque and Maximum Torque

Let s_f be the slip corresponding to full-load torque, then

$$T_f \propto \frac{s_f\, R_2}{R_2^2 + (s_f\, X_2)^2} \quad \text{and} \quad T_{max} \propto \frac{1}{2 \times X_2} \qquad \text{...Art 34.20}$$

$$\therefore \quad \frac{T_f}{T_{max}} = \frac{2s_f\, R_2\, X_2}{R_2^2 + (s_f\, X_2)^2}$$

Dividing both the numerator and the denominator by X_2^2, we get

$$\frac{T_f}{T_{max}} = \frac{2s_f \cdot R_2 / X_2}{(R_2/X_2)^2 + s_f^2} = \frac{2as_f}{a^2 + s_f^2}$$

where $a = R_2/X_2$ = resistance/standstill reactance*

* In fact $a = s_m$ —slip corresponding to maximum torque. In that case, the relation becomes

$$\frac{T_f}{T_{max}} = \frac{2s_m s_f^2}{s_m^2 + s_f^2} \text{ — where } s_f = \text{full-load slip.}$$

In general, $\dfrac{\text{operating torque at any slip } s}{\text{maximum torque}} = \dfrac{2\,as}{a^2 + s_f^{\,2}}$

34.26. Starting Torque and Maximum Torque

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2} \qquad \text{...Art 34.13}$$

$$T_{max} \propto \frac{1}{2\,X_2} \qquad \text{...Art. 34.20}$$

$$\therefore \quad \frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2/X_2}{1+(R_2/X_2)^2} = \frac{2a}{1+a^2}$$

where $\quad a = \dfrac{R_2}{X_2} = \dfrac{\text{rotor resistance}}{\text{stand still reactance}}$ per phase*

Example. 34.13(*a*). *A 3-φ induction motor is driving full-load torque which is independent of speed. If line voltage drops to 90% of the rated value, find the increase in motor copper losses.*

Solution. As seen from Art. 34.23, when T remains constant, $s_1 V_1^{\,2} = s_2 V_2^{\,2}$

$$\therefore \quad \frac{s_2}{s_1} = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{1}{0.9}\right)^2 = 1.23$$

Again from Art. 34-19, $I_2 \propto sV$ $\therefore$ $\dfrac{I_2'}{I_2} = \dfrac{s_2V_2}{s_1V_1} = 1.2 \times 0.9 = 1.107$

Now, Cu losses are nearly proportional to $I_2^{\,2}$

$$\therefore \quad \frac{\text{Cu loss in the 2nd case}}{\text{Cu loss in the 1st case}} = \frac{(I_2')^2}{I_2^2} = 1.107^2 = 1.23$$

Thus a reduction of 10% in line voltage causes about 23% increase in Cu losses.

Example. 34.13 (*b*). *A 230-V, 6-pole, 3-φ, 50-Hz, 15-kW induction motor drives a constant torque load at rated frequency, rated voltage and rated kW output and has a speed of 980 rpm and an efficiency of 93%. Calculate (i) the new operating speed if there is a 10% drop in voltage and 5% drop in frequency and (ii) the new output power. Assume all losses to remain constant.*

Solution. (*i*) $V_2 = 0.9 \times 230 = 207$ V; $f_2 = 0.95 \times 50 = 47.5$ Hz; $N_{s1} = 120 \times 50/6 = 1000$ rpm; $N_{s2} = 120 \times 47.5/6 = 950$ rpm; $s_1 = (1000 - 980)/1000 = 0.02$

Since the load torque remains constant, the product (sV^2/f) remains constant.

$$\therefore \quad s_1V_1^2/f_1 = \frac{s_2V_2^2}{f_2} \text{ or } s_2 = s_1\left(\frac{V_1}{V_2}\right)^2 \cdot f_2/f_1 = 0.02\,(230/230 \times 0.9)^2 \times 47.5/50 = 0.2234$$

$$\therefore \quad N_2 = Ns_2(1 - s_2) = 950\,(1 - 0.0234) = 928 \text{ rpm}$$

(*ii*) $P \propto TN$. Since torque remains constant, $P \propto N$

$\therefore$ $P_1 \propto N_1$; $P_2 \propto N_2$; or $P_2 = P_1 \times N_2/N_1 = 15 \times 928/980 =$ **14.2 kW**

* Similarly, the relation becomes $\dfrac{T_f}{T_{max}} = \dfrac{2s_m}{1+s_m^2}$

Example 34.14 (a). *A 3-phase, 400/200-V, Y-Y connected wound-rotor induction motor has 0.06Ω rotor resistance and 0.3 Ω standstill reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque of the motor.* **(Electrical Technology, Bombay Univ. 1990)**

Solution. $\dfrac{T_{st}}{T_{max}} = \dfrac{2a}{1+a^2}$; Since $T_{st} = T_{max}$

$\therefore \quad 1 = \dfrac{2a}{1+a^2}$ or $a = 1$ Now, $a = \dfrac{R_2 + r}{X_2}$

where r = external resistance per phase added to the rotor circuit

$\therefore \quad 1 = \dfrac{0.06 + r}{0.3} \quad \therefore \quad r = 0.3 - 0.06 = \mathbf{0.24\ \Omega}$

Example 34.14 (b). *3-phase, 50-Hz, 8-pole, induction motor has full-load slip of 2%. The rotor-resistance and stand still rotor-reactance per phase are 0.001 ohm and 0.005 ohm respectively. Find the ratio of the maximum to full-load torque and the speed at which the maximum torque occurs.* **(Amravati University, 1999)**

Solution. Synchronous speed, $N_s = 120 \times 50/8 = 750$ rpm

Slip at maximum torque, $s_{mT} = r_2/x_2$

Thus, let a $= \dfrac{r_2}{x_2} = \dfrac{0.001}{0.005} = 0.2$

Corresponding speed $= (1 - 0.2) \times 750 = 600$ rpm

$$\frac{\text{Full} - \text{load torque}}{\text{Maximum torque}} = \frac{2\, s_{mT}\, s_{fL}^2}{s_{mT}^2 + s_{fL}^2} \therefore \frac{T_{fL}}{T_{max}} = \frac{2 \times 0.2 \times 0.02^2}{0.20^2 + 0.02^2} = \frac{1.6 \times 10^{-4}}{0.0404}$$

$$\therefore \quad \frac{T_{max}}{T_f} = 252.5$$

$$= \mathbf{3.96 \times 10^{-3}}$$

Example 34.14 (c). *A 12-pole, 3-phase, 600-V, 50-Hz, star-connected, induction motor has rotor-resistance and stand-still reactance of 0.03 and 0.5 ohm per phase respectively. Calculate:*

(a) Speed of maximum torque. (b) ratio of full-load torque to maximum torque, if the full-load speed is 495 rpm. **(Nagpur University, April 1999)**

Solution. For a 12-pole, 50 Hz motor,

Synchronous speed = $120 \times 50 / 12 = 500$ rpm

For $r = 0.03$ and $x = 0.5$ ohm, the slip for maximum torque is related as :

$S_{mT} = a = r/x = 0.03/0.5 = 0.06$

(a) Corresponding speed $= 500\,(1 - s_{mT}) = 470$ rpm

(b) Full-load speed $= 495$ rpm, slip $s = 0.01$, at full load.

$$\frac{\text{Full-load torque}}{\text{Maximum torque}} = \frac{2\,as}{a^2 + s^2} = \frac{2 \times 0.06 \times 0.01}{0.06^2 + 0.01^2} = \mathbf{0.324}$$

Example 34.15. *A 746-kW, 3-phase, 50-Hz, 16-pole induction motor has a rotor impedance of (0.02 + j 0.15) W at standstill. Full-load torque is obtained at 360 rpm. Calculate (i) the ratio of maximum to full-load torque (ii) the speed of maximum torque and (iii) the rotor resistance to be added to get maximum starting torque.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. Let us first find out the value of full-load slip s_f

$N_s = 120 \times 50/16 = 375$ rpm.; F.L. Speed = 360 rpm.

$s_f = (375 - 360)/375 = 0.04$; $a = R_2/X_2 = 0.02/0.15 = 2/15$

(i) $$\frac{T_f}{T_{max}} = \frac{2as_f}{a^2 + s_f^2} = \frac{2 \times (2/15) \times 0.04}{(2/15)^2 + (0.04)^2} = 0.55 \text{ or } \frac{T_{max}}{T_f} = \frac{1}{0.55} = \mathbf{1.818}$$

(ii) At maximum torque, $a = s_m = R_2/X_2 = 0.02/0.15 = 2/15$

$$N = N_s(1 - s) = 375\,(1 - 2/15) = \mathbf{325 \text{ r.p.m.}}$$

(iii) For maximum starting torque, $R_2 = X_2$. Hence, total rotor resistance per phase = **0.15Ω**

∴ external resistance required/phase = 0.15 − 0.02 = **0.13 Ω**

Example 34.16. *The rotor resistance and reactance per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.025 ohm and 0.12 ohm respectively. Make simplifying assumptions, state them and :*

(i) *find speed at maximum torque*

(ii) *find value of additional rotor resistance per phase required to give three-fourth of maximum torque at starting. Draw the equivalent circuit of a single-phase induction motor.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. (i) At maximum torque, $s = R_2/X_2 = 0.025/0.12 = 0.208$.

$$N_s = 120 \times 50/4 = 1500 \text{ rpm} \qquad \therefore N = 1500\,(1 - 0.208) = \mathbf{1188 \text{ rpm}}$$

(ii) It is given that $T_{st} = 0.75\,T_{max}$ Now, $\dfrac{T_{st}}{T_{max}} = \dfrac{2a}{1+a^2} = \dfrac{3}{4}$

$$\therefore \quad 3a^2 - 8a + 3 = 0; \qquad a = \frac{8 \pm \sqrt{64 - 36}}{6} = 0.45\,\Omega^*$$

Let, r = additional rotor resistance reqd., then

$$a = \frac{R_2 + r}{R_2} \text{ or } 0.45 = \frac{0.025 + r}{0.12} \qquad \therefore r = \mathbf{0.029\Omega}$$

Example. 34.17. *A 50-Hz, 8-pole induction motor has F.L. slip of 4%. The rotor resistance/phase = 0.01 ohm and standstill reactance/phase = 0.1 ohm. Find the ratio of maximum to full-load torque and the speed at which the maximum torque occurs.*

Solution. $$\frac{T_f}{T_{max}} = \frac{2as_f}{a^2 + s_f^2}$$

Now, $a = R_2/X_2 = 0.01/0.1 = 0.1$, $s_f = 0.04$

$$\therefore \quad \frac{T_f}{T_{max}} = \frac{2 \times 0.1 \times 0.04}{0.1^2 + 0.04^2} = \frac{0.008}{0.0116} = 0.69 \quad \therefore \quad \frac{T_{max}}{T_f} = \frac{1}{0.69} = \mathbf{1.45}$$

$$N_s = 120 \times 50/8 = 750 \text{ rpm}, \; s_m = 0.1$$
$$N = (1 - 0.1) \times 750 = 675 \text{ rpm}$$

Example 34.18. *For a 3-phase slip-ring induction motor, the maximum torque is 2.5 times the full-load torque and the starting torque is 1.5 times the full-load torque. Determine the percentage reduction in rotor circuit resistance to get a full-load slip of 3%. Neglect stator impedance.*

(Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution. Given, $T_{max} = 2.5\,T_f$; $T_{st} = 1.5\,T_f$; $T_{st}/T_{max} = 1.5/2.5 = 3/5$.

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* The larger value of 2.214 Ω has been rejected.

Now, $$\frac{T_{st}}{T_f} = \frac{3}{5} = \frac{2a}{1+a^2} \quad \text{or} \quad 3a^2 - 10a + 3 = 0 \quad \text{or} \quad a = 1/3$$

Now, $$a = R_2/X_2 \text{ or } R_2 = X_2/3$$

When F.L. slip is 0.03

$$\frac{T_f}{T_{st}} = \frac{2as}{a^2+s^2} \quad \text{or} \quad \frac{2}{2.5} = \frac{2a \times 0.03}{a^2 + 0.03^2}$$

$$a^2 - 0.15\,a + 0.009 = 0 \quad \text{or} \quad a = 0.1437$$

If R'_2 is the new rotor circuit resistance, then $0.1437 = R'_2/X_2$ or $R'_2 = 0.1437\,X_2$

% reduction in rotor resistance is

$$= \frac{(X_2/3) - 0.1437 \times X_2}{(X_2/3)} \times 100 = \mathbf{56.8\%}$$

Example 34.19. *An 8-pole, 50-Hz, 3-phase slip-ring induction motor has effective rotor resistance of 0.08 Ω/phase. Stalling speed is 650 r.p.m. How much resistance must be inserted in the rotor phase to obtain the maximum torque at starting? Ignore the magnetising current and stator leakage impedance.* **(Elect. Machines-I, Punjab Univ. 1991)**

Solution. It should be noted that stalling speed corresponds to maximum torque (also called stalling torque) and to maximum slip ***under running conditions***

$$N_s = 120 \times 50/8 = 750 \text{ r.p.m.; stalling speed is} = 650 \text{ r.p.m.}$$

$$s_b = (750 - 650)/750 = 2/15 = 0.1333 \text{ or } 13.33\%$$

Now, $$s_b = R_2/X_2 \quad \therefore \quad X_2 = 0.08 \times 15/2 = 0.6\,\Omega$$

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}. \text{ Since } \quad T_{st} = T_{max} \quad \therefore \quad 1 = \frac{2a}{1+a^2} \text{ or } a = 1$$

Let r be the external resistance per phase added to the rotor circuit. Then

$$a = \frac{R_2 + r}{X_2} \quad \text{or} \quad 1 = \frac{0.08 + r}{0.6} \quad \therefore \quad \mathbf{r = 0.52\,\Omega \text{ per phase.}}$$

Example 34.20. *A 4-pole, 50-Hz, 3-φ induction motor develops a maximum torque of 162.8 N-m at 1365 r.p.m. The resistance of the star-connected rotor is 0.2 Ω/phase. Calculate the value of the resistance that must be inserted in series with each rotor phase to produce a starting torque equal to half the maximum torque.*

Solution. $$N_s = 120 \times 50/4 = 1500 \text{ r.p.m. } N = 1365 \text{ r.p.m.}$$

$\therefore$ Slip corresponding to maximum torque is

$$s_b = (1500 - 1365)/1500 = 0.09 \quad \text{But} \quad s_b = R_2/X_2 \qquad \therefore X_2 = 0.2/0.09 = 2.22\,\Omega$$

Now, $$T_{max} = \frac{k\,\Phi\,E_2}{2\,X_2} = \frac{K}{2X_2} \quad (\text{where } K = k\Phi E_2) \qquad \text{...Art 34.20}$$

$$= \frac{K}{2 \times 2.22} = 0.225\,\text{K}$$

Let 'r' be the external resistance introduced per phase in the rotor circuit, then

Starting torque $$T_{st} = \frac{k\,\Phi\,E_2\,(R_2 + r)}{(R_2+r)^2 + (X_2)^2} = \frac{K\,(0.2 + r)}{(0.2+r)^2 + (0.2/0.09)^2}$$

$$\because \quad T_{st} = \frac{1}{2}.T_{max} \quad \therefore \frac{K\,(0.2+r)}{(0.2+r)^2 + (2.22)^2} = \frac{0.225\,K}{2}$$

Solving the quadratic equation for 'r', we get $r = \mathbf{0.4\,\Omega}$

Example 34.21. *A 4-pole, 50-Hz, 7.46.kW motor has, at rated voltage and frequency, a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Determine (i) full-load speed (ii) speed at maximum torque.* **(Electrical Technology-I, Osmania Univ. 1990)**

Solution. $\dfrac{T_{st}}{T_f} = 1.6$ and $\dfrac{T_{max}}{T_f} = 2 \quad \therefore \quad \dfrac{T_{st}}{T_{max}} = \dfrac{1.6}{2} = 0.8$

Now, $\dfrac{T_{st}}{T_{max}} = \dfrac{2a}{1+a^2} \quad \therefore \quad \dfrac{2a}{1+a^2} = 0.8$

or $0.8a^2 - 2a + 0.8 = 0 \quad a = 0.04 \quad \therefore \quad a = R_2/X_2 = 0.04$ or $R_2 = 0.04X_2$

Also, $\dfrac{T_f}{T_{max}} = \dfrac{2as_f}{a^2+s_f^2} = \dfrac{1}{2}$ or $\dfrac{2 \times 0.04\, s_f}{0.0016 + s_f^2} = \dfrac{1}{2}$ or $s_f = 0.01$

(*i*) full-load speed occurs at a slip of 0.01 or 1 per cent. Now,

$$N_s = 120 \times 50/4 = 1500 \text{ r.p.m.}; N = 1500 - 15 = \mathbf{1485\ r.p.m.}$$

(*ii*) Maximum torque occurs at a slip given by $s_b = R_2/X_2$. As seen from above slip corresponding to maximum torque is 0.04.

$$\therefore \quad N = 1500 - 1500 \times 0.04 = \mathbf{1440\ r.p.m.}$$

Example 34.22. *A 3-phase induction motor having a 6-pole, star-connected stator winding runs on 240-V, 50-Hz supply. The rotor resistance and standstill reactance are 0.12 ohm and 0.85 ohm per phase. The ratio of stator to rotor turns is 1.8. Full load slip is 4%.*

Calculate the developed torque at full load, maximum torque and speed at maximum torque. **(Elect. Machines, Nagpur Univ. 1993)**

Solution. Here,
$$K = \frac{\text{rotor turns/phase}}{\text{stator turns/phase}} = \frac{1}{1.8}$$
$$E_2 = KE_1 = \frac{1}{1.8} \times \frac{240}{\sqrt{3}} = 77 \text{ V}; \; s = 0.04;$$
$$N_s = 120 \times 50/6 = 1000 \text{ rpm} = 50/3 \text{ rps}$$
$$T_f = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \qquad \text{...Art. 34.19}$$
$$= \frac{3}{2\pi(50/3)} \cdot \frac{0.04 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 52.4 \text{ N-m}$$

For maximum torque, $s = R_2/X_2 = 0.12/0.85 = 0.14$

$$\therefore \quad T_{max} = \frac{3}{2\pi(50/3)} \cdot \frac{0.14 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 99.9 \text{ N-m}$$

Alternatively, as seen from Art 34.20.
$$T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2\,X_2}$$
$$\therefore \quad T_{max} = \frac{3}{2\pi(50/3)} \cdot \frac{77^2}{2 \times 0.85} = 99.9 \text{ N-m}$$

Speed corresponding to maximum torque, $N = 1000(1 - 0.14) = $ **860 rpm**

Example 34.23. *The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at half full-load speed. Also, calculate the power factor.* **(Adv. Elect. Machines, A.M.I.E. 1989)**

Solution. Let $N_s = 100$ r.p.m. F.L. speed $= (1 - 0.03)100 = 97$ r.p.m.

Let the normal voltage be V_1 volts.

Speed in second case $= 97/2 = 48.5$ r.p.m.

$\therefore$ slip $= (100 - 48.5)/100 = 0.515$ or 51.5%

Now, $$T = \frac{k\,\Phi\, sE_2R_2}{R_2^2 + (sX_2)^2} = \frac{ksV^2R_2}{R_2^2 + (sX_2)^2} \qquad (\because E_2 \propto \Phi \propto V)$$

Since torque is the same in both cases,

$$\frac{kV_1^2\, s_1R_2}{R_2^2 + (s_1X_2)^2} = \frac{kV_2^2\, s_2R_2}{R_2^2 + (s_2X_2)^2} \qquad \text{where } V_2 = \text{stator voltage in second case}$$

$$\therefore \quad \left(\frac{V_1}{V_2}\right)^2 = \frac{s_2}{s_1}\cdot\frac{R_2^2 + (s_1X_2)^2}{R_2^2 + (s_2X_2)^2}$$

$$= \frac{51.3}{3}\cdot\frac{0.015^2 + (0.03\times0.09)^2}{0.015^2 + (0.515\times0.09)^2} = 1.68$$

$$\therefore \quad \frac{V_1}{V_2} = \sqrt{1.68} = 1.296 \quad \text{or} \quad \frac{V_1 - V_2}{V_1} = \frac{0.296}{1.296}$$

Hence, percentage, reduction in stator (or supply voltage) is

$$= \frac{V_1 - V_2}{V_1}\times100 = \frac{0.296\times100}{1.296} = \mathbf{22.84\%}$$

In the second case, $\tan\phi = s_2X_2/R_2 = 0.515\times0.09/0.015 = 3.09$

$\therefore$ $\phi = \tan^{-1}(3.09) = 72°4'$ and p.f. $= \cos\phi = \cos 72°4' =$ **0.31**

34.27. Torque/Speed Curve

The torque developed by a conventional 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easier to show the relationship in the form of a curve (Fig. 34.22). In this diagram, T represents the nominal full-load torque of the motor. As seen, the starting torque (at $N = 0$) is $1.5\,T$ and the maximum torque (also called breakdown torque) is $2.5\,T$.

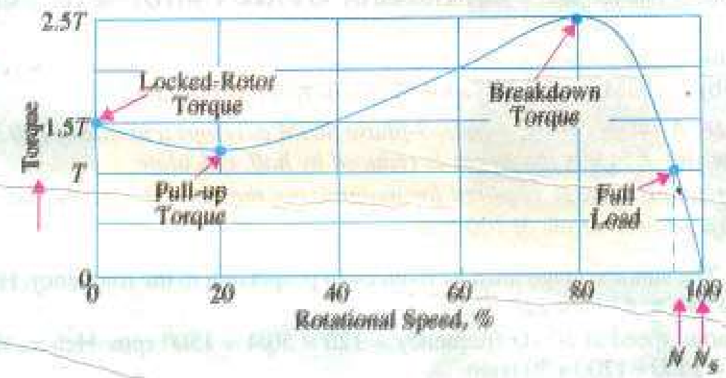


Fig. 34.22

At full-load, the motor runs at a speed of N. When mechanical load increases, motor speed decreases till the motor torque again becomes equal to the load torque. As long as the two torques are in balance, the motor will run at constant (but lower) speed. However, if the load torque exceeds $2.5\,T$, the motor will suddenly stop.

34.28. Shape of Torque/Speed Curve

For a squirrel-cage induction motor (SCIM), shape of its torque/speed curve depends on the voltage and frequency applied to its stator. If f is fixed, $T \propto V^2$ (Art 34.22). Also, synchronous speed

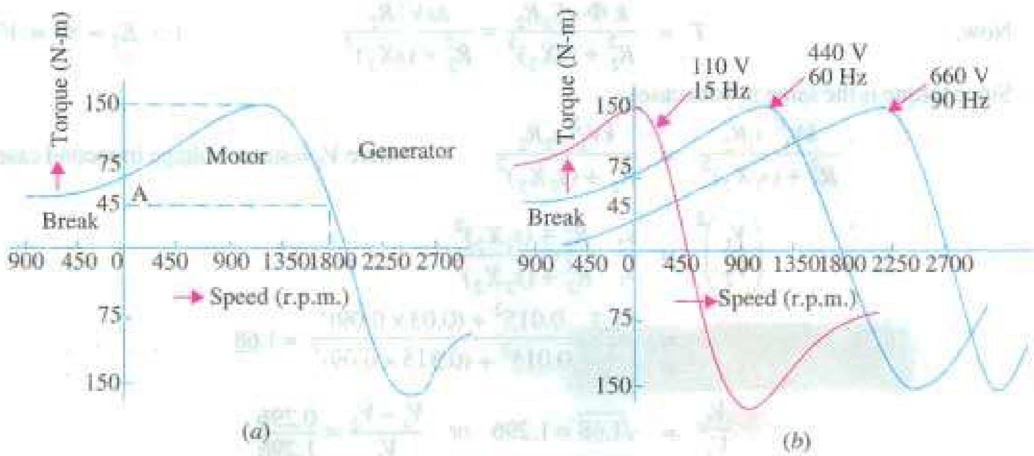


Fig. 34.23

depends on the supply frequency. Now, let us see what happens when *both* stator voltage and frequency are changed. In practice, supply voltage and frequency are varied in the ***same** proportion* in order to maintain a constant flux in the air-gap. For example, if voltage is doubled, then frequency is also doubled. Under these conditions, shape of the torque/speed curve remains the same but its position along the X-axis (*i.e.* speed axis) shifts with frequency.

Fig. 34.23 (*a*) shows the torque/speed curve of an 11 kW, 440-V, 60-Hz 3-φ SCIM. As seen, full-load speed is 1728 rpm and full-load torque is 45 N-m (point-A) whereas breakdown torque is 150 N-m and locked-rotor torque is 75 N–m.

Suppose, we now reduce both the voltage and fequency to ***one-fourth*** their original values *i.e.* to 110 V and 15 Hz respectively. As seen in Fig. 34.23 (*b*), the torque/speed curve shifts to the left. Now, the curve crosses the X-axis at the synchronous speed of $120 \times 15/4 = 450$ rpm (*i.e.* $1800/4 = 450$ rpm). Similarly, if the voltage and frequency are increased by 50% (660 V 90 Hz), the curve shifts to the right and cuts the X-axis at the synchronous speed of 2700 rpm.

Since the *shape* of the torque/speed curve remains the same at all ***frequencies***, it follows that torque developed by a SCIM is the same ***whenever slip-speed is the same***

Exampel 34.26. *A 440-V, 50-Hz, 4-pole, 3-phase SCIM develops a torque of 100 N-m at a speed of 1200 rpm. If the stator supply frequency is reduced by half, calculate*

(*a*) *the stator supply voltage required for maintaining the same flux in the machine.*

(*b*) *the new speed at a torque of 100 N-m.*

Solution. (*a*) The stator voltage must be reduced in proportion to the frequency. Hence, it should also be reduced by half to 440/2 = **220 V.**

(*b*) Synchronous speed at 50 Hz frequency = $120 \times 50/4 = 1500$ rpm. Hence, slip speed for a torque of 100 N-m = $1500 - 1200 = 300$ rpm.

Now, synchronous speed at 25 Hz = $1500/2 = 750$ rpm.

Since slip-speed has to be the same for the same torque irrespective of the frequency, the new speed at 100 N-m is = $750 + 300$ = **1050 rpm.**

34.29. Current/Speed Curve of an Induction Motor

It is a V-shaped curve having a minimum value at synchronous speed. This minimum is equal to the magnetising current which is needed to create flux in the machine. Since flux is purposely kept constant, it means that magnetising current is the same at all synchronous speeds.

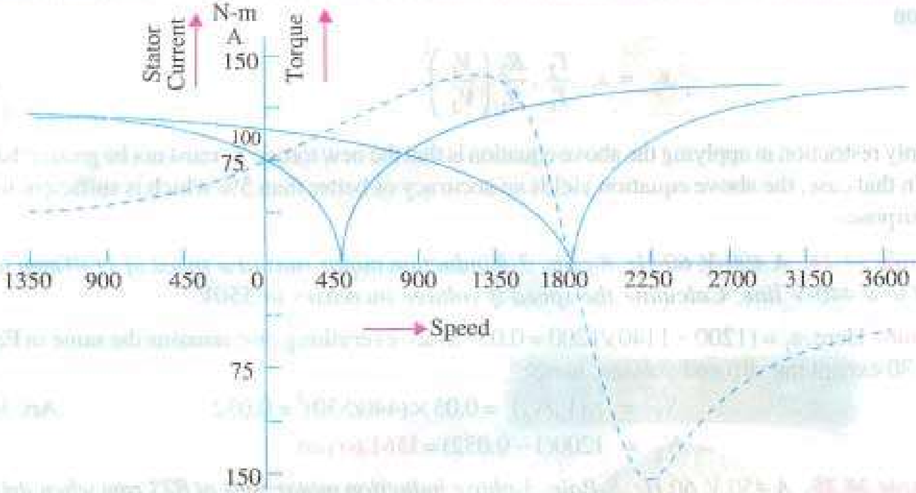


Fig. 34.24

Fig. 34.24. shows the current/speed curve of the SCIM discussed in Art. 34.28 above. Refer Fig. 34.23(*b*) and Fig. 34.24, As seen, locked rotor current is 100 A and the corresponding torque is 75 N-m. If stator voltage and frequency are varied in the same proportion, current/speed curve has the same shape, but shifts along the speed axis. Suppose that voltage and frequency are reduced to one-fourth of their previous values *i.e.* to 110 V, 15 Hz respectively. Then, locked rotor current decreases to 75 A but corresponding torque ***increases*** to 150 N-m which is equal to full breakdown torque (Fig. 34.25). It means that by reducing frequency, we can obtain ***a larger torque with a reduced current***. This is one of the big advantages of frequency control method. By progressively increasing the voltage and current during the start-up period, a SCIM can be made to develop close to its breakdown torque all the way from zero to rated speed.

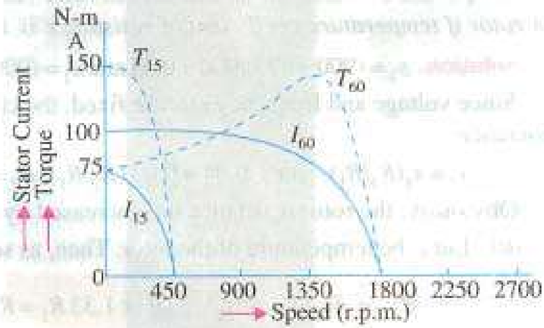


Fig. 34.25

Another advantage of frequency control is that it permits regenerative braking of the motor. In fact, the main reason for the popularity of frequency-controlled induction motor drives is their ability to develop high torque from zero to full speed together with the economy of regenerative braking.

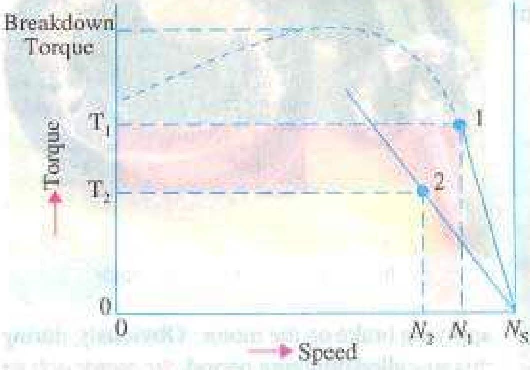


Fig. 34.26

34.30. Torque/Speed Characteristic Under Load

As stated earlier, stable operation of an induction motor lies over the linear portion of its torque/speed curve. The slope of this straight line depends mainly on the rotor resistance. Higher the resistance, sharper the slope. This linear relationship between torque and speed (Fig. 34.26) enables us to establish a very simple equation between different parameters of an induction motor. The parameters under two different load conditions are related by the equation

$$s_2 = s_1 \cdot \frac{T_2}{T_1} \cdot \frac{R_2}{R_1}\left(\frac{V_1}{V_2}\right)^2 \qquad ...(i)$$

The only restriction in applying the above equation is that the new torque T_2 must not be greater than T_1 $(V_2/V_1)^2$. In that case, the above equation yields an accuracy of better than 5% which is sufficient for all practical purposes.

Example 34.24. *A 400-V, 60-Hz, 8-pole, 3-φ induction motor runs at a speed of 1140 rpm when connected to a 440-V line. Calculate the speed if voltage increases to 550V.*

Solution. Here, $s_1 = (1200 - 1140)/1200 = 0.05$. Since everything else remains the same in Eq. (*i*) of Art. 34.30 except the slip and voltage, hence

$$S_2 = s_1(V_1/V_2)^2 = 0.05 \times (440/550)^2 = 0.032 \qquad \text{...Art. 34.23}$$

$$\therefore \quad N_2 = 1200(1 - 0.032) = \mathbf{1161.6\ rpm.}$$

Example 34.25. *A 450.V, 60.Hz, 8-Pole, 3-phase induction motor runs at 873 rpm when driving a fan. The initial rotor temperature is 23°C. The speed drops to 864 rpm when the motor reaches its final temperature. Calculate (i) increase in rotor resistance and (ii) approximate temperature of the hot rotor if temperature coefficient of resistance is 1/234 per °C.*

Solution. $s_1 = (900 - 873)/900 = 0.03$ and $s_2 = (900 - 864)/900 = 0.04$

Since voltage and frequency etc. are fixed, the change in speed is entirely due to change in rotor resistance.

(*i*) $s_2 = s_1(R_2/R_1)$ or $0.04 = 0.03\,(R_2/R_1)$; $R_2 = 1.33\,R_1$

Obviously, the rotor resistance has increased by **33 percent.**

(*ii*) Let t_2 be temperature of the rotor. Then, as seen from Art.1-11,

$$R_2 = R_1\,[1 + \alpha(t_2 - 23)] \text{ or } 1.33\,R_1 = R_1\left[1 + \frac{1}{234}(t_2 - 23)\right] \qquad \therefore t_2 = \mathbf{100.2°C}$$

34.31. Plugging of an Induction Motor

An induction motor can be quickly stopped by simply inter-changing any of its two stator leads. It reverses the direction of the revolving flux which produces a torque in the reverse direction, thus applying brake on the motor. Obviously, during this so-called plugging period, ***the motor acts as a brake***. It absorbs kinetic energy from the still

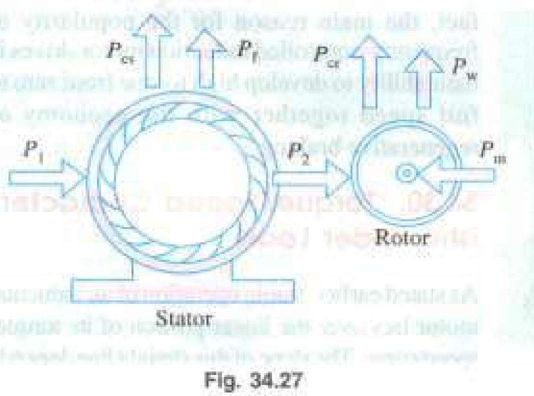


Fig. 34.27


Induction asynchronous motor

revolving load causing its speed to fall. The associated Power P_m is dissipated as heat in the rotor. At the same time, the rotor also continues to receive power P_2 from the stator (Fig. 34.27) which is also dissipated as heat. Consequently, plugging produces rotor I^2R losses which even exceed those when the rotor is locked.

34.32. Induction Motor Operating as a Generator

When run *faster than* its synchronous speed, an induction motor runs as a generator called a ***Induction generator***. It converts the mechanical energy it receives into electrical energy and this energy is released by the stator (Fig. 34.29). Fig. 34.28 shows an ordinary squirrel-cage motor which is driven by a petrol engine and is connected to a 3-phase line. As soon as motor speed exceeds its synchronous speed, it starts delivering ***active*** power P to the 3-phase line. However, for creating its own magnetic field, it absorbs *reactive* power Q from the line to which it is connected. As seen, Q flows in the ***opposite*** direction to P.

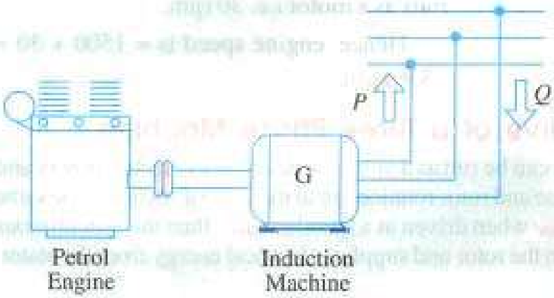


Fig. 34.28

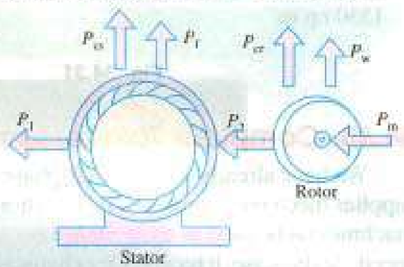


Fig. 34.29

The active power ***is directly proportional to the slip*** above the synchronous speed. The reactive power required by the machine can also be supplied by a group of capacitors connected across its terminals (Fig. 34.30). This arrangement can be used to supply a 3-phase load without using an external source. The frequency generated is slightly less than that corresponding to the speed of rotation.

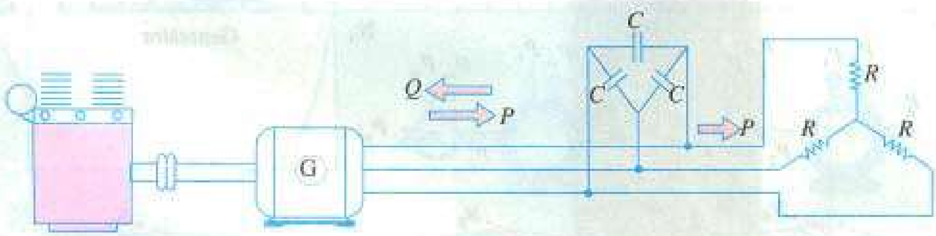


Fig. 34.30

The terminal voltage increases with capacitance. If capacitance is insufficient, the generator voltage will not build up. Hence, capacitor bank must be large enough to supply the reactive power normally drawn by the motor.

Example 34.26. *A 440-V, 4-pole, 1470 rpm, 30-kW, 3-phase induction motor is to be used as an asynchronous generator. The rated current of the motor is 40 A and full-load power factor is 85%. Calculate*

(a) capacitance required per phase if capacitors are connected in delta.

(b) speed of the driving engine for generating a frequency of 50 Hz.

Solution. (*i*) $S = \sqrt{3}\,VI = 1.73 \times 440 \times 40 = 30.4$ kVA

$$P = S\cos\phi = 30.4 \times 0.85 = 25.8 \text{ kW}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{30.4^2 - 25.8^2} = 16 \text{ kVAR}$$

Hence, the Δ-connected capacitor bank (Fig. 32.31) must provide 16/3 = 5.333 kVAR per phase.

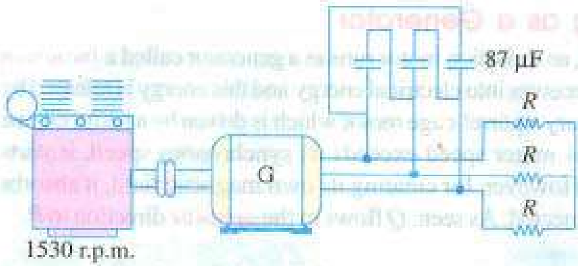


Fig. 34.31

Capacitor current per phase is = 5,333/440 = 12 A. Hence X_c = 440/12 = 36.6 Ω. Now,

$$C = \frac{1}{2\pi_f X_C} = 1/2\pi \times 50 \times 36.6 = \mathbf{87\mu F}$$

(*ii*) The driving engine must run at slightly more than synchronous speed. The slip speed is usually the same as that when the machine runs as a motor *i.e.* 30 rpm.

Hence, engine speed is = 1500 + 30 = **1530 rpm.**

34.33. Complete Torque/Speed Curve of a Three-Phase Machine

We have already seen that a 3-phase machine can be run as a ***motor***, when it takes electric power and supplies mechanical power. The directions of torque and rotor rotation are in the ***same*** direction. The same machine can be used as an ***asynchronous generator*** when driven at a speed ***greater*** than the synchronous speed. In this case, it receives mechanical energy in the rotor and supplies electrical energy from the stator. The torque and speed are ***oppositely-directed***.

The same machine can also be used as a ***brake*** during the plugging period (Art. 34.31). The three modes of operation are depicted in the torque/speed curve shown in Fig. 34.32.

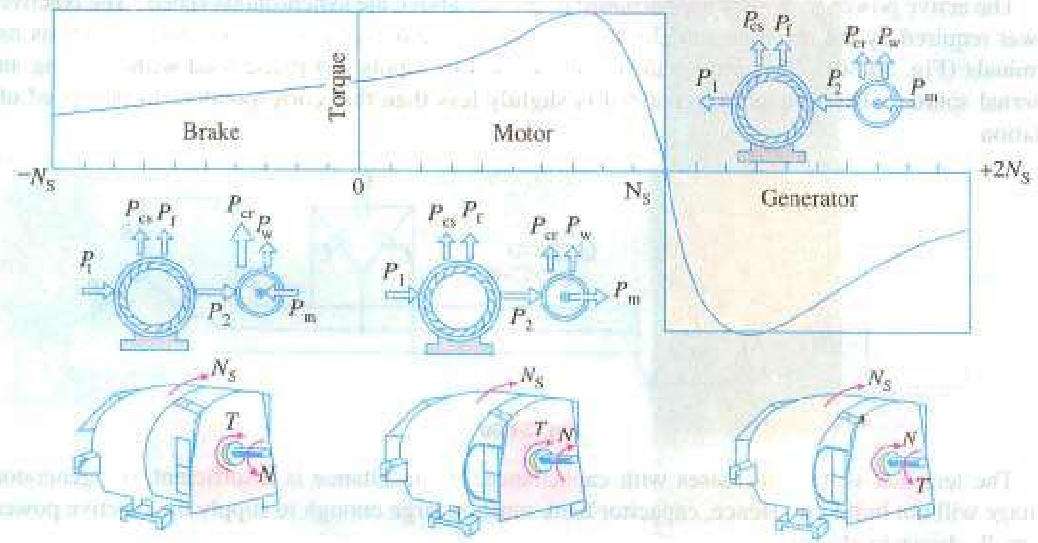


Fig. 34.32

Tutorial Problem No. 34.2

1. In a 3-phase, slip-ring induction motor, the open-circuit voltage across slip-rings is measured to be 110 V with normal voltage applied to the stator. The rotor is star-connected and has a resistance of 1 Ω and reactance of 4 Ω at standstill condition. Find the rotor current when the machine is (*a*) at standstill with slip-rings joined to a star-connected starter with a resistance of 2Ω per phase and negligible reactance (*b*) running normally with 5% slip. State any assumptions made.
[12.7 A ; 3.11 A] (*Electrical Technology-I, Bombay Univ. 1978*)

2. The star-connected rotor of an induction motor has a standstill impedance of $(0.4 + j4)$ ohm per phase and the rheostat impedance per phase is $(6+j2)$ ohm. The motor has an induced e.m.f. of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find (*a*) rotor current at standstill with the rheostat in the circuit (*b*) when the slip-rings are short-circuited and the motor is running with a slip of 3%. **[5.27 A ; 3.3 A]**
3. A 4-pole, 50-Hz induction motor has a full-load slip of 5%. Each rotor phase has a resistance of 0.3 Ω and a standstill reactance of 1.2 Ω. Find the ratio of maximum torque to full-load torque and the speed at which maximum torque occurs. **[2.6 ; 1125 r.p.m.]**
4. A 3-phase, 4-pole, 50-Hz induction motor has a star-connected rotor. The voltage of each rotor phase at standstill and on open-circuit is 121 V. The rotor resistance per phase is 0.3 Ω and the reactance at standstill is 0.8 Ω. If the rotor current is 15 A, calculate the speed at which the motor is running. Also, calculate the speed at which the torque is a maximum and the corresponding value of the input power to the motor, assuming the flux to remain constant. **[1444 r.p.m.; 937.5 r.p.m.]**
5. A 4-pole, 3-phase, 50 Hz induction motor has a voltage between slip-rings on open-circuit of 520 V. The star-connected rotor has a standstill reactance and resistance of 2.0 and 0.4 Ω per phase respectively. Determine :
   (*a*) the full-load torque if full-load speed is 1,425 r.p.m.
   (*b*) the ratio of starting torque to full-load torque
   (*c*) the additional rotor resistance required to give maximum torque at standstill
   [(*a*) 200 N-m (*b*) 0.82 (*c*) 1.6 Ω] ***(Elect. Machines-II, Vikram Univ. Ujjain 1977)***
6. A 50-Hz, 8-pole induction motor has a full-load slip of 4 per cent. The rotor resistance is 0.001 Ω per phase and standstill reactance is 0.005 Ω per phase. Find the ratio of the maximum to the full-load torque and the speed at which the maximum torque occurs.
   [2.6; 600 r.p.m.] ***(City & Guilds, London)***
7. A 3-ϕ, 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between slip-rings on open circuit. Calculate the current and power factor in each phase of the rotor windings at standstill when joined to a star-connected circuit, each limb of which has a resistance of 10 Ω and an inductance of 0.03 H. The resistance per phase of the rotor windings is 0.2 Ω and inductance 0.03 H. Calculate also the current and power factor in each rotor phase when the rings are short-circuited and the motor is running with a slip of 4 per cent.
   [13.6 A, 0.48; 27.0 A, 0.47] ***(London University)***
8. A 4-pole, 50-Hz, 3-phase induction motor has a slip-ring rotor with a resistance and standstill reactance of 0.04 Ω and 0.2 Ω per phase respectively. Find the amount of resistance to be inserted in each rotor phase to obtain full-load torque at starting. What will be the approximate power factor in the rotor at this instant ? The slip at full-load is 3 per cent. **[0.084 Ω, 0.516 p.f.]** ***(London University)***
9. A 3-ϕ induction motor has a synchronous speed of 250 r.p.m. and 4 per cent slip at full-load. The rotor has a resistance of 0.02 Ω/phase and a standstill leakage reactance of 0.15 Ω/phase. Calculate (*a*) the ratio of maximum and full-load torque (*b*) the speed at which the maximum torque is developed. Neglect resistance and leakage of the stator winding.
   [(*a*) 1.82 (*b*) 217 r.p.m.] ***(London University)***
10. The rotor of an 8-pole, 50-Hz, 3-phase induction motor has a resistance of 0.2 Ω/phase and runs at 720 r.p.m. If the load torque remains unchanged. Calculate the additional rotor resistance that will reduce this speed by 10% **[0.8 Ω]** ***(City & Guilds, London)***
11. A 3-phase induction motor has a rotor for which the resistance per phase is 0.1 Ω and the reactance per phase when stationary is 0.4 Ω. The rotor induced e.m.f. per phase is 100 V when stationary. Calculate the rotor current and rotor power factor (*a*) when stationary (*b*) when running with a slip of 5 per cent. **[(*a*) 242.5 A; 0.243 (*b*) 49 A; 0.98]**
12. An induction motor with 3-phase star-connected rotor has a rotor resistance and standstill reactance of 0.1 Ω and 0.5 Ω respectively. The slip-rings are connected to a star-connected resistance of 0.2 Ω per phase. If the standstill voltage between slip-rings is 200 volts, calculate the rotor current per phase when the slip is 5%, the resistance being still in circuit. **[19.1 A]**
13. A 3-phase, 50-Hz induction motor has its rotor windings connected in star. At the moment of starting

the rotor, induced e.m.f. between each pair of slip-rings is 350 V. The rotor resistance per phase is 0.2 Ω and the standstill reactance per phase is 1 Ω. Calculate the rotor starting current if the external starting resistance per phase is 8 Ω and also the rotor current when running with slip-rings short-circuited, the slip being 3 per cent. **[24.5 A ; 30.0 A]**

14. In a certain 8-pole, 50-Hz machine, the rotor resistance per phase is 0.04 Ω and the maximum torque occurs at a speed of 645 r.p.m. Assuming that the air-gap flux is constant at all loads, determine the percentage of maximum torque (a) at starting (b) when the slip is 3%. **[(a) 0.273 (b) 0.41]** *(London University)*

15. A 6-pole, 3-phase, 50-Hz induction motor has rotor resistance and reactance of 0.02 Ω and 0.1 Ω respectively per phase. At what speed would it develop maximum torque ? Find out the value of resistance necessary to give half of maximum torque at starting. **[800 rpm; 0.007 Ω]** *(Elect.Engg. Grad I.E.T.E. June 1988)*

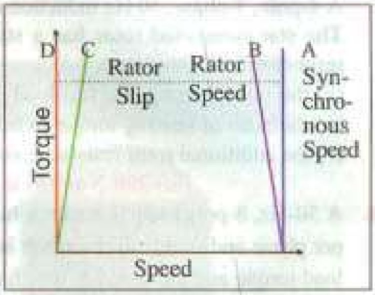
34.34. Measurement of Slip

Following are some of the methods used for finding the slip of an induction motor whether squirrel-cage or slip-ring type.

(i) **By actual measurement of motor speed**

This method requires measurement of actual motor speed N and calculation of synchronous speed N_s. N is measured with the help of a speedometer and N_s calculated from the knowledge of supply frequency and the number of poles of the motor.* Then slip can be calculated by using the equation.

$$s = (N_s - N) \times 100/N_s$$



The speed curve of an induction motor. The slip is the difference in rotor speed relative to that of the synchronous speed. CD = AD - BD = AB.

(ii) **By comparing rotor and stator supply frequencies**

This method is based on the fact that $s = f_r/f$

Since f is generally known, s can be found if frequency of rotor current can be measured by some method. In the usual case, where f is 50 Hz, f_r is so low that individual cycles can be easily counted. For this purpose, a d.c. moving-coil millivoltmeter, preferably of centre-zero, is employed as described below :

(a) In the case of a slip-ring motor, the leads of the millivoltmeter are lightly pressed against the adjacent slip-rings as they revolve (Fig. 34.33). Usually, there is sufficient voltage drop in the brushes and their short-circuiting strap to provide an indication on the millivoltmeter. The current in the millivoltmeter follows the variations of the rotor current and hence the pointer oscillates about its mean zero position. The number of complete cycles made by the pointer per second can be easily counted (it is worth remembering that one cycle consists of a movement from zero to a maximum to the right, back to zero and on to a maximum to the left and then back to zero).

As an example, consider the case of a 4-pole motor fed from a 50-Hz supply and running at 1,425 r.p.m. Since

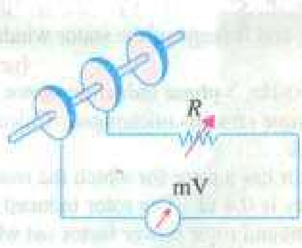


Fig. 34.33

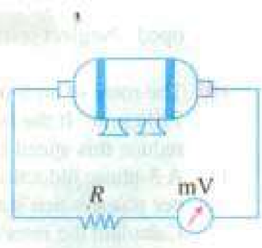


Fig. 34.34

* Since an induction motor does not have salient poles, the number of poles is usually inferred from the no-load speed or from the rated speed of the motor.

N_s = 1,500 r.p.m., its slip is 5% or 0.05. The frequency of the rotor current would be $f_r = sf = 0.05 \times 50 =$ 2.5 Hz which (being slow enough) can be easily counted.

(*b*) For squirrel-cage motors (which do not have slip-rings) it is not possible to employ the millivoltmeter so *directly*, although it is sometimes possible to pick up some voltage by connecting the millivoltmeter across the ends of the motor shaft (Fig. 34.34)

Another method, sometime employed, is as follows :

A large flat search coil of many turns is placed centrally against the end plate on the non-driving end of the motor. Quite often, it is possible to pick up sufficient voltage by induction from the leakage fluxes to obtain a reading on the millivoltmeter. Obviously, a large 50-Hz voltage will also be induced in the search coil although it is too rapid to affect the millivoltmeter. Commercial slip-indicators use such a search coil and, in addition, contain a low-pass filter amplifier for eliminating fundamental frequency and a bridge circuit for comparing stator and rotor current frequencies.

(*iii*) Stroboscopic Method

In this method, a circular metallic disc is taken and painted with alternately black and white segments. The number of segments (both black and white) is equal to the number of poles of the motor. For a 6-pole motor, there will be six segments, three black and three white, as shown in Fig. 34.35(*a*). The painted disc is mounted on the end of the shaft and illuminated by means of a neon-filled stroboscopic lamp, which may be supplied preferably with a combined d.c. and a.c. supply although only a.c. supply will do*. The connections for combined supply are shown in Fig. 34.36 whereas Fig. 34.35 (*b*) shows the connection for a.c. supply only. It must be noted that with combined d.c. and a.c. supply, the lamp will flash once per cycle**. But with a.c. supply, it will flash twice per cycle.

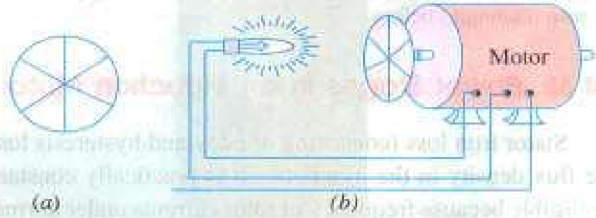


Fig. 34.35

Consider the case when the revolving disc is seen in the flash light of the bulb which is fed by the combined d.c. and a.c. supply.

If the disc were to rotate at synchronous speed, it would appear to be stationary. Since, in actual practice, its speed is slightly less than the synchronous speed, it appears to rotate slowly backwards. The reason for this apparent backward movement is as follows :

Fig. 34.36

* When combined d.c. and a.c. supply is used, the lamp should be tried both ways in its socket to see which way it gives better light.

** It will flash only when the two voltages add and remain extinguished when they oppose.

Let Fig. 34.37(*a*) represent the position of the white lines when they are illuminated by the first flash. When the next flash comes, they have ***nearly*** reached positions 120° ahead (but not quite), as shown in Fig. 34.37(*b*). Hence, line No. 1 has ***almost*** reached the position previously occupied by line No. 2 and one flash still later [Fig. 34.37 (*c*)] it has ***nearly*** reached the position previously occupied by line No. 3 in Fig. 34.37(*a*).

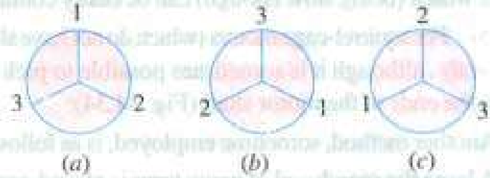


Fig. 34.37

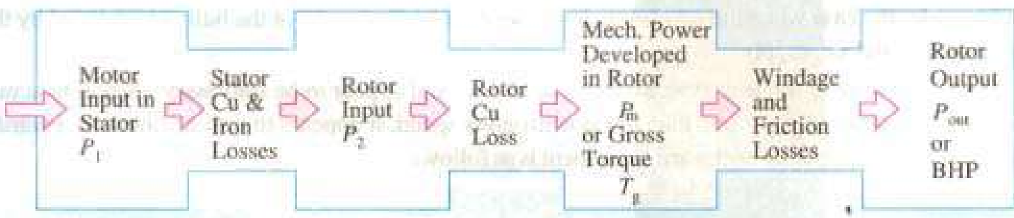
By counting the number of lines passing a fixed point in, say, a minute and dividing by the number of lines seen (*i.e.* three in the case of a 6-pole motor and so on) the apparent backward speed in r.p.m. can be found. This gives slip-speed in r.p.m. *i.e.* $N_s - N$. The slip may be found from the relation $s = \frac{N_S - N}{N_S} \times 100$

Note. If the lamp is fed with a.c. supply alone, then it will flash twice per cycle and twice as many lines will be seen rotating as before.

34.35. Power Stages in an Induction Motor

Stator iron loss (consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core. It is practically constant. The iron loss of the rotor is, however, negligible because frequency of rotor currents under normal running conditions is always small. Total rotor Cu loss = $3\ I_2^2 R_2$.

Different stages of power development in an induction motor are as under :



A better visual for power flow, within an induction motor, is given in Fig. 34.38.

34.36. Torque Developed by an Induction Motor

An induction motor develops gross torque T_g due to gross rotor output P_m (Fig 34.38). Its value can be expressed either in terms of rotor input P_2 or rotor gross output P_m as given below.

$$T_g = \frac{P_2}{\omega_s} = \frac{P_2}{2\pi N_s} \qquad \text{...in terms of rotor input}$$

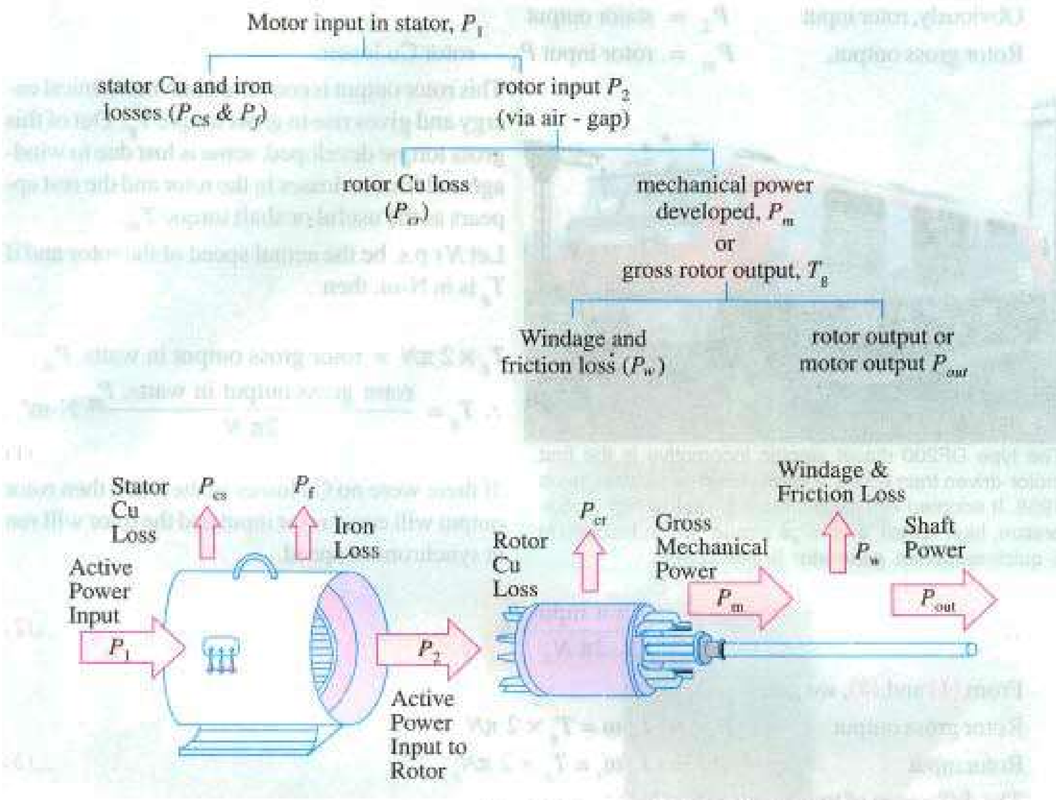


Fig. 34.38

$$T_g = \frac{P_m}{\omega} = \frac{P_m}{2\pi N} \quad \text{... in terms of rotor output}$$

The shaft torque T_{sh} is due to output power P_{out} which is less than P_m because of rotor friction and windage losses.

$$\therefore \quad T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

The difference between T_g and T_{sh} equals the torque lost due to friction and windage loss in the motor.

In the above expressions, N and N_s are in r.p.s. However, if they are in r.p.m., the above expressions for motor torque become

$$T_g = \frac{P_2}{2\pi N_s/60} = \frac{60}{2\pi} \cdot \frac{P_2}{N_s} = 9.55 \frac{P_2}{N_s} \text{ N-m}$$

$$= \frac{P_m}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_m}{N} = 9.55 \frac{P_m}{N} \text{ N-m}$$

$$T_{sh} = \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_{out}}{N} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$

34.37. Torque, Mechanical Power and Rotor Output

Stator input $\quad P_1$ = stator output + stator losses

The stator output is transferred entirely inductively to the rotor circuit.

Obviously, rotor input P_2 = stator output

Rotor gross output, P_m = rotor input P_2 — rotor Cu losses

The type DF200 diesel electric locomotive is the first motor-driven train equipped with diesel generators since 1958. It adopted induction motors to realize high acceleration, high speed, and large torque, which resulted in a quick-response generator brake system.

This rotor output is converted into mechanical energy and gives rise to gross torque T_g. Out of this gross torque developed, some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh}.

Let N r.p.s. be the actual speed of the rotor and if T_g is in N-m, then

$$T_g \times 2\pi N = \text{rotor gross output in watts, } P_m$$

$$\therefore T_g = \frac{\text{rotor gross output in watts, } P_m}{2\pi N} \text{ N-m*} \quad ...(1)$$

If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed.

$$\therefore \quad T_g = \frac{\text{rotor input } P_2}{2\pi N_S} \quad ...(2)$$

From (1) and (2), we get,

Rotor gross output $P_m = T_g\,\omega = T_g \times 2\pi N$

Rotor input $P_2 = T_g\,\omega_s = T_g \times 2\pi N_s$ ...(3)

The difference of two equals rotor Cu loss.

$\therefore$ rotor Cu loss $= P_2 - P_m = T_g \times 2\pi(N_s - N)$ ...(4)

From (3) and (4), $\dfrac{\text{rotor Cu loss}}{\text{rotor input}} = \dfrac{N_s - N}{N_s} = s$

$\therefore$ rotor Cu loss $= s \times \text{rotor input} = s \times \text{power across air-gap} = s\,P_2$ ...(5)

Also, rotor input = rotor Cu loss/s

Rotor gross output, P_m = input P_2 − rotor Cu loss = input − s × rotor input

$= (1 - s)$ input P_2 ...(6)

$\therefore$ rotor gross output $P_m = (1 - s)$ rotor input P_2

or $\dfrac{\text{rotor gross output, } P_m}{\text{rotor input, } P_2} = 1 - s = \dfrac{N}{N_s}$; $\dfrac{P_m}{P_2} = \dfrac{N}{N_s}$

$\therefore$ rotor efficiency $= \dfrac{N}{N_s}$ Also, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s}$

Important Conclusion

If some power P_2 is delivered to a rotor, then a part sP_2 is lost in the rotor itself as copper loss (and appears as heat) and the remaining $(1 - s)P_2$ appears as gross mechanical power P_m (including friction and windage losses).

$\therefore$ $P_2 : P_m : I^2R :: 1 : (1 - s) : s$ or $P_2 : P_m : P_{cr} :: 1 : (1 - s) : s$

---

* The value of gross torque in kg-m is given by

$$T_g = \frac{\text{rotor gross output in watts}}{9.81 \times 2\pi N} \text{ kg-m.} = \frac{P_m}{9.81 \times 2\pi N} \text{ kg-m}$$

The rotor input power will always divide itself in this ratio, hence it is advantageous to run the motor with as small a slip as possible.

Example 34.27. *The power input to the rotor of 440 V, 50 Hz, 6-pole, 3-phase, induction motor is 80 kW. The rotor electromotive force is observed to make 100 complete alterations per minute. Calculate (i) the slip, (ii) the rotor speed, (iii) rotor copper losses per phase.* **[Madras University, 1997]**

Solution. 100 alterations/minute $= \dfrac{100}{60}$ cycles/sec

$$1.6667\text{ Hz} = sf$$

Hence, the slip, $s = \dfrac{1.6667}{50} =$ **0.3333 P.u. or 3.333%**

(ii) rotor speed, $N = (1-s)N_S = (1-0.03333)\times 1000$

Since $N_s = \dfrac{120\times 50}{6} = 1000$ rpm, $N =$ **966.67 rpm**

(iii) rotor copper losses/phase $= \dfrac{1}{3}\times(s\times\text{rotor input})$

total rotor power input = 80 kW

rotor power input per phase = 80/3 kW

rotor copper losses per phase

$$= \frac{0.0333\times 80}{3}\text{ kW} = \mathbf{0.8888\ kW}$$

Example 34.28. *A 440-V, 3-ϕ, 50-Hz, 4-pole, Y-connected induction motor has a full-load speed of 1425 rpm. The rotor has an impedance of (0.4 + J 4) ohm and rotor/stator turn ratio of 0.8. Calculate (i) full-load torque (ii) rotor current and full-load rotor Cu loss (iii) power output if windage and friction losses amount to 500 W (iv) maximum torque and the speed at which it occurs (v) starting current and (vi) starting torque.*

Solution. $N_s = 120\times 50/4 = 1500$ rpm $= 25$ rps, $s = 75/1500 = 0.05$

$$E_1 = 440/1.73 = 254\text{ V/phase}$$

(i) Full-load

$$T_f = \frac{3}{2\pi\times 25}\times\frac{0.05\,(0.8\times 254)^2\times 0.4}{(0.4)^2+(0.05\times 4)^2} = \mathbf{78.87\ N\text{-}m}$$

(ii)

$$I_r = \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}} = \frac{sKE_1}{\sqrt{R_2^2+(sX_2)^2}} = \frac{0.05\times(0.8\times 254)}{\sqrt{(0.4)^2+(0.05\times 4)^2}} = \mathbf{22.73\ A}$$

Total Cu loss $= 3I_r^2R = 3\times 22.73^2\times 0.4 =$ **620 W**

(iii) Now, $P_m = 2\pi NT = 2\,\pi\times(1425/60)\times 78.87 = 11{,}745\ \Omega$

∴ $P_{out} = P_m -$ windage and friction loss $= 11{,}745 - 500 =$ **11,245 W**

(iv) For maximum torque, $s = R_2/X_2 = 0.4/4 = 0.1$

$$\therefore\quad T_{max} = \frac{3}{2\pi\times 25}\times\frac{0.1\times(0.8\times 254)^2\times 0.4}{(0.4)^2+(0.1\times 4)^2} = \mathbf{98.5\ N\text{-}m}$$

Since $s = 0.1$, slip speed $= s\,N_s = 0.1\times 1500 = 150$ rpm.

∴ Speed for maximum torque = 1500 − 150 = **1350 rpm.**

(v) starting current $= \dfrac{E_2}{\sqrt{R_2^2 + X_2^2}} = \dfrac{KE_1}{\sqrt{R_2^2 + X_2^2}} = \dfrac{0.8 \times 254}{\sqrt{0.4^2 + 4^2}} =$ **50.5 A**

(vi) At start, $s = 1$, hence

$$T_{st} = \frac{3}{2\pi \times 25} \times \frac{(0.8 \times 254)^2 \times 0.4}{(0.4)^2 + 4^2} = \textbf{19.5 N-m}$$

It is seen that as compared to full-load torque, the starting torque is much less-almost 25 per cent.

34.38. Induction Motor Torque Equation

The gross torque T_g developed by an induction motor is given by

$$T_g = P_2 / 2\,\pi N_s \qquad \text{—}N_s \text{ in r.p.s.}$$
$$= 60\,P_2 / 2\pi N_s = 9.55\,P_2 / N_s \qquad \text{—}N_s \text{ in r.p.m.}$$

Now, $P_2 = \text{rotor Cu loss}/s = 3 I_2^2 R_2 / s$

As seen from Art. 34.19, $I_2 = \dfrac{sE_2}{\sqrt{R_2^2 + (s\,X_2)^2}} = \dfrac{sKE_1}{\sqrt{R_2^2 + (s\,X_2)^2}}$

where K is rotor/stator turn ratio per phase.

$$\therefore \quad P_2 = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s\,X_2)^2} \times \frac{1}{s} = \frac{3\,s\,E_2^2 R_2}{R_2^2 + (s\,X_2)^2}$$

Also,
$$P_2 = 3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s\,X_2)^2} \times \frac{1}{s} = \frac{3\,s\,K^2 E_1^2 R_2}{R_2^2 + (s\,X_2)^2}$$

$$\therefore \quad T_g = \frac{P_2}{2\pi N_s} = \frac{3}{2\,\pi N_S} \times \frac{s\,E_2^2 R_2}{R_2^2 + (s\,X_2)^2} \qquad \text{— in terms of } E_2$$

or
$$= \frac{3}{2\,\pi N_S} \times \frac{s\,K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \qquad \text{— in terms of } E_1$$

Here, E_1, E_2, R_2 and X_2 represent phase values.

In fact, $3\,K^2/2\,\pi N_s = k$ is called the constant of the given machine. Hence, the above torque equation may be simplified to

$$T_g = k \frac{s\,E_1^2 R_2}{R_2^2 + (sX_2)^2} \qquad \text{— in terms of } E_1$$

34.39. Synchronous Watt

It is clear from the above relations that torque is proportional to rotor input. By defining a new unit of torque (instead of the force-at-radius unit), we can say that the rotor torque *equals* rotor input. The new unit is synchronous watt. When we say that a motor is developing a torque of 1,000 synchronous watts, we mean that the rotor input is 1,000 watts and that the torque is such that power developed would be 1,000 watts provided the rotor were running synchronously and developing the same torque.

Or

Synchronous watt is that torque which, at the synchronous speed of the machine under consideration, would develop a power of 1 watt.

$$\text{rotor input} = T_{sw} \times 2\,\pi N_S \qquad \therefore \quad T_{sw} = \frac{\text{rotor input, } P_2}{2\pi \times \text{synch. speed}}$$

$$= \frac{1}{\omega_s} \cdot \frac{N_S}{N} \cdot P_g = \frac{1}{\omega_s} \cdot \frac{N_S}{N} \cdot P_m$$

Synchronous wattage of an induction motor equals the power transferred across the air-gap to the rotor.

∴ torque in synchronous watt

$$= \text{rotor input} = \frac{\text{rotor Cu loss}}{s} = \frac{\text{gross output power, } P_m}{(1-s)}$$

Obviously, at $s = 1$, torque in synchronous watt equals the total rotor Cu loss because at standstill, entire rotor input is lost as Cu loss.

Suppose a 23-kW, 4-pole induction motor has an efficiency of 92% and a speed of 1440 r.p.m. at rated load. If mechanical losses are assumed to be about 25 per cent of the total losses, then

motor input = 23/0.92 = 25 kW, total loss = 25 − 23 = 2 kW.

Friction and windage loss = 2/4 = 0.5 kW

∴ $P_m = 23 + 0.5 = 23.5$ kW

Power in synchronous watts $P_{sw} = P_2 = 23.5 \times 1500/1440 = 24.5$ kW

synchronous speed $w_S = 2\pi(1500/60) = 157$ rad / s

∴ synchronous torque, $T_{sw} = 24.5 \times 10^3/157 = 156$ N-m

or $$T_{sw} = P_m \frac{N_S}{N} \cdot \frac{1}{\omega_S} = 23.5 \times \frac{1500}{1440} \times \frac{1}{2\pi(1500/60)}$$

$= 156$ N–m —Art 34.39

34.40. Variations in Rotor Current

The magnitude of the rotor current varies with load carried by the motor.

As seen from Art. 34.37

$$\frac{\text{rotor output}}{\text{rotor input}} = \frac{N}{N_S} \text{ or rotor output} = \text{rotor input} \times \frac{N}{N_S}$$

∴ rotor input = rotor output × N_s / N

Also, rotor output $\propto 2\pi NT = kNT$

∴ rotor input = $kNT \times N_s / N = kN_s T$

Now, $$\frac{\text{rotor Cu loss}}{\text{rotor input}} = s \text{ or } \frac{3I_2^2 R_2}{s} = \text{rotor input}$$

∴ $3I_2^2 R_2/s = kN_sT$ or $T \propto I_2^2 R_2/s$

$T_{st} \propto I_{2st}^2 R_2$ —since $s = 1$

$T_f \propto I_{2f}^2 R_2 / s_f$ —s_f = full-load slip

∴ $$\frac{T_{st}}{T_f} = s_f \left(\frac{I_{2st}}{I_2 f}\right)^2$$

where I_{2st} and I_{2f} are the rotor currents for starting and full-load running conditions.

34.41. Analogy with a Mechanical Clutch

We have seen above that, rotor Cu loss = slip × rotor input

This fact can be further clarified by considering the working of a mechanical clutch (though it is not meant to be a proof for the above) similar to the one used in automobiles. A plate clutch is shown in Fig. 34.39. It is obvious that the torque on the driving shaft must exactly equal the torque on the driven shaft. In fact, these two torques are actually one and the same torque, because the torque is caused by friction between the discs and it is true whether the clutch is slipping or not. Let ω_1 and ω_2 be the angular velocities of the shaft when the clutch is slipping.

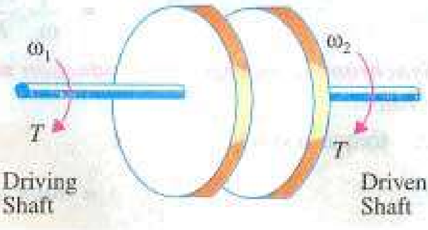


Fig. 34.39

Then, input $= T\,\omega_1$ and output $= T\,\omega_2 = T\,\omega_1(1-s)$; [$\omega_2 = \omega_1(1-s)$]

loss $= T\,\omega_1 - T\,\omega_2 = T\,\omega_1 - T\,\omega_1(1-s) = sT\,\omega_1 =$ slip × input

34.42. Analogy with a D.C. Motor

The above relations could also be derived by comparing an induction motor with a d.c. motor. As shown in Art 29.3, in a d.c. shunt motor, the applied voltage is always opposed by a back e.m.f. E_b. The power developed in the motor armature is $E_b\,I_a$ where I_a is armature current. This power, as we know, is converted into mechanical power in the armature of the motor.

Now, in an induction motor, it is seen that the induced e.m.f in the rotor decreases from its standstill value of E_2 to sE_2 when in rotation. Obviously, the difference $(1-s)\,E_2$ is the e.m.f. called forth by the rotation of the rotor similar to the back e.m.f. in a d.c. motor. Hence, gross power P_m developed in the rotor is given by the product of the back e.m.f., armature current and rotor power factor.

$$P_m = (1-s)E_2 \times I_2 \cos\phi_2\ ; \qquad \text{Now } I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \text{ and } \cos\phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\therefore \quad P_m = (1-s)E_2 \times \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \times \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} = \frac{s(1-s)E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Multiplying the numerator and the denominator by s, we get

$$P_m = \left(\frac{1-s}{s}\right) R_2 \times \frac{s^2 E_2^2}{[R_2^2 + (sX_2)^2]} = \left(\frac{1-s}{s}\right) I_2^2 R_2 \qquad \left(\because\ I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}\right)$$

Now, $I_2^2 R_2 =$ rotor Cu loss/phase $\qquad \therefore \dfrac{\text{Cu loss}}{\text{rotor output}} = \dfrac{s}{1-s}$

This is the same relationship as derived in Art. 34-37.

Example 34.30. *The power input to a 3-phase induction motor is 60 kW. The stator losses total 1 kW. Find the mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.*

(Elect. Machines AMIE Sec. E Summer 1991)

Solution. Rotor input, P_2 = stator input − stator losses = 60 − 1 = 59 kW

$P_m = (1-s)P_2 = (1-0.03) \times 59 = 57.23$ kW

Total rotor Cu loss $= sP_2 = 0.03 \times 59 = 1.77$ kW = 1770 W

Rotor Cu loss/phase = 1770/3 = 590 W

Example 34.31. *The power input to the rotor of a 400 V, 50.Hz, 6-pole, 3-phase induction motor is 20 kW. The slip is 3%. Calculate (i) the frequency of rotor currents (ii) rotor speed (iii) rotor copper losses and (iv) rotor resistance per phase if rotor current is 60 A.*

(Elect. Engg. Punjab Univ. 1991)

Solution. *(i)* Frequency of rotor current = $sf = 0.03 \times 50 = 1.5$ Hz

(ii) $N_s = 120 \times 50/6 = 1000$ rpm; $N = 1000(1 - 0.03) = 700$ rpm

(iii) rotor Cu loss = $s \times$ rotor input = $0.03 \times 20 = 0.6$ kW = 600 W

(iv) rotor Cu loss/phase = 200 W; $\therefore\ 60^2 R_2 = 200$; $R_2 = 0.055\ \Omega$

Example 34.32. *A 3-phase, 6-pole, 50-Hz induction motor develops 3.73 kW at 960 rpm. What will be the stator input if the stator loss is 280 W?* **(Madurai Kamraj Univ. 1999)**

Solution. As seen from Art. 34.37, $\dfrac{\text{power developed in rotor}}{\text{rotor input}} = \dfrac{N}{N_s}$

Now, mechanical power developed in rotor = 3.73 kW., $N_s = 120 \times 50/6 = 1000$ r.p.m.

$\therefore$ 3,730/rotor input = 960/1000 $\therefore$ rotor input = 3,885 W

Stator input = rotor input + stator losses = 3885 + 280 = **4,156 W**

Example 34.33. *The power input to the rotor of a 400 V, 50-Hz, 6-pole, 3-ϕ induction motor is 75 kW. The rotor electromotive force is observed to make 100 complete alteration per minute. Calculate: (i) slip (ii) rotor speed (iii) rotor copper losses per phase (iv) mechanical power developed.*

(Elect. Engg. I, Nagpur Univ. 1993)

Solution. Frequency of rotor emf, $f' = 100/60 = 5/3$ Hz

(i) Now, $f' = sf$ or $5/3 = s \times 50$; $s = 1/30 =$ **0.033 or 3.33%**

(ii) $N_s = 120 \times 50/6 = 1000$ rpm ; $N = N_s(1 - s) = 1000\,(1 - 1/30) = 966.7$ rpm

(iii) $P_2 = 75\ kW$; total rotor Cu loss = $sP_2 = (1/30) \times 75 = 2.5$ kW

rotor Cu loss/phase = 2.5/3 = **0.833 kW**

(iv) $P_m = (1 - s)P_2 = (1 - 1/30) \times 75 =$ **72.5 kW**

Example 34.34. *The power input to a 500 V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate : (i) the slip (ii) the rotor copper loss (iii) shaft power and (iv) the efficiency.*

(Elect. Engg. - II, Pune Univ. 1989)

Solution. *(i)* $N_s = 120 \times 50/6 = 1000$ rpm ; $s = (100 - 975)/1000 = 0.025$ or 2.5%

(ii) Motor input P_1=40 kW ; stator loss = 1 kW; rotor input $P_2 = 40 - 1 = 39$ kW

$\therefore$ rotor Cu loss = $s \times$ rotor input = $0.025 \times 39 =$ **0.975 kW**

(iii) $P_m = P_2 -$ rotor Cu loss = $39 - 0.975 = 38.025$ kW

$P_{out} = P_m -$ friction and windage loss = $38.025 - 2 =$ **36.025 kW**

(iv) $\eta = P_{out}/P_1 = 36.025/40 = 0.9$ or **90%**

Example 34.35. *A 100-kW (output), 3300-V, 50-Hz, 3-phase, star-connected induction motor has a synchronous speed of 500 r.p.m. The full-load slip is 1.8% and F.L. power factor 0.85. Stator copper loss = 2440 W. Iron loss = 3500 W. Rotational losses = 1200 W. Calculate (i) the rotor copper loss (ii) the line current (iii) the full-load efficiency.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. P_m = output + rotational loss = 100 + 1.2 = 101.2 kW

(*i*) $$\text{rotor Cu loss} = \frac{s}{1-s} \times P_m = \frac{0.018}{1-0.018} \times 101.2 = \mathbf{1.855\ kW}$$

(*ii*) $$\text{rotor input, } P_2 = P_m + \text{rotor Cu loss} = 101.2 + 1.855 = 103.055 \text{ kW}$$

$$\text{Stator input} = P_2 + \text{stator Cu and iron losses}$$

$$= 103.055 + 2.44 + 3.5 = 108.995 \text{ kW}$$

$\therefore$ $$108{,}995 = \sqrt{3} \times 3300 \times I_L \times 0.85; \quad I_L = \mathbf{22.4\ A}$$

The entire power flow in the motor is given below.

Stator input 108,995 W — Rotor input 103, 055 W — Mechanical power developed in rotor 101,200 W — Motor output 100,000 W or 100 kW

Stator Cu & Iron losses 5940 W — Rotor Cu loss 1855 W — Rotational losses 1200 W

(*iii*) F.L. efficiency = 100,000/108,995 = 0.917 or **91.7%**

Example 34.36. *The power input to the rotor of a 440 V, 50-Hz, 6-pole, 3-phase induction motor is 100 kW. The rotor electromotive force is observed to make 120 cycles per minute. Calculate (i) the slip (ii) the rotor speed (iii) mechanical power developed (iv) the rotor copper loss per phase and (v) speed of stator field with respect to rotor.* **(Elect. Engg. AMIETE Sec. A June 1991)**

Solution. (*i*) $f' = sf$ or $(120/60) = s \times 50$; $s = \mathbf{0.01}$

(*ii*) $N_s = 120 \times 50/6 = 1000$ rpm; $N = 1000(1 - 0.01) = \mathbf{990\ rpm}$

(*iii*) $P_m\ (1 - s)P_2 = (1 - 0.01) \times 100 = \mathbf{99\ kW}$

(*iv*) total rotor Cu loss = $sP_2 = 0.01 \times 100 = 1$ kW ; Cu loss/phase = **1/3 kW**

(*v*) $N_s = 1000$ rpm; $N = 990$ rpm. Hence, speed of stator field with respect to rotor is $= 1000 - 990 = \mathbf{10\ rpm.}$

Example 34.37. *An induction motor has an efficiency of 0.9 when delivering an output of 37 kW. At this load, the stator Cu loss and rotor Cu loss each equals the stator iron loss. The mechanical losses are one-third of the no-load loss. Calculate the slip.*

(Adv. Elect. Machines, A.M.I.E. Sec. B Winter 1993)

Solution. Motor input = 37,000/0.9 = 41,111 W

$\therefore$ total loss = 41,111 − 37,000 = 4,111 W

This includes (*i*) stator Cu and iron losses (*ii*) rotor Cu loss (its iron loss being negligibly small) and (*iii*) rotor mechanical losses.

Now, no-load loss of an induction motor consists of (*i*) stator iron loss and (*ii*) mechanical losses provided we neglect the small amount of stator Cu loss under no-load condition. Moreover, these two losses are independent of the load on the motor.

$$\text{no-load loss} = W_i + W_m = 3W_m$$

$\therefore$ $$W_m = W_i/2$$

where W_i is the stator iron loss and W_m is the rotor mechanical losses.

Let, stator iron loss = x; then stator Cu loss = x; rotor Cu loss = x; mechanical loss = $x/2$

$\therefore \quad 3x + x/2 = 4{,}111$ or $x = 1175$ W

Now, rotor input = gross output + mechanical losses + rotor Cu loss

$$= 37{,}000 + (1175/2) + 1175 = 38{,}752 \text{ W}$$

$$s = \frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{1175}{38{,}752} = 0.03 \text{ or } \mathbf{3\%}.$$

Example 34.38. *A 400 V, 50-Hz, 6-pole, Δ-connected, 3-φ induction motor consumes 45 kW with a line current of 75 A and runs at a slip of 3%. If stator iron loss is 1200 kW, windage and fricition loss is 900 W and resistance between two stator terminals is 0.12 Ω, calculate (i) power supplied to the rotor P_2 (ii) rotor Cu loss P_{cr} (iii) power supplied to load P_{out} (iv) efficiency and (v) shaft torque developed.*

Solution. $\cos\phi = \dfrac{45 \times 1000}{\sqrt{3} \times 400 \times 75} = 0.866$ lag

A line current of 75 amp means a phase-current of $75/\sqrt{3}$ *i.e.* 43.3 amp

Next, winding resistance has to be worked out

Refer to Fig. 34.40.

r and $2r$ in parallel have an equivalent resistance measured at a and b in delta connected motor as $r \times 2r/3r = 2r/3$ ohms

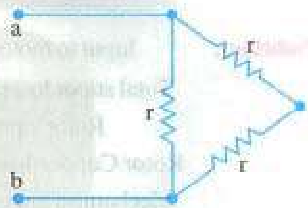


Fig. 34.40

From the data given $\quad \dfrac{2r}{3} = 0.12, \; r = 0.18$

Total stator copper loss $= 3 \times 43.3^2 \times 0.18 = 1012$ Watts

Total input to stator = 45,000 Watts

Stator copper loss = 1012 Watts, stator core loss = 1200 Watts

Stator output = Rotor input = 42,788 Watts

Rotor copper loss = Slip × Rotor input = 0.03 × 42,788 = 1284 Watts

Rotor mech. output power = 42,788 − 1284 = 41,504 Watts

Shaft output = Mech. output of rotor − Mech losses

= 41504 − 900 = 40604 Watts

$$\text{Efficiency} = \frac{40604}{45000} \times 100\% = 92.23\%$$

$$\text{Shaft output torque, } T = \frac{40604 \times 60}{2\pi \times 970} = \mathbf{400 \text{ Nw-m}}$$

Example 34.39(*a*). *A 3-phase induction motor has a 4-pole, star-connected stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance per phase is 0.1 Ω and reactance 0.9 Ω. The ratio of stator to rotor turns is 1.75. The full-load slip is 5%. Calculate for this load:*

(a) the load torque in kg-m

(b) speed at maximum torque

(c) rotor e.m.f. at maximum torque. **(Electrical Machines-I, South Gujarat Univ. 1985)**

Solution. (*a*) K = rotor turns/stator turns = 1/1.75

stator voltage/phase, $E_1 = 220/\sqrt{3}$ V

$\therefore$ standstill rotor e.m.f./phase, $E_2 = K E_1 = \dfrac{220}{\sqrt{3}} \times \dfrac{1}{1.75} = \mathbf{72.6 \text{ V}}$

$$Z_r = \sqrt{R_2^2 + (sX_2^2)} = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.11 \, \Omega$$

$$I_2 = sE_2/Z_r = 0.05 \times 72.6/0.11 = 33\text{ A}$$

Rotor Cu loss $\quad P_{cr} = 3I_2^2R_2 = 3 \times 33^2 \times 0.1 = 327\text{ W}$

$$\frac{\text{rotor Cu loss}}{\text{mech. power developed}} = \frac{s}{1-s}; \quad \frac{327}{P_m} = \frac{0.05}{1-0.05}; \quad P_m = 6213\text{ W}$$

$T_g = 9.55\, P_m / N; \quad N = N_s(1-s) = 1500\,(1-0.05) = 1425$ rpm

$\therefore \quad T_g = 9.55 \times 6213/1425 = 41.6$ N-m $= 41.6/9.81 =$ **4.24 kg.m**

(*b*) For maximum torque, $s_m = R_2/X_2 = 0.1/0.81 =$ **1/9**

$\therefore \quad N = N_s(1-s) = 1500(1-1/9) =$ **1333 r.p.m.**

(*c*) rotor e.m.f./phase at maximum torque $= (1/9) \times 72.6 =$ **8.07 V**

Example 34.39 (*b*). *A 400 V, 3-phase, 50 Hz, 4-pole, star-connected induction-motor takes a line current of 10 A with 0.86 p. f. lagging. Its total stator losses are 5 % of the input. Rotor copper losses are 4 % of the input to the rotor, and mechanical losses are 3 % of the input of the rotor. Calculate (i) slip and rotor speed, (ii) torque developed in the rotor, and (iii) shaft-torque.*

[Nagpur University, April 1998]

Solution. Input to motor $= \sqrt{3} \times 400 \times 10 \times 0.86 = 5958$ Watts

Total stator losses = 5% of 5958 = 298 Watts

Rotor input = Stator Output = 5958 − 298 = 5660 Watts

Rotor Copper-loss = 4 % of 5660 = 226.4 Watts

Mechanical losses = 3% of 5660 = 169.8 Watts

Shaft output = 5660 − 226.4 − 169.8 = 5264 Watts

(*i*) slip, s = Rotor-copper-loss / rotor-input = 4 %, as given

Synchronous speed, $N_s = 120 \times f/P = 1500$ rpm

Rotor speed, $N = N_s(1-s) = 1500\ (1-0.04) = 1440$ rpm

(*ii*) Let the torque developed in the rotor $= T_r$

ω_r, Angular speed of rotor $= 2\pi N/60 = 150.72$ radians/sec

Rotor output = Rotor input − Rotor-copper-loss

= 5660 − 226.4 = 5433.6 watts

$T_r\omega_r = 5433.6$, giving $T_r = 5433.6 / 150.72 = 36.05$ Nw-m

(Alternatively, synchronous angular speed $= \omega_s = 2\pi \times 1500/60 = 157$ rad / sec

$T_r\omega_s$ = rotor input in Watts = 5660

$T_r = 5660 / 157 = 36.05$ Nw − m

(*iii*) Shaft-torque $= T_m$ = Shaft output in Watts $/\omega_r$

$= 5264 / 150.72 = 34.93$ Nw − m.

Example 34.40. *A 3-phase, 440-V, 50-Hz, 40-pole, Y-connected induction motor has rotor resistance of 0.1 Ω and reactance 0.9 Ω per phase. The ratio of stator to rotor turns is 3.5. Calculate:*

(*a*) *gross output at a slip of 5%*

(*b*) *the maximum torque in synchronous watts and the corresponding slip.*

Solution. (*a*) Phase voltage $= \dfrac{440}{\sqrt{3}}\text{ V}; \; K = \dfrac{\text{rotor turns}}{\text{stator turns}} = \dfrac{1}{3.5}$

Standstill e.m.f. per rotor phase is $E_2 = KE_1 = \dfrac{440}{\sqrt{3}} \times \dfrac{1}{3.5} = 72.6\text{ V}$

$$E_r = s.E_2 = 0.05 \times 72.6 = 3.63 \text{ V} ; Z_r = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.1096\ \Omega$$

$$I_2 = \frac{E_r}{Z_r} = \frac{3.63}{0.1096} = 33.1 \text{ A} ; \quad \text{Total Cu loss} = 3\, I_2^2 R = 3 \times (33.1)^2 \times 0.1 = 330\text{W}$$

$$\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s} \quad \therefore \text{ rotor gross output} = \frac{330 \times 0.95}{0.05} = \mathbf{7{,}250\ W}$$

(b) For Maximum Torque

$$R_2 = s_m \,.\, X_2 \qquad \therefore\ s_m = R_2/X_2 = 0.1/0.9 = \mathbf{1/9};\ E_r = (1/9) \times 72.6 = 8.07 \text{ V}$$

$$Z_r = \sqrt{0.1^2 + (0.9/9)^2} = 0.1414\ \Omega ; I_2 = 8.07/0.1414 = 57.1 \text{ A}$$

$$\text{Total rotor Cu loss} = 3 \times 57.1^2 \times 0.1 = 978 \text{ W}$$

$$\text{rotor input} = \frac{\text{rotor Cu loss}}{S} = \frac{978}{1/9} = 8{,}802 \text{ W}$$

$\therefore$ Maximum torque in synchronous watts = rotor input = **8,802 W**

Example 34.41. *An 18.65-kW, 4-pole, 50-Hz, 3-phase induction motor has friction and windage losses of 2.5 per cent of the output. The full-load slip is 4%. Compute for full load* **(a)** *the rotor Cu loss* **(b)** *the rotor input* **(c)** *the shaft torque* **(d)** *the gross electromagnetic torque.*

(Elect. Machines-II, Indore Univ. 1987)

Solution. Motor output $P_{out} = 18{,}650 \text{ W}$

Friction and windage loss $P_w = 2.5\%$ of $18{,}650 = 466$ W

Rotor gross output $P_m = 18{,}650 + 466 = 19{,}116$ W

(a) $$\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s}, \text{ rotor Cu loss} = \frac{0.04}{(1-0.04)} \times 19{,}116 = \mathbf{796.6\ W}$$

(b) Rotor input $$P_2 = \frac{\text{rotor Cu loss}}{s} = \frac{796.5}{0.04} = \mathbf{19{,}912.5\ W}$$

(or rotor input = 19,116 + 796.6 = 19,913 W)

(c) $T_{sh} = 9.55\, P_{out}/N$: $N_s = 120 \times 50/4 = 1500$ r.p.m.

$N = (1 - 0.04) \times 1500 = 1440$ r.p.m

$\therefore$ $T_{sh} = 9.55 \times 18{,}650/1440 = \mathbf{123.7\ N\text{-}m}$

(d) Gross torque $T_g = 9.55\, P_m/N = 9.55 \times 19{,}116/1440 = \mathbf{126.8\ N\text{-}m}$

(or $T_g = P_2/2\,\pi N_s = 19{,}913/2\,\pi \times 25 = 127$ N-m)

Example 34.42. *An 8-pole, 3-phase, 50 Hz, induction motor is running at a speed of 710 rpm with an input power of 35 kW. The stator losses at this operating condition are known to be 1200 W while the rotational losses are 600 W. Find* **(i)** *the rotor copper loss,* **(ii)** *the gross torque developed,* **(iii)** *the gross mechanical power developed,* **(iv)** *the net torque and* **(v)** *the mechanical power output.*

(Elect. Engg. AMIETE Sec. A 1991 & Rajiv Gandhi Techn. Univ., Bhopal, 2000)

Solution. (i) $P_2 = 35 - 1.2 = 33.8$ kW ; $N_s = 120 \times 50/8 = 750$ rpm ; $N = 710$ rpm; $s = (750 - 710)/750 = 0.0533$

$\therefore$ Rotor Cu loss $= sP_2 = \mathbf{1.8\ kW}$

(ii) $P_m = P_2 -$ rotor Cu loss $= 33.8 - 1.8 = \mathbf{32\ kW}$

(iii) $T_g = 9.55\, P_m/N = 9.55 \times 32000/710 = 430.42$ N-m

(iv) $P_{out} = P_m -$ rotational losses $= 32000 - 600 = \mathbf{31400\ W}$

(*v*) $T_{sh} = 9.55\,P_{out}/N = 9.55 \times 31400/710 =$ **422.35 N-m**

Example 34.43. *A 6-pole, 50-Hz, 3-phase, induction motor running on full-load with 4% slip develops a torque of 149.3 N-m at its pulley rim. The friction and windage losses are 200 W and the stator Cu and iron losses equal 1,620 W. Calculate (a) output power (b) the rotor Cu loss and (c) the efficiency at full-load.* **(Elect. Technology, Mysore Univ. 1989)**

Solution. $N_s = 120 \times 50/6 = 1{,}000$ r.p.m; $N = (1 - 0.04) \times 1{,}000 = 960$ r.p.m.

Out put power $= T_{sh} \times 2\,\pi N = 2\,\pi \times (960/60) \times 149.3 =$ **15 kW**

Now, output = 15,000 W

Friction and windage losses = 200 W ; Rotor gross output = 15,200 W

$$\frac{P_m}{P_2} = \frac{N}{N_S} \quad \therefore \text{ rotor input } P_2 = 15{,}200 \times 1{,}000/960 = 15{,}833 \text{ W}$$

(*b*) $\therefore$ rotor Cu loss = 15,833 − 15,200 = **633 W**

$$\left(\text{rotor Cu loss is given by : } \frac{\text{rotor Cu loss}}{\text{rotor output}} = \frac{s}{1-s}\right)$$

(*c*) stator output = rotor input = 15,833 W

Stator Cu and iron losses = 1,620 W

$\therefore$ Stator input P_1 = 15,833 + 1,620 = 17,453 W

overall efficiency, $\eta = 15{,}000 \times 100/17{,}453 =$ **86%**

Example 34.44. *An 18.65-kW, 6-pole, 50-Hz, 3-φ slip-ring induction motor runs at 960 r.p.m. on full-load with a rotor current per phase of 35 A. Allowing 1 kW for mechanical losses, find the resistance per phase of 3-phase rotor winding.* **(Elect. Engg-I, Nagpur Univ. 1992)**

Solution. Motor output = 18.65 kW; Mechanical losses = 1 kW

$\therefore$ Mechanical power developed by rotor, $P_m = 18.65 + 1 = 19.65$ kW

Now, $N_s = 120 \times 50/6 = 1000$ r.p.m. ; $s = (1000 - 960)/1000 = 0.04$

$$\text{rotor Cu loss} = \frac{s}{1-s} \times P_m = \frac{0.04}{(1-0.04)} \times 19.65 = 0.819 \text{ kW} = 819 \text{ W}$$

$\therefore$ $3I_2^{\,2}R_2 = 819$ or $3 \times 35^2 \times R_2 = 819$ $\qquad R_2 =$ **0.023 Ω/phase**

Example 34.45. *A 400 V, 4-pole, 3-phase, 50-Hz induction motor has a rotor resistance and reactance per phase of 0.01 Ω and 0.1 Ω respectively. Determine (a) maximum torque in N-m and the corresponding slip (b) the full-load slip and power output in watts, if maximum torque is twice the full-load torque. The ratio of stator to rotor turns is 4.*

Solution. Applied voltage/phase $E_1 = 400/\sqrt{3} = 231$ V

Standstill e.m.f. induced in rotor, $E_2 = KE_1 = 231/4 = 57.75$ V

(*a*) Slip for maximum torque, $s_m = R_2/X_2 = 0.01/0.1 = 0.1$ or **10%**

$$T_{max} = \frac{3}{2\pi N_S} \times \frac{E_2^2}{2\,X_2} \qquad \text{... Art. 34.20}$$

$$N_s = 120 \times 50/4 = 1500 \text{ r.p.m.} = 25 \text{ r.p.s.}$$

$$\therefore \quad T_{max} = \frac{3}{2\pi \times 25} \times \frac{57.75^2}{2 \times 0.1} = \mathbf{320\ N\text{-}m}$$

(*b*)
$$\frac{T_f}{T} = \frac{2as_f}{a^2 + s^2} = \frac{1}{2}\text{; Now, } a = R_2/X_2 = 0.01/0.1 = 0.1$$

$\therefore \quad 2 \times 0.1 \times s_f/(0.1^2 + s_f^2) = 1/2 \qquad \therefore s_f = 0.027$ or 0.373

Since $s_f = 0.373$ is not in the operating region of the motor, we select $s_f = 0.027$.

Hence, $s_f =$ **0.027.** $N = 1500\,(1 - 0.027) = 1459.5$ r.p.m.

Full-load torque $T_f = 320/2 = 160$ N-m

F.L. Motor output $= 2\,\pi\, N\, T_f/60 = 2\,\pi \times 1459.5 \times 160/60 =$ **24,454 W**

Example 34.46. *A 3-phase induction motor has a 4-pole, Y-connected stator winding. The motor runs on 50-Hz supply with 200V between lines. The motor resistance and standstill reactance per phase are 0.1 Ω and 0.9 Ω respectively. Calculate*

(a) the total torque at 4% slip (b) the maximum torque

(c) the speed at maximum torque if the ratio of the rotor to stator turns is 0.67. Nelgect stator impedance.

(Elect. Machinery, Mysore Univ. 1987)

Solution. (*a*) Voltage/phase $= \dfrac{200}{\sqrt{3}}$ V; $K = \dfrac{\text{rotor turns}}{\text{stator turns}} = 0.67$

Standstill rotor e.m.f. per phase is $E_2 = \dfrac{200}{\sqrt{3}} \times 0.67 = 77.4$ V and $s = 0.04$

$$Z_r = \sqrt{R_2^2 + \left(sX_2^2\right)} = \sqrt{0.1^2 + (0.04 \times 0.9)^2} = 0.106\,\Omega$$

$$I_2 = \frac{sE_2}{Z_r} = \frac{77.4 \times 0.04}{0.106} = 29.1 \text{ A}$$

Total Cu loss in rotor $= 3\,I_2^2 R_2 = 3 \times 29.1^2 \times 0.1 = 255$ W

Now, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s} \quad \therefore \quad P_m = 255 \times 0.96/0.04 = 6{,}120$ W

$N_s = 120 \times 50/4 = 1{,}500$ r.p.m; $N = (1 - 0.04) \times 1{,}500 = 1440$ r.p.m.

Gross torque developed, $T_g = 9.55\, P_m/N = 9.55 \times 6120/1440 =$ **40.6 N-m**

(*b*) For maximum torque, $s_m = R_2/X_2 = 0.1/0.9 = 1/9$ and $E_r = sE_2 = 77.4 \times 1/9 = 8.6$ V

$Z_r = \sqrt{0.1^2 + (0.9 \times 1/9)^2} = 0.1414\,\Omega; \quad I_2 = 8.6/0.1414 = 60.8$ A

Total rotor Cu loss $= 3 \times 60.8^2 \times 0.1 = 1{,}110$ W

Rotor gross output $= \dfrac{1{,}100 \times (1 - 1/9)}{1/9} = 8{,}800$ W

$N = (1 - 1/9) \times 1{,}500 = 1{,}333$ rpm

$\therefore \quad T_{max} = 9.55 \times 8{,}800/1333 =$ **63 N-m**

(*c*) Speed at maximum torque, as found above, is **1333.3 r.p.m.**

Example 34.47. *The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase.*

(i) What is the p.f. of the motor at start?

(ii) What is the p.f. at a slip of 4 percent ?

(iii) If the number of poles is 4, the supply frequency is 50-Hz and the standstill e.m.f. per rotor phase is 110 V, find out the full-load torque. Take full-load slip as 4 per cent.

(Electrical Technology-I, Osmania Univ. 1990)

Solution. (*i*) rotor impedance/phase $= \sqrt{0.015^2 + 0.09^2} = 0.0912\,\Omega$

p.f. $= 0.015/0.0912 =$ **0.164**

(*ii*) reactance/phase $= sX_2 = 0.04 \times 0.09 = 0.0036\,\Omega$

rotor impedance/phase $= \sqrt{0.015^2 + 0.0036^2} = 0.0154\ \Omega$

p.f. $= 0.015/0.0154 =$ **0.974**

(iii) $N_s = 120 \times 50/4 = 1{,}500$ r.p.m.; $N = 1{,}500 - (0.04 \times 1{,}500) = 1{,}440$ r.p.m.

$E_r = sE_2 = 0.04 \times 110 = 4.4$ V ; $Z_r = 0.0154\ \Omega$ ...found above

$I_2 = 4.4/0.0154 = 286$ A

Total rotor Cu loss $= 3I_2^2 R_2 = 3 \times 286^2 \times 0.015 = 3{,}650$ W

Now, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s}$

$\therefore$ Rotor gross output $P_m = 3{,}650 \times 0.96/0.04 = 87{,}600$ W

If T_g is the gross torque developed by the rotor, then

$$T_g = 9.55\ P_m/N = 9.55 \times 87{,}600/1440 = \mathbf{581\ N\text{-}m}$$

Example 34.48. *The useful full load torque of 3-phase, 6-pole, 50-Hz induction motor is 162.84 N-m. The rotor e.m. f. is observed to make 90 cycles per minute. Calculate (a) motor output (b) Cu loss in rotor (c) motor input and (d) efficiency if mechanical torque lost in windage and friction is 20.36 N-m and stator losses are 830 W.* **(Elect. Machines-II, Indore Univ. 1988)**

Solution. $N_s = 120 \times 50/6 = 1{,}000$ r.p.m.

Frequency of rotor e.m.f. $= 90/60 = 1.5$ Hz; $s = f_r/f = 1.5/50 = 0.03$

Rotor speed $= 1{,}000(1 - 0.03) = 970$ r.p.m. ; Useful F.L. torque $= 162.84$ N-m

(a) motor output $= T_{sh}\dfrac{2\pi N}{60} = \dfrac{2\pi \times 970 \times 162.84}{60} =$ **16,540 W**

(b) *gross torque* $T_g = 162.84 + 20.36 = 183.2$ N-m

Now, $T_g = 9.55\dfrac{P_2}{N_s}$ $\therefore$ $183.2 = 9.55 \times \dfrac{P_2}{1000}$

$\therefore$ rotor input, $P_2 = 183.2 \times 1000/9.55 = 19{,}170$ W

$\therefore$ rotor Cu loss $= s \times$ rotor input $= 0.03 \times 19{,}170 =$ **575.1 W**

(c) Motor input, $P_1 = 19{,}170 + 830 =$ **20,000 W**

(d) $\eta = (16{,}540/20{,}000) \times 100 =$ **82.27%**

Example 34.49. *Estimate in kg-m the starting torque exerted by an 18.65-kW, 420-V, 6-pole, 50-Hz, 3-phase induction motor when an external resistance of 1 Ω is inserted in each rotor phase.*

stator impedance : (0.25 + j 0.75) Ω *rotor impedance : (0.173 + J 0.52) Ω*

stato/rotor voltage ratio: 420/350 *connection: Star-Star*

Solution. $K = E_2/E_1 = 350/420 = 5/6$

Equivalent resistance of the *motor* as referred to rotor is

$$R_{02} = R_2 + K^2 R_1 = 0.173 + (5/6)^2 \times 0.25 = 0.346\ \Omega/\text{phase}$$

Similarly, $X_{02} = 0.52 + (5/6)^2 \times 0.75 = 1.04\ \Omega/\text{phase}$

When an external resistance of 1 ohm/phase is added to the rotor circuit, the equivalent motor impedance as referred to rotor circuit is

$$Z_{02} = \sqrt{(1 + 0.346)^2 + 1.04^2} = 1.7\ \Omega$$

Short-circuit rotor current is $I_2 = \dfrac{350/\sqrt{3}}{1.7} = 119$ A

Rotor Cu loss per phase on short-circuit $= 119^2 \times 1.173 = 16{,}610$ W

Now, rotor power input $= \dfrac{\text{rotor Cu loss}}{s}$; On short-circuit, $s = 1$

$\therefore$ rotor power input = rotor Cu loss on short-circuit

= 16,610 W/phase = 49,830 W for 3 phases

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m.}$$

If T_{st} is the starting torque in newton-metres, then

$$T_{st} = 9.55 \quad P_2 / N_s = 9.55 \times 49{,}830/1000 = 476 \text{ N-m} = 476/9.81 = \mathbf{46.7 \text{ kg-m.}}$$

Example 34.50. *An 8-pole, 37.3-kW, 3-phase induction motor has both stator and rotor windings connected in star. The supply voltage is 280 V per phase at a frequency of 50 Hz. The short-circuit current is 200 A per phase at a short-circuit power factor of 0.25. The stator resistance per phase is 0.15 Ω. If transformation ratio between the stator and rotor windings is 3, find*

(i) the resistance per phase of the rotor winding

(ii) the starting torque of the motor.

Solution. Under short circuit, all the power supplied to the motor is dissipated in the stator and rotor winding resistances. Short-circuit power supplied to the motor is

$$W_{sc} = 3V_1 I_{sc} \cos \phi_{sc} = 3(280 \times 200 \times 0.25) \text{ W}$$

Power supplied/phase $= 280 \times 200 \times 0.25 = 14{,}000$ W

Let r_2' be the rotor resistance per phase as referred to stator. If r_1 is the stator resistance per phase, then

$$I_{sc}^2 (r_1 + r_2') = 14{,}000 \qquad \therefore \quad r_1 + r_2' = 14{,}000 / 200^2 = 0.35\ \Omega$$

$\therefore \quad r_2' = 0.35 - 0.15 = 0.2\ \Omega$; Now, $r_2' = r_2 / K^2$ where $K = 1/3$

(i) $\therefore \quad r_2 = K^2 r_2' = 0.2/9 = \mathbf{0.022\ \Omega}$ per phase

(ii) Power supplied to the rotor circuit is

$$= 3I_{sc}^2 r_2' = 3 \times 200^2 \times 0.2 = 24{,}000 \text{ W}$$

$$N_s = 120 f/P = 120 \times 50/8 = 750 \text{ r.p.m.} = 12.5 \text{ r.p.s.}$$

$\therefore$ Starting torque $= 9.55\ P_2 / N_s = 9.55 \times 24{,}000/750 = \mathbf{305.6 \text{ N-m}}$

Example 34.51. *A 3-phase induction motor, at rated voltage and frequency has a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Neglecting stator resistance and rotational losses and assuming constant rotor resistance, determine*

(a) the slip at full-load

(b) the slip at maximum torque

(c) the rotor current at starting in terms of F.L. rotor current.

(Electrical Machine - II, Bombay Univ. 1987)

Solution. As seen from Example 34.22 above,

(a) $s_f = 0.01$ or **1** %

(b) From the same example it is seen that at maximum torque, $a = s_b = 0.04$ or **4** %

(c) As seen from Art. 34.40.

$$\frac{I_{2st}}{I_{2f}} = \sqrt{\frac{T_{st}}{s_f \cdot T_f}} = \sqrt{\frac{1.6}{0.01}} = 12.65$$

$\therefore$ Starting rotor current = 12.65 × F.L. rotor current

34.43. Sector Induction Motor

Consider a standard 3-phase, 4-pole, 50-Hz, Y-connected induction motor. Obviously, its $N_s = 1500$ rpm. Suppose we cut the stator in half *i.e.* we remove half the stator winding with the result that only two complete *N* and *S* poles are left behind. Next, let us star the three phases without making any other changes in the existing connections. Finally, let us mount the original rotor above this ***sector stator*** leaving a small air-gap between the two. When this stator is energised from a 3-phase 50-Hz source, the rotor is found to run at almost 1500 rpm. In order to prevent saturation, the stator voltage should be reduced to half its original value because the sector stator winding has only half the original number of turns. It is found that under these conditions, this half-truncated sector motor still develops about 30% of its original rated power.

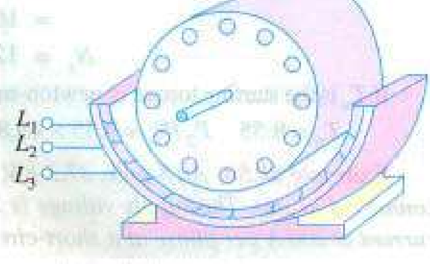


Fig. 34.41

The stator flux of the sector motor revolves at the same peripheral speed as the flux in the original motor. But instead of making a complete round, the flux in the sector motor simply travels ***continuously from one end of the stator to the other***.

34.44. Linear Induction Motor

If, in a sector motor, the sector is laid out flat and a flat squirrel-cage winding is brought near to it, we get a linear induction motor (Fig. 34.42). In practice, instead of a flat squirrel-cage winding, an aluminium or copper or iron plate is used as a 'rotor'. The flat stator produces a flux that moves in a straight line from its one end to the other at a linear synchronous speed given by

$$\upsilon_s = 2\, w.\, f.$$

where υ_s = linear synchronous speed (m/s)

w = width of one pole-pitch (m)

f = supply frequency (Hz)

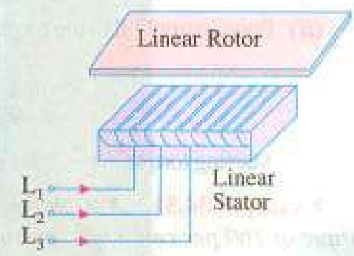


Fig. 34.42

It is worth noting that speed ***does not depend on the number of poles***, but only on the pole-pitch and stator supply frequency. As the flux moves linearly, it drags the rotor plate along with it in the ***same direction***. However, in many practical applications, the 'rotor' is stationary, while the stator moves. For example, in high-speed trains, which utilize magnetic levitation (Art. 34.46), the rotor is composed of thick aluminium plate that is fixed to the ground and extends over the full length of the track. The linear stator is bolted to the undercarriage of the train.

34.45. Properties of a Linear Induction Motor

These properties are almost identical to those of a standard rotating machine.

1. ***Synchronous speed***. It is given by $\upsilon_s = 2wf$
2. ***Slip***. It is given by $s = (v_s - v)\,/\,v_s$

where v is the actual speed.

3. ***Thrust or Force***. It is given by $F = P_2/v_s$

where P_2 is the active power supplied to the rotor.

4. ***Active Power Flow***. It is similar to that in a rotating motor.

(*i*) $P_{cr} = sP_2$ and (*ii*) $P_m = (1 - s)\, P_2$

Example 34.52. *An electric train, driven by a linear motor, moves with 200 km/h, when stator frequency is 100 Hz. Assuming negligible slip, calculate the pole-pitch of the linear motor.*

Solution. $\upsilon_s = 2\,\omega f \qquad \omega = \dfrac{(200 \times 5/18)}{2 \times 100} = \mathbf{277.8\ mm}$

Example 34.53. *An overhead crane in a factory is driven horizontally by means of two similar linear induction motors, whose 'rotors' are the two steel I-beams, on which the crane rolls. The 3-phase, 4-pole linear stators, which are mounted on opposite sides of the crane, have a pole-pitch of 6 mm and are energised by a variable-frequency electronic source. When one of the motors was tested, it yielded the following results:*

Stator frequency	*= 25 Hz;*	*Power to stator*	*= 6 kW*
Stator Cu and iron loss	*= 1.2 kW;*	*crane speed*	*= 2.4 m/s*

Calculate (i) synchronous speed and slip (ii) power input to rotor (iii) Cu losses in the rotor (iv) gross mechanical power developed and (v) thrust.

Solution. (*i*) $v_s = 2\,\omega f = 2 \times 0.06 \times 25 = \mathbf{3\ m/s}$

$s = (\upsilon_s - \upsilon)/\upsilon_s = (3 - 2.4)/3 = 0.2$ or $\mathbf{20\%}$

(*ii*) $P_2 = 6 - 1.2 = \mathbf{4.8\ kW}$

(*iii*) $P_{cr} = sP_2 = 0.2 \times 4.8 = \mathbf{0.96\ kW}$

(*iv*) $P_m = P_2 - P_{cr} = 4.8 - 0.96 = \mathbf{3.84\ kW}$

(*v*) $F = P_2/\upsilon_s = 4.8 \times 10^3/3 = 1600\ \text{N} = \mathbf{1.60\ kN}$

34.46. Magnetic Levitation

As shown in Fig. 34.43 (*a*), when a moving permanent magnet sweeps across a conducting ladder, it tends to drag the ladder along with, because it applies a horizontal tractive force $F = BIl$. It will now be shown that this horizontal force is also accompanied by a ***vertical*** force (particularly, at high magnet speeds), which tends to push the magnet away from the ladder in the upward direction.

Magnetic levitation

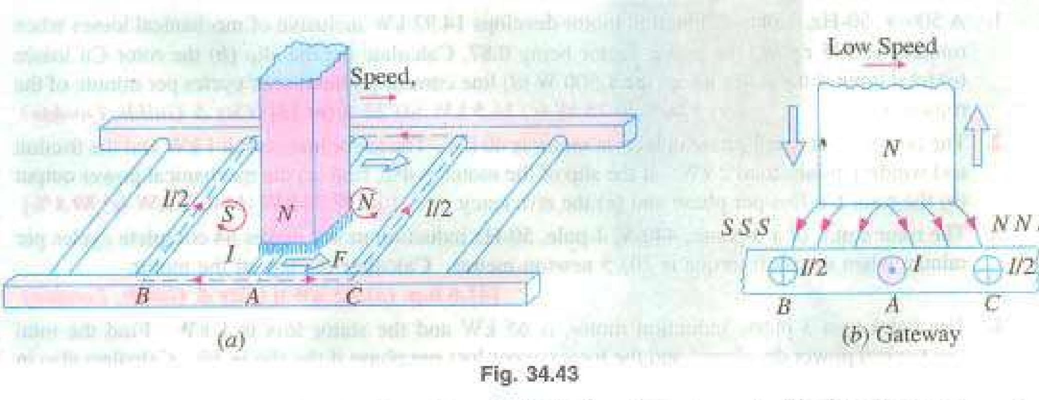


Fig. 34.43

A portion of the conducting ladder of Fig. 34.43 (*a*) has been shown in Fig. 34.43 (*b*). The voltage induced in conductor (or bar) *A* is maximum because flux is greatest at the centre of the *N* pole. If the magnet speed is very low, the induced current reaches its maximum value in *A* at virtually the ***same time*** (because delay due to conductor inductance is negligible). As this current flows via conductors *B* and *C*, it produces induced *SSS* and *NNN* poles, as shown. Consequently, the front half of the magnet is pushed upwards while the rear half is pulled downwards. Since distribution of *SSS* and *NNN* pole is symmetrical with respect to the centre of the magnet, the vertical forces of attraction and repulsion, being equal and opposite, cancel each other out, leaving behind only horizontal tractive force.

Now, consider the case when the magnet sweeps over the conductor *A* with a *very high speed*, as shown in Fig. 34.44. Due to conductor inductance, current in A reaches its maximum value a fraction of a second (Δt) after voltage reaches its maximum value. Hence, by the time I in conductor *A* reaches its maximum value, the centre of the magnet is already ahead of *A* by a distance $= \upsilon . \Delta t$ where υ is the magnet velocity. The induced poles *SSS* and *NNN* are produced, as before, by the currents returning via conductors *B* and *C* respectively. But, by now, the *N* pole of the permanent magnet lies over the induced *NNN* pole, which pushes it upwards with a strong vertical force. * This forms the basis of ***magnetic levitation*** which literally means 'floating in air'.

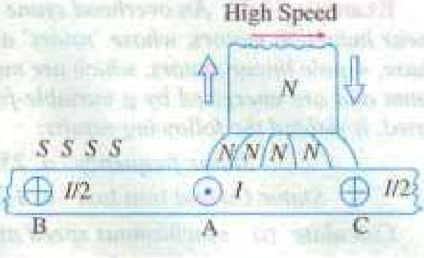


Fig. 34.44

Magnetic levitation is being used in ultrahigh speed trains (upto 300 km/h) which float in the air about 100 mm to 300 mm above the metallic track. They do not have any wheels and do not require the traditional steel rail. A powerful electromagnet (whose coils are cooled to about 4 °K by liquid helium) fixed underneath the train moves across the conducting rail, thereby inducing current in the rail. This gives rise to vertical force (called force of levitation) which keeps the train pushed up in the air above the track. Linear motors are used to propel the train.

A similar magnetic levitation system of transit is being considered for connecting Vivek Vihar in East Delhi to Vikaspuri in West Delhi. The system popularly known as Magneto-Bahn (*M*-Bahn) completely eliminates the centuries-old 'steel-wheel-over steel rail' traction. The *M*-Bahn train floats in the air through the principle of magnetic levitation and propulsion is by linear induction motors. There is 50% decrease in the train weight and 60% reduction in energy consumption for propulsion purposes. The system is extraordinarily safe (even during an earthquake) and the operation is fully automatic and computer-based.

Tutorial Problem No. 34.3

1. A 500-V, 50-Hz, 3-phase induction motor develops 14.92 kW inclusive of mechanical losses when running at 995 r.p.m., the power factor being 0.87. Calculate (*a*) the slip (*b*) the rotor Cu losses (*c*) total input if the stator losses are 1,500 W (*d*) line current (*e*) number of cycles per minute of the rotor e.m.f. **[(*a*) 0.005 (*b*) 75 W (*c*) 16.5 kW (*d*) 22 A (*e*) 15] (*City & Guilds, London*)**
2. The power input to a 3-phase induction motor is 40 kW. The stator losses total 1 kW and the friction and winding losses total 2 kW. If the slip of the motor is 4%, find (*a*) the mechanical power output (*b*) the rotor Cu loss per phase and (*c*) the efficiency. **[(*a*) 37.74 kW (*b*) 0.42 kW (*c*) 89.4%]**
3. The rotor e.m.f. of a 3-phase, 440-V, 4-pole, 50-Hz induction motor makes 84 complete cycles per minute when the shaft torque is 203.5 newton-metres. Calculate the h.p. of the motor. **[41.6 h.p. (31.03 kW)] (*City & Guilds, London*)**
4. The input to a 3-phase induction motor, is 65 kW and the stator loss is 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the slip is 3%. Calculate also in

* The induced current is always delayed (even at low magnet speeds) by an interval of time Δt which depends on the L/R time-constant of the conductor circuit. This delay is so brief at slow speed that voltage and the current reach their maximum value virtually at the same *time* and *place*. But at high speed, the same delay Δt is sufficient to produce large shift *in space between* the points where the voltage and current achieve their maximum values.

terms of the mechanical power developed the input to the rotor when the motor yields full-load torque at half speed. **[83.2 h.p. (62.067 kW) : 640 W, Double the output]** *(City & Guilds, London)*

5. A 6-pole, 50-Hz, 8-phase induction motor, running on full-load, develops a useful torque of 162 N-m and it is observed that the rotor electromotive force makes 90 complete cycles per min. Calculate the shaft output. If the mechanical torque lost in friction be 13.5 Nm, find the copper loss in the rotor windings, the input to the motor and the efficiency. Stator losses total 750 W. **[16.49 kW; 550 W; 19.2 kW; 86%]**

6. The power input to a 500-V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate (*a*) the slip (*b*) the rotor copper loss (*c*) shaft output (*d*) the efficiency. **[(*a*) 0.025 (*b*) 975 W (*c*) 36.1 kW (*d*) 90%]**

7. A 6-pole, 3-phase induction motor develops a power of 22.38kW, including mechanical losses which total 1.492 kW at a speed of 950 rpm on 550-V, 50-Hz mains. The power factor is 0.88. Calculate for this load (*a*) the slip (*b*) the rotor copper loss (*c*) the total input if the stator losses are 2000 W (*d*) the efficiency (*e*) the line current (*f*) the number of complete cycles of the rotor electromotive force per minute. **[(*a*) 0.05 (*b*) 1175 (*c*) 25.6 kW (*d*) 82% (*e*) 30.4 A (*f*) 150]**

8. A 3-phase induction motor has a 4-pole, star-connected stator winding. The motor runs on a 50-Hz supply with 200 V between lines. The rotor resistance and standstill reactance per phase are 0.1 Ω and 0.9 Ω respectively. The ratio of rotor to stator turns is 0.67. Calculate (*a*) total torque at 4% slip (*b*) total mechanical power at 4% slip (*c*) maximum torque (*d*) speed at maximum torque (*e*) maximum mechanical power. Prove the formulae employed, neglecting stator impedance. **[(*a*) 40 Nm (*b*) 6 kW (*c*) 63.7 Nm (*d*) 1335 rpm (*e*) 8.952 kW.]**

9. A 3-phase induction motor has a 4-pole, star-connected, stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance is 0.1 Ω and reactance 0.9. The ratio of stator to rotor turns is 1.75. The full load slip is 5%. Calculate for this load (*a*) the total torque (*b*) the shaft output. Find also (*c*) the maximum torque (*d*) the speed at maximum torque. **[(*a*) 42 Nm (*b*) 6.266 kW (*c*) 56 Nm (*d*) 1330 rpm]**

10. A 3000-V, 24-pole, 50-Hz 3-phase, star-connected induction motor has a slip-ring rotor of resistance 0.016 Ω and standstill reactance 0.265 Ω per phase. Full-load torque is obtained at a speed of 247 rpm.
Calculate (*a*) the ratio of maximum to full-load torque (*b*) the speed at maximum torque. Neglect stator impedance. **[(*a*) 2.61 (*b*) 235 rpm]**

11. The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at one-half of full-load speed. What is then the power factor ? **[22.5%; 0.31]**

12. The power input to a 3-phase, 50-Hz induction motor is 60 kW. The total stator loss is 1000 W. Find the total mechanical power developed and rotor copper loss if it is observed that the rotor e.m.f. makes 120 complete cycles per minute. **[56.64 kW; 2.36 kW]** *(AMIE Sec. B Elect. Machine (E-3) Summer 1990)*

13. A balanced three phase induction motor has an efficiency of 0.85 when its output is 44.76 kW. At this load both the stator copper loss and the rotor copper loss are equal to the core losses. The mechanical losses are one-fourth of the no-load loss. Calculate the slip. **[4.94%]** *(AMIE Sec. B Elect. Machines (E-3) Winter 1991)*

14. An induction motor is running at 20% slip, the output is 36.775 kW and the total mechanical losses are 1500 W. Estimate Cu losses in the rotor circuit. If the stator losses are 3 kW, estimate efficiency of the motor. **[9,569 W, 72.35%]** *(Electrical Engineering-II, Bombay Univ. 1978)*

15. A 3- ϕ, 50-Hz, 500-V, 6-pole induction motor gives an output of 37.3 kW at 955 r.p.m. The power factor is 0.86, frictional and windage losses total 1.492 kW; stator losses amount to 1.5 kW. Determine (*i*) line current (*ii*) the rotor Cu loss for this load. **[(*i*) 56.54 A (*ii*) 88.6% (*iii*) 1.828 kW]** *(Electrical Technology, Kerala Univ. 1977)*

16. Determine the efficiency and the output horse-power of a 3-phase, 400-V induction motor running

on load with a slip of 4 per cent and taking a current of 50 A at a power factor of 0.86. When running light at 400 V, the motor has an input current of 15 A and the power taken is 2,000 W, of which 650 W represent the friction, windage and rotor core loss. The resistance per phase of the stator winding (delta-connected) is 0.5 Ω.

[85.8 per cent; 34.2 h.p. (25.51 kW)] *(Electrical Engineering-II, M.S. Univ. Baroda 1977)*

17. The power input to the rotor of a 440-V, 50-Hz, 3-phase, 6-pole induction motor is 60 kW. It is observed that the rotor e.m.f. makes 90 complete cycles per minute. Calculate (*a*) the slip (*b*) rotor speed (*c*) rotor Cu loss per phase (*c*) the mechanical power developed and (*e*) the rotor resistance per phase if rotor current is 60 A. **[(*a*) 0.03 (*b*) 970 r.p.m. (*c*) 600 W (*d*) 58.2 kW (*e*) 0.167 Ω]**

18. An induction motor is running at 50% of the synchronous speed with a useful output of 41.03 kW and the mechanical losses total 1.492 kW. Estimate the Cu loss in the rotor circuit of the motor. If the stator losses total 3.5 kW, at what efficiency is the motor working ?

[42.52 kW; 46.34%] *(Electrical Engineering-II, Bombay Univ. 1975)*

19. Plot the torque/speed curve of a 6-pole, 50-Hz, 3-phase induction motor. The rotor resistance and reactance per phase are 0.02 Ω and 0.1 Ω respectively. At what speed is the torque a maximum? What must be the value of the external rotor resistance per phase to give two-third of maximum torque at starting ? **[(*a*) 800 rpm (*b*) 0.242 Ω or 0.018 Ω]**

34.47. Induction Motor as a Generalized Transformer

The transfer of energy from stator to the rotor of an induction motor takes place entirely *inductively*, with the help of a flux mutually linking the two. Hence, an induction motor is essentially a transformer with stator forming the primary and rotor forming (the short-circuited) rotating secondary (Fig. 34.45). The vector diagram is similar to that of a transformer (Art. 32.15).

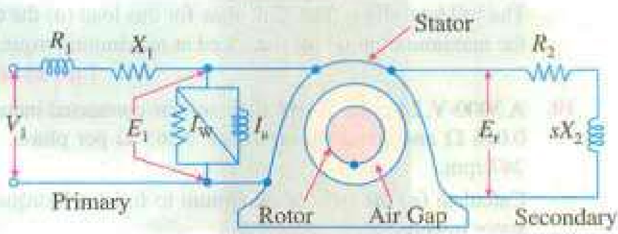


Fig. 34.45

In the vector diagram of Fig. 34.46, V_1 is the applied voltage per stator phase, R_1 and X_1 are stator resistance and leakage reactance per phase respectively, shown external to the stator winding in Fig. 34.45. The applied voltage V_1 produces a magnetic flux which links both primary and secondary thereby producing a counter e.m.f of self-induction E_1 in primary (*i.e.* stator) and a mutually-induced e.m.f. $E_r (= sE_2)$ in secondary (*i.e.* rotor). There is no secondary terminal voltage V_2 in secondary because whole of the induced e.m.f. E_r is used up in circulating the rotor current as the rotor is closed upon itself (which is equivalent to its being short-circuited).

Obviously $$\mathbf{V_1} = \mathbf{E_1} + I_1R_1 + j\,I_1X_1$$

The magnitude of $\mathbf{E_r}$ depends on voltage transformation ratio K between stator and rotor and the slip. As it is wholly absorbed in the rotor impedance.

$$\therefore \quad \mathbf{E_r} = \mathbf{I_2 Z_2} = I_2\,(R_2 + jsX_2)$$

In the vector diagram, I_0 is the no-load primary current. It has two components (*i*) the working or iron loss components I_w which supplies the no-load motor losses and (*ii*) the magnetising component I_μ which sets up magnetic flux in the core and the air-gap.

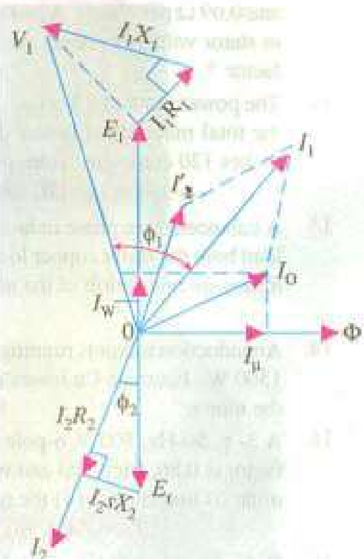


Fig. 34.46

Obviously $I_0 = \sqrt{\left(I_\omega^2 + I_\mu^2\right)}$ I_n Fig. 34.45, I_ω and I_μ are taken care of by an exciting circuit containing $R_0 = E_1/I_\omega$ and $X_0 = E_1/I_\mu$ respectively.

It should be noted here, in passing that in the usual two-winding transformer, I_0 is quite small (about 1 % of the full-load current). The reason is that the magnetic flux path lies almost completely in the steel core of low reluctance, hence I_μ is small, with the result that I_0 is small But in an induction motor, the presence of an air-gap (of high reluctance) necessitates a large I_μ hence I_0 is very large (approximately 40 to 50% of the full-load current).

In the vector diagram, I_2' is the equivalent load current in primary and is equal to KI_2. Total primary current is the *vector* sum of I_0 and I_2'.

At this place, a few words may be said to justify the representation of the stator and rotor quantities on the same vector diagram, even though the frequency of rotor current and e.m.f. is only a fraction of that of the stator. We will now show that even though the frequencies of stator and rotor currents are different, yet magnetic fields due to them are synchronous with each other, when seen by an observer stationed in space—both fields rotate at synchronous speed N_s (Art. 34.11).

The current flowing in the short-circuited rotor produces a magnetic field, which revolves round the rotor in the same direction as the stator field. The speed of rotation of the rotor field is

$$= \frac{120\, f_r}{P} = \frac{120\, sf}{P} = sN_s = N_s \times \frac{N_S - N}{N_s} = (N_S - N)$$

Rotor speed $N = (1 - s)\, N_s$

Hence, speed of the rotating field of the rotor with respect to the stationary stator or space is

$$= sN_S + N = (N_S - N) + N = N_s$$

34.48. Rotor Output

Primary current I_1 consists of two parts, I_0 and I_2'. It is the latter which is transferred to the rotor, because I_0 is used in meeting the Cu and iron losses in the stator itself. Out of the applied primary voltage V_1, some is absorbed in the primary itself ($= I_1 Z_1$) and the remaining E_1 is transferred to the rotor. If the angle between E_1 and I_2' is ϕ, then

$$\text{Rotor input / phase} = E_1 I_2' \cos\phi\,; \quad \text{Total rotor input} = 3E_1 I_2' \cos\phi$$

The electrical input to the rotor which is wasted in the form of heat is

$$= 3I_2\, E_r \cos\phi \qquad (\text{or} = 3\, I_2^2\, R_2)$$

Now $\quad I_2' = KI_2 \quad$ or $\quad I_2 = I_2'/K$

$E_r = s\,E_2 \quad$ or $\quad E_2 = KE_1$

$\therefore \quad E_r = sKE_1$

$\therefore$ electrical input wasted as heat

$$= 3 \times (I_2'/K) \times sK\,E_1 \times \cos\phi = 3E_1 I_2' \cos\phi \times s = \text{rotor input} \times s$$

$$\text{Now, rotor output} = \text{rotor input} - \text{losses} = 3E_1\, I_2' \cos\phi - 3E_1 I_2' \cos\phi \times s$$

$$= (1 - s)\, 3E_1 I_2' \cos\phi = (1 - s) \times \text{rotor input}$$

$$\therefore \quad \frac{\text{rotor output}}{\text{rotor input}} = 1 - s \quad \therefore \quad \text{rotor Cu loss} = s \times \text{rotor input}$$

$$\boxed{\text{rotor efficiency} = 1 - s = \frac{N}{N_S} = \frac{\text{actual speed}}{\text{synchronous speed}}}$$

In the same way, other relation similar to those derived in Art. 34.37 can be found.

34.49. Equivalent Circuit of the Rotor

When motor is loaded, the rotor current I_2 is given by

$$I_2 = s\frac{E_2}{\sqrt{\left[R_2^2 + (sX_2)^2\right]}} = \frac{E_2}{\sqrt{\left[(R_2/s)^2 + X_2^2\right]}}$$

From the above relation it appears that the rotor circuit which actually consists of a fixed resistance R_2 and a variable reactance sX_2 (proportional to slip) connected across $E_r = sE_2$ [Fig. 34.47 (*a*)] can be looked upon as equivalent to a rotor circuit having a fixed reactance X_2 connected in series with a variable resistance R_2/s (inversely proportional to slip) and supplied with constant voltage E_2, as shown in Fig. 34.47 (*b*).

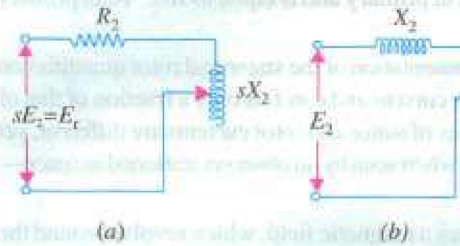


Fig. 34.47

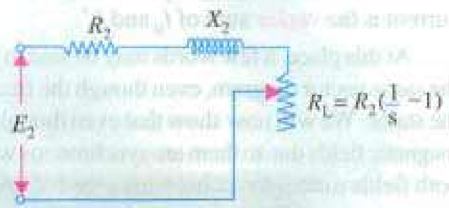


Fig. 34.48

Now, the resistance $\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$. It consists of two parts :

(*i*) the first part R_2 is the rotor resistance itself and represents the rotor Cu loss.

(*ii*) the second part is $R_2\left(\frac{1}{s} - 1\right)$

This is known as the load resistance R_L and is the electrical equivalent of the mechanical load on the motor. In other words, the mechanical load on an induction motor can be represented by a non-inductive resistance of the value $R_2\left(\frac{1}{s} - 1\right)$. The equivalent rotor circuit along with the load resistance R_L may be drawn as in Fig. 34.48.

34.50. Equivalent Circuit of an Induction Motor

As in the case of a transformer (Fig. 32.14), in this case also, the secondary values may be transferred to the primary and *vice versa*. As before, it should be remembered that when shifting impedance or resistance from secondary to primary, it should be ***divided*** by K^2 whereas current should be ***multiplied*** by K. The equivalent circuit of an induction motor where all values have been referred to primary *i.e.* stator is shown in Fig. 34.49.

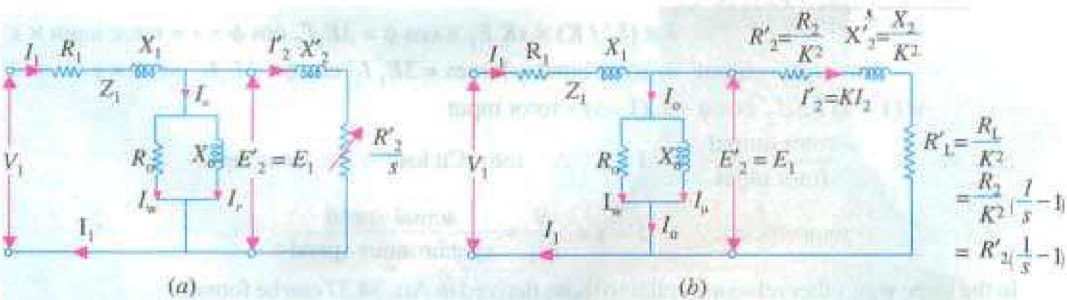


Fig. 34.49

As shown in Fig. 34.50, the exciting circuit may be transferred to the left, because inaccuracy involved is negligible but the circuit and hence the calculations are very much simplified. This is known as the ***approximate equivalent*** circuit of the induction motor.

If transformation ratio is assumed unity *i.e.* $E_2/E_1 = 1$, then the equivalent circuit is as shown in Fig. 34.51 instead of that in Fig. 34.49.

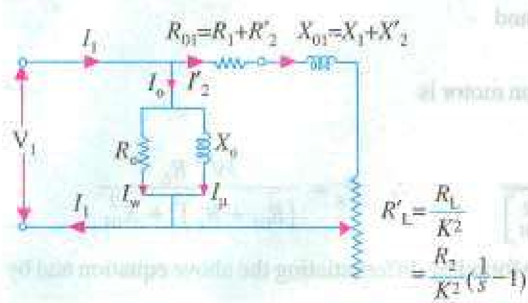


Fig. 34.50

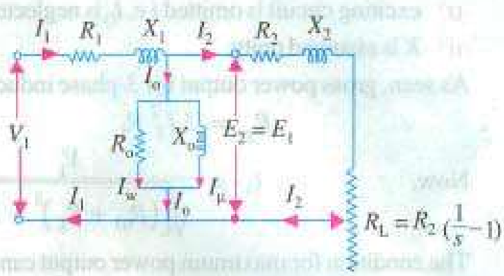


Fig. 34.51

34.51. Power Balance Equations

With reference to Fig. 34.49 (*a*), following power relations in an induction motor can be deduced:

Input power $= 3V_1 I_1 \cos\phi_1$; stator core loss $= I_\omega^2 R_0$; stator Cu loss $= 3 I_1^{'2} R_1$

Power transferred to rotor $= 3 I_2^{'2} R_2'/s$; Rotor Cu loss $= 3 I_2^{'2} R_2'$

Mechanical power developed by rotor (P_m) or gross power developed by rotor (P_g)

$$= \text{rotor input} - \text{rotor Cu losses}$$

$$= 3 I_2^{'2} R_2'/s - 3 I_2^{'2} R_2' = 3 I_2^{'2} R_2' \left(\frac{1-s}{s}\right) \text{ Watt}$$

If T_g is the gross torque* developed by the rotor, then

$$T_g \times \omega = T_g \times 2\pi \frac{N}{60} = 3 I_2^{'2} R_2' \left(\frac{1-s}{s}\right)$$

$$\therefore \quad T_g = \frac{3 I_2^{'2} R_2' \left(\dfrac{1-s}{s}\right)}{2\pi N/60} \text{ N-m}$$

Now, $N = N_S(1-s)$. Hence gross torque becomes

$$T_g = \frac{3 I_2^{'2} R_2'/s}{2\pi N_s/60} \text{ N-m} = 9.55 \times \frac{3 I_2^{'2} R_2'/s}{N_s} \text{ N-m}$$

Since gross torque in ***synchronous watts*** is equal to the power transferred to the rotor across the air-gap.

$$\therefore \quad T_g = 3 I_2^{'2} R_2'/s \text{ synch. watt.}$$

It is seen from the approximate circuit of Fig. 34.50 that

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$

$$T_g = \frac{3}{2\pi N_s/60} \times \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s} \text{ N-m}$$

* It is different from shaft torque, which is less than T_g by the torque required to meet windage and frictional losses.

34.52. Maximum Power Output

Fig. 34.52 shows the approximate equivalent circuit of an induction motor with the simplification that :

(*i*) exciting circuit is omitted *i.e.* I_0 is neglected and

(*ii*) K is assumed unity.

As seen, gross power output for 3-phase induction motor is

$$P_g = 3I_1^2 R_L$$

Now, $$I_1 = \frac{V_1}{\sqrt{\left[(R_{01}+R_L)^2 + X_{01}^2\right]}} \qquad \therefore \quad P_g = \frac{3V_1^2 R_L}{(R_{01}+R_L)^2 + X_{01}^2}$$

The condition for maximum power output can be found by differentiating the above equation and by equating the first derivative to zero. If it is done, it will be found that

$$R_L^2 = R_{01}^2 + X_{01}^2 = Z_{01}^2$$

where Z_{01} = leakage impedance of the motor as referred to primary

$$\therefore \quad R_L = Z_{01}$$

Hence, the power output is maximum ***when the equivalent load resistance is equal to the standstill leakage impedance of the motor***

Fig. 34.52

34.53. Corresponding Slip

Now $\quad R_L = R_2[(1/s) - 1] \qquad \therefore \quad Z_{01} = R_L = R_2\,[(1/s) - 1]$ or $s = \dfrac{R_2}{R_2 + Z_{01}}$

This is the slip corresponding to maximum gross power output. The vlaue of P_{gmax} is obtained by substituting R_L by Z_{01} in the above equation.

$$\therefore \quad P_{gmax} = \frac{3V_1^2 Z_{01}}{(R_{01}+Z_{01})^2 + X_{01}^2} = \frac{3V_1^2 Z_{01}}{R_{01}^2 + Z_{01}^2 + 2R_{01}Z_{01} + X_{01}^2} = \frac{3V_1^2}{2(R_{01}+Z_{01})}$$

It should be noted that V_1 is voltage/phase of the motor and K has been taken as unity.

Example 34.54. *The maximum torque of a 3-phase induction motor occurs at a slip of 12%. The motor has an equivalent secondary resistance of 0.08 Ω/phase. Calculate the equivalent load resistance R_L, the equivalent load voltage V_L and the current at this slip if the gross power output is 9,000 watts.*

Solution. $\quad R_L = R_2[(1/s) - 1] = 0.08\,[(1/0.12) - 1]$ = **0.587 Ω /phase.**

As shown in the equivalent circuit of the rotor in Fig. 34.53, V is a fictitious voltage drop equivalent to that consumed in the load connected to the secondary *i.e.* rotor. The value of $V = I_2R_L$

Now, gross power $P_g = 3I_2^2\,R_L = 3V^2/R_L$

$$V = \sqrt{(P_g \times R_L/3)}$$

$$= \sqrt{(0.587 \times 9000/3)} = \mathbf{42\ V}$$

Equivalent load current $= V/R_L = 42/0.587 =$ **71.6A**

Fig. 34.53

Example 34.55. *A 3-phase, star-connected 400 V, 50-Hz, 4-pole induction motor has the following per phase parameters in ohms, referred to the stators.*

$$R_1 = 0.15,\ X_1 = 0.45,\ R_2' = 0.12,\ X_2' = 0.45,\ X_m = 28.5$$

Compute the stator current and power factor when the motor is operated at rated voltage and frequency with s = 0.04. **(Elect. Machines, A.M.I.E. Sec. B, 1990)**

Solution. The equivalent circuit with all values referred to stator is shown in Fig. 34.54.

$$R_L' = R_2'\left(\frac{1}{s}-1\right)$$

$$= 0.12\left(\frac{1}{0.04}-1\right) = 2.88\ \Omega$$

$$I_2' = \frac{V}{(R_{01}+R_L')+j\,X_{01}}$$

$$= \frac{400/\sqrt{3}}{(0.15+0.12+2.88)+j(0.45+0.45)}$$

$$= 67.78 - j\,19.36$$

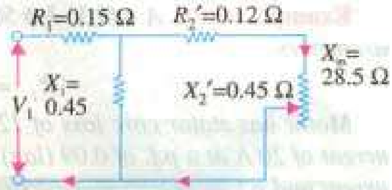


Fig. 34.54

$$I_0 = \frac{400/\sqrt{3}}{X_m} = \frac{400}{\sqrt{3}\times j\,28.5} = -j\,8.1$$

Stator current, $I_1 = I_0 + I_2' = 67.78 - j\,19.36 - j\,8.1 = \mathbf{73.13 \angle -22°}$

p.f. $= \cos\phi = \cos 22° = 0.927$ (lag)

Example 34.56. *A 220-V, 3-φ, 4-pole, 50-Hz, Y-connected induction motor is rated 3.73 kW. The equivalent circuit parameters are:*

$$R_1 = 0.45\ \Omega,\ X_1 = 0.8\ \Omega;\ R_2' = 0.4\ \Omega,\ X_2' = 0.8\ \Omega,\ B_0 = -1/30 \text{ mho}$$

The stator core loss is 50 W and rotational loss is 150 W. For a slip of 0.04, find (i) input current (ii) p.f. (iii) air-gap power (iv) mechanical power (v) electro-magnetic torque (vi) output power and (vii) efficiency.

Solution. The exact equivalent circuit is shown in Fig. 34.55. Since R_0 (or G_0) is negligible in determining I_1, we will consider B_0 (or X_0) only

$$Z_{AB} = \frac{j\,X_m\,[(R_2'/s)+j\,X_2']}{(R_2'/s)+j\,(X_2'+X_m)} = \frac{j\,30\,(10+j0.8)}{10+j\,30.8} = 8.58 + j3.56 = 9.29\angle 22.5°$$

$$\mathbf{Z}_{01} = \mathbf{Z}_1 + \mathbf{Z}_{AB} = (0.45+j0.8)+(8.58+j3.56) = 9.03+j4.36 = 10\angle 25.8°$$

$$V_{ph} = \frac{220}{\sqrt{3}}\angle 0° = 127\angle 0°$$

(i) $\therefore\ \mathbf{I}_1 = V_1/Z_{01} = 127\angle 0°/10\angle 25.8°$

$$= \mathbf{12.7\angle -25.8°\ A}$$

(ii) p.f. $= \cos 25.8° = \mathbf{0.9}$

(iii) air-gap power, $P_2 = 3\,I_2'^2\,(R_2'/s) = 3I_1^2 R_{AB}$

$$= 3\times 12.7^2 \times 8.58 = \mathbf{4152\ W}$$

(iv) $P_m = (1-s)\,P_2 = 0.96\times 4152 = \mathbf{3{,}986\ W}$

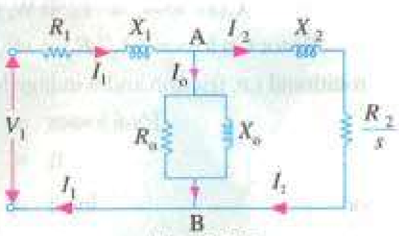


Fig. 34.55

(v) Electromagnetic torque (*i.e.* gross torque)

$$T_g = \frac{P_m}{2\pi N/60} = 9.55\frac{P_m}{N}\ \text{N-m}$$

Now, $N_s = 1500$ r.p.m., $N = 1500\,(1-0.04) = 1440$ r.p.m.

$$T_g = 9.55 \times 3986/1440 = \textbf{26.4 N-m}$$

$$(\text{or } T_g = 9.55\frac{P_2}{N_s} = 9.55 \times \frac{4152}{1500} = \textbf{26.4 N-m}$$

(*vi*) output power $= 3986 - 150 =$ **3836 W**

(*vii*) stator core loss $= 50$ W; stator Cu loss $= 3\,I_1^2 R_1 = 3 \times 12.7^2 \times 0.45 = 218$ W

Rotor Cu loss $= 3\,I_2'^2\,R_2' = sP_2 = 0.04 \times 4152 = 166$ W; Rotational losses $= 150$ W

Total loss $= 50 + 218 + 166 + 150 = 584$ W; $\eta = 3836/(3836 + 584) = 0.868$ or **86.8%**

Example 34.57. *A 440-V, 3-φ 50-Hz, 37.3 kW, Y-connected induction motor has the following parameters:*

$$R_1 = 0.1\,\Omega,\ X_1 = 0.4\,\Omega,\ R_2' = 0.15\,\Omega,\ X_2' = 0.44\,\Omega$$

Motor has stator core loss of 1250 W and rotational loss of 1000 W. It draws a no-load line current of 20 A at a p.f. of 0.09 (lag). When motor operates at a slip of 3%, calculate (i) input line current and p.f. (ii) electromagnetic torque developed in N-m (iii) output and (iv) efficiency of the motor. **(Elect. Machines-II, Nagpur Univ. 1992)**

Solution. The equivalent circuit of the motor is shown in Fig. 34.49 (*a*). Applied voltage per phase $= 440/\sqrt{3} = 254$ V.

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j\,(X_1 + X_2')} = \frac{254\angle 0^\circ}{(0.1 + 0.15/0.03) + j\,(0.4 + 0.44)}$$

$$= \frac{254\angle 0^\circ}{5.1 + j\,0.84} = \frac{254\angle 0^\circ}{5.17\angle 9.3^\circ} = 49.1\angle -9.3^\circ = 48.4 - j\,7.9$$

For all practical purposes, no-load motor current may be taken as equal to magnetising current I_0. Hence, $I_0 = 20\angle -84.9^\circ = 1.78 - j\,19.9$.

(*i*) $\mathbf{I_1} = \mathbf{I_0} + \mathbf{I_2'} = (48.4 - j\,7.9) + (1.78 - j\,19.9) = 50.2 - j\,27.8 = \mathbf{57.4\angle -29^\circ}$

∴ p.f. $= \cos 29^\circ =$ **0.875 (lag).**

(*ii*) $P_2 = 3\,I_2'^2\,(R_2'/s) = 3 \times 49.1^2 \times (0.15/0.03) = 36{,}160$ W

$N_s = 1500$ r.p.m.

∴ $T_g = 9.55 \times 36{,}160/1500 =$ **230 N-m**

(*iii*) $P_m = (1 - s)\,P_2 = 0.97 \times 36{,}160 = 35{,}075$ W

Output power $= 35{,}075 - 1000 =$ **34,075 W**

Obviously, motor is delivering less than its rated output at this slip.

(*iv*) Let us total up the losses.

Core loss $= 1250$ W, stator Cu loss $= 3\,I_1^2\,R_1 = 3 \times 57.4^2 \times 0.1 = 988$ W

Rotor Cu loss $= 3\,I_2'^2\,R_2' = sP_2 = 0.03 \times 36{,}160 = 1085$ W

rotational *i.e.* friction and windage losses $= 1000$ W

Total losses $= 1250 + 988 + 1085 + 1000 = 4323$ W

$\eta = 34{,}075/(34{,}075 + 4323) = 0.887$ or **88.7%**

or input $= \sqrt{3} \times 440 \times 57.4 \times 0.875 = 38{,}275$ W

∴ $\eta = 1 - (4323/38{,}275) = 0.887$ or **88.7%**

Example 34.58. *A 400 V, 3-φ, star-connected induction motor has a stator exciting impedance of (0.06 + j 0.2) Ω and an equivalent rotor impedance of (0.06 + j 0.22) Ω. Neglecting exciting current, find the maximum gross power and the slip at which it occurs.* **(Elect. Engg.-II, Bombay Univ. 1987)**

Solution. The equivalent circuit is shown in Fig. 34.56.

$$R_{01} = R_1 + R_2' = 0.06 + 0.06 = 0.12\ \Omega$$
$$X_{01} = X_1 + X_2' = 0.2 + 0.22 = 0.42\ \Omega$$
$$\therefore \quad Z_{01} = \sqrt{(0.12^3 + 0.42^2)} = 0.44\ \Omega$$

As shown in Art. 34.53, slip corresponding to maximum gross power output is given by

$$s = \frac{R_2}{R_2 + Z_{01}} = \frac{0.06}{0.06 + 0.44}$$
$$= 0.12 \text{ or } \mathbf{12\%}$$

Voltage/phase, $V_1 = 400/\sqrt{3}\,V$

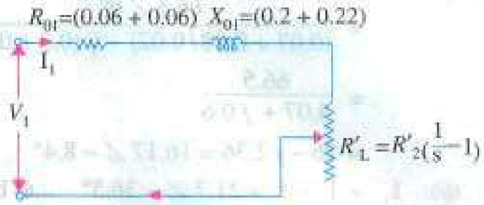


Fig. 34.56

$$P_{g\,max} = \frac{3V_1^2}{2(R_{01} + Z_{01})} = \frac{3\,(400/\sqrt{3})^2}{2\,(0.12 + 0.44)} = \mathbf{142{,}900\ W.}$$

Example 34.59. *A 115V, 60-Hz, 3-phase, Y-Connected, 6-pole induction motor has an equivalent T-circuit consisting of stator impedance of (0.07 + j 0.3) Ω and an equivalent rotor impedance at standstill of (0.08 + j 0.3) Ω. Magnetising branch has G_o = 0.022 mho, B_o = 0.158 mho. Find (a) secondary current (b) primary current (c) primary p.f. (d) gross power output (e) gross torque (f) input (g) gross efficiency by using approximate equivalent circuit. Assume a slip of 2% .*

Solution. The equivalent circuit is shown in Fig. 34.57 $R_L' = R_2'\,[(1/s) - 1]$

$$= 0.88\left(\frac{1}{0.02} - 1\right) = 3.92\ \Omega/\text{phase}$$

The impedance to the right of terminals *c* and *d* is

$$Z_{cd} = R_{01} + R_L' + j\,X_{01}$$
$$= (0.07 + 0.08) + 3.92 + j\,0.6$$
$$= 4.07 + j\,0.6$$
$$= 4.11 \angle 8.4°\ \Omega/\text{phase}$$
$$V = 115/\sqrt{3} = 66.5\ \text{V}$$

(a) Secondary current $\mathbf{I_2' = I_2}$

$$= \frac{66.5}{4.11 \angle 8.4°} = 16.17 \angle -j\,8.4°$$
$$= 16 - j\,2.36\ \text{A}$$

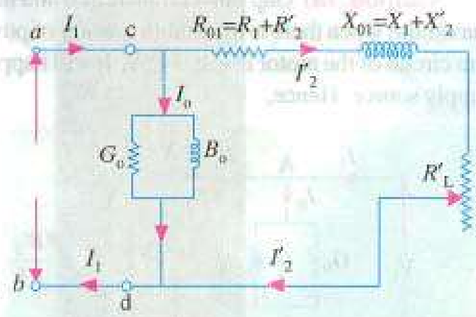


Fig. 34.57

The exciting current $\mathbf{I_0} = V(G_0 - j\,B_0) = 66.5\,(0.022 - j\,0.158) = 1.46 - j\,10.5$ A

(b) $$\mathbf{I_1} = \mathbf{I_0} + \mathbf{I_2'} = \mathbf{I_0} + \mathbf{I_2} = (1.46 - j\,10.5) + (16 - j\,2.36)$$
$$= 17.46 - j\,12.86 = 21.7 \angle -36.5°$$

(c) Primary p.f. $= \cos 36.5° = \mathbf{0.804}$

(d) $P_g = 3\,I_2^2 R_L' = 3 \times 16.17^2 \times 3.92 = \mathbf{3{,}075\ W}$

(e) Synchronous speed $N_s = 120 \times 60/6 = 1{,}200$ r.p.m.

Actual rotor speed $N = (1 - s)\,N_s = (1 - 0.02) \times 1200 = 1{,}176$ r.p.m.

$$\therefore \quad T_g = 9.55.\frac{P_m}{N} = 9.55 \times \frac{3075}{1176} = \mathbf{24.97\ N-m}$$

(f) Primary power input $= \sqrt{3}\,V_1\,I_1 \cos\phi = \sqrt{3} \times 115 \times 21.7 \times 0.804 = \mathbf{3{,}450\ W}$

(g) Gross efficiency $= 3{,}075 \times 100/3{,}450 = \mathbf{89.5\ \%}$

Alternative Solution

Instead of using the equivalent circuit of Fig. 34.57, we could use that shown in Fig. 34.49 which is reproduced in Fig. 34.58.

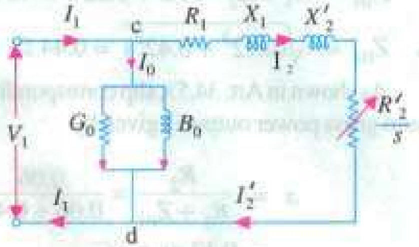


Fig. 34.58

(*a*) $$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$
$$= \frac{66.5 \angle 0°}{(0.07 + 0.08/0.02) + j(0.3 + 0.3)}$$
$$= \frac{66.5}{4.07 + j\,0.6}$$
$$= 16 - j\,2.36 = 16.17 \angle -8.4°$$

(*b*) $\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_2 = 21.7 \angle -36.5°$ ...as before

(*c*) primary p.f. = **0.804** ...as before

(*d*) gross power developed, $P_g = 3\, I_2'^2\, R_2' \left(\frac{1-s}{s}\right)$

$$= 3 \times 16.17^2 \times 0.08 \left(\frac{1 - 0.02}{0.02}\right) = \mathbf{3075\ W}$$

The rest of the solution is the same as above.

Example 34.60. *The equivalent circuit of a 400 V, 3-phase induction motor with a star-connected winding has the following impedances per phase referred to the stator at standstill:*

Stator : (0.4 + j 1) ohm; Rotor : (0.6 + j 1) ohm; Magnetising branch : (10 + j 50) ohm.

Find (i) maximum torque developed (ii) slip at maximum torque and (iii) p.f. at a slip of 5%. Use approximate equivalent circuit. **(Elect. Machinery-III, Bangalore Univ. 1987)**

Solution. (*ii*) Gap power transferred and hence the mechanical torque developed by rotor would be maximum when there is maximum transfer of power to the resistor R_2'/s shown in the approximate equivalent circuit of the motor in Fig. 34.59. It will happen when R_2'/s equals the impedance looking back into the supply source. Hence,

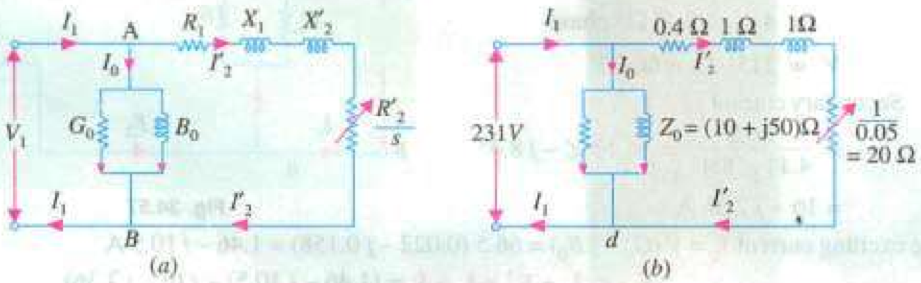


Fig. 34.59

$$\frac{R_2'}{s_m} = \sqrt{R_1^2 + (X_1 + X_2')^2}$$

or $$s_m = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.6}{\sqrt{0.4^2 + 2^2}} = 0.29 \text{ or } \mathbf{29\%}$$

(*i*) Maximum value of gross torque developed by rotor

$$T_{g\,max} = \frac{P_{g\,max}}{2\pi N_s/60} = \frac{3\, I_2'^2\, R_2'/s_m}{2\pi N_s/60} \text{ N-m}$$

Now, $$I_2' = \frac{V_1}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \frac{400/\sqrt{3}}{\sqrt{(0.4+0.6)^2 + (1+1)^2}} = 103.3 \text{ A}$$

$$\therefore \quad T_{g\text{-}max} = \frac{3 \times 103.3^2 \times 1/0.29}{2\pi \times 1500/60} = \mathbf{351 \text{ N-m}} \quad \text{... assuming } N_S = 1500 \text{ r.p.m.}$$

(iii) The equivalent circuit for one phase for a slip of 0.05 is shown in Fig. 32.59 (*b*).

$$I_2' = 231/[(20 + 0.4) + j\,2] = 11.2 - j\,1.1$$

$$I_0 = 231/(10 + j\,50) = 0.89 - j\,4.4$$

$$I_1 = I_0 + I_2' = 12.09 - j\,5.5 = 13.28 \angle -24.4°;\ \text{p.f.} = \cos 24.4° = \mathbf{0.91\ (lag)}$$

Tutorial Problem No. 34.4.

1. A 3-phase, 115-volt induction motor has the following constants : $R_2 = 0.07\ \Omega$; $R_2' = 0.08\ \Omega$, $X_1 = 0.4\ \Omega$ and $X_2' = 0.2\ \Omega$. All the values are for one phase only. At which slip the gross power output will be maximum and the value of the gross power output ? **[11.4% ; 8.6 kW]**
2. A 3-phase, 400-V, Y-connected induction motor has an equivalent *T*-circuit consisting of $R_1 = 1\ \Omega$, $X_1 = 2\ \Omega$, equivalent rotor values are $R_2' = 1.2\ \Omega$, $X_2' = 1.5\ \Omega$. The exciting branch has an impedance of $(4 + j\,40)\ \Omega$. If slip is 5% find (*i*) current (*ii*) efficiency (*iii*) power factor (*iv*) output. Assume friction loss to be 250 W. *[(i) 10.8 A (ii) 81% (iii) 0.82 (iv) 5 kW]*
3. A 50 HP, 440 Volt, 3-phase, 50 Hz Induction motor with star-connected stator winding gave the following test results:

   (*i*)No load test: Applied line voltage 440 V, line current 24 A, wattmeter reading 5150 and 3350 watts.

   (*ii*)Blocked rotor test: applied line voltage 33.6 volt, line current 65 A, wattmeter reading 2150 and 766 watts.

   Calculate the parameters of the equivalent circuit.

   [Rajiv Gandhi Technical University, Bhopal, 2000]

   [(*i*) Shunt branch : R_o = 107.6 ohms, X_m = 10.60 ohms (*ii*) Series branch : r = 0.23 ohm, x = 0.19 ohm]

OBJECTIVE TESTS – 34

1. Regarding skewing of motor bars in a squirrel-cage induction motor, (SCIM) which statement is false ?
   (*a*) it prevents cogging
   (*b*) it increases starting torque
   (*c*) it produces more uniform torque
   (*d*) it reduces motor 'hum' during its operation.
2. The principle of operation of a 3-phase. Induction motor is most similar to that of a
   (*a*) synchronous motor
   (*b*) repulsion-start induction motor
   (*c*) transformer with a shorted secondary
   (*d*) capacitor-start, induction-run motor.
3. The magnetising current drawn by transformers and induction motors is the cause of their ....power factor.
   (*a*) zero (*b*) unity
   (c) lagging (*d*) leading.
4. The effect of increasing the length of air-gap in an induction motor will be to increase the
   (*a*) power factor
   (*b*) speed
   (*c*) magnetising current
   (*d*) air-gap flux.

   (Power App-II, Delhi Univ. Jan. 1987)
5. In a 3-phase induction motor, the relative speed of stator flux with respect to ....is zero.
   (*a*) stator winding (*b*) rotor
   (c) rotor flux (*d*) space.
6. An eight-pole wound rotor induction motor operating on 60 Hz supply is driven at 1800 r.p.m. by a prime mover in the opposite direction of revolving magnetic field. The frequency of rotor current is
   (*a*) 60 Hz (*b*) 120 Hz
   (c) 180 Hz (*d*) none of the above.

(Elect. Machines, A.M.I.E. Sec. B, 1993)

7. A 3-phase, 4-pole, 50-Hz induction motor runs at a speed of 1440 r.p.m. The rotating field produced by the rotor rotates at a speed of ....r.p.m. with respect to the rotor.
   (*a*) 1500 (*b*) 1440
   (*c*) 60 (*d*) 0.

8. In a 3-ϕ induction motor, the rotor field rotates at synchronous speed with respect to
   (*a*) stator (*b*) rotor
   (*c*) stator flux (*d*) none of the above.

9. Irrespective of the supply frequency, the torque developed by a SCIM is the same whenever .... is the same.
   (*a*) supply voltage (*b*) external load
   (*c*) rotor resistance (*d*) slip speed.

10. In the case of a 3-ϕ induction motor having $N_s = 1500$ rpm and running with $s = 0.04$
    (*a*) revolving speed of the stator flux is space is .....rpm
    (*b*) rotor speed is ....rpm
    (*c*) speed of rotor flux relative to the rotor is ....rpm
    (*d*) speed of the rotor flux with respect to the stator is ....rpm.

11. The number of stator poles produced in the rotating magnetic field of a 3-ϕ induction motor having 3 slots per pole per phase is
    (*a*) 3 (*b*) 6
    (*c*) 2 (*d*) 12

12. The power factor of a squirrel-cage induction motor is
    (*a*) low at light loads only
    (*b*) low at heavy loads only
    (*c*) low at light and heavy loads both
    (*d*) low at rated load only.

(Elect. Machines, A.M.I.E. Sec.B, 1993)

13. Which of the following rotor quantity in a SCIM does NOT depend on its slip ?
    (*a*) reactance (*b*) speed
    (*c*) induced emf (*d*) frequency.

14. A 6-pole, 50-Hz, 3-ϕ induction motor is running at 950 rpm and has rotor Cu loss of 5 kW. Its rotor input is ......kW.
    (*a*) 100 (b) 10
    (c) 95 (*d*) 5.3.

15. The efficiency of a 3-phase induction motor is approximately proportional to
    (*a*) (1– s) (*b*) s
    (*c*) N (*d*) N_s.

16. A 6-pole, 50-Hz, 3-ϕ induction motor has a full-load speed of 950 rpm. At half-load, its speed would be ......rpm.
    (*a*) 475 (*b*) 500
    (*c*) 975 (*d*) 1000

17. If rotor input of a SCIM running with a slip of 10% is 100 kW, gross power developed by its rotor is ...... kW.
    (*a*) 10 (*b*) 90
    (*c*) 99 (*d*) 80

18. Pull-out torque of a SCIM occurs at that value of slip where rotor power factor equals
    (*a*) unity (*b*) 0.707
    (*c*) 0.866 (*d*) 0.5

19. Fill in the blanks.
    When load is placed on a 3-phase induction motor, its
    (*i*) speed ....
    (*ii*) slip ......
    (*iii*) rotor induced emf ......
    (*iv*) rotor current ......
    (*v*) rotor torque ......
    (*vi*) rotor continues to rotate at that value of slip at which developed torque equals ...... torque.

20. When applied rated voltage per phase is reduced by one-half, the starting torque of a SCIM becomes ...... of the starting torque with full voltage.
    (*a*) 1/2 (*b*) 1/4
    (*c*) $1/\sqrt{2}$ (*d*) $\sqrt{3}/2$

21. If maximum torque of an induction motor is 200 kg-m at a slip of 12%, the torque at 6% slip would be ...... kg-m.
    (*a*) 100 (*b*) 160
    (*c*) 50 (*d*) 40

22. The fractional slip of an induction motor is the ratio
    (*a*) rotor Cu loss/rotor input
    (*b*) stator Cu loss/stator input
    (*c*) rotor Cu loss/rotor output
    (*d*) rotor Cu loss/stator Cu loss

23. The torque developed by a 3-phase induction motor depends on the following three factors:
    (*a*) speed, frequency, number of poles
    (*b*) voltage, current and stator impedance
    (*c*) synchronous speed, rotor speed and frequency

(*d*) rotor emf, rotor current and rotor p.f.

24. If the stator voltage and frequency of an induction motor are reduced proportionately, its
(*a*) locked rotor current is reduced
(*b*) torque developed is increased
(*c*) magnetising current is decreased
(*d*) both (*a*) and (*b*)

25. The efficiency and p.f. of a SCIM increases in proportion to its
(*a*) speed
(*b*) mechanical load
(*c*) voltage
(*d*) rotor torque

26. A SCIM runs at *constant* speed only so long as
(*a*) torque developed by it remains constant
(*b*) its supply voltage remains constant
(*c*) its torque exactly equals the mechanical load
(*d*) stator flux remains constant

27. The synchronous speed of a linear induction motor does NOT depend on
(*a*) width of pole pitch
(*b*) number of poles
(*c*) supply frequency
(*d*) any of the above

28. Thrust developed by a linear induction motor depends on
(*a*) synchronous speed
(*b*) rotor input
(*c*) number of poles
(*d*) both (*a*) and (*b*)

ANSWERS

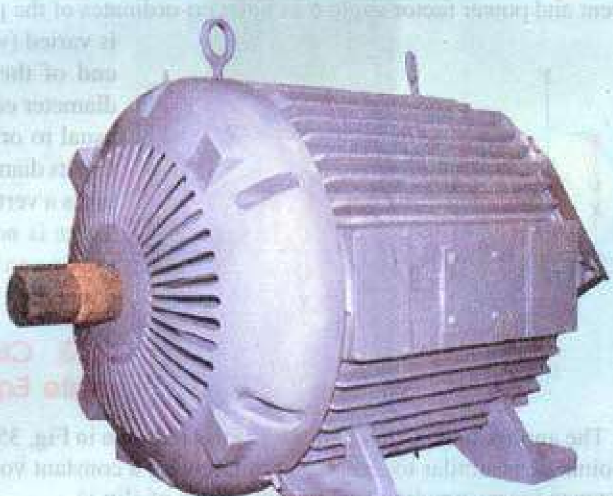
1. *b* **2.** *c* **3.** *c* **4.** *c* **5.** *c* **6.** *c* **7.** *c* **8.** *a* **9.** *d* **10.** (*i*) 1500 (*ii*) 1440 (*iii*) 60 (*iv*) 1500 **11.** *b* **12.** *a* **13.** *b* **14.** *a* **15.** *a* **16.** *c* **17.** *b* **18.** *b* **19.** (*i*) decreases (*ii*) increases (*iii*) increases (*iv*) increases (*v*) increases (*vi*) applied **20.** *b* **21.** *b* **22.** *a* **23.** *d* **24.** *d* **25.** *b* **26.** *c* **27.** *b* **28.** *d*

C H A P T E R 35

COMPUTATIONS AND CIRCLE DIAGRAMS

Learning Objectives

- General
- Circle Diagram for a Series Circuit
- Circle Diagram of the Approximate Equivalent Circle
- Determination of G0 and B0
- No-load Test
- Blocked Rotor Test
- Construction of the Circle Diagram
- Maximum Quantities
- Starting of Induction Motors
- Direct-Switching or Line Starting of Induction Motors
- Squirrel-cage Motors
- Starting of Slip-ring Motors
- Starter Steps
- Crawling
- Cogging or Magnetic Locking
- Double Sqiurrel-cage Motor
- Equivalent circuit
- Speed Control of Induction Motor
- Three-Phase A.C. Commutator Motors
- Schrage Motor
- Motor Enclosures
- Standard type of Squirrel-cage Motors
- Class A Motors
- Class B Motors
- Class C Motors
- Class D Motors
- Class E Motors
- Class F Motors
- Questions and Answer on Induction Motors



This chapter explains you how to derive performance characteristics of induction motors using circular diagrams

35.1. General

In this chapter, it will be shown that the performance characteristics of an induction motor are derivable from a circular locus. The data necessary to draw the circle diagram may be found from no-load and blocked-rotor tests, corresponding to the open-circuit and short-circuit tests of a transformer. The stator and rotor Cu losses can be separated by drawing a torque line. The parameters of the motor, in the equivalent circuit, can be found from the above tests, as shown below.

35.2. Circle Diagram for a Series Circuit

It will be shown that the end of the current vector for a series circuit with constant reactance and voltage, but with a variable resistance is a circle. With reference to Fig. 35.1, it is clear that

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R^2 + X^2)}}$$

$$= \frac{V}{X} \times \frac{X}{\sqrt{(R^2 + X^2)}} = \frac{V}{X} \sin\phi$$

$$\because \sin\phi = \frac{X}{\sqrt{(R^2 + X^2)}} \qquad \text{– Fig. 35.2}$$

$$\therefore \quad I = (V/X)\sin\phi$$

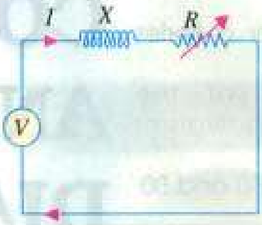


Fig. 35.1

Fig. 35.2

It is the equation of a circle in polar co-ordinates, with diameter equal to V/X. Such a circle is drawn in Fig. 35.3, using the magnitude of the current and power factor angle ϕ as polar co-ordinates of the point A. In other words, as resistance R is varied (which means, in fact, ϕ is changed), the end of the current vector lies on a circle with diameter equal to V/X. For a lagging current, it is usual to orientate the circle of Fig. 35.3 (*a*) such that its diameter is horizontal and the voltage vector takes a vertical position, as shown in Fig. 35.3 (*b*). There is no difference between the two so far as the magnitude and phase relationships are concerned.

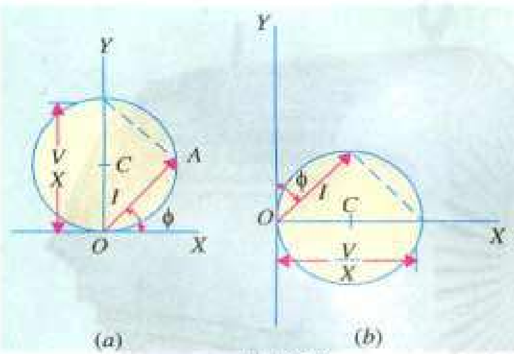


Fig. 35.3

35.3. Circle Diagram for the Approximate Equivalent Circuit

The approximate equivalent diagram is redrawn in Fig. 35.4. It is clear that the circuit to the right of points *ab* is similar to a series circuit, having a constant voltage V_1 and reactance X_{01} but variable resistance (corresponding to different values of slip s).

Hence, the end of current vector for I_2' will lie on a circle with a diameter of V/X_{01}. In Fig. 35.5, I_2' is the rotor current referred to stator, I_0 is no-load current (or exciting current) and I_1 is the total stator current and is the vector sum of the first two. When I_2' is lagging and $\phi_2 = 90°$, then the position of vector for I_2' will be along OC *i.e.* at right angles to the voltage vector OE. For any other value of ϕ_2, point A will move along the circle shown dotted. The exciting current I_0 is drawn lagging V by an angle ϕ_0. If conductance G_0 and susceptance

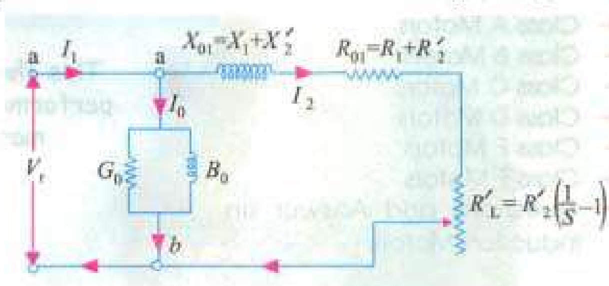


Fig. 35.4

B_0 of the exciting circuit are assumed constant, then I_0 and ϕ_0 are also constant. The end of current vector for I_1 is also seen to lie on another circle which is displaced from the dotted circle by an amount I_0. Its diameter is still V/X_{01} and is parallel to the horizontal axis OC. Hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.

35.4. Determination of G_0 and B_0

If the total leakage reactance X_{01} of the motor, exciting conductance G_0 and exciting susceptance B_0 are found, then the position of the circle $O'BC'$ is determined uniquely. One method of finding G_0 and B_0 consists in running the motor synchronously so that slip $s = 0$. In practice, it is impossible for an induction motor to run at synchronous speed, due to the inevitable presence of friction and windage losses. However, the induction motor may be run at synchronous speed by another machine which supplies the friction and windage losses. In that case, the circuit to the right of points ab behaves like an open circuit, because with $s = 0$, $R_L = \infty$ (Fig. 35.6). Hence, the current drawn by the motor is I_0 only. Let

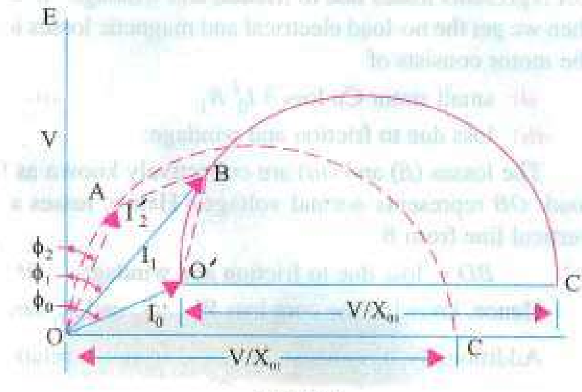


Fig. 35.5

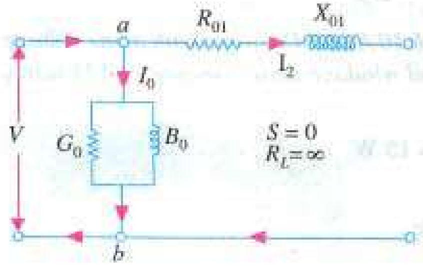


Fig. 35.6

V = applied voltage/phase; I_0 = motor current / phase

W = wattmeter reading *i.e.* input in watt ; Y_0 = exciting admittance of the motor. Then, for a 3-phase induction motor

$$W = 3G_0V^2 \quad \text{or} \quad G_0 = \frac{W}{3V^2} \quad \text{Also, } I_0 = VY_0 \text{ or } Y_0 = I_0/V$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{[(I_0/V)^2 - G_0^2]}$$

Hence, G_0 and B_0 can be found.

35.5. No-load Test

In practice, it is neither necessary nor feasible to run the induction motor synchronously for getting G_0 and B_0. Instead, the motor is run without any external mechanical load on it. The speed of the rotor would not be synchronous, but very much near to it ; so that, for all practical purposes, the speed may be assumed synchronous. The no load test is carried out with different values of applied voltage, below and above the value of normal voltage. The power input is measured by two wattmeters,

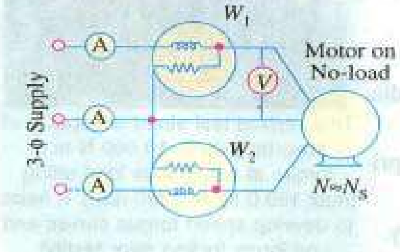


Fig. 35.7

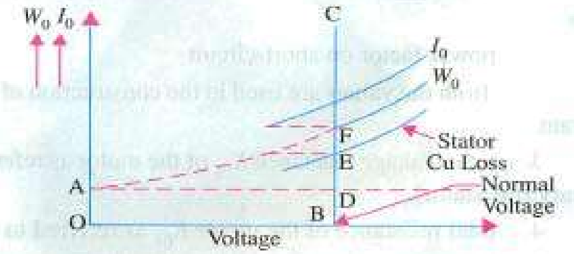


Fig. 35.8

I_0 by an ammeter and V by a voltmeter, which are included in the circuit of Fig. 35.7. As motor is running on light load, the p.f. would be low *i.e.* less than 0.5, hence total power input will be the difference of the two wattmeter readings W_1 and W_2. The readings of the total power input W_0, I_0 and voltage V are plotted as in Fig. 35.8. If we extend the curve for W_0, it cuts the vertical axis at point A. OA represents losses due to friction and windage. If we subtract loss corresponding to OA from W_0, then we get the no-load electrical and magnetic losses in the machine, because the no-load input W_0 to the motor consists of

(*i*) small stator Cu loss $3\,I_0^2\,R_1$ (*ii*) stator core loss $W_{CL} = 3G_0\,V^2$

(*iii*) loss due to friction and windage.

The losses (*ii*) and (*iii*) are collectively known as fixed losses, because they are independent of load. OB represents normal voltage. Hence, losses at normal voltage can be found by drawing a vertical line from B.

BD = loss due to friction and windage DE = stator Cu loss EF = core loss

Hence, knowing the core loss W_{CL}, G_0 and B_0 can be found, as discussed in Art. 35.4.

Additionally, ϕ_0 can also be found from the relation $W_0 = \sqrt{3}\,V_L\,I_0\cos\phi_0$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{\sqrt{3}\,V_L\,I_0}$$ where V_L = line voltage and W_0 is no-load stator input.

Example 35.1. *In a no-load test, an induction motor took 10 A and 450 watts with a line voltage of 110 V. If stator resistance/phase is 0.05 Ω and friction and windage losses amount to 135 watts, calculate the exciting conductance and susceptance/phase.*

Solution. stator Cu loss $= 3\,I_0^2\,R_1 = 3 \times 10^2 \times 0.05 = 15$ W

$\therefore$ stator core loss $= 450 - 135 - 15 = 300$ W

Voltage/phase $V = 110/\sqrt{3}$ V ; Core loss $= 3\,G_0\,V^2$

$$300 = 3\,G_0 \times (110/\sqrt{3})^2 \; ; \; G_0 = \frac{300}{3 \times (110/\sqrt{3})^2}$$

$= \mathbf{0.025}$ **siemens/phase**

$Y_0 = I_0 / V = (10 \times \sqrt{3})/110 = 0.158$ siemens/phase

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{(0.158^2 - 0.025^2)}$$

$= \mathbf{0.156}$ **siemens/phase.**

35.6. Blocked Rotor Test

It is also known as locked-rotor or short-circuit test. This test is used to find–

1. short-circuit current with ***normal*** voltage applied to stator
2. power factor on short-circuit

   Both the values are used in the construction of circle diagram
3. total leakage reactance X_{01} of the motor as referred to primary (*i.e.* stator)
4. total resistance of the motor R_{01} as referred to primary.

In this test, the rotor is locked (or allowed very slow rotation)

This vertical test stand is capable of absorbing up to 10,000 N-m of torque at continuous load rating (max 150.0 hp at 1800 rpm). It helps to develop speed torque curves and performs locked rotor testing and the rotor windings are short-circuited at slip-rings, if the motor has a wound rotor. Just as in the case of a short-circuit test on a transformer, a reduced voltage (up to 15 or 20 per cent of normal value) is applied to the stator terminals and is so adjusted that full-load current flows in the stator. As in this case $s = 1$, the equivalent circuit of the motor is exactly like a transformer, having a short-circuited secondary. The values of current, voltage and power input on short-circuit are measured by the ammeter, voltmeter and wattmeter connected in the circuits as before. Curves connecting the above quantities may also be drawn by taking two or three additional sets of readings at progressively reduced voltages of the stator.

(*a*) It is found that relation between the short-circuit current and voltage is approximately a straight line. Hence, if V is normal stator voltage, V_s the short-circuit voltage (a fraction of V), then short-circuit or standstill rotor current, if normal voltage were applied to stator, is found from the relation

$$I_{SN} = I_s \times V/V_s$$

where I_{SN} = short-circuit current obtainable with normal voltage

I_s = short-circuit current with voltage V_S

(*b*) Power factor on short-circuit is found from

$$W_S = \sqrt{3}\, V_{SL}\, I_{SL} \cos \phi_S\,; \qquad \therefore \quad \cos \phi_S = W_S / (\sqrt{3}\, V_{SL}\, I_{SL})$$

where W_S = total power input on short-circuit

V_{SL} = line voltage on short-circuit

I_{SL} = line current on short-circuit.

(*c*) Now, the motor input on short-circuit consists of

(*i*) mainly stator and rotor Cu losses

(*ii*) core-loss, which is small due to the fact that applied voltage is only a small percentage of the normal voltage. This core-loss (if found appreciable) can be calculated from the curves of Fig. 35.8.

$$\therefore \qquad \text{Total Cu loss} = W_S - W_{CL}$$

$$3\, I_s^2\, R_{01} = W_s - W_{CL}\,; \qquad R_{01} = (W_s - W_{CL}) / 3\, I_s^2$$

(*d*) With reference to the approximate equivalent circuit of an induction motor (Fig. 35.4), motor leakage reactance per phase X_{01} as referred to the stator may be calculated as follows :

$$Z_{01} = V_S / I_S \qquad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$$

Usually, X_1 is assumed equal to X_2' where X_1 and X_2 are stator and rotor reactances per phase respectively as referred to stator. $X_1 = X_2' = X_{01} / 2$

If the motor has a wound rotor, then stator and rotor resistances are separated by dividing R_{01} in the ratio of the d.c. resistances of stator and rotor windings.

In the case of squirrel-cage rotor, R_1 is determined as usual and after allowing for 'skin effect' is subtracted from R_{01} to give R_2' – the effective rotor resistance as referred to stator.

$$\therefore \qquad R_2' = R_{01} - R_1$$

Example 35.2. *A 110-V, 3-ϕ, star-connected induction motor takes 25 A at a line voltage of 30 V with rotor locked. With this line voltage, power input to motor is 440 W and core loss is 40 W. The d.c. resistance between a pair of stator terminals is 0.1 Ω. If the ratio of a.c. to d.c. resistance is 1.6, find the equivalent leakage reactance/phase of the motor and the stator and rotor resistance per phase.* **(Electrical Technology, Madras Univ. 1987)**

Solution. S.C. voltage/phase, $V_s = 30 / \sqrt{3} = 17.3$ V ; $I_s = 25$ A per phase

$Z_{01} = 17.3/25 = 0.7\ \Omega$ (approx.) per phase

$$\text{Stator and rotor Cu losses} = \text{input} - \text{core loss} = 440 - 40 = 400 \text{ W}$$

$$\therefore \quad 3 \times 25^2 \times R_{01} = 400 \qquad \therefore R_{01} = 400/3 \times 625 = \mathbf{0.21\ \Omega}$$

where R_{01} is equivalent resistance/phase of motor as referred to stator.

$$\text{Leakage reactance/phase} \qquad X_{01} = \sqrt{(0.7^2 - 0.21^2)} = 0.668\ \Omega$$

$$\text{d.c. resistance/phase of stator} = 0.1/2 = 0.05\ \Omega$$

$$\text{a.c. resistance/phase} \qquad R_1 = 0.05 \times 1.6 = 0.08\ \Omega$$

Hence, effective resistance/phase of rotor as referred to stator

$$R_2' = 0.21 - 0.08 = \mathbf{0.13\ \Omega}$$

35.7. Construction of the Circle Diagram

Windings inside a motor

Circle diagram of an induction motor can be drawn by using the data obtained from **(1) no-load (2) short-circuit test** and **(3) stator resistance test**, as shown below.

Step No. 1

From no-load test, I_0 and ϕ_0 can be calculated. Hence, as shown in Fig. 35.9, vector for I_0 can be laid off lagging ϕ_0 behind the applied voltage V.

Step No. 2

Next, from blocked rotor test or short-circuit test, short-circuit current I_{SN} ***corresponding to normal voltage*** and ϕ_S are found. The vector ***OA*** represents $I_{SN} = (I_S V/V_S)$ in magnitude and phase. Vector $O'A$ represents rotor current I_2' as referred to stator.

Clearly, the two points O' and A lie on the required circle. For finding the centre C of this circle, chord $O'A$ is bisected at right angles–its bisector giving point C. The diameter $O'D$ is drawn perpendicular to the voltage vector.

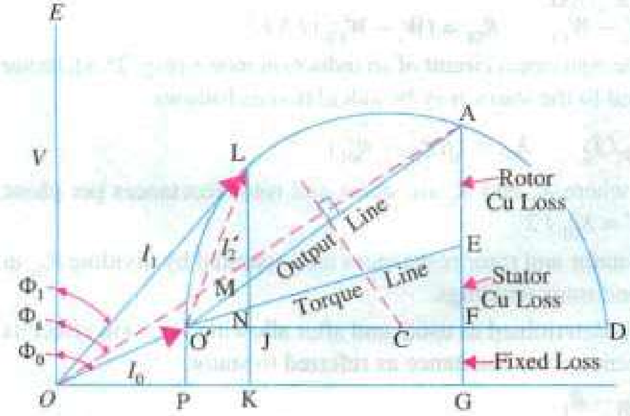


Fig. 35.9

As a matter of practical contingency, it is recommended that the scale of current vectors should be so chosen that the diameter is more than 25 cm, in order that the performance data of the motor may be read with reasonable accuracy from the circle diagram. With centre C and radius = CO', the circle can be drawn.* The line $O'A$ is known as **out-put line.**

It should be noted that as the voltage vector is drawn vertically, all vertical distances represent the active or power or energy components of the currents. For example, the vertical component $O'P$ of no-load current OO' represents the no-load input, which supplies core loss, friction and windage loss and a negligibly small amount of stator I^2R loss. Similarly, the vertical component AG of short-circuit current OA is proportional to the motor input on short-circuit or if measured to a proper scale, may be said to equal power input.

Step No. 3

Torque line. ***This is the line which separates the stator and the rotor copper losses.*** When the rotor is locked, then all the power supplied to the motor goes to meet core losses and Cu losses in the stator and rotor windings. The power input is proportional to AG. Out of this, FG (= $O'P$) represents fixed losses *i.e.* stator core loss and friction and windage losses. AF is proportional to the sum of the stator and rotor Cu losses. The point E is such that

$$\frac{AE}{EF} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

As said earlier, line $O'E$ is known as torque line.

How to locate point E ?

(i) Squirrel-cage Rotor. Stator resistance/phase *i.e.* R_1 is found from stator-resistance test. Now, the short-circuit motor input W_s is approximately equal to motor Cu losses (neglecting iron losses).

Stator Cu loss = $3I_S^2 R_1$ $\quad\therefore$ rotor Cu loss = $W_S - 3\,I_S^2\,R_1$ $\quad\therefore \dfrac{AE}{EF} = \dfrac{W_S - 3I_S^2 R_1}{3\,I_S^2 R_1}$

(ii) Wound Rotor. In this case, rotor and stator resistances per phase r_2 and r_1 can be easily computed. For any values of stator and rotor currents I_1 and I_2 respectively, we can write

$$\frac{AE}{EF} = \frac{I_2^2 r_2}{I_1^2 r_1} = \frac{r_2}{r_1}\left(\frac{I_2}{I_1}\right)^2; \qquad \text{Now, } \frac{I_1}{I_2} = K = \text{transformation ratio}$$

$$\frac{AE}{EF} = \frac{r_2}{r_1}\times\frac{1}{K^2} = \frac{r_2/K^2}{r_1} = \frac{r_2'}{r_1} = \frac{\text{equivalent rotor resistance per phase}}{\text{stator resistance per phase}}$$

Value of K may be found from short-circuit test itself by using two ammeters, both in stator and rotor circuits.

Let us assume that the motor is running and taking a current OL (Fig. 35.9). Then, the perpendicular JK represents fixed losses, JN is stator Cu loss, NL is the rotor input, NM is rotor Cu loss, ML is rotor output and LK is the total motor input.

From our knowledge of the relations between the above-given various quantities, we can write :

$\sqrt{3}\,.\,V_L\,.\,LK$ = motor input $\qquad \sqrt{3}\,.\,V_L\,.\,JK$ = fixed losses

$\sqrt{3}\,.\,V_L\,.\,JN$ = stator copper loss $\qquad \sqrt{3}\,.\,V_L\,.\,MN$ = rotor copper loss

$\sqrt{3}\,.\,V_L\,.\,MK$ = total loss $\qquad \sqrt{3}\,.\,V_L\,.\,ML$ = mechanical output

$\sqrt{3}\,.\,V_L\,.\,NL$ = rotor input $\propto$ torque

1. ML/LK = output/input = efficiency
2. MN/NL = (rotor Cu loss)/(rotor input) = slip, s.
3. $\dfrac{ML}{NL} = \dfrac{\text{rotor output}}{\text{rotor input}} = 1 - s = \dfrac{N}{N_S} = \dfrac{\text{actual speed}}{\text{synchronous speed}}$
4. $\dfrac{LK}{OL}$ = power factor

Hence, it is seen that, at least, theoretically, it is possible to obtain all the characteristics of an induction motor from its circle diagram. As said earlier, for drawing the circle diagram, we need *(a)* stator-resistance test for separating stator and rotor Cu losses and *(b)* the data obtained from *(i)* no-load test and *(ii)* short-circuit test.

35.8. Maximum Quantities

It will now be shown from the circle diagram (Fig. 35.10) that the maximum values occur at the positions stated below :

(i) Maximum Output

It occurs at point M where the tangent is parallel to output line $O'A$. Point M may be located by drawing a line CM from point C such that it is perpendicular to the output line $O'A$. Maximum output is represented by the vertical MP.

(ii) Maximum Torque or Rotor Input

It occurs at point N where the tangent is parallel to torque line $O'E$. Again, point N may be found by drawing CN perpendicular to the torque line. Its value is represented by NQ. Maximum torque is also known as stalling or pull-out torque.

(iii) Maximum Input Power

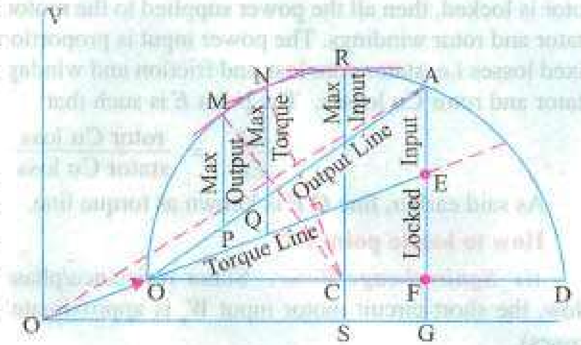


Fig. 35.10

It occurs at the highest point of the circle *i.e.* at point R where the tangent to the circle is horizontal. It is proportional to RS. As the point R is beyond the point of maximum torque, the induction motor will be unstable here. However, the maximum input is a measure of the size of the circle and is an indication of the ability of the motor to carry short-time over-loads. Generally, RS is twice or thrice the motor input at rated load.

Example 35.3. *A 3-ph, 400-V induction motor gave the following test readings;*
No-load : 400 V, 1250 W, 9 A, Short-circuit : 150 V, 4 kW, 38 A
Draw the circle diagram.
If the normal rating is 14.9 kW, find from the circle diagram, the full-load value of current, p.f. and slip. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution.
$$\cos\phi_0 = \frac{1250}{\sqrt{3}\times 400\times 9} = 0.2004\ ; \quad \phi_0 = 78.5°$$

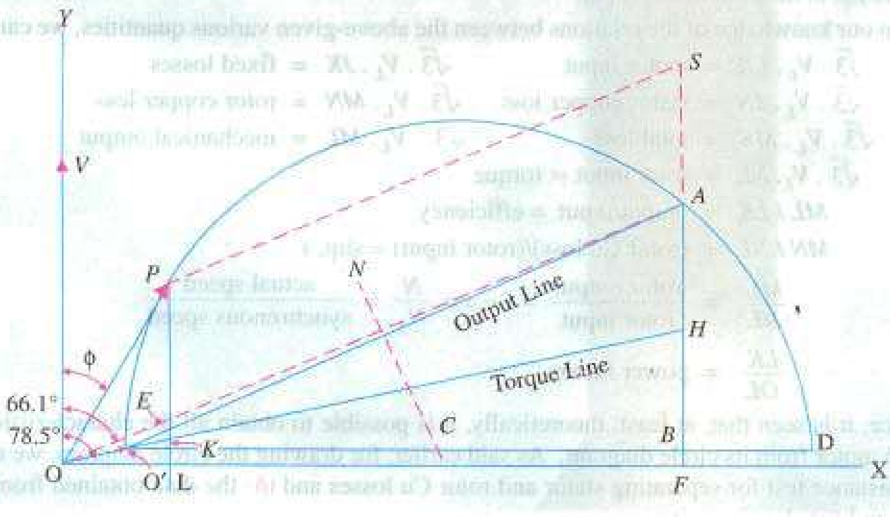


Fig. 35.11

$$\cos\phi_S = \frac{4000}{\sqrt{3}\times 150\times 38} = 0.405\ ; \quad \phi_S = 66.1°$$

Short-circuit current with normal voltage is $I_{SN} = 38\,(400/150) = 101.3$ A. Power taken would be $= 4000\,(400/150)^2 = 28{,}440$ W. In Fig. 33.11, OO' represents I_0 of 9 A. If current scale is 1 cm = 5 A, then vector $OO' = 9/5 = 1.8$ cm* and is drawn at an angle of $\phi_0 = 78.5°$ with the vertical OV (which represents voltage). Similarly, OA represents I_{SN} (S.C. current with normal voltage applied) equal to 101.3 A. It measures $101.3/5 = 20.26$* cm and is drawn at an angle of 66.1°, with the vertical OV.

Line $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$. The semi-circle $O'AD$ is drawn with C as the centre. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit. Now, AF represents 28,440 W and measures 8.1 cm. Hence, power scale becomes : 1 cm = 28,440/8.1 = 3,510 W. Now, full-load motor output $= 14.9 \times 10^3 = 14{,}900$ W. According to the above calculated power scale, the intercept between the semi-circle and output line $O'A$ should measure = 14,900/3510 = 4.25 cm. For locating full-load point P, BA is extended. AS is made equal to 4.25 cm and SP is drawn parallel to output line $O'A$. PL is perpendicular to OX.

$$\text{Line current} = OP = 6 \text{ cm} = 6 \times 5 = \mathbf{30\ A}\,;\ \phi = 30° \text{ (by measurement)}$$

$$\text{p.f.} = \cos 30° = \mathbf{0.886} \text{ (or } \cos\phi = PL/OP = 5.2/6 = 0.865)$$

Now,
$$\text{slip} = \frac{\text{rotor Cu loss}}{\text{rotor input}}$$

In Fig. 35.11, EK represents rotor Cu loss and PK represents rotor input.

$$\therefore \qquad \text{slip} = \frac{EK}{PK} = \frac{0.3}{4.5} = 0.067 \text{ or } \mathbf{6.7\%}$$

Example 35.4. *Draw the circle diagram for a 3.73 kW, 200-V, 50-Hz, 4-pole, 3-φ star-connected induction motor from the following test data :*

No-load : Line voltage 200 V, line current 5 A; total input 350 W

Blocked rotor : Line voltage 100 V, line current 26 A; total input 1700 W

Estimate from the diagram for full-load condition, the line current, power factor and also the maximum torque in terms of the full-load torque. The rotor Cu loss at standstill is half the total Cu loss. **(Electrical Engineering, Bombay Univ. 1987)**

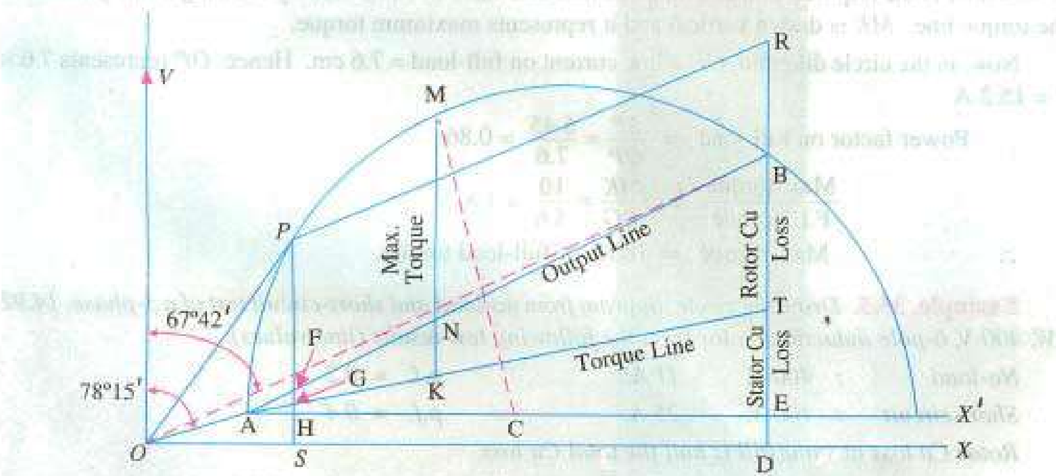


Fig. 35.12

Solution. No-load test

$$I_0 = 5 \text{ A}, \cos\phi_0 = \frac{350}{\sqrt{3} \times 200 \times 5} = 0.202\ ;\ \phi_0 = 78°15'$$

* The actual lengths are different from these values, due to reduction in block making.

Blocked-rotor test :

$$\cos \phi_s = \frac{1700}{\sqrt{3} \times 100 \times 26} = 0.378 \; ; \phi_s = 67°42'$$

Short-circuit current with normal voltage, $I_{SN} = 26 \times 200/100 = 52$ A

Short-circuit/blocked rotor input with normal voltage $= 1700(52/26)^2 = 6{,}800$ W

In the circle diagram of Fig. 35.12, voltage is represented along OV which is drawn perpendicular to OX. Current scale is 1 cm = 2 A

Line OA is drawn at an angle of $\phi_0 = 78°15'$ with OV and 2.5 cm in length. Line AX' is drawn parallel to OX. Line OB represents short-circuit current with normal voltage *i.e.* 52 A and measures 52/2 = 26 cm. AB represents output line. Perpendicular bisector of AB is drawn to locate the centre C of the circle. With C as centre and radius = CA, a circle is drawn which passes through points A and B. From point B, a perpendicular is drawn to the base. BD represents total input of 6,800 W for blocked rotor test. Out of this, ED represents no-load loss of 350 W and BE represents 6,800 − 350 = 6,450 W. Now BD = 9.8 cm and represents 6,800 W

$\therefore$ power scale = 6,800/9.8 = 700 watt/cm or 1 cm = **700 W**

BE which represents total copper loss in rotor and stator, is bisected at point T to separate the two losses. AT represents torque line.

Now, motor output = 3,730 watt. It will be represented by a line = 3,730/700 = 5.33 cm

The output point P on the circle is located thus :

DB is extended and BR is cut = 5.33 cm. Line RP is drawn parallel to output line AB and cuts the circle at point P. Perpendicular PS is drawn and P is joined to origin O.

Point M corresponding to maximum torque is obtained thus :

From centre C, a line CM is drawn such that it is perpendicular to torque line AT. It cuts the circle at M which is the required point. Point M could also have been located by drawing a line parallel to the torque line. MK is drawn vertical and it represents maximum torque.

Now, in the circle diagram, OP = line current on full-load = 7.6 cm. Hence, OP represents 7.6 × 2 = 15.2 A

$$\text{Power factor on full-load} = \frac{SP}{OP} = \frac{6.45}{7.6} = 0.86$$

$$\frac{\text{Max. torque}}{\text{F.L. torque}} = \frac{MK}{PG} = \frac{10}{5.6} = 1.8$$

$\therefore$ Max. torque = 180% of full-load torque.

Example. 35.5. *Draw the circle diagram from no-load and short-circuit test of a 3-phase, 14.92 kW, 400-V, 6-pole induction motor from the following test results (line values).*

No-load	*: 400-V,*	*11 A,*	*p.f. = 0.2*
Short-circuit	*: 100-V,*	*25 A,*	*p.f. = 0.4*

Rotor Cu loss at standstill is half the total Cu loss.

From the diagram, find (a) line current, slip, efficiency and p.f. at full-load (b) the maximum torque. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. No-load p.f. $= 0.2; \phi_0 = \cos^{-1}(0.2) = 78.5°$

Short-circuit p.f. $= 0.4; \phi_s = \cos^{-1}(0.4) = 66.4°$

S.C. current I_{SN} if normal voltage were applied = 25 (400/100) = 100 A

S.C. power input with this current $= \sqrt{3} \times 400 \times 100 \times 0.4 = 27{,}710$ W

Assume a current scale of 1 cm = 5 A.* The circle diagram of Fig. 35.13 is constructed as follows :

(i) No-load current vector OO' represents 11 A. Hence, it measures 11/5 = 2.2 cm and is drawn at an angle of 78.5° with OY.

(ii) Vector OA represents 100 A and measures 100/5 = 20 cm. It is drawn at an angle of 66.4° with OY.

(iii) $O'D$ is drawn parallel to OX. NC is the right angle bisector of $O'A$.

(iv) With C as the centre and CO' as radius, a semicircle is drawn as shown.

(v) AF represents power input on short-circuit with normal voltage applied. It measures 8 cm and (as calculated above) represents 27,710 W. Hence, power scale becomes

$$1 \text{ cm} = 27{,}710/8 = 3{,}465 \text{ W}$$

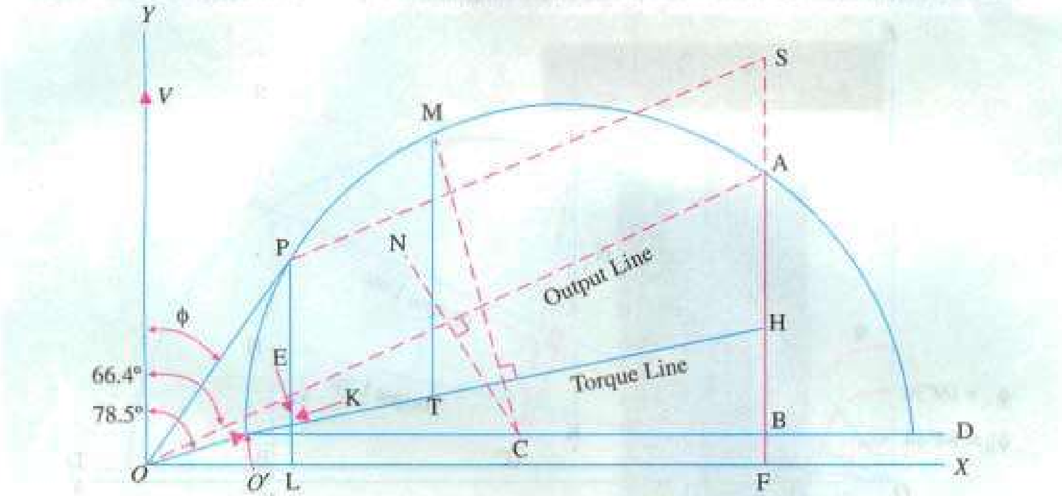


Fig. 35.13

(a) F.L. motor output = 14,920 W. According to the above power scale, the intercept between the semicircle and the output line $O'A$ should measure = 14,920/3,465 = 4.31 cm. Hence, vertical line PL is found which measures 4.31 cm. Point P represents the full-load operating point.**

(a) Line current = OP = 6.5 cm which means that full-load line current = 6.5 × 5 = **32.5 A.** ϕ = 32.9° (by measurement)

∴ cos 32.9° = **0.84** (or cos $\phi = PL/OP = 5.4/6.5 = 0.84$)

$$\text{slip} = \frac{EK}{PK} = \frac{0.3}{5.35} = 0.056 \text{ or } \mathbf{5.6\%}\ ; \quad \eta = \frac{PE}{PL} = \frac{4.3}{5.4} = 0.8 \text{ or } \mathbf{80\%}$$

(b) For finding maximum torque, line CM is drawn ⊥ to torque line $O'H$. MT is the vertical intercept between the semicircle and the torque line and represents the maximum torque of the motor in synchronous watts

Now, MT = 7.8 cm (by measurement) ∴ T_{max} = 7.8 × 3465 = **27,030 synch. watt**

Example 35.6. *A 415-V, 29.84 kW, 50-Hz, delta-connected motor gave the following test data :*

No-load test	*: 415 V,*	*21 A,*	*1,250 W*
Locked rotor test	*: 100 V,*	*45 A,*	*2,730 W*

Construct the circle diagram and determine

---

* The actual scale of the book diagram is different because it has been reduced during block making.

** The operating point may also be found by making AS = 4.31 cm and drawing SP parallel to $O'A$.

(*a*) *the line current and power factor for rated output* (*b*) *the maximum torque. Assume stator and rotor Cu losses equal at standstill.* **(A.C. Machines-I, Jadavpur Univ. 1990)**

Solution. Power factor on no-load is $= \dfrac{1250}{\sqrt{3} \times 415 \times 21} = 0.0918$

$\therefore \qquad \phi_0 = \cos^{-1}(0.0918) = 84°44'$

Power factor with locked rotor is $= \dfrac{2,730}{\sqrt{3} \times 100 \times 45} = 0.3503$

$\therefore \qquad \phi_S = \cos^{-1}(0.3503) = 69°30'$

The input current I_{SN} on short-circuit if normal voltage were applied = 45 (415/100) = 186.75 A and power taken would be = 2,730 $(415/100)^2$ = 47,000 W.

Let the current scale be 1 cm = 10 A. The circle diagram of Fig. 35.14 is constructed as follows :

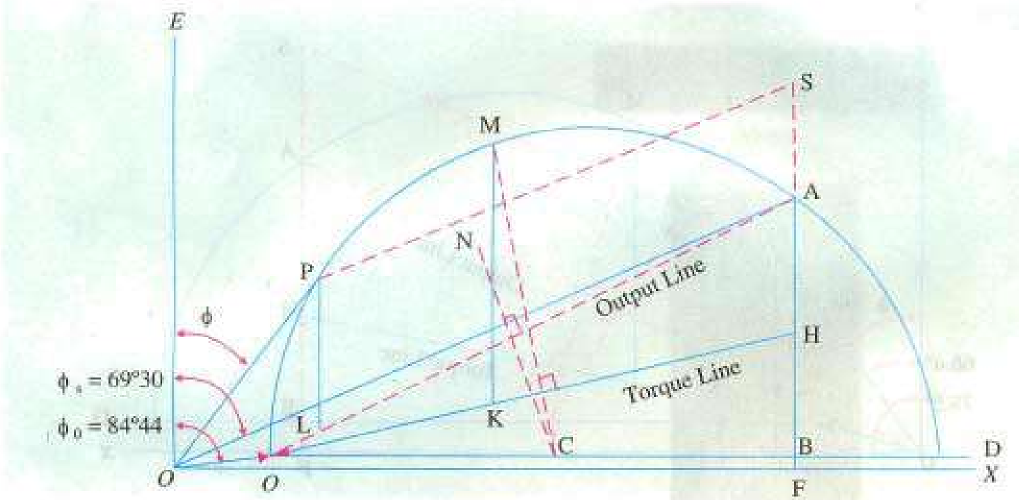


Fig. 35.14

(*i*) Vector OO' represents 21 A so that it measures 2.1 cm and is laid at an angle of 84°44′ with OE (which is vertical *i.e.* along *Y*-axis).

(*ii*) Vector OA measures 186.75/10 = 18.675 cm and is drawn at an angle of 69°30′ with OE.

(*iii*) $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$

(*iv*) With *C* as the centre and CO' as radius, a semi-circle is drawn as shown. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit.

(*v*) The vertical AF represents power input on short-circuit with normal voltage applied. AF measures 6.6 cm and (as calculated above) represents 47,000 W. Hence, power scale becomes, 1 cm = 47,000/6.6 = 7,120 W

(*a*) Full-load output = 29,840 W. According to the above power scale, the intercept between the semicircle and output line $O'A$ should measure 29,840/7,120 = 4.19 cm. Hence, line PL is found which measures 4.19 cm. Point P represents the full-load operating point.*

Phase current $= OP = 6 \text{ cm} = 6 \times 10 = 60$ A; Line current $= \sqrt{3} \times 60$ = **104 A**

Power factor $= \cos \angle POE = \cos 35° =$ **0.819**

(*b*) For finding the maximum torque, line *CM* is drawn ⊥ to the torque line $O'H$. Point *H* is such that

$$\frac{AH}{BH} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

* The operating point may also by found be making AS = 4.19 cm and drawing SP parallel to $O'A$.

Since the two Cu losses are equal, point H is the mid-point of AB.

Line MK represents the maximum torque of the motor in synchronous watts

$$MK = 7.3 \text{ cm (by measurement)} = 7.3 \times 7{,}120 = \mathbf{51{,}980 \text{ synch. watt.}}$$

Example 35.7. *Draw the circle diagram for a 5.6 kW, 400-V, 3-ϕ, 4-pole, 50-Hz, slip-ring induction motor from the following data :*

No-load readings : 400 V, 6 A, cos ϕ_0 = 0.087 : Short-circuit test : 100 V, 12 A, 720 W.

The ratio of primary to secondary turns = 2.62, stator resistance per phase is 0.67 Ω and of the rotor is 0.185 Ω. Calculate

(i) *full-load current* (ii) *full-load slip*

(iii) *full-load power factor*

(iv) $\dfrac{\textit{maximum torque}}{\textit{full-load torque}}$ (v) *maximum power.*

Solution. No-load condition

$$\phi_0 = \cos^{-1}(0.087) = 85°$$

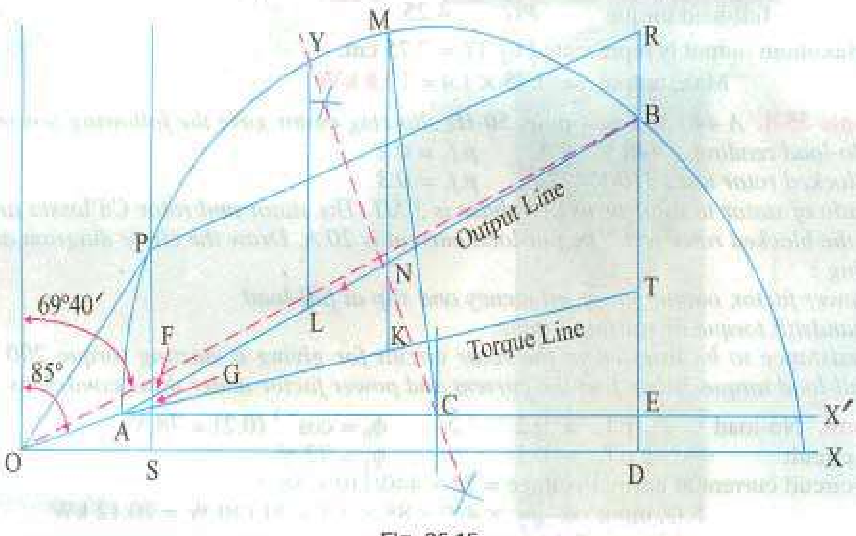


Fig. 35.15

Short-circuit condition

Short-circuit current with normal voltage = 12 × 400/100 = 48 A

$$\text{Total input} = 720 \times (48/12)^2 = 11.52 \text{ kW}$$

$$\cos\phi_s = \frac{720}{\sqrt{3}\times 100 \times 12} = 0.347 \quad \text{or} \quad \phi_s = 69°\,40'$$

Current scale is, 1 cm = 2 A

In the circle diagram of Fig. 35.15, OA = 3 cm and inclined at 85° with OV. Line OB represents short-circuit current with normal voltage. It measures 48/2 = 24 cm and represent 48 A. BD is perpendicular to OX.

For Drawing Torque Line

$K = 2.62$ $R_1 = 0.67\ \Omega$ $R_2 = 0.185\ \Omega$

(in practice, an allowance of 10% is made for skin effect)

$$\therefore \quad \frac{\text{rotor Cu loss}}{\text{stator Cu loss}} = 2.62^2 \times \frac{0.185}{0.67} = 1.9 \qquad \therefore \quad \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{1.9}{2.9} = 0.655$$

Now BD = 8.25 cm and represents 11.52 kW

power scale = 11.52/8.25 = 1.4 kW/cm

$\therefore$ 1 cm = 1.4 kW

BE represents total Cu loss and is divided at point *T* in the ratio 1.9 : 1.

$$BT = BE \times 1.9/2.9 = 0.655 \times 8 = 5.24 \text{ cm}$$

AT is the torque line

Full-load output = 5.6 kW

It is represented by a line = 5.6/1.4 =4 cm

DB is produced to *R* such that *BR* = 4 cm. Line *RP* is parallel to output line and cuts the circle at *P*. OP represents full-load current.

PS is drawn vertically. Points *M* and *Y* represent points of maximum torque and maximum output respectively.

(*i*) F.L. current = $OP = 5.75 \text{ cm} = 5.75 \times 2 =$ **11.5 A**

(*ii*) $$\text{F.L. slip} = \frac{FG}{PG} = \frac{0.2}{4.25} = 0.047 \quad \text{or} \quad \mathbf{4.7\%}$$

(*iii*) $$\text{p.f.} = \frac{SP}{OP} = \frac{4.6}{5.75} = \mathbf{0.8}$$

(*iv*) $$\frac{\text{max. torque}}{\text{full-load torque}} = \frac{MK}{PG} = \frac{10.05}{4.25} = \mathbf{2.37}$$

(*v*) Maximum output is represented by *YL* = 7.75 cm.

$\therefore$ Max. output = $7.75 \times 1.4 =$ **10.8 kW**

Example 35.8. *A 440-V, 3-φ, 4-pole, 50-Hz slip-ring motor gave the following test results :*

No-load reading : 440 V, 9 A, p.f. = 0.2

Blocked rotor test : 110 V, 22 A, p.f. = 0.3

The ratio of stator to rotor turns per phase is 3.5/1. The stator and rotor Cu losses are divided equally in the blocked rotor test. The full-load current is 20 A. Draw the circle diagram and obtain the following :

(*a*) *power factor, output power, efficiency and slip at full-load*

(*b*) *standstill torque or starting torque.*

(*c*) *resistance to be inserted in the rotor circuit for giving a starting torque 200 % of the full-load torque. Also, find the current and power factor under these conditions.*

Solution. No-load p.f. = 0.2 $\therefore$ $\phi_0 = \cos^{-1}(0.2) = 78.5°$

Short-circuit p.f. = 0.3 $\therefore$ $\phi_S = 72.5°$

Short-circuit current at normal voltage = 22 × 440/110 = 88 A

$$\text{S.C. input} = \sqrt{3} \times 440 \times 88 \times 0.3 = 20{,}120 \text{ W} = 20.12 \text{ kW}$$

Take a current scale of 1 cm = 4 A

In the circle diagram of Fig. 35.16, *OA* = 2.25 cm drawn at an angle of 78.5° behind *OV*. Similarly, *OB* = 88/4 = 22 cm and is drawn at an angle of 72.50 behind OV. The semi-circle is drawn as usual. Point *T* is such that *BT* = *TD*. Hence, torque line *AT* can be drawn. *BC* represents 20.12 kW. By measurement *BC* = 6.6 cm.

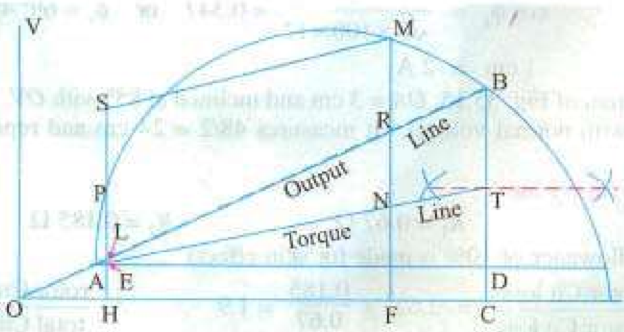


Fig. 35.16

$\therefore$ power scale $= 20.12/6.6 = 3.05$

$\therefore$ 1 cm $= 3.05$ kW

Full-load current = 20 A. Hence, it is represented by a length of 20/4 = 5 cm. With O as centre and 5 cm as radius, an arc is drawn which cuts the semi-circle at point P. This point represents full-load condition. PH is drawn perpendicular to the base OC.

(*a*) (*i*) p.f. $= \cos\phi = PH/OP = 4.05/5 =$ **0.81**

(*ii*) Torque can be found by measuring the input.

$$\text{Rotor input} = PE = 3.5 \text{ cm} = 3.5 \times 3.05 = 10.67 \text{ kW}$$

Now $N_s = 120 \times 50/4 = 1500$ r.p.m.

$\therefore$ $T_g = 9.55\, P_2/N_s = 9.55 \times 10{,}670/1500 =$ **61 N-m**

(*iii*) output $= PL = 3.35 \times 3.05 =$ **10.21 kW**

(*iv*) $$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{PL}{PH} = \frac{3.35}{4.05} = 0.83 \quad \text{or} \quad \mathbf{83\%}$$

(*v*) $$\text{slip } s = \frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{LE}{PE} = \frac{0.1025}{3.5} = 0.03 \quad \text{or} \quad \mathbf{3\%}$$

(*b*) Standstill torque is represented by BT.

$$BT = 3.1 \text{ cm} = 3.1 \times 3.05 = 9.45 \text{ kW} \qquad \therefore \quad T_{st} = 9.55 \times \frac{9.45 \times 10^3}{1500} = \mathbf{60.25 \text{ N-m}}$$

(*c*) We will now locate point M on the semi-circle which corresponds to a starting torque twice the full-load torque *i.e.* 200% of F.L. torque.

Full-load torque = PE. Produce EP to point S such that $PS = PE$. From point S draw a line parallel to torque line AT cutting the semi-circle at M. Draw MN perpendicular to the base.

At starting when rotor is stationary, MN represents total rotor copper losses.

NR = Cu loss in rotor itself as before ; RM = Cu loss in external resistance

$RM = 4.5 \text{ cm} = 4.5 \times 3.05 = 13.716 \text{ kW} = 13{,}716$ watt.

Cu loss/phase = 13,716/3 =4,572 watt

Rotor current $AM = 17.5 \text{ cm} = 17.5 \times 4 = 70$ A

Let r_2' be the additional external resistance in the rotor circuit (as referred to stator) then

$$r_2' \times 70^2 = 4{,}572 \quad \text{or} \quad r_2' = 4{,}572/4{,}900 = 0.93\ \Omega$$

Now $K = 1/3.5$

$\therefore$ rotor resistance/phase, $r_2 = r_2' \times K^2 = 0.93/3.5^2 =$ **0.076 Ω**

Stator current $= OM = 19.6 \times 4 =$ **78.4 A** ; power factor $= \dfrac{MF}{OM} = \dfrac{9.75}{18.7} =$ **0.498**

Example 35.9. *Draw the circle diagram of a 7.46 kW, 200-V, 50-Hz, 3-phase slip-ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of 0.38 ohm/phase and a rotor resistance of 0.24 ohm/phase. The following are the test readings ;*

No-load : 200 V, 7.7 A, $\cos\phi_0 = 0.195$

Short-circuit : 100 V, 47.6 A, $\cos\phi_s = 0.454$

Find (a) starting torque and (b) maximum torque, both in synchronous watts

(c) the maximum power factor (d) the slip for maximum torque

(e) the maximum output **(Elect. Tech.-II, Madras Univ. 1989)**

Solution. $\phi_0 = \cos^{-1}(0.195) = 78°45'$; $\phi_S = \cos^{-1}(0.454) = 63°$

The short-circuit I_{SN} with normal voltage applied is $= 47.6 \times (200/100) = 95.2$ A

The circle diagram is drawn as usual and is shown in fig. 35.17.

With a current scale of 1 cm = 5 A, vector OO' measures 7.7/5 =1.54 cm and represents the no-load current of 7.7 A.

Similarly, vector OA represents I_{SN} *i.e.* short-circuit current with normal voltage and measures 95.2/5 = 19.04 cm

Both vectors are drawn at their respective angles with OE.

The vertical line AF measures the power input on short-circuit with normal voltage and is $= \sqrt{3} \times 200 \times 95.2 \times 0.454 = 14{,}970$ W.

Since AF measures 8.6 cm, the power scale is 1 cm = 14,970/8.6 = 1740 W

The point H is such that

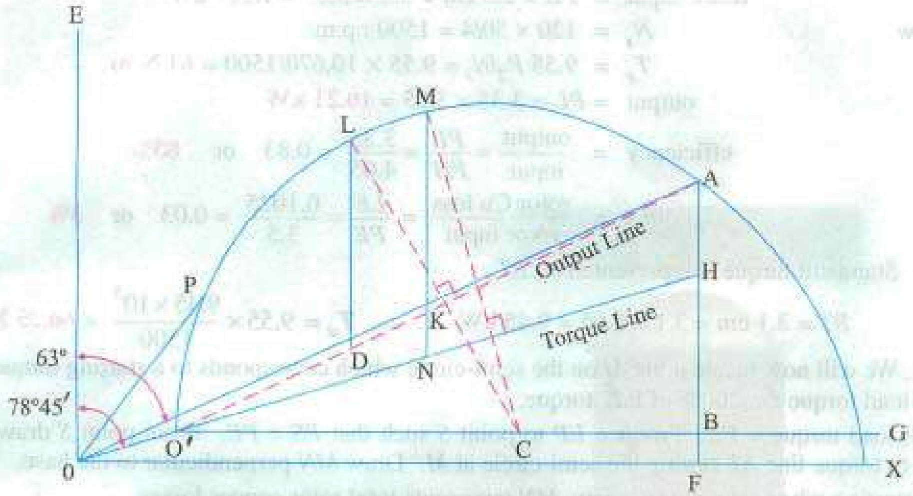


Fig. 35.17

$$\frac{AH}{AB} = \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{\text{rotor resistance}^*}{\text{rotor + stator resistance}} = \frac{0.24}{0.62}$$

Now AB = 8.2 cm (by measurement) $\therefore AH = 8.2 \times 0.24/0.62 = 3.2$ cm

(*a*) Starting torque = AH = 3.2 cm = 3.2 × 1740 = **5,570 synch. watt.**

(*b*) Line CM is drawn perpendicular to the torque line $O'H$. The intercept MN represents the maximum torque in synchronous watts.

Maximum torque = MN = 7.15 cm = 7.15 × 1740 = **12,440 synch. watts.**

(*c*) For finding the maximum power, line OP is drawn tangential to the semi-circle.

$\angle POE$ = 28.5°

$\therefore$ maximum p.f. = cos 28.5° = **0.879**

(*d*) The slip for maximum torque is = KN/MN= 1.4/7.15 = **0.195**

(*e*) Line CL is drawn perpendicular to the output line $O'A$. From L is drawn the vertical line LD. It measures 5.9 cm and represents the maximum output.

$\therefore$ maximum output = 5.9 × 1740 = **10,270 W**

Tutorial Problems 35.1

1. A 300 h.p. (223.8 kW), 3000-V, 3-ϕ, induction motor has a magnetising current of 20 A at 0.10 p.f. and a short-circuit (or locked) current of 240 A at 0.25 p.f. Draw the circuit diagram, determine the p.f. at full-load and the maximum horse-power. **[0.85 p.f. 621 h.p. (463.27 kW)]** (*I.E.E. London*)
2. The following are test results for a 18.65 kW, 3-ϕ, 440.V slip-ring induction motor :

   Light load : 440-V, 7.5 A, 1350 W (including 650 W friction loss).

* Because $K = 1$, otherwise it should be $R_2' = R_2/K^2$.

S.C. test : 100 V, 32 A, 1800 W

Draw the locus diagram of the stator current and hence obtain the current, p.f. and slip on full-load. On short-circuit, the rotor and stator copper losses are equal. **[30 A, 0.915, 0.035] (*London Univ*)**

3. Draw the circle diagram for 20 h.p. (14.92 kW), 440-V, 50-Hz, 3-ϕ induction motor from the following test figures (line values) :

No-load : 440 V, 10A, p.f. 0.2 Short-circuit : 200 V, 50 A, p.f. 0.4

From the diagram, estimate (*a*) the line current and p.f. at full-load (*b*) the maximum power developed (*c*) the starting torque. Assume the rotor and stator I^2R losses on short-circuit to be equal.

[(*a*) 28.1 A at 0.844 p.f. (*b*) 27.75 kW (*c*) 11.6 synchronous kW/phase] (*London Univ.*)

4. A 40 h.p. (29.84.kW), 440-V, 50-Hz, 3-phase induction motor gave the following test results

No. load : 440 V, 16 A, p.f. = 0.15 S.C. test : 100 V, 55 A, p.f. = 0.225

Ratio of rotor to stator losses on short-circuit = 0.9. Find the full-load current and p.f., the pull-out torque and the maximum output power developed.

[49 A at 0.88 p.f. ; 78.5 synch. kW or 2.575 times F.L. torque ; 701.2 kW] (*I.E.E. London*)

5. A 40 h.p. (29.84 kW), 50-Hz, 6-pole, 420-V, 3-ϕ, slip-ring induction motor furnished the following test figures :

No-load : 420 V, 18 A, p.f. = 0.15 S.C. test : 210 V, 140 A, p.f. = 0.25

The ratio of stator to rotor Cu losses on short-circuit was 7 : 6. Draw the circle diagram and find from it (*a*) the full-load current and power factor (*b*) the maximum torque and power developed.

[(*a*) 70 A at 0.885 p.f. (*b*) 89.7 kg.m ; 76.09 kW] (*I.E.E. London*)

6. A 500 h.p. (373 kW), 8-pole, 3-ϕ, 6,000-V, 50-Hz induction motor gives on test the following figures :

Running light at 6000 V, 14 A/phase, 20,000 W ; Short-circuit at 2000 V, 70 A/phase, 30,500 W

The resistance/phase of the star-connected stator winding is 1.1 Ω, ratio of transformation is 4 :1. Draw the circle diagram of this motor and calculate how much resistance must be connected in each phase of the rotor to make it yield full-load torque at starting. **[0.138 Ω] (*London Univ.*)**

7. A 3-phase induction motor has full-load output of 18.65 kW at 220 V, 720 r.p.m. The full-load p.f. is 0.83 and efficiency is 85%. When running light, the motor takes 5 A at 0.2 p.f. Draw the circle diagram and use it to determine the maximum torque which the motor can exert (*a*) in N-m (*b*) in terms of full-load torque and (*c*) in terms of the starting torque.

[(*a*) 268.7 N-m (*b*) 1.08 (*c*) 7.2 approx.] (*London Univ.*)

8. A 415-V, 40 h.p. (29.84 kW), 50.Hz, Δ-connected motor gave the following test data :

No-load test : 415 V, 21 A, 1250 W ; Locked rotor test : 100 V, 45 A, 2,730 W

Construct the circle diagram and determine

(*a*)the line current and power factor for rated output (*b*) the maximum torque. Assume stator and rotor Cu losses equal at standstill.

[(*a*) 104 A : 0.819 (*b*) 51,980 synch watt] (*A.C. Machines-I, Jadavpur Univ. 1978*)

9. Draw the no-load and short circuit diagram for a 14.92 kW., 400-V, 50-Hz, 3-phase star-connected induction motor from the following data (line values) :

No load test : 400 V, 9 A, cos ϕ = 0.2

Short circuit test : 200 V, 50 A, cos ϕ = 0.4

From the diagram find (*a*) the line current and power factor at full load, and (*b*) the maximum output power. **[(*a*) 32.0 A, 0.85 (*b*) 21.634 kW]**

35.9. Starting of Induction Motors

It has been shown earlier that a plain induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary. Therefore, if normal supply voltage is applied to the stationary motor, then, as in the case of a transformer, a very large initial current is taken by the primary, at least, for a short while. It would be remembered that exactly similar conditions exist in the case of a d.c. motor, if it is thrown directly across the supply lines, because at the time of starting it, there is no back e.m.f. to oppose the initial inrush of current.

Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque. This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines. Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

It was seen in Art. 34.15 that the starting torque of an induction motor can be improved by increasing the resistance of the rotor circuit. This is easily feasible in the case of slip-ring motors but not in the case of squirrel-cage motors. However, in their case, the initial in-rush of current is controlled by applying a reduced voltage to the stator during the starting period, full normal voltage being applied when the motor has run up to speed.

35.10. Direct-switching or Line starting of Induction Motors

It has been shown earlier that

Rotor input $= 2\pi N_s T = kT$ —Art. 34.36

Also, rotor Cu loss $= s \times$ rotor input

$\therefore \quad 3 I_2^2 R_2 = s \times kT \qquad \therefore \quad T \propto I_2^2/s$ (if R_2 is the same)

Now $\quad I_2 \propto I_1 \qquad \therefore \quad T \propto I_1^2/s \quad$ or $\quad T = K I_1^2/s$

At starting moment $\quad s = 1 \qquad \therefore \quad T_{st} = K I_{st}^2$ where I_{st} = starting current

If $\quad I_f$ = normal full-load current and $\quad s_f$ = full-load slip

then $T_f = K I_f^2 / s_f \qquad \therefore \quad \dfrac{T_{st}}{T_f} = \left(\dfrac{I_{st}}{I_f}\right)^2 . s_f$

When motor is direct-switched onto normal voltage, then starting current is the short-circuit current I_{sc}.

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 . s_f = a^2 . s_f \quad \text{where } a = I_{sc} / I_f$$

Suppose in a case, $\quad I_{sc} = 7 I_f,\ s_f = 4\% = 0.04$, the $T_{st} / T_f = 7^2 \times 0.04 = 1.96$

$\therefore \quad$ starting torque $= 1.96 \times$ full-load torque

Hence, we find that with a current as great as seven times the full-load current, the motor develops a starting torque which is only 1.96 times the full-load value.

Some of the methods for starting induction motors are discussed below :

Squirrel-cage Motors

(*a*) Primary resistors (or rheostat) or reactors
(*b*) Auto-transformer (or autostarter)
(*c*) Star-delta switches

In all these methods, terminal voltage of the squirrel-cage motor is reduced during starting.

Slip-ring Motors

(*a*) Rotor rheostat

35.11. Squirrel-cage Motors

(*a*) Primary resistors

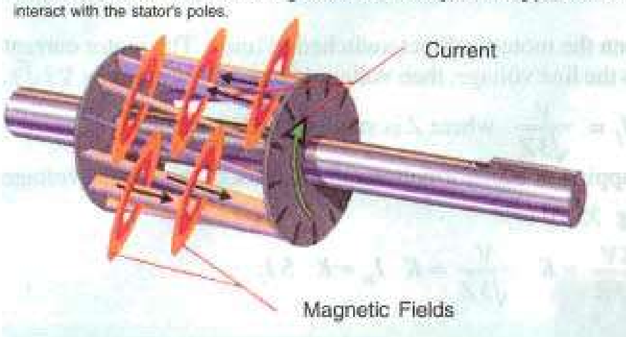
Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way, the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as the voltage, the torque varies as square of applied voltage*

* When applied voltage is reduced, the rotating flux Φ is reduced which, in turn, decreases rotor e.m.f. and hence rotor current I_2. Starting torque, which depends both on Φ and I_2 suffers on two counts when impressed voltage is reduced.

Squirrel Cage Rotor

When the stator's moving magnetic field cuts across the rotor's conductor bars, it induces voltage in them.

This voltage produces current, which circulates through the bars and around the rotor end ring. This current in turn produces magnetic fields around each rotor bar. The continuously changing stator magnetic field results in a continuously changing rotor field. The rotor becomes an electromagnet with continuously alternating poles, which interact with the stator's poles.



Squirrel cage rotor

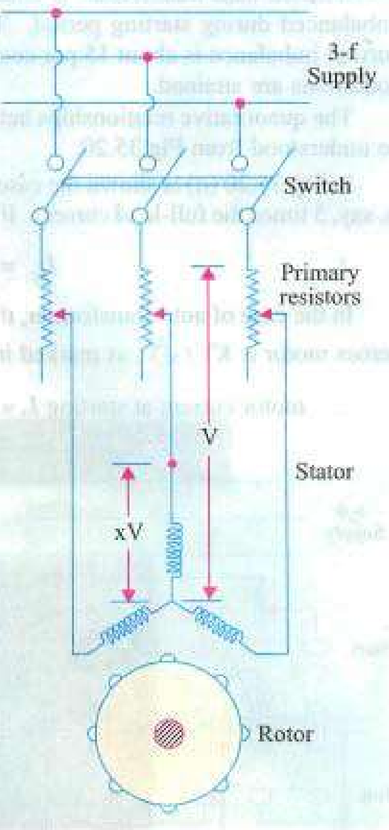


Fig. 35.18

(Art 34.17). If the voltage applied across the motor terminals is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

By using primary resistors (Fig. 35.18), the applied voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x I_{sc} \quad \text{and} \quad T_{st} = x^2 T_{sc}$$

As seen from Art 35.10, above,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . s_f = \left(\frac{x I_{sc}}{I_f}\right)^2 s_f$$

$$= x^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = x^2 . a^2 . s_f$$

It is obvious that the ratio of the starting torque to full-load torque is x^2 of that obtained with direct switching or across-the-line starting. This method is useful for the smooth starting of small machines only.

(*b*) Auto-transformers

Variable auto-transformer

Such starters, known variously as ***auto-starters or compensators***, consist of an auto-transformer, with necessary switches. We may use either two auto-transformers connected as usual [Fig. 35.19 (*b*)] or 3 auto-transformers connected in open delta [Fig. 35.19 (*a*)]. ***This method can be used both for star-and delta-connected motors***. As shown in Fig. 35.20 with starting connections, a reduced voltage is applied across the motor terminals. When the motor has ran up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor. The switch making these changes from 'start' to 'run' may be airbreak (for small motors) or may be oil-immersed (for large motors) to reduce sparking. There is also provision for no-voltage and over-load protection, along with a time-delay device, so that momentary interruption of voltage or momentary over-load do not disconnect the motor from supply line. Most of the auto-starters are provided with 3 sets of taps, so as to reduce voltage to 80, 65 or 50 per cent of the line voltage, to suit the local conditions of supply. The

V-connected auto-transformer is commonly used, because it is cheaper, although the currents are unbalanced during starting period. This is, however, not much objectionable firstly, because the current imbalance is about 15 per cent and secondly, because balance is restored as soon as running conditions are attained.

The quantitative relationships between the motor current, line current, and torque developed can be understood from Fig.35.20.

In Fig 35.20 (*a*) is shown the case when the motor is direct-switched to lines. The motor current is, say, 5 times the full-load current. If *V* is the line voltage, then voltage/phase across motor is $V/\sqrt{3}$.

$$\therefore \qquad I_{sc} = 5\,I_f = \frac{V}{\sqrt{3}Z} \quad \text{where } Z \text{ is stator impedance/phase.}$$

In the case of auto-transformer, if a tapping of transformation ratio *K* is used, then phase voltage across motor is $KV/\sqrt{3}$, as marked in Fig. 35.20 (*b*).

$$\therefore \quad \text{motor current at starting } I_2 = \frac{KV}{\sqrt{3}Z} = K\,.\,\frac{V}{\sqrt{3}Z} = K\,.\,I_{sc} = K\,.\,5\,I_f$$

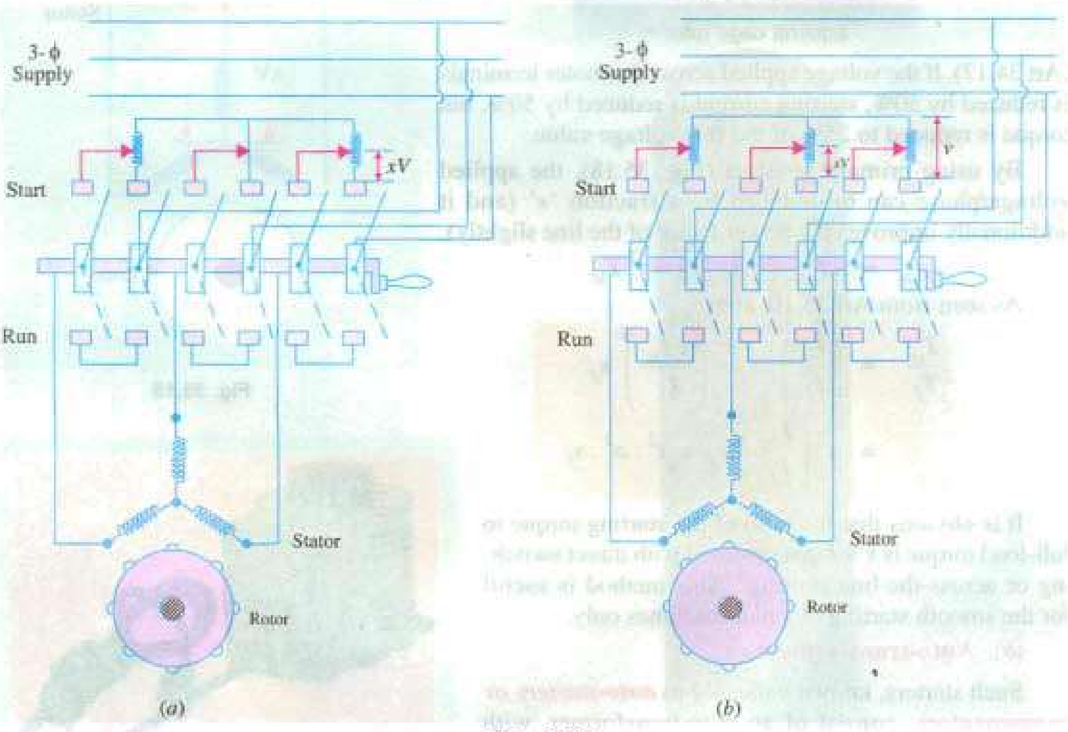


Fig. 35.19

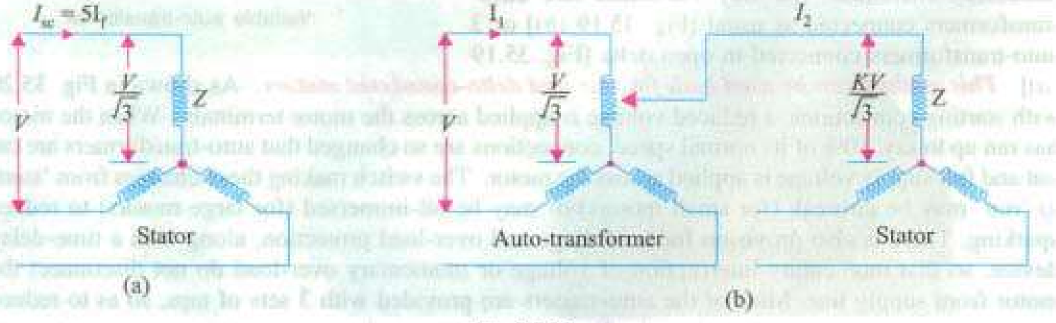


Fig. 35.20

The current taken from supply or by auto-transformer is $I_1 = K I_2 = K^2 \times 5 I_f = K^2 I_{sc}$ if magnetising current of the transformer is ignored. Hence, we find that although motor current per phase is reduced only K times the direct-switching current ($\because K < 1$), the current taken by the line is reduced K^2 times.

Now, remembering that torque is proportional to the square of the voltage, we get

With direct-switching, $T_1 \propto (V/\sqrt{3})^2$; With auto-transformer, $T_2 \propto (KV/\sqrt{3})^2$

$$\therefore \quad T_2/T_1 = (KV/\sqrt{3})^2/(V/\sqrt{3})^2 \quad \text{or} \quad T_2 = K^2 T_1 \quad \text{or} \quad T_{st} = K^2 . T_{sc}$$

$\therefore$ torque with auto-starter $= K^2 \times$ torque with direct-switching.

Relation Between Starting and F.L. Torque

It is seen that voltage across motor phase on direct-switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. With auto-starter, voltage across motor phase is $KV/\sqrt{3}$ and $I_{st} = K I_{sc}$

$$\text{Now,} \quad T_{st} \propto I_{st}^2 \, (s = 1) \quad \text{and} \quad T_f \propto \frac{I_f^2}{s_f^2}$$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f \quad \text{or} \quad \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = K^2 . a^2 . s_f \qquad (\because I_{st} = K I_{sc})$$

Note that this expression is similar to the one derived in Art. 34.11. (*a*) except that x has been replaced by transformation ratio K.

Example 35.10. *Find the percentage tapping required on an auto-transformer required for a squirrel-cage motor to start the motor against 1/4 of full-load torque. The short-circuit current on normal voltage is 4 times the full-load current and the full-load slip is 3%.*

Solution. $\dfrac{T_{st}}{T_f} = \dfrac{1}{4}, \quad \dfrac{I_{sc}}{I_f} = 4, \quad s_f = 0.03$

$$\therefore \text{ Using} \quad \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f \text{, we get } \frac{1}{4} = K^2 \times 4^2 \times 0.03$$

$$\therefore \quad K^2 = \frac{1}{64 \times 0.03} \qquad \therefore \; K = 0.722 \quad \text{or} \quad K = 72.2\%$$

Example 35.11. *A 20 h.p. (14.92 kW), 400-V, 950 r.p.m., 3-φ, 50-Hz, 6-pole cage motor with 400 V applied takes 6 times full-load current at standstill and develops 1.8 times full-load running torque. The full-load current is 30 A.*

(*a*) *what voltage must be applied to produce full-load torque at starting ?*
(*b*) *what current will this voltage produce ?*
(*c*) *if the voltage is obtained by an auto-transformer, what will be the line current ?*
(*d*) *if starting current is limited to full-load current by an auto-transformer, what will be the starting torque as a percentage of full-load torque ?*

Ignore the magnetising current and stator impedance drops.

Solution. (*a*) Remembering that $T \propto V^2$, we have

In the first case, $1.8\, T_f \propto 400^2$, ; In the second case, $T_f \propto V^2$

$$\therefore \quad \left(\frac{V}{400}\right)^2 = \frac{1}{1.8} \quad \text{or} \quad V = \frac{400}{\sqrt{1.8}} = \mathbf{298.1\ V}$$

(*b*) Currents are proportional to the applied voltage.

$$\therefore \quad 6 I_f \propto 400 \, ; \, I \propto 298.1 \quad \therefore \quad I = 6 \times \frac{298.1}{400}, I_f = \frac{6 \times 298.1 \times 30}{400} = \mathbf{134.2\ A}$$

(*c*) Here $K = 298.1/400$

Line current $= K^2 I_{sc} = (298.1/400)^2 \times 6 \times 30 =$ **100 A**

(*d*) We have seen in Art. 33.11 (*b*) that line current $= K^2 I_{sc}$

Now, line current $=$ full-load current I_f (given)

$\therefore \quad 30 = K^2 \times 6 \times 30 \quad \therefore \quad K^2 = 1/6$

Now, using $\dfrac{T_{st}}{T_f} = K^2\left(\dfrac{I_{sc}}{I_f}\right)^2 \times s_f$ we get $\dfrac{T_{st}}{T_f} = \dfrac{1}{6}\times\left(\dfrac{6 I_f}{I_f}\right)^2 \times 0.05 = 0.3$

Here $N_s = 120 \times 50/6 = 1000$ r.p.m. $N = 950$ r.p.m.; $s_f = 50/1000 = 0.05$

$\therefore \quad T_{st} = 0.3\, T_f$ or **30%** F.L. torque

Example 35.12. *Determine the suitable auto-transformation ratio for starting a 3-phase induction motor with line current not exceeding three times the full-load current. The short-circuit current is 5 times the full-load current and full-load slip is 5%.*

Estimate also the starting torque in terms of the full-load torque.

(Elect. Engg.II, Bombay Univ. 1987)

Solution. Supply line current $= K^2 I_{sc}$

It is given that supply line current at start equals $3 I_f$ and short-circuit current $I_{sc} = 5 I_f$ where I_f is the full-load current

$\therefore \quad 3 I_f = K^2 \times 5 I_f$ or $K^2 = 0.6 \quad \therefore \quad K = 0.775$ or **77.5%**

In the case of an auto starter,

$$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 \times s_f \quad \therefore \quad \frac{T_{st}}{T_f} = 0.6 \times \left(\frac{5 I_f}{I_f}\right)^2 \times 0.05 = 0.75$$

$\therefore \quad T_{st} = 0.75\, T_f =$ **75%** of full-load torque.

Example 35.13. *The full-load slip of a 400-V, 3-phase cage induction motor is 3.5% and with locked rotor, full-load current is circulated when 92 volt is applied between lines. Find necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque.*

(Elect. Machines, Banglore Univ. 1991)

Solution. Short-circuit current with full normal voltage applied is

$$I_{sc} = (400/92)\, I_f = (100/23)\, I_f$$

Supply *line* current $= I_{st} = 2 I_f$

Now, line current $I_{st} = K^2 I_{sc}$

$\therefore \quad 2 I_f = K^2 \times (100/23)\, I_f \qquad \therefore K^2 = 0.46$; $K = 0.678$ or **67.8%**

Also, $\dfrac{T_{st}}{T_f} = K^2\left(\dfrac{I_{sc}}{I_f}\right)^2 \times s_f = 0.46 \times (100/23)^2 \times 0.035 = 0.304$

$\therefore \quad T_{st} =$ **30.4%** of full-load torque

Tutorial Problems 35.2

1. A 3-ϕ motor is designed to run at 5% slip on full-load. If motor draws 6 times the full-load current at starting at the rated voltage, estimate the ratio of starting torque to the full-load torque.

[1.8] *(Electrical Engineering Grad, I.E.T.E. Dec. 1986)*

2. A squirrel-cage induction motor has a short-circuit current of 4 times the full-load value and has a full-load slip of 5%. Determine a suitable auto-transformer ratio if the supply line current is not to

exceed twice the full-load current. Also, express the starting torque in terms of the full-load torque. Neglect magnetising current. **[70.7%, 0.4]**

3. A 3-ϕ, 400-V, 50-Hz induction motor takes 4 times the full-load current and develops twice the full-load torque when direct-switched to 400-V supply. Calculate in terms of full-load values (*a*) the line current, the motor current and starting torque when started by an auto-starter with 50% tap and (*b*) the voltage that has to be applied and the motor current, if it is desired to obtain full-load torque on starting. **[(*a*) 100%, 200%, 50% (*b*) 228 V, 282%]**

(*c*) Star-delta Starter

This method is used in the case of motors which are built to run normally with a delta-connected stator winding. It consists of a two-way switch which connects the motor in star for starting and then in delta for normal running. The usual connections are shown in Fig. 35.21. When star-connected, the applied voltage over each motor phase is reduced by a factor of $1/\sqrt{3}$ and hence the torque developed becomes 1/3 of that which would have been developed if motor were directly connected in delta. The line current is reduced to 1/3. Hence, during starting period when motor is *Y*-connected, it takes 1/3rd as much starting current and develops 1/3rd as much torque as would have been developed were it directly connected in delta.

Relation Between Starting and F.L. Torque

$$I_{st} \text{ per phase } = \frac{1}{\sqrt{3}} I_{sc} \text{ per phase}$$

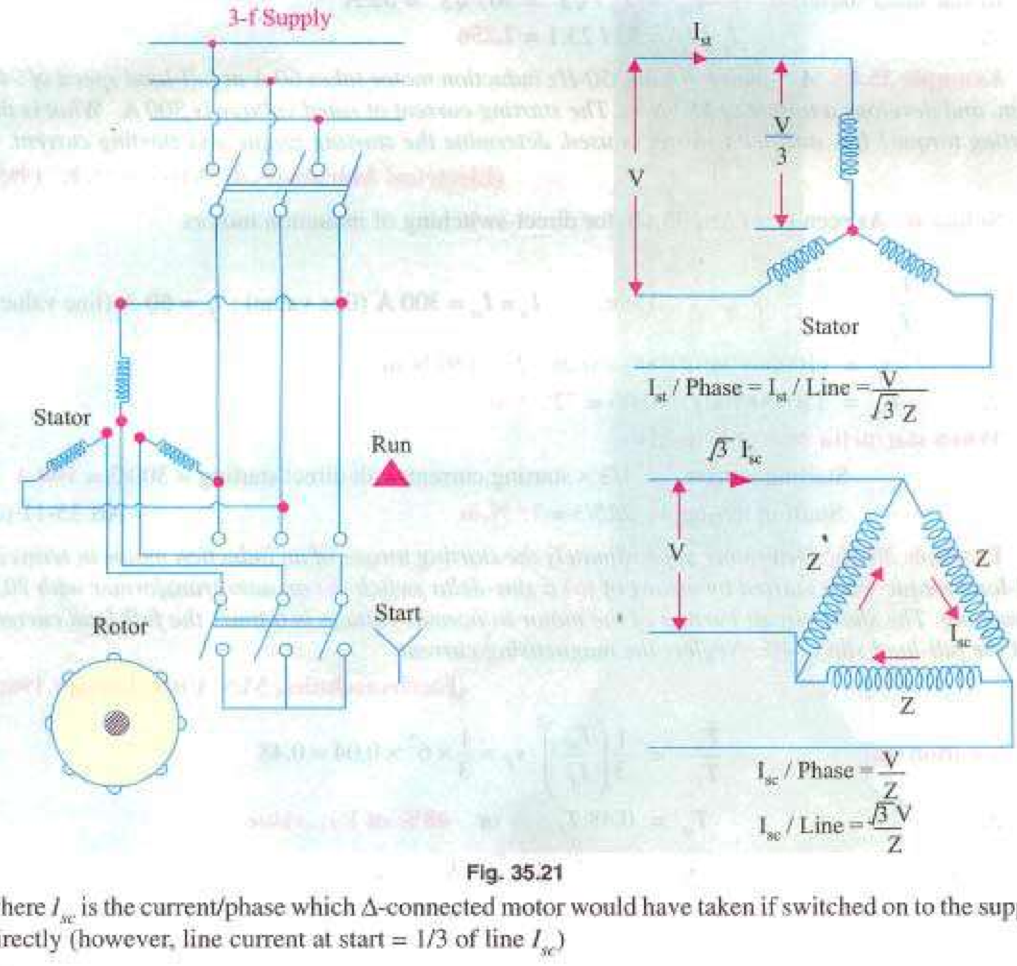


Fig. 35.21

where I_{sc} is the current/phase which Δ-connected motor would have taken if switched on to the supply directly (however, line current at start = 1/3 of line I_{sc})

Now $\qquad T_{st} \propto I_{st}^2 \qquad (s = 1) \qquad$ – Art. 35.10

$$T_f \propto I_f^2 / s_f$$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{I_{sc}}{\sqrt{3} I_f}\right)^2 s_f = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} a^2 s_f$$

Here, I_{st} and I_{sc} represent phase values.

It is clear that the star-delta swith is equivalent* to an auto-transformer of ratio $1/\sqrt{3}$ or 58% approximately.

This method is cheap and effective provided the starting torque is required not to be more than 1.5 times the full-load torque. Hence, it is used for machine tools, pumps and motor-generators etc.

Example 35.14. *The full-load efficiency and power factor of a 12-kW, 440-V, 3-phase induction motor are 85% and 0.8 lag respectively. The blocked rotor line current is 45 A at 220 V. Calculate the ratio of starting to full-load current, if the motor is provided with a star-delta starter. Neglect magnetising current.* **(Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. Blocked rotor current with full voltage applied

$$I_{sc} = 45 \times 440/220 = 90 \text{ A}$$

Now, $\quad \sqrt{3} \times 440 \times I_f \times 0.8 = 12{,}000/0.85, \quad \therefore \quad I_f = 23.1 \text{ A}$

In star-delta starter, $\quad I_{st} = I_{sc}/\sqrt{3} = 90/\sqrt{3} = 52 \text{ A}$

$\therefore \quad I_{st}/I_f = 52/23.1 = \mathbf{2.256}$

Example 35.15. *A 3-phase, 6-pole, 50-Hz induction motor takes 60 A at full-load speed of 940 r.p.m. and develops a torque of 150 N-m. The starting current at rated voltage is 300 A. What is the starting torque? If a star/delta starter is used, determine the starting torque and starting current.*

(Electrical Machinery-II, Mysore Univ. 1988)

Solution. As seen from Art. 33.10, for direct-switching of induction motors

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f$$

Here, $I_{st} = I_{sc} = 300$ A (line value) ; $I_f = 60$ A (line value).

$$s_f = (1000 - 940)/1000 = 0.06 \; ; \; T_f = 150 \text{ N-m}$$

$$\therefore \quad T_{st} = 150(300/60)^2 \times 0.06 = \mathbf{225 \text{ N-m}}$$

When star/delta starter is used

Starting current = 1/3 × starting current with direct starting = 300/3 = **100 A**

Starting torque = 225/3 = **75 N-m** $\qquad$ – Art 35-11 (*c*)

Example 35.16. *Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (a) a star-delta switch (b) an auto-transformer with 70.7 % tapping. The short-circuit current of the motor at normal voltage is 6 times the full-load current and the full-load slip is 4%. Neglect the magnetising current.*

(Electrotechnics, M.S. Univ. Baroda 1986)

Solution. (*a*)

$$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} \times 6^2 \times 0.04 = 0.48$$

$\therefore \quad T_{st} = 0.48\, T_f \quad$ or $\quad$ **48% of F.L. value**

(*b*) Here $\quad K = 0.707 = 1/\sqrt{2}\; ; \; K^2 = 1/2$

* By comparing it with the expression given in Art. 35.11 (*b*)

Now, $$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{2}\times 6^2 \times 0.04 = 0.72$$

$\therefore$ $T_{st} = 0.72\,T_f$ or **72% of T_f**

Example 35.17. *A 15 h.p. (11.2 kW), 3-φ, 6-pole, 50-HZ, 400-V, Δ-connected induction motor runs at 960 r.p.m. on full-load. If it takes 86.4 A on direct starting, find the ratio of starting torque to full-load torque with a star-delta starter. Full-load efficiency and power factor are 88% and 0.85 respectively.*

Solution. Here, I_{sc}/phase $= 86.4/\sqrt{3}$ A

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}}\cdot I_{sc} \text{ per phase} = \frac{86.4}{\sqrt{3}\times\sqrt{3}} = 28.8 \text{ A}$$

Full-load input line current may be found from

$$\sqrt{3}\times 400\times I_L\times 0.85 = 11.2\times 10^3/0.88 \qquad \therefore \text{ Full-load } I_L = 21.59 \text{ A}$$

$$\text{F.L. } I_{ph} = 21.59/\sqrt{3} \text{ A}; \qquad I_f = 21.59/\sqrt{3} \text{ A per phase}$$

$$N_s = 120\times 50/6 = 1000 \text{ r.p.m.}, \qquad N = 950;\ s_f = 0.05$$

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{28.8\times\sqrt{3}}{21.59}\right)^2\times 0.05 \quad \therefore \quad T_{st} = 0.267\,T_f$$ or **26.7% F.L. torque**

Example 35.18. *Find the ratio of starting to full-load current in a 10 kW (output), 400-V, 3-phase induction motor with star/delta starter, given that full-load p.f. is 0.85, the full-load efficiency is 0.88 and the blocked rotor current at 200 V is 40 A. Ignore magnetising current.*

(Electrical Engineering, Madras Univ. 1985)

Solution. F.L. line current drawn by the Δ-connected motor may be found from

$$\sqrt{3}\times 400\times I_L\times 0.85 = 10\times 1000/0.88 \qquad \therefore \quad I_L = 19.3 \text{ A}$$

Now, with 200 V, the line value of S.C. current of the Δ-connected motor is 40 A. If full normal voltage were applied, the line value of S.C. current would be = 40 × (400/200) = 80 A.

$\therefore$ I_{sc} (line value) = 80 A ; I_{sc} (phase value) = $80/\sqrt{3}$ A

When connected in star across 400 V, the starting current per phase drawn by the motor stator during starting is

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}}\times I_{sc} \text{ per phase} = \frac{1}{\sqrt{3}}\times\frac{80}{\sqrt{3}} = \frac{80}{3} \text{ A} \qquad \text{– Art. 35.10}$$

Since during starting, motor is star-connected, I_{st} per phase = line value of I_{sc} = 80/3A

$$\therefore \quad \frac{\text{line value of starting current}}{\text{line value of F.L. current}} = \frac{80/3}{19.3} = \mathbf{1.38}$$

Example 35.19. *A 5 h.p. (3.73 kW), 400-V, 3-φ, 50-Hz cage motor has a full-load slip of 4.5%. The motor develops 250% of the rated torque and draws 650% of the rated current when thrown directly on the line. What would be the line current, motor current and the starting torque if the motor were started (i) be means of a star/delta starter and (ii) by connecting across 60% taps of a starting compensator.* **(Elect. Machines-II, Indore Univ. 1989)**

Solution. (*i*) Line current = (1/3) × 650 = **216.7%**

Motor being star-connected, line current is equal to phase current.

$\therefore$ motor current = 650/3 = **216.7%**

As shown earlier, starting torque developed for star-connection is one-third of that developed on direct switching with delta-connection $\therefore$ T_{st} = 250/3 = **83.3%**

(*ii*) Line current = $K^2\times I_{sc} = (60/100)^2\times 650$ = **234%**

$$\text{Motor current} = K \times I_{sc} = (60/100) \times 650 = \mathbf{390\%}$$

$$T_{st} = K^2 \times T_{sc} = (60/100)^2 \times 250 = \mathbf{90\%}$$

Example. 35.20. *A squirrel-cage type induction motor when started by means of a star/delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculated the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed.* **(Utilization of Elect. Power, A.M.I.E. 1987)**

Solution. With star-delta starter, $\dfrac{T_{st}}{T_f} = \dfrac{1}{3}\left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$

Line current on line-start $I_{sc} = 3 \times 180\%$ of $I_f = 3 \times 1.8\, I_f = 5.4\, I_f$

Now, $T_{st}/T_f = 0.35$ (given) ; $I_{sc}/I_f = 5.4$

$\therefore \quad 0.35 = (1/3) \times 5.4^2\, s_f \quad$ or $\quad 5.4^2\, s_f = 1.05$

Autostarter : Here, $K = 0.75$

Line starting current $= K^2 I_{sc} = (0.75)^2 \times 5.4\, I_f = \mathbf{3.04\, I_f = 304\%\ of\ F.L.\ current}$

$$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f\,;\quad \frac{T_{st}}{T_f} = (0.75)^2 \times 5.4^2\, s_f$$

$$= (0.75)^2 \times 1.05 = 0.59$$

$$T_{st} = 0.59\, T_f = \mathbf{59\%\ F.L.\ torque}$$

Example. 35.21. *A 10 h.p. (7.46 kW) motor when started at normal voltage with a star-delta switch in the star position is found to take an initial current of 1.7 × full-load current and gave an initial starting torque of 35% of full-load torque. Explain what happens when the motor is started under the following conditions : (a) an auto-transformer giving 60% of normal voltage (b) a resistance in series with the stator reducing the voltage to 60% of the normal and calculate in each case the value of starting current and torque in terms of the corresponding quantities at full-load.*

(Elect. Machinery-III, Kerala Univ. 1987)

Solution. If the motor were connected in delta and direct-switched to the line, then it would take a line current three times that which it takes when star-connected.

$\therefore$ line current on line start or $I_{sc} = 3 \times 1.7\, I_f = 5.1\, I_f$

We know $\dfrac{T_{st}}{T_f} = \dfrac{1}{3}\left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$

Now $\dfrac{T_{st}}{T_f} = 0.35$ ...given ; $\dfrac{I_{sc}}{I_f} = 5.1$ ...calculated

$\therefore \quad 0.35 = (1/3) \times 5.1^2 \times s_f$

We can find s_f from $5.1^2 \times s_f = 1.05$

(*a*) When it is started with an auto-starter, then $K = 0.6$

Line starting current $= K^2 \times I_{sc} = 0.6^2 \times 5.1\, I_f = \mathbf{0.836\, I_f}$

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.6^2 \times 1.05 = 0.378 \qquad \therefore\ T_{st} = \mathbf{37.8\%}$ of F.L. torque

(*b*) Here, voltage across motor is reduced to 60% of normal value. In this case *motor* current is the same as line current but it decreases in proportion to the decrease in voltage.

As voltage across motor $= 0.6$ of normal voltage

$\therefore$ line starting current $= 0.6 \times 5.1\, I_f = \mathbf{3.06\, I_f}$

Torque at starting would be the same as before.

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.378 \qquad \therefore\ T_{st} = \mathbf{37.8\%}$ of F.L. torque.

Tutorial Problems 35.3

1. A 3-phase induction motor whose full-load slip is 4 per cent, takes six times full-load current when switched directly on to the supply. Calculate the approximate starting torque in terms of the full-load torque when started by means of an auto-transformer starter, having a 70 percent voltage tap. **[0.7 T_f]**

2. A 3-phase, cage induction motor takes a starting current at normal voltage of 5 times the full-load value and its full-load slip is 4 per cent. What auto-transformer ratio would enable the motor to be started with not more than twice full-load current drawn from the supply ?

   What would be the starting torque under these conditions and how would it compare with that obtained by using a stator resistance starter under the same limitations of line current ?

   [63.3% tap ; 0.4 T_f ; 0.16 T_f]

3. A 3-phase, 4-pole, 50-Hz induction motor takes 40 A at a full-load speed of 1440 r.p.m. and develops a torque of 100 N-m at full-load. The starting current at rated voltage is 200 A. What is the starting torque ? If a star-delta starter is used, what is the starting torque and starting current ? Neglect magnetising current.

   [100 N-m; 33.3 N-m; 66.7 A] *(Electrical Machines-IV, Bangalore Univ. Aug. 1978)*

4. Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (*a*) a star-delta switch (*b*) an auto-transformer with 50% tapping. Ignore magnetising current. The short-circuit current of the motor at normal voltage is 5 times the full-load current and the full-load slip is 4 per cent. **[(*a*) 0.33 (*b*) 0.25]** *(A.C. Machines, Madras Univ. 1976)*

5. Find the ratio of starting to full-load current for a 7.46 kW, 400-V, 3-phase induction motor with star/delta starter, given that the full-load efficiency is 0.87, the full-load p.f. is 0.85 and the short-circuit current is 15 A at 100 V. **[1.37]** *(Electric Machinery-II, Madras Univ. April 1978)*

6. A four-pole, 3-phase, 50-Hz, induction motor has a starting current which is 5 times its full-load current when directly switched on. What will be the percentage reduction in starting torque if (*a*) star-delta switch is used for starting (*b*) auto-transformer with a 60 per cent tapping is used for starting ? *(Electrical Technology-III, Gwalior Univ. Nov. 1917)*

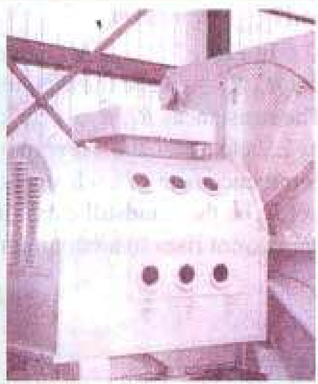
7. Explain how the performance of induction motor can be predicted by circle diagram. Draw the circle diagram for a 3-phase, mesh-connected, 22.38 kW, 500-V, 4-pole, 50-Hz induction motor. The data below give the measurements of line current, voltage and reading of two wattmeters connected to measure the input :

No load	500 V	8.3 A	2.85 kW	− 1.35 kW
Short circuit	100 V	32 A	− 0.75 kW	2.35 kW

   From the diagram, find the line current, power factor, efficiency and the maximum output.

   [83 A, 0.9, 88%, 50.73 kW] *(Electrical Machines-II, Vikram Univ. Ujjan 1977)*

35.12. Starting of Slip-ring Motors


Slip-ring electric motor

These motors are practically always started with full line voltage applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. The controlling resistance is in the form of a rheostat, connected in star (Fig. 35.22), the resistance being gradually cut-out of the rotor circuit, as the motor gathers speed. It has been already shown that by increasing the rotor resistance, not only is the rotor (and hence stator) current reduced at starting, but at the same time, the starting torque is also increased due to improvement in power factor.

The controlling rheostat is either of stud or contactor type and

Rheostat

may be hand-operated or automatic. The starter unit usually includes a line switching contactor for the stator along with no-voltage (or low- voltage) and over-current protective devices. There is some form of interlocking to ensure proper sequential operation of the line contactor and the starter. This interlocking prevents the closing of stator contactor unless the starter is 'all in'.

As said earlier, the introduction of additional external resistance in the rotor circuit enables a slip-ring motor to develop a high starting torque with reasonably moderate starting current. Hence, such motors can be started under load. This additional resistance is for starting purpose only. It is gradually cut out as the motor comes up to speed.

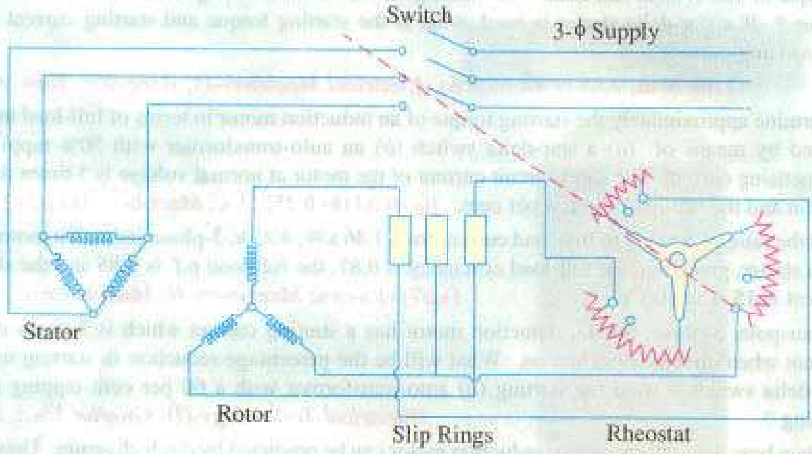


Fig. 35.22

The rings are, later on, short-circuited and brushes lifted from them when motor runs under normal conditions.

35.13. Starter Steps

Let it be assumed, as usually it is in the case of starters, that (*i*) the motor starts against a constant torque and (*ii*) that the rotor current fluctuates between fixed maximum and minimum values of I_{2max} and I_{2min} respectively.

In Fig. 35.23 is shown one phase of the 3-phase rheostat *AB* having *n* steps and the rotor circuit. Let R_1, R_2 ....etc. be the *total* resistances of the rotor circuit on the first, second step...etc. respectively. The resistances R_1, R_2 ..., etc. consist of rotor resistance per phase r_2 and the external resistances ρ_1, ρ_2 .... etc. Let the corresponding values of slips be s_1, s_2 ...etc. at stud No.1, 2...etc. At the commencement of each step, the current is I_{2max} and at the instant of leaving it, the current is I_{2min}. Let E_2 be the standstill e.m.f. induced in each phase of the rotor. When the handle touches first stud, the current rises to a maximum value I_{2max}, so that

$$I_{2max} = \frac{s_1 E_2}{\sqrt{[R_1^2 + (s_1 X_2)^2]}} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}}$$

where s_1 = slip at starting *i.e.* unity and X_2 = rotor reactance/phase

Then, *before* moving to stud No. 2, the current is reduced to I_{2min} and slip changes to s_2 such that

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}}$$

As we now move to stud No. 2, the speed momentarily remains the same, but current rises to I_{2max} because some resistance is cut out.

Hence, $I_{2max} = \dfrac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}}$

After some time, the current is again reduced to I_{2min} and the slip changes to s_3 such that

$$I_{2min} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}}$$

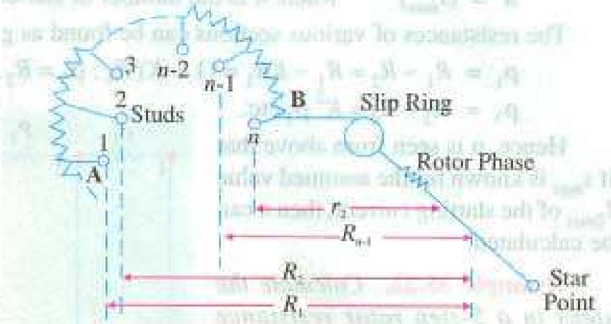


Fig. 35.23

As we next move over to stud No.3, again current rises to I_{2max} although speed remains momentarily the same.

$$\therefore \quad I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} \qquad \text{Similarly} \quad I_{2min} = \frac{E_2}{\sqrt{[(R_3/s_4)^2 + X_2^2]}}$$

At the last stud *i.e.* nth stud, $I_{2max} = \dfrac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$ where s_{max} = slip under normal running conditions, when external resistance is completely cut out.

It is found from above that

$$I_{2max} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}} = ... = \frac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$$

$$\text{or} \quad \frac{R_1}{s_1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = ... = \frac{R_{n-1}}{s_{n-1}} = \frac{R_n}{s_n} = \frac{r_2}{s_{max}} \qquad ...(i)$$

Similarly,

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} = ... = \frac{E_2}{\sqrt{[(R_{n-1}/s_{max})^2 + X_2^2]}}$$

$$\text{or} \quad \frac{R_1}{s_2} = \frac{R_2}{s_3} = \frac{R_3}{s_4} = ... = \frac{R_{n-1}}{s_{max}} \qquad ...(ii)$$

From *(i)* and *(ii)*, we get

$$\frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = ... = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = ... = \frac{r_2}{R_{n-1}} = K \text{ (say)} \qquad ...(iii)$$

Now, from *(i)* it is seen that $R_1 = \dfrac{s_1 \times r_2}{s_{max}}$.

Now, $s_1 = 1$ at starting, when rotor is stationary.

$\therefore \quad R_1 = r_2/s_{max}$. Hence, R_1 becomes known in terms of rotor resistance/phase and normal slip. From *(iii)*, we obtain

$$R_2 = KR_1 \,;\, R_3 = KR_2 = K^2R_1 \,;\, R_4 = KR_3 = K^3R_1 \text{ and } r_2 = KR_{n-1} = K^{n-1} \,.\, R_1$$

$$\text{or} \quad r_2 = K^{n-1} \,.\, \frac{r_2}{s_{max}} \qquad \text{(putting the value of } R_1\text{)}$$

$K = (s_{max})^{1/n-1}$ where n is the number of starter studs.

The resistances of various sections can be found as given below :

$$\rho_1 = R_1 - R_2 = R_1 - KR_1 = (1-K)R_1 ; \rho_2 = R_2 - R_3 = KR_1 - K^2R_1 = K\rho_1$$

$$\rho_3 = R_3 - R_4 = K^2\rho_1 \text{ etc.}$$

Hence, it is seen from above that if s_{max} is known for the assumed value I_{2max} of the starting current, then n can be calculated.

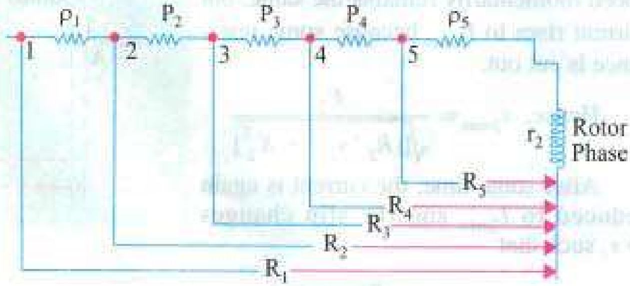


Fig. 35.24

Example 35.22. *Calculate the steps in a 5-step rotor resistance starter for a 3 phase induction motor. The slip at the maximum starting current is 2% with slip-ring short-circuited and the resistance per rotor phase is 0.02 Ω.*

Solution. Here,

$$s_{max} = 2\% = 0.02;\ r_2 = 0.02\ \Omega,\ n = 6$$

$$R_1 = \text{total resistance in rotor circuit/phase on first stud} = r_2/s_{max} = 0.02/0.02 = 1\ \Omega$$

Now,

$$K = (s_{max})^{1/n-1} = (0.02)^{1/5} = 0.4573$$

$$R_1 = 1\ \Omega;\ R_2 = KR_1 = 0.4573 \times 1 = 0.4573\ \Omega$$

$$R_3 = KR_2 = 0.4573 \times 0.4573 = 0.2091\ \Omega;\ R_4 = KR_3 = 0.4573 \times 0.2091 = 0.0956\ \Omega$$

$$R_5 = KR_4 = 0.4573 \times 0.0956 = 0.0437\ \Omega;\ r_2 = KR_5 = 0.4573 \times 0.0437 = 0.02\ \Omega \text{ (as given)}$$

The resistances of various starter sections are as found below :

$$\rho_1 = R_1 - R_2 = 1 - 0.4573 = \mathbf{0.5427\ \Omega};\quad \rho_2 = R_2 - R_3 = 0.4573 - 0.2091 = \mathbf{0.2482\ \Omega}$$

$$\rho_3 = R_3 - R_4 = 0.2091 - 0.0956 = \mathbf{0.1135\ \Omega};\quad \rho_4 = R_4 - R_5 = 0.0956 - 0.0437 = \mathbf{0.0519\ \Omega}$$

$$\rho_5 = R_5 - r_2 = 0.0437 - 0.02 = \mathbf{0.0237\ \Omega}$$

The resistances of various sections are shown in Fig. 35.24.

35.14. Crawling

It has been found that induction motors, particularly the squirrel-cage type, sometimes exhibit a tendency to run stably at speeds as low as one-seventh of their synchronous speed N_s. This phenomenon is known as crawling of an induction motor.

This action is due to the fact that the a.c. winding of the stator produces a flux wave, which is not a pure sine wave. It is a complex wave consisting of a fundamental wave, which revolves synchronously and odd harmonics like 3rd, 5th, and 7th etc. which rotate either in the forward or backward direction at $N_s/3$, $N_s/5$ and $N_s/7$ speeds respectively. As a result, in addition to the fundamental torque, harmonic torques are also developed, whose synchronous speeds are $1/n$th of the speed for the fundamental torque *i.e.* N_s/n, where n is the order of the harmonic torque. Since 3rd harmonic currents are absent in a balanced 3-phase system, they produce no rotating field and, therefore, no torque. Hence, total motor torque has three components : (*i*) the fundamental torque, rotating with the synchronous speed N_s (*ii*) 5th harmonic torque* rotating at $N_s/5$ speed and (*iii*) 7th harmonic torque, having a speed of $N_s/7$.

* The magnitude of the harmonic torques is $1/n^2$ of the fundamental torque.

Now, the 5th harmonic currents have a phase difference of $5 \times 120° = 600° = -120°$ in three stator windings. The revolving field, set up by them, rotates in the *reverse* direction at $N_s / 5$. The forward speed of the rotor corresponds to a slip greater than 100%. The small amount of 5th harmonic reverse torque produces a braking action and may be neglected.

The 7th harmonic currents in the three stator windings have a phase difference of $7 \times 120° = 2 \times 360° + 120° = 120°$. They set up a forward rotating field, with a synchronous speed equal to 1/7th of the synchronous speed of the fundamental torque.

If we neglect all higher harmonics, the resultant torque can be taken as equal to the sum of the fundamental torque and the 7th harmonic torque, as shown in Fig. 35.25. It is seen that the 7th harmonic torque reaches its maximum positive value just before 1/7th synchronous speed N_s, beyond which it becomes negative in value. Consequently, the resultant torque characteristic shows a dip which may become very pronounced with certain slot combinations. If the mechanical load on the shaft involves a constant load torque, it is possible that the torque developed by the motor may fall below this load torque. When this happens, the motor will not accelerate upto its normal speed but will remain running at a speed, which is nearly 1/7th of its full-speed. This is referred to as crawling of the motor.

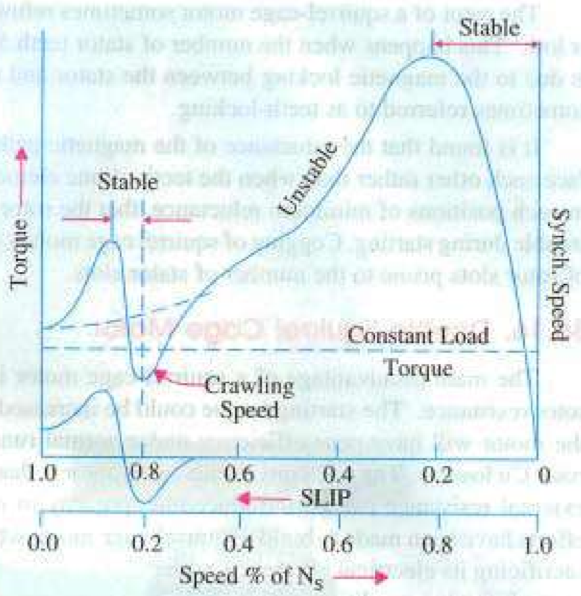


Fig. 35.25

35.15. Cogging or Magnetic Locking

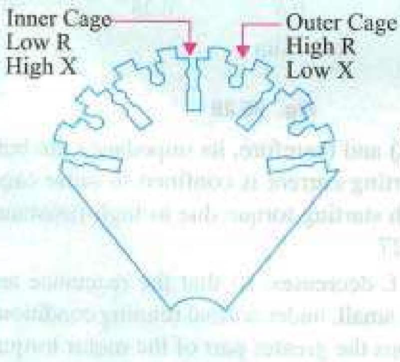


Fig. 35.26

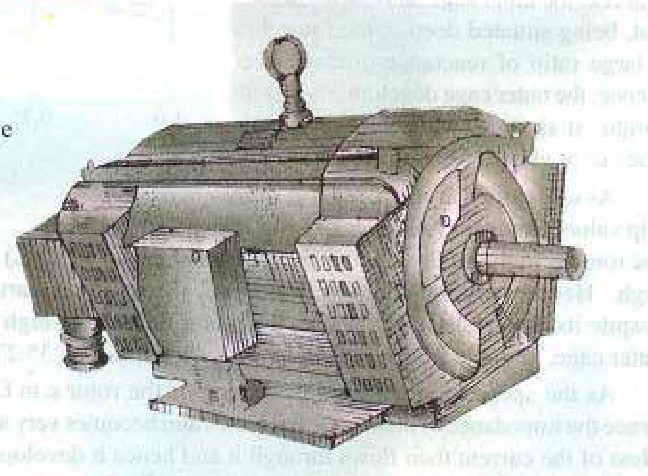


Fig. 35.27

Double-cage, 30-kW, 400/440-V, 3-f, 960 r.p.m. squirrel-cage motor. (*Courtesy : Jyoti Ltd., Baroda*)

The rotor of a squirrel-cage motor sometimes refuses to start at all, particularly when the voltage is low. This happens when the number of stator teeth S_1 is equal to the number of rotor teeth S_2 and is due to the magnetic locking between the stator and rotor teeth. That is why this phenomenon is sometimes referred to as teeth-locking.

It is found that the reluctance of the magnetic path is minimum when the stator and rotor teeth face each other rather than when the teeth of one element are opposite to the slots on the other. It is in such positions of minimum reluctance, that the rotor tends to remain fixed and thus cause serious trouble during starting. Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

35.16. Double Squirrel Cage Motor

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance. The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions (because there will be more rotor Cu losses). The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period. Many efforts have been made to build a squirrel-cage motor which should have a high starting torque without sacrificing its electrical efficiency, under normal running conditions. The result is a motor, due to Boucheort, which has two independent cages on the same rotor, one inside the other. A punching for such a double cage rotor is shown in Fig. 35.26.

The *outer* cage consists of bars of a *high-resistance* metal, whereas the inner cage has low-resistance copper bars.

Hence, outer cage has high resistance and low ratio of reactance-to-resistance, whereas the inner cage has low resistance but, being situated deep in the rotor, has a large ratio of reactance-to-resistance. Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

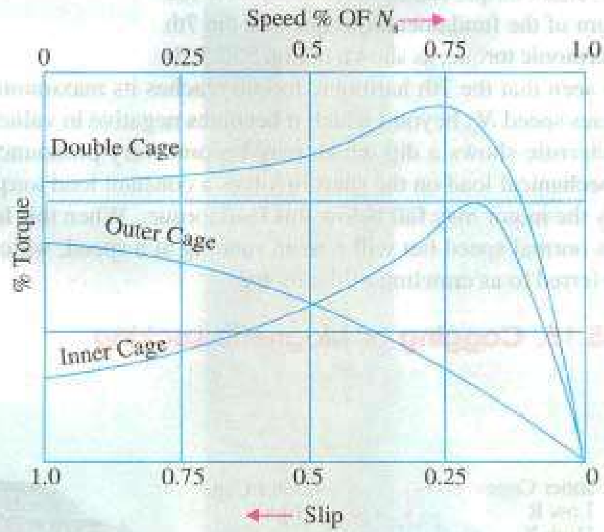


Fig. 35.28

As said earlier, at starting and at large slip values, frequency of induced e.m.f in the rotor is high. So the reactance of the inner cage ($= 2\pi f L$) and therefore, its impedance are both high. Hence, very little current flows in it. Most of the starting current is confined to outer cage, despite its high resistance. Hence, the motor develops a high starting torque due to high-resistance outer cage. Double squirrel-cage motor is shown in Fig. 35.27.

As the speed increases, the frequency of the rotor e.m.f. decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions. Most of the current then flows through it and hence it develops the greater part of the motor torque.

In fact, when speed is normal, frequency of rotor e.m.f. is so small that the reactance of both cages is practically negligible. The current is carried by two cages in parallel, giving a low combined resistance.

Hence, it has been made possible to construct a single machine, which has a good starting torque with reasonable starting current and which maintains high efficiency and good speed regulation, under normal operating conditions.

The torque-speed characteristic of a double cage motor may be approximately taken to be the sum of two motors, one having a high-resistance rotor and the other a low-resistance one (Fig. 35.28).

Such motors are particularly useful where frequent starting under heavy loads is required.

35.17. Equivalent Circuit

The two rotor cages can be considered in parallel, if it is assumed that both cages completely link the main flux. The equivalent circuit for one phase of the rotor, as referred to stator, is shown in Fig. 35.29. If the magnetising current is neglected, then the figure is simplified to that shown in Fig. 35.30. Hence, R_0'/s and R_i'/s are resistances of outer and inner rotors as referred to stator respectively and X_0' and X_i' their reactances

Total impedance as referred to primary is given by

$$Z_{01} = R_1 + jX_1 + \frac{1}{1/Z_i' + 1/Z_0'} = R_1 + jX_1 + \frac{Z_i' Z_0'}{Z_0' + Z_i'}$$

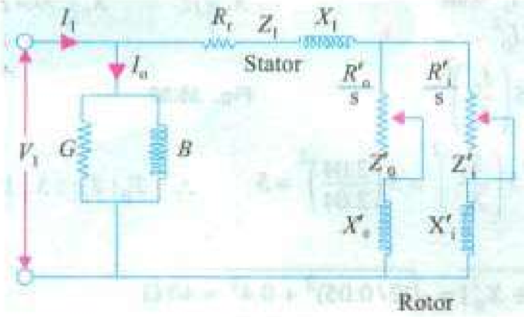


Fig. 35.29

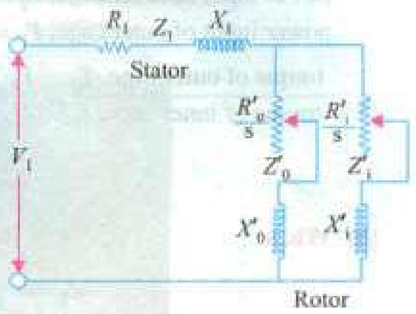


Fig. 35.30

Example 35.23. *A double-cage induction motor has the following equivalent circuit parameters, all of which are phase values referred to the primary :*

Primary	$R_1 = 1\,\Omega$	$X_1 = 3\,\Omega$
Outer cage	$R_0' = 3\,\Omega$	$X_0' = 1\,\Omega$
Inner cage	$R_i' = 0.6\,\Omega$	$X_i' = 5\,\Omega$

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4%. The magnetising current may be neglected.

Solution.

Refer to Fig. 35.31

(i) At start, $s = 1$

$$\mathbf{Z}_{01} = 1 + j3 + \frac{1}{1/(3 + j1) + 1/(0.6 + j5)}$$

$$= 2.68 + j\,4.538\ \Omega$$

$$\text{Current / phase} = 440 / (2.68^2 + 4.538^2)^{1/2} = 83.43\ \text{A}$$

$$\text{Torque} = 3 \times 83.43^2 \times (2.68 - 1)$$

$$= \mathbf{35{,}000\ synch\ watt.}$$

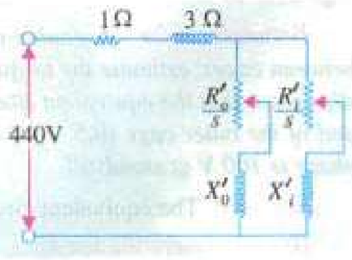


Fig. 35.31

(ii) when s = 4 per cent

In this case, approximate torque may be found by neglecting the outer cage impedance altogether. However, it does carry some current, which is almost entirely determined by its resistance.

$$Z_{01} = 1 + j\,3 + \frac{1}{1/(3/0.04) + 1/(0.6/0.04 + j5)} = 13.65 + j\,6.45$$

$$\text{Current/phase} = 440/(13.65^2 + 6.45^2)^{1/2} = 29.14 \text{ A}$$

$$\text{Torque} = 3 \times 29.14^2 \times (13.65 - 1) = \mathbf{32,000 \text{ synch watt.}}$$

Example 35.24. *At standstill, the equivalent impedance of inner and outer cages of a double-cage rotor are (0.4 + j2) Ω and (2+ j0.4) Ω respectively. Calculate the ratio of torques produced by the two cages (i) at standstill (ii) at 5% slip.* **(Elect. Machines-II, Punjab Univ. 1989)**

Solution. The equivalent circuit for one phase is shown in Fig. 35.32.

***(i)* At standstill, $s = 1$**

Impedance of inner cage $= Z_i = \sqrt{0.4^2 + 2^2} = 2.04\ \Omega$

Impedance of outer cage $= Z_0 = \sqrt{2^2 + 0.4^2} = 2.04\ \Omega$

If I_0 and I_i are the current inputs of the two cages, then

power input of inner cage, $P_i = I_i^2 R_i = 0.4\ I_i^2$ watt

power input of outer cage, $P_0 = I_0^2 R_0 = 2\ I_0^2$

$$\therefore \quad \frac{\text{torque of outer cage, } T_0}{\text{torque of inner cage, } T_i} = \frac{P_0}{P_i} = \frac{2I_0^2}{0.4\,I_i^2} = 5\left(\frac{I_0}{I_i}\right)^2$$

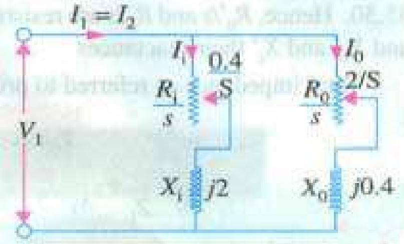


Fig. 35.32

$$= 5\left(\frac{Z_i}{Z_0}\right)^2 = 5\left(\frac{2.04}{2.04}\right)^2 = 5 \qquad \therefore \quad T_0 : T_i :: 5 : 1$$

***(ii)* When $\quad s = 0.05$**

$$Z_0 = \sqrt{[(R_0/s)^2 + X_0^2]} = \sqrt{(2/0.05)^2 + 0.4^2} = 40\ \Omega$$

$$Z_i = \sqrt{[(R_i/s)^2 + X_i^2]} = \sqrt{(0.4/0.05)^2 + 2^2} = 8.25\ \Omega$$

$$\therefore \quad \frac{I_o}{I_i} = \frac{Z_i}{Z_o} = \frac{8.25}{40} = 0.206$$

$$P_0 = I_0^2 R_0 / s = 40\ I_0^2;\ P_i = I_i^2 R_i/s = 8\ I_i^2$$

$$\therefore \quad \frac{T_o}{T_i} = \frac{P_o}{P_i} = \frac{40\,I_o^2}{8\,I_i^2} = 5\left(\frac{I_o}{I_i}\right)^2 = 5\ (0.206)^2 = 0.21$$

$\therefore$ $\mathbf{T_0 : T_i :: 0.21 : 1}$

It is seen from above, that the outer cage provides maximum torque at starting, whereas inner cage does so later.

Example 35.25. *A double-cage rotor has two independent cages. Ignoring mutual coupling between cages, estimate the torque in synchronous watts per phase (i) at standstill and at 5 per cent slip, given that the equivalent standstill impedance of the inner cage is (0.05 + j 0.4) ohm per phase and of the outer cage (0.5 + j 0.1) ohm per phase and that the rotor equivalent induced e.m.f. per phase is 100 V at standstill.*

Solution. The equivalent circuit of the double-cage rotor is shown in Fig. 35.33.

***(i)* At standstill, $s = 1$**

The combined impedance of the two cages is

$$Z = \frac{Z_0 Z_i}{Z_0 + Z_i}$$

where Z_0 = impedance of the outer cage

Z_i = impedance of the inner cage

$$Z = \frac{(0.5 + j\,0.1)\,(0.05 + j\,0.4)}{(0.55 + j\,0.5)}$$

$$= 0.1705 + j\,0.191 \text{ ohm}$$

$$\therefore \quad Z = \sqrt{0.1705^2 + 0.191^2} = 0.256\ \Omega$$

Fig. 35.33

Rotor current $I_2 = 100/0.256$ A; Combined resistance $R_2 = 0.1705\ \Omega$

Torque at standstill in synchronous watts per phase is

$$= I_2^2\,R_2 = \left(\frac{100}{0.256}\right)^2 \times 0.1705 = \textbf{26,000 synch watts}$$

(ii) Here s = 0.05

$$Z = \frac{\left(\frac{0.5}{0.05} + j\,0.1\right)\left(\frac{0.05}{0.05} + j\,0.4\right)}{\left(\frac{0.05}{0.5} + \frac{0.5}{0.05} + j\,0.5\right)} = 1.01 + j\,0.326 \text{ ohm}$$

$$Z = \sqrt{(1.01)^2 + 0.326^2} = 1.06\ \Omega$$

Combined resistance $R_2 = 1.01\ \Omega$; rotor current = 100/1.06 A

Torque in synchronous watts per phase is

$$= I_2^2\,R_2 = (100/1.06)^2 \times 1.1 = \textbf{9,000 synch.watt.}$$

Example 35.26. *In a double-cage induction motor, if the outer cage has an impedance at standstill of (2 + j 1.2) ohm, determine the slip at which the two cages develop equal torques if the inner cage has an impedance of (0.5 + j 3.5) ohm at standstill.*

(Electric Machines, Osmania Univ. 1991)

Solution. Let s be the slip at which two cage develop equal torques.

Then $Z_1 = \sqrt{(2/s)^2 + 1.2^2}$ and $Z_2 = \sqrt{(0.5/s)^2 + 3.5^2}$

$$\therefore \quad \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{Z_2}{Z_1}\right)^2 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44}$$

Power input to outer cage $P_1 = I_1^2\,R_1/s$

$$\therefore \quad P_1 = I_1^2 \times \frac{2}{s}\ ; \qquad P_2 = I_2^2 \times \frac{0.5}{s}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{P_1}{P_2} = \left(\frac{I_1}{I_2}\right)^2 \times 4 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44} \times 4$$

As $T_1 = T_2$

$$\therefore \quad \frac{4}{s^2} + 1.44 = \left(\frac{0.25}{s^2} + 12.25\right) \times 4 \qquad \therefore \quad s = 0.251 = \textbf{25.1\%}$$

Example 35.27. *The resistance and reactance (equivalent) values of a double-cage induction motor for stator, outer and inner cage are 0.25, 1.0 and 0.15 ohm resistance and 3.5, zero and 3.0 ohm reactance respectively. Find the starting torque if the phase voltage is 250 V and the synchronous speed is 1000 r.p.m.*

(I.E.E. London)

Solution. The equivalent circuit is shown in Fig. 35.34 where magnetising current has been neglected. At starting, $s = 1$

Impedance of outer cage $Z_0' = (1 + j\,0)$

Impedance of inner cage $Z_i' = (0.15 + j\,3)$

The two impedances are in parallel. Hence, their equivalent impedance

$$\mathbf{Z_2'} = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(1 + j\,0)(0.15 + j\,3)}{(1 + j\,0) + (0.15 + j\,3)}$$

$$= 0.889 + j\,0.29$$

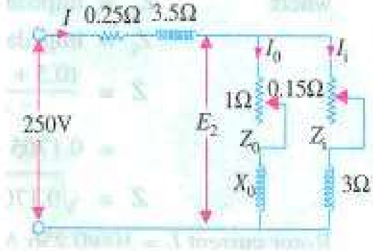


Fig. 35.34

Stator impedance = $(0.25 + j\,3.5)$

$\therefore$ total impedance $Z_{01} = (0.889 + j\,0.29) + (0.25 + j\,3.5)$
$= (1.14 + j\,3.79)\ \Omega$ (approx.)

$$\text{Current } I = \frac{\text{phase voltage}}{\text{total phase impedance}} = \frac{250 + j\,0}{1.14 + j3.79} = 18.2 - j\,60.5 = 66.15\text{ A}$$

Rotor Cu loss/phase = $(\text{current})^2 \times$ total resistance/phase of two rotors

$$= 66.15^2 \times 0.889 = 3{,}890\text{ W}$$

Total Cu loss in 3-phases = $3 \times 3890 = 11{,}670$ W

Now, rotor input = $\dfrac{\text{rotor Cu loss}}{s}$; At starting, $s = 1$ $\quad\therefore\quad$ rotor input = 11,670 W

$\therefore \qquad T_{start} = 11{,}670$ synchronous watts

Also $T_{start} \times 2\pi\, N_S = 11{,}670$ or $T_{start} = \dfrac{11{,}670}{2\pi \times (1000/60)}$ = **111.6 N-m (approx.)**

Note. If torques developed by the two rotors separately are required, then find E_2 (Fig. 35.34), then I_1 and I_2. Knowing these values, T_1 and T_2 can be found as given in previous example.

Example 35.28. *A double-cage induction motor has the following equivalent circuit parameters all of which are phase values referred to the primary :*

Primary : $\quad R_1 = 1$ *ohm* $\quad X_1 = 2.8$ *ohm*

Outer cage : $\quad R_0' = 3$ *ohm* $\quad X_0' = 1.0$ *ohm*

Inner cage : $\quad R_i' = 0.5$ *ohm* $\quad X_i' = 5$ *ohm*

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4 per cent. The magnetizing branch can be assumed connected across the primary terminals.

(Electrical Machines-II, South Gujarat Univ. 1987)

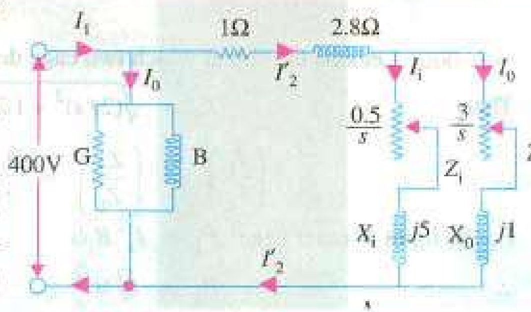


Fig. 35.35

Solution. The equivalent circuit for one phase is shown in Fig. 35.35. It should be noted that magnetising impedance Z_0 has no bearing on the torque and speed and hence, can be neglected so far as these two quantities are concerned

(i) At standstill s = 1

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(3 + j1.0)(0.5 + j5)}{(3.5 + j6)}$$

$$= 1.67 + j\,1.56$$

$$Z_{01} = Z_1 + Z_2' = (1 + j\,2.8) + (1.67 + j\,1.56)$$

$$= (2.67 + j\,4.36) = 5.1 \angle 58.5^\circ$$

Voltage per phase, $V_1 = 440$ V. Since stator is delta-connected.

$$I_2' = V_1/Z_{01} = 440/5.1 \angle 58.5° = 86.27 \angle -58.5°$$

Combined resistance $R_2 = 1.6\ \Omega$

$\therefore$ starting torque per phase $= I_2'^2 R_2 = 86.27^2 \times 1.67 =$ **12,430 synch. watt.**

(ii) when s = 0.04

$$Z_0' = (3/0.04) + j\,1.0 = 75 + j\,1.0; \quad Z_i' = (0.5/0.04) + j\,5 = 12.5 + j\,5$$

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(75 + j1.0)(12.5 + j5)}{(87.5 + j6)} = 10.3 + j\,3.67$$

$$Z_{01} = Z_1 + Z_2' = (1 + j\,2.8) + (10.3 + j\,6.47) = 11.3 + j\,6.47 = 13.03 \angle 29.8°$$

$$I_2' = V_1/Z_{01} = 440/13.03 \angle 29.8° = 33.76 \angle -29.8°$$

combined resistance $R_2 = 10.3\ \Omega$

$\therefore$ full-load torque per phase $= I_2' R_2 = 33.76^2 \times 10.3 =$ **11,740 synch. watts**

Obviously, starting torque is higher than full-load torque.

Tutorial Problems 35.4

1. Calculate the steps in a 5-section rotor starter of a 3-phase induction motor for which the starting current should not exceed the full-load current, the full-load slip is 0.018 and the rotor resistance is 0.015 Ω per phase
$\rho_1 = 0.46\ \Omega$; $\rho_2 = 0.206\ \Omega$; $\rho_3 = 0.092\ \Omega$; $\rho_4 = 0.042\ \Omega$; $\rho_5 = 0.0185\ \Omega$
(Electrical Machinery-III, Kerala Univ. Apr. 1976)
2. The full-load slip of a 3-phase double-cage induction motor is 6% and the two cages have impedances of $(3.5 + j\,1.5)\ \Omega$ and $(0.6 + j\,7.0)\ \Omega$ respectively. Neglecting stator impedances and magnetising current, calculate the starting torque in terms of full-load torque. **[79%]**
3. In a double-cage induction motor, if the outer cage has an impedance at standstill of $(2 + j\,2)$ ohm and the inner cage an impedance of $(0.5 + j\,5)\ \Omega$, determine the slip at which the two cages develop equal torques. **[17.7%]**
4. The two independent cages of a rotor have the respective standstill impedance of $(3 + j\,1)$ ohm and $(1 + j\,4)$ ohm. What proportion of the total torque is due to the outer cage (*a*) at starting and (*b*) at a fractional slip of 0.05 ? **[(*a*) 83.6% (*b*) 25.8%]** *(Principle of Elect. Engg.I, Jadavpur Univ. 1975)*
5. An induction motor has a double cage rotor with equivalent impedance at standstill of $(1.0 + j\,1.0)$ and $(0.2 + j\,4.0)$ ohm. Find the relative value of torque given by each cage at a slip of 5%.
[(*a*) 40.1 (*b*) 0.4 : 1] *(Electrical Machines-I, Gwalior Univ. Nov. 1977)*

35.18. Speed Control of Induction Motors*

A 3-phase induction motor is practically a constant-speed machine, more or less like a d.c. shunt motor. The speed regulation of an induction motor (having low resistance) is usually less than 5% at full-load. However, there is one difference of practical importance between the two. Whereas d.c. shunt motors can be made to run at any speed within wide limits, with good efficiency and speed regulation, merely by manipulating a simple field rheostat, the same is not possible with induction motors. In their case, speed reduction is accompanied by a corresponding loss of efficiency and good speed regulation. That is why it is much easier to build a good adjustable-speed d.c. shunt motor than an adjustable speed induction motor.

Different methods by which speed control of induction motors is achieved, may be grouped under two main headings :

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* For Electronic Control of AC Motors, please consult the relevant chapter of this book, in vol. III.

1. **Control from stator side**
   - (*a*) by changing the applied voltage
   - (*b*) by changing the applied frequency
   - (*c*) by changing the number of stator poles
2. **Control from rotor side**
   - (*d*) rotor rheostat control
   - (*e*) by operating two motors in concatenation or cascade
   - (*f*) by injecting an e.m.f. in the rotor circuit.

A brief description of these methods would be given below :

(*a*) Changing Applied Voltage

This method, though the cheapest and the easiest, is rarely used because

(*i*) a large change in voltage is required for a relatively small change in speed

(*ii*) this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

(*b*) Changing the Applied Frequency

This method is also used very rarely. We have seen that the synchronous speed of an induction motor is given by $N_s = 120\, f/P$. Clearly, the synchronous speed (and hence the running speed) of an induction motor can be changed by changing the supply frequency f. However, this method could only be used in cases where the induction motor happens to be the only load on the generators, in which case, the supply frequency could be controlled by controlling the speed of the prime movers of the generators. But, here again the range over which the motor speed may be varied is limited by the economical speeds of the prime movers. This method has been used to some extent on electrically-driven ships.

(*c*) Changing the Number of Stator Poles

This method is *easily applicable to squirrel-cage motors* because the squirrel-cage rotor adopts itself to any reasonable number of stator poles.

From the above equation it is also clear that the synchronous (and hence the running)speed of an induction motor could also be changed by changing the number of stator poles. This change of number of poles is achieved by having two or more entirely independent stator windings in the same slots. Each winding gives a different number of poles and hence different synchronous speed. For example, a 36-slot stator may have two 3-ϕ windings, one with 4 poles and the other with 6-poles. With a supply frequency of 50-Hz, 4-pole winding will give $N_s = 120 \times 50/4 = 1500$ r.p.m. and the 6-pole winding will give $N_s = 120 \times 50/6 = 1000$ r.p.m. Motors with four independent stator winding are also in use and they give four different synchronous (and hence running) speeds. Of course, one winding is used at a time, the others being entirely disconnected.

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Speeds in the ratio of 2:1 can be produced by a single winding if wound on the consequent-pole principle. In that case, each of the two stator windings can be connected by a simple switch to give two speeds, each, which means four speeds in all. For example, one stator winding may give 4 or 8-poles and the other 6 or 12-poles. For a supply frequency of 50-Hz, the four speeds will be 1500, 750, 1000 and 500 r.p.m. Another combination, commonly used, is to group 2- and 4-pole winding with a 6- and 12-pole winding, which gives four synchronous speeds of 3000, 1500, 1000 and 500 r.p.m.

(*d*) Rotor Rheostat Control

In this method (Fig. 35.36), which is applicable to slip-ring motors alone, the motor speed is reduced by introducing an external resistance in the rotor circuit. For this purpose, the rotor starter may be used, provided it is continuously rated. ***This method is, in fact, similar to the armature rheostat control method of d.c. shunt motors.***

It has been shown in Art 34.22 that near synchronous speed (*i.e.* for very small slip value), $T \propto s/R_2$.

It is obvious that for a given torque, slip can be increased *i.e.* speed can be decreased by increasing the rotor resistance R_2.

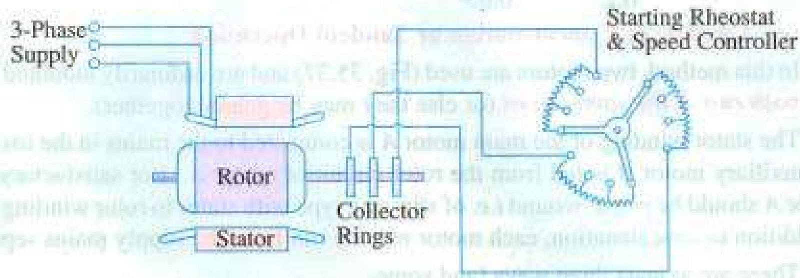

Fig. 35. 36

One serious disadvantage of this method is that with increase in rotor resistance, I^2R losses also increase which decrease the operating efficiency of the motor. In fact, the loss is directly proportional to the reduction in the speed.

The second disadvantage is the double dependence of speed, not only on R_2 but on load as well.

Because of the wastefulness of this method, it is used where speed changes are needed for short periods only.

Example 35.29. *The rotor of a 4-pole, 50-Hz slip-ring induction motor has a resistance of 0.30 Ω per phase and runs at 1440 rpm. at full load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before.*

(Advanced Elect. Machines AMIE Sec.E1992)

Solution. The motor torque is given by $T = \dfrac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2}$

Since, X_2 is not given, $T = \dfrac{K\,s\,R_2}{R_2^2} = \dfrac{K\,s}{R_2}$

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In the first case, $T_1 = Ks_1/R_2$; in the second case, $T_2 = Ks_2 / (R_2 + r)$

where r is the external resistance per phase, added to the rotor circuit

Since $T_1 = T_2$ $\quad\therefore\quad K\,s_1/R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r) / R_2 = s_2 / s_1$

Now, $N_s = 120 \times 50/4 = 1500$ rpm; $N_1 = 1440$ rpm; $N_2 = 1320$ rpm

$\therefore s_1 = (1500 - 1440)/1500 = 0.04$; $s_2 = (1500 - 1320)/1500 = 0.12$

$\therefore \quad \dfrac{0.3 + r}{0.3} = \dfrac{0.12}{0.04} \qquad \therefore \quad r = \mathbf{0.6\ \Omega}$

Example 35.30. *A certain 3-phase, 6-pole, 50-Hz induction motor when fully-loaded, runs with a slip of 3%. Find the value of the resistance necessary in series per phase of the rotor to reduce the speed by 10%. Assume that the resistance of the rotor per phase is 0.2 ohm.*

(Electrical Engineering-II (M), Bangalore Univ. 1989)

Solution. $$T = \frac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2} = \frac{K_s\,R_2}{R_2^2} = \frac{K\,s}{R_2} \quad \text{- neglecting } (s\,X_2)$$

$\therefore\quad T_1 = Ks_1 / R_2$ and $T_2 = K\,s_2 / (R_2 + r)$ where r is the external resistance per phase added to the rotor circuit.

Since $\quad T_1 = T_2$, $K\,s_1 / R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r)/R_2 = s_2 / s_1$

Now, $\quad N_s = 120 \times 50/6 = 1000$ rpm., $s_1 = 0.03$, $N_1 = 1000\,(1 - 0.030) = 970$ rpm.

$$N_2 = 970 - 10\% \text{ of } 970 = 873 \text{ rpm.}, s_2 = (1000 - 873)/1000 = 0.127$$

$$\therefore \quad \frac{0.2 + r}{0.2} = \frac{0.127}{0.03}; \quad r = \mathbf{0.65\ \Omega.}$$

(e) Cascade or Concatenation or Tandem Operation

In this method, two motors are used (Fig. 35.37) and are ordinarily mounted on the same shaft, *so that both run at the same speed* (or else they may be geared together).

The stator winding of the main motor *A* is connected to the mains in the usual way, while that of the auxiliary motor *B* is fed from the rotor circuit of motor *A*. For satisfactory operation, the main motor *A* should be phase-wound *i.e.* of slip-ring type with stator to rotor winding ratio of 1 : 1, so that, in addition to concatenation, each motor may be run from the supply mains separately.

There are at least three ways (and sometimes four ways) in which the combination may be run.

1. Main motor *A* may be run separately from the supply. In that case, the synchronous speed is $N_{sa} = 120\, f / P_a$ where P_a = Number of stator poles of motor *A*.

2. Auxiliary motor *B* may be run separately from the mains (with motor *A* being disconnected). In that case, synchronous speed is $N_{sb} = 120 \times f / P_b$ where P_b = Number of stator poles of motor *B*.

3. The combination may be connected in cumulative cascade *i.e.* in such a way that the phase rotation of the stator fields of both motors is in the same direction. The synchronous speed of the cascaded set, in this case, is $N_{sc} = 120\, f/(P_a + P_b)$.

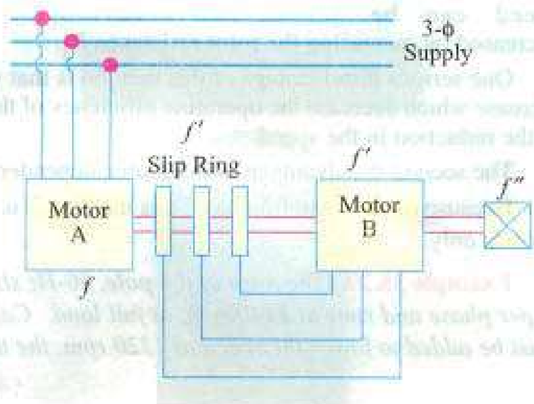

Fig. 35. 37

Proof

Let $\quad N$ = actual speed of concatenated set ;

N_{sa} = synchronous speed of motor *A*, it being independent of *N*.

Clearly, the relative speed of rotor *A* with respect to its stator field is $(N_{sa} - N)$. Hence, the frequency f' of the induced e.m.f. in rotor *A* is given by

$$f' = \frac{N_{sa} - N}{N_{sa}} \times f$$

This is also the frequency of the e.m.f. applied to the stator of motor B. Hence, the synchronous speed of motor *B* with this input frequency is

$$N' = 120\,\frac{f'}{P_b} = \frac{120\,(N_{sa} - N)\, f}{P_b \times N_{sa}} \qquad \ldots(i)$$

(Note that N' is not equal to N_{sb} which is the synchronous speed of motor *B* with supply frequency f).

This will induce an e.m.f. of frequency, say, f'' in the rotor *B*. Its value is found from the fact that the stator and rotor frequencies are proportional to the speeds of stator field and the rotor

$$\therefore \quad f'' = \frac{N' - N}{N'}\, f$$

Now, on no-load, the speed of rotor *B* is almost equal to its synchronous speed, so that the frequency of induced e.m.f. is, to a first approximation, zero.

$$f'' = 0, \text{ or } \frac{N' - N}{N'} f = 0 \text{ or } N' = N \qquad ...(ii)$$

From (*i*) above $$N' = \frac{120\, f\, (N_{sa} - N)}{P_b \times N_{sa}} = \frac{120\, f}{P_b}\left(1 - \frac{N}{N_{sa}}\right)$$

Hence, from (*ii*) above,

$$\frac{120\, f}{P_b}\left(1 - \frac{N}{N_{sa}}\right) = N \text{ or } \frac{120\, f}{P_b} = N\left(1 + \frac{1}{N_{sa}} \times \frac{120\, f}{P_b}\right)$$

Putting $N_{sa} = 120\, f/P_a$, we get

$$\frac{120 f}{P_b} = \text{N}\left(1 + \frac{P_a}{120 f} \times \frac{120\, f}{P_b}\right) = N\left(1 + \frac{P_a}{P_b}\right) \therefore N = \frac{120\, f}{(P_a + P_b)}$$

Concatenated speed of the set = $120\, f\, /\, (P_a + P_b)$

How the Set Starts ?

When the cascaded set is started, the voltage at frequency f is applied to the stator winding of machine *A*. An induced e.m.f. of the same frequency is produced in rotor *A* which is supplied to auxiliary motor *B*. Both the motors develop a forward torque. As the shaft speed rises, the rotor frequency of motor *A* falls and so does the synchronous speed of motor *B*. The set settles down to a stable speed when the shaft speed becomes equal to the speed of rotating field of motor *B*.

Considering load conditions, we find that the electrical power taken in by stator *A* is partly used to meet its I^2R and core losses and the rest is given to its rotor. The power given to rotor is further divided into two parts : one part, proportional to the speed of set *i.e.* N is converted into mechanical power and the other part proportional to $(N_{sa} - N)$ is developed as electrical power at the slip frequency, and is passed on to the auxliary motor *B*, which uses it for producing mechanical power and losses. Hence, approximately, the mechanical outputs of the two motors are in the ratio $N : (N_{sa} - N)$. ***In fact, it comes to that the mechanical outputs are in the ratio of the number of poles of the motors***.

It may be of interest to the reader to know that it can be proved that

(*i*) $s = f''\, /\, f$ where s = slip of the set referred to its synchronous speed N_{sc}.

$= (N_{sc} - N)\, /\, N_{sc}$

(*ii*) $s = s_a\, s_b$

where s_a and s_b are slips of two motors, referred to their respective stators *i.e*

$$s_a = \frac{N_{sa} - N}{N_{sa}} \quad \text{and} \quad s_b = \frac{N' - N}{N}$$

Conclusion

We can briefly note the main conclusions drawn from the above discussion :

(*a*) the mechanical outputs of the two motors are in the ratio of their number of poles.

(*b*) $s = f''/f$ (*c*) $s = s_a . s_b$

4. The fourth possible connection is the ***differential cascade***. In this method, the phase rotation of stator field of the motor *B* is opposite to that of the stator of motor *A*. This reversal of phase rotation of stator of motor *B* is obtained by interchanging any of its two leads. It can be proved in the same way as above, that for this method of connection, the synchronous speed of the set is

$$N_{sc} = 120\, f\, /\, (P_a - P_b)$$

As the differentially-cascaded set has a very small or zero starting torque, this method is rarely used. Moreover, the above expression for synchronous speed becomes meaningless for $P_a = P_b$.

Example 35.31. *Two 50-Hz, 3-φ induction motors having six and four poles respectively are cumulatively cascaded, the 6-pole motor being connected to the main supply. Determine the* *frequencies of the rotor currents and the slips referred to each stator field if the set has a slip of 2 per cent.* **(Elect. Machinery-II Madras Univ. 1987)**

Solution. Synchronous speed of set $N_{sc} = 120 \times 50/10 = 600$ r.p.m.

Actual rotor speed $N = (1 - s)\, N_{sc} = (1 - 0.02)\, 600 = 588$ r.p.m.

Synchronous speed of the stator field of 6-pole motor, $N_{sa} = 120 \times 50/6 = 1000$ r.p.m.
Slip referred to this stator field is

$$s_a = \frac{N_{sa} - N}{N_{sa}} = \frac{1000 - 588}{1000} = 0.412 \quad \text{or} \quad \mathbf{41.2\%}$$

Frequency of the rotor currents of 6-pole motor $f' = s_a f = 0.412 \times 50 =$ **20.6 Hz**

This is also the frequency of stator currents of the four pole motor. The synchronous speed of the stator of 4-pole motor is

$$N' = 120 \times 20.6/4 = 618 \text{ r.p.m.}$$

This slip, as referred to the 4-pole motor, is $s_b = \dfrac{N' - N}{N'} = \dfrac{618 - 588}{618}$

$= 0.0485$ or **4.85 %**

The frequency of rotor current of 4-pole motor is

$$f'' = s_b f' = 0.0485 \times 20.6 = \textbf{1.0 Hz (approx)}$$

As a check, $f'' = sf = 0.02 \times 50 = 1.0$ Hz

Example 35.32. *A 4-pole induction motor and a 6-pole induction motor are connected in cumulative cascade. The frequency in the secondary circuit of the 6-pole motor is observed to be 1.0 Hz. Determine the slip in each machine and the combined speed of the set. Take supply frequency as 50 Hz.* **(Electrical Machinery-II, Madras Univ. 1986)**

Solution. With reference to Art. 35.18 (*e*) and Fig. 35.38

$N_{sc} = 120 \times 50/(4 + 6) = 600$ r.p.m.

$s = f''/f = 1/50 = 0.02$

N = actual speed of the concatenated set

$\therefore \quad 0.02 = \dfrac{600 - N}{600}$ or $N =$ **588 r.p.m.**

$N_{sa} = 120 \times 50/\, 4 = 1500$ r.p.m.

$s_a = (1500 - 588)/1500$

$= 0.608$ or **60.8%**

$f' = s_a f = 0.608 \times 50 = 30.4$ Hz

N' = synchronous speed of 6-pole motor with frequency f'

$= 120 \times 30.4/6 = 608$ r.p.m.

$$s_b = \frac{N' - N}{N'} = \frac{608 - 588}{608} = 0.033 \quad \text{or} \quad \mathbf{3.3\%}$$

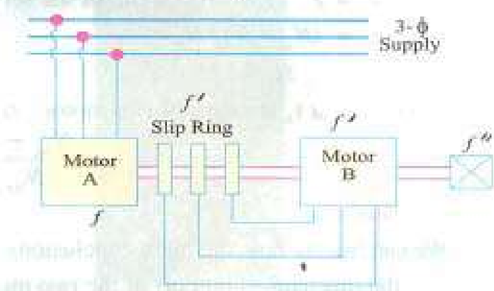

Fig. 35.38

Example 35.33. *The stator of a 6-pole motor is joined to a 50-Hz supply and the machine is mechanically coupled and joined in cascade with a 4-pole motor. Neglecting all losses, determine the speed and output of the 4-pole motor when the total load on the combination is 74.6 kW.*

Solution. As all losses are neglected, the actual speed of the rotor is assumed to be equal to the synchronous speed of the set.

Now, $N_{sc} = 120 \times 50/10 =$ **600 r.p.m.**

As said earlier, mechanical outputs are in the ratio of the number of poles of the motors.

$\therefore$ output of 4-pole motor = 74.6 × 4/10 = **29.84 kW**

Example 35.34. *A cascaded set consists of two motors A and B with 4 poles and 6 poles respectively. The motor A is connected to a 50-Hz supply. Find*

(*i*) the speed of the set

(*ii*) *the electric power transferred to motor B when the input to motor A is 25 kW. Neglect losses.* **(Electric Machines-I, Utkal Univ. 1990)**

Solution. Synchronous speed of the set is*

$N_{sc} = 120\,f/(P_a + P_b) = 120 \times 50/(6 + 4) =$ **600 r.p.m.**

(*ii*) The outputs of the two motors are proportional to the number of their poles.

$\therefore$ output of 4-pole motor B = 25 × 4/10 = **10 kW**

(*f*) Injecting an e.m.f. in the Rotor Circuit

In this method, the speed of an induction motor is controlled by injecting a voltage in the rotor circuit, it being of course, necessary for the injected voltage to have the same frequency as the slip frequency. There is, however, no restriction as to the phase of the injected e.m.f.

When we insert a voltage which is in ***phase opposition*** to the induced rotor e.m.f., it amounts to increasing the rotor resistance, whereas inserting a voltage which is in phase with the induced rotor e.m.f., is equivalent to ***decreasing*** its resistance. Hence, by changing the phase of the injected e.m.f. and hence the rotor resistance, the speed can be controlled.

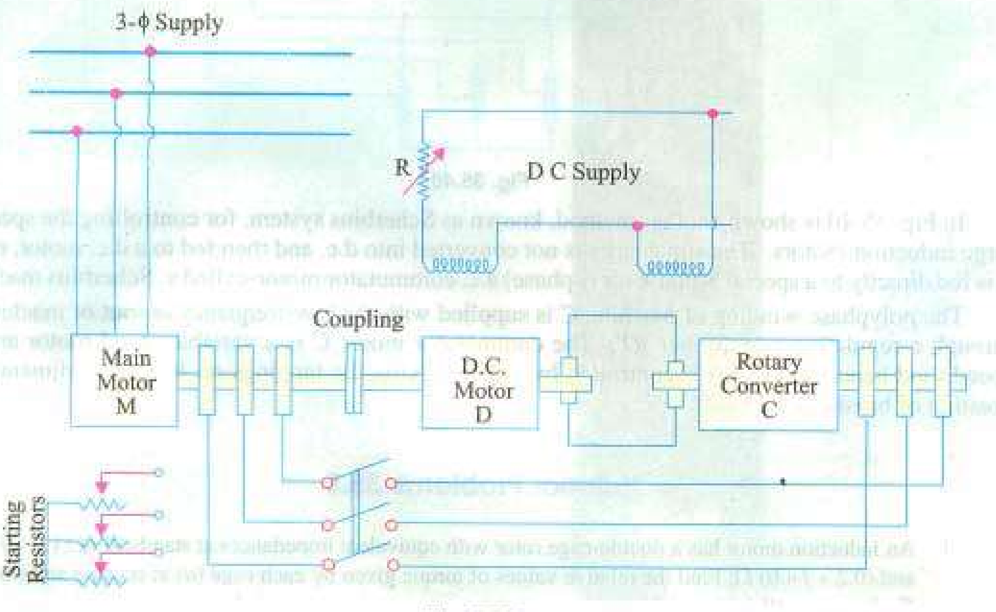

Fig. 35.39

One such practical method of this type of speed control is Kramer system, as shown in Fig. 35.39, which is used in the case of large motors of 4000 kW or above. It consists of a rotary converter *C* which converts the low-slip frequency a.c. power into d.c. power, which is used to drive a d.c. shunt motor *D*, mechanically coupled to the main motor *M*.

The main motor is coupled to the shaft of the d.c. shunt motor *D*. The slip-rings of *M* are connected to those of the rotary converter *C*. The d.c. output of *C* is used to drive *D*. Both *C* and *D*

---

* It is assumed that the two motors are connected in cumulative cascade.

are excited from the d.c. bus-bars or from an exciter. There is a field regulator which governs the back e.m.f. E_b of D and hence the d.c. potential at the commutator of C which further controls the slip-ring voltage and therefore, the speed of M.

One big advantage of this method is that any speed, within the working range, can be obtained instead of only two or three, as with other methods of speed control.

Yet another advantage is that if the rotary converter is over-excited, it will take a leading current which compensates for the lagging current drawn by main motor M and hence improves the power factor of the system.

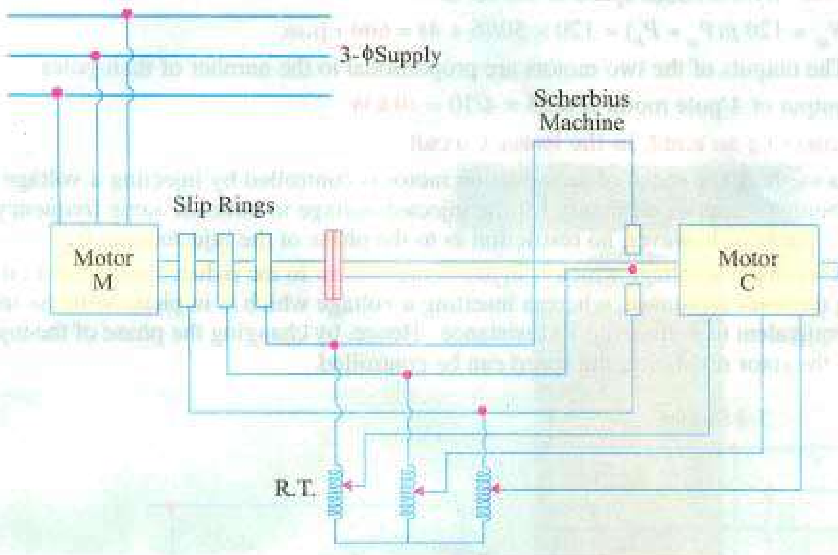

Fig. 35.40

In Fig. 35.40 is shown another method, known as Scherbius system, for controlling the speed of large induction motors. The slip energy is not converted into d.c. and then fed to a d.c. motor, rather it is fed directly to a special 3-phase (or 6-phase) a.c. commutator motor-called a, Scherbius machine.

The polyphase winding of machine C is supplied with the low-frequency output of machine M through a regulating transformer RT. The commutator motor C is a variable-speed motor and its speed (and hence that of M) is controlled by either varying the tappings on RT or by adjusting the position of brushes on C.

Tutorial Problems 35.5

1. An induction motor has a double-cage rotor with equivalent impedances at standstill of $(1.0 + j\,1.0)$ and $(0.2 + j\,4.0)\ \Omega$. Find the relative values of torque given by each cage (*a*) at starting and (*b*) at 5 % slip **[(*a*) 40:1 (*b*) 0.4:1]** *(Adv. Elect. Machines AMIE Sec. B 1991)*
2. The cages of a double-cage induction motor have standstill impedances of $(3.5 + j\,1.5)\ \Omega$ and $(0.6 + j\,7.0)\ \Omega$ respectively. The full-load slip is 6%. Find the starting torque at normal voltage in terms of full-load torque. Neglect stator impedance and magnetizing current.
   [300%] *(Elect. Machines-I, Nagpur Univ. 1993)*
3. The rotor of a 4 pole, 50 Hz, slip ring induction motor has a resistance of 0.25 ohm per phase and runs at 1440 rpm at full-load. Calculate the external resistance per phase, which must be added to lower the speed to 1200 rpm, the torque being same as before.
   [1 Ω] *(Utilisation of Electric Power (E-8) AMIE Sec. B Summer 1992)*

35.19. Three-phase A.C. Commutator Motors

Such motors have shunt speed characteristics *i.e.* change in their speed is only moderate, as compared to the change in the load. They are ideally suited for drives, requiring a uniform accelerating torque and continuously variable speed characteristics over a wide range. Hence, they find wide use in high-speed lifts, fans and pumps and in the drives for cement kilns, printing presses, pulverised fuel plants, stokers and many textile machines. Being more complicated, they are also more expensive than single-speed motors. Their efficiency is high over the whole speed range and their power factor varies from low value at synchronous speed to unity at maximum (supersynchronous) speed.

The speed control is obtained by injecting a variable voltage at *correct* frequency into the secondary winding of the motor via its commutator. If injected voltage assists the voltage induced in the secondary winding, the speed is increased but if it is in the opposing direction, then motor speed is reduced. The commutator acts as a frequency changer because it converts the supply frequency of the regulating voltage to the slip frequency corresponding to the speed required.

Following are the two principal types of such motors :

(*i*) Schrage or rotor-fed or brush shift motor and (*ii*) stator-fed or induction-regulator type motor.

35.20. Schrage Motor*

It is a rotor-fed, shunt-type, brush-shifting, 3-phase commutator induction motor which has built-in arrangement both for ***speed control and power factor improvement***. In fact, it is an induction motor with a built-in slip-regulator. It has **three** windings:two in rotor and one in stator as shown in Fig. 35.41 and 35.42 (*a*) The three windings are as under:

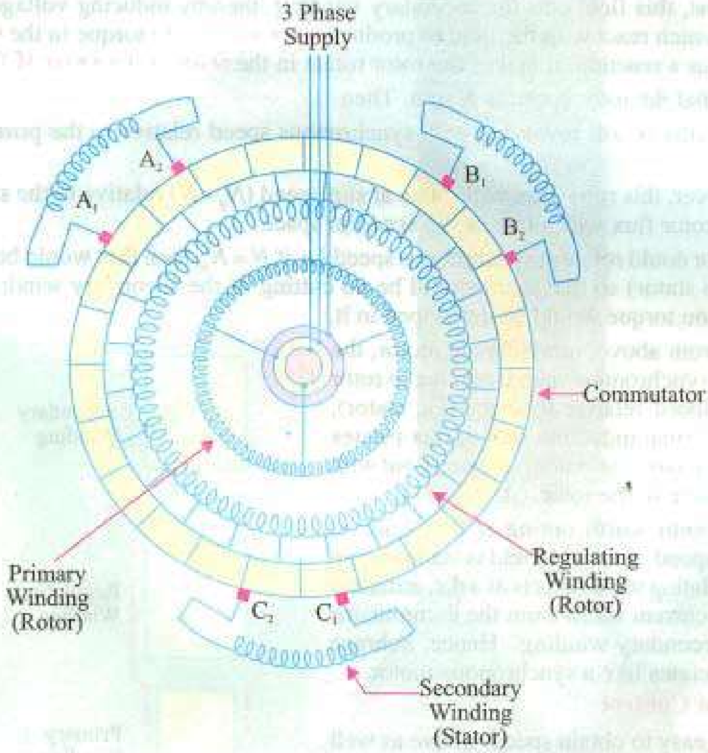

Fig. 35.41

(*i*) **Primary winding.** It is housed in the lower part of the rotor slots (not stator) and is supplied through slip-rings and brushes at *line* frequency. It generates the working flux in the machine.

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* After the name of its inventor K.H. Schrage of Sweden.

(*ii*) **Regulating winding.** It is variously known as *compensating* winding or *tertiary* winding. It is also housed in *rotor slots* (in the upper part) and is connected to the commutator in a manner similar to the armature of a d.c. motor.

(*iii*) **Secondary winding.** It is contained in the *stator slots*, but end of each phase winding is connected to one of the pair of brushes arranged on the commutator. These brushes are mounted on two separate brush rockers, which are designed to move in opposite directions relative to the centre line of the corresponding stator phase (usually by a rack and pinion mechanism). Brushes A_1, B_1, and C_1 move together and are 120 electrical degrees apart. Similarly, brushes A_2, B_2 and C_2 move together and are also 120 electrical degrees apart. A sectional drawing of the motor is shown in Fig. 35.44.

(*a*) Working

When primary is supplied at line frequency, there is transformer action between primary and regulating winding and normal induction motor action between primary and secondary winding. Hence, voltage at line frequency is induced in the regulating winding by transformer action. The commutator, acting as a frequency changer, converts this line-frequency voltage of the regulating winding to the slip frequency for feeding it into the secondary winding on the stator. The voltage across brush pairs A_1A_2, B_1B_2 and C_1C_2 increases as brushes are separated. In fact, magnitude of the voltage injected into the secondary winding depends on the angle of separation of the brushes A_1 and A_2, B_1 and B_2 and C_1 and C_2. How slip-frequency e.m.f. is induced in secondary winding is detailed below:

When 3-ϕ power is connected to slip-rings, synchronously rotating field is set up in the rotor core. Let us suppose that this field revolves in the clockwise direction. Let us further suppose that brush pairs are on *one commutator segment*, which means that secondary is short-circuited. With rotor still at rest, this field cuts the secondary winding, thereby inducing voltage and so producing currents in it which react with the field to produce *clockwise (CW)* torque in the *stator*. Since stator cannot rotate, as a reaction, it makes the rotor rotate in the *counterclockwise (CCW)* direction.

Suppose that the rotor speed is N rpm. Then

1. rotor flux is still revolving with synchronous speed relative to the primary and regulating winding.
2. however, this rotor flux will rotate at slip speed $(N_s - N)$ relative to the stator. It means that the revolving rotor flux will rotate at slip speed in space.
3. if rotor could rotate at synchronous speed *i.e.* if $N = N_s$, then flux would be stationary in space (*i.e.* relative to stator) so that there would be no cutting of the secondary winding by the flux and, consequently, no torque would be developed in it.

As seen from above, in a Schrage motor, the flux rotates at synchronous speed, relative to rotor but with slip speed relative to space (*i.e.* stator), whereas in a normal induction motor, flux rotates synchronously relative to stator (*i.e.* space) but with slip speed relative to the rotor. (Art. 34.11).

Another point worth noting is that since at *synchronous* speed, magnetic field is stationary in space, the regulating winding acts as a d.c. armature and the direct current taken from the commutator flows in the secondary winding. Hence, Schrage motor then operates like a synchronous motor.

(*b*) Speed Control

It is quite easy to obtain speeds above as well as below synchronism in a Schrage motor. As shown in Fig. 35.42 (*b*) (*i*) when brush pairs are together on the same commutator segment (*i.e.* are electrically connected via commutator), the secondary winding is short-circuited and the machine

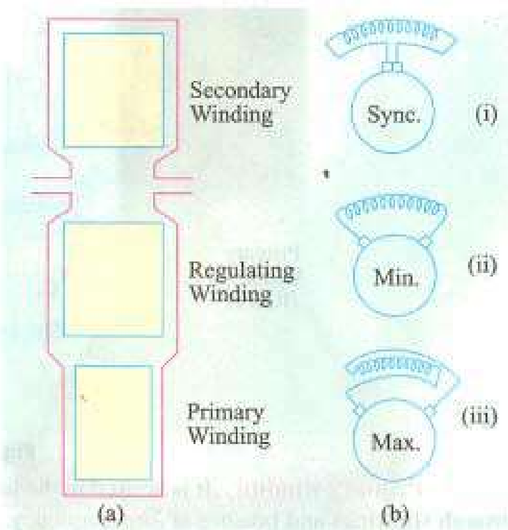


Fig. 35.42

operates as an inverted* plain squirrel-cage induction motor, running with a small positive slip. Parting the brushes in one direction, as shown in Fig. 35.42 (*b*) (*ii*) produces subsynchronous speeds, because in this case, regulating voltage injected into the secondary winding ***opposes*** the voltage induced in it from primary winding. However, when movement of brushes is reversed and they are parted in opposite directions, the direction of the regulating voltage is reversed and so motor speed ***increases*** to super synchronous (maximum) value, as shown in Fig. 35.41 (*b*) (*iii*) The commutator provides maximum voltage when brushes are separated by one pole pitch.

No-load motor speed is given by $N \cong N_s(1 - \text{K} \sin 0.5\beta)$ where β is brush separation in electrical degrees and K is a constant whose value depends on turn ratio of the secondary and regulating windings.

Maximum and minimum speeds are obtained by changing the magnitude of the regulating voltage. Schrage motors are capable of speed variations from zero to nearly twice the synchronous speed, though a speed range of 3:1 is sufficient for most applications. It is worth noting that Schrage motor is essentially a shunt machine, because for a particular brush separation, speed remains approximately constant as the load torque is increased as happens with dc shunt motors (Art 29.14).

(*c*) Power Factor Improvement

Power factor improvement can be brought about by changing the phase angle of the voltage injected into the secondary winding. As shown in Fig. 35.43, when one set of brushes is advanced more rapidly than the other is retarted, then injected voltage has a quadrature component which leads the rotor induced voltage. Hence, it results in the improvement of motor power factor. This differential movement of brush sets is obtained by coupling the racks driving the brush rockers to the hand wheel with gears having differing ratios. In Schrage motor, speed depends on angular distance between the individual brush sets (A_1 and A_2 in Fig. 35.41) but p.f. depends on the angular positions of the brushes as a whole.

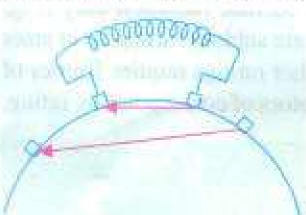


Fig. 35.43

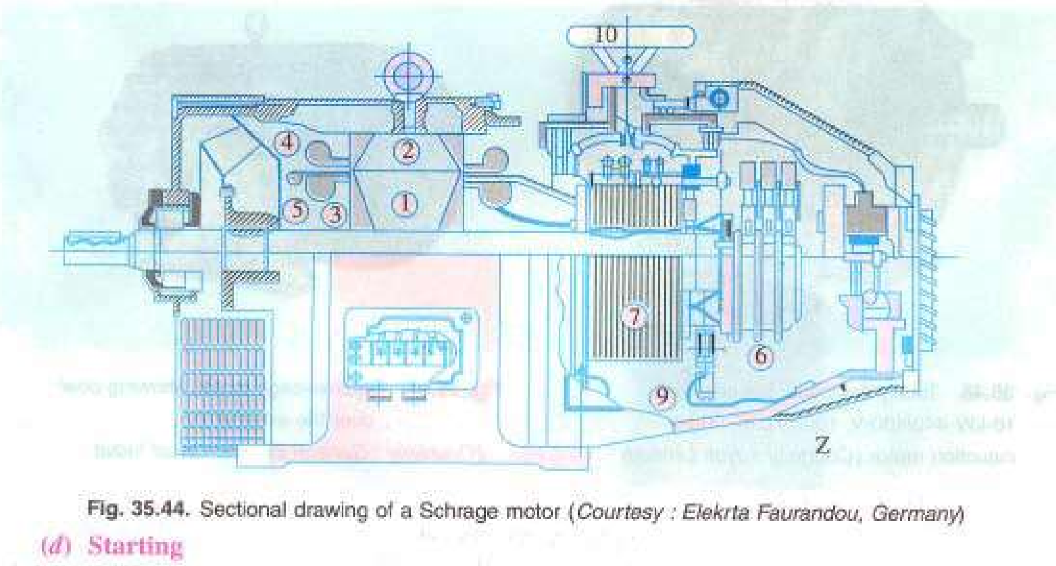


Fig. 35.44. Sectional drawing of a Schrage motor (*Courtesy : Elekrta Faurandou, Germany*)

(*d*) Starting

Schrage motors are usually started with brushes in the lowest speed position by direct-on contactor starters. Usually, interlocks are provided to prevent the contactor getting closed on the line when brushes are in any other position. One major disadvantage of this motor is that its operating voltage is limited to about 700 V because a.c. power has to be fed through slip-rings. It is available in sizes upto 40 kW and is designed to operate on 220, 440 and 550 V. It is ordinarily wound for four or six poles.

* Inverted in the sense that primary is in the rotor and secondary in the stator - just opposite of that in the normal induction motor (Art. 34.3).

Fig. 35.44 shows a sectional drawing of a Schrage motor. The details of different parts labelled in the diagram are as under:
1. rotor laminations **2.** stator laminations **3.** primary winding **4.** secondary winding **5.** regulating winding **6.** slip-ring unit **7.** commutator **8.** cable feed for outer brush yoke **9.** cable feed for inner brush yoke **10.** hand wheel.

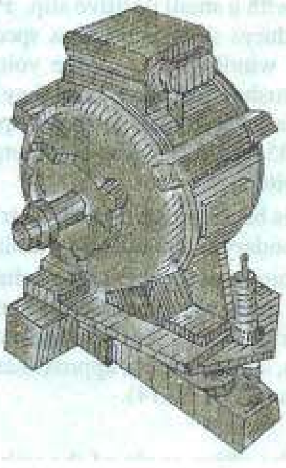


Fig. 35.45. Totally-enclosed surface cooled 750 W, 750 r.p.m. high torque induction motor (*Courtesy : Jyoti Limited*)

35.21. Motor Enclosures

Enclosed and semi-enclosed motors are practically identical with open motors in mechanical construction and in their operating characteristics. Many different types of frames or enclosures are available to suit particular requirements. Some of the common type enclosures are described below:

(i) Totally-enclosed, Non-ventilated Type

Such motors have solid frames and end- shields, but no openings for ventilation. They get cooled by surface radiation only (Fig. 35.45). Such surface-cooled motors are seldom furnished in sizes above two or three kW, because higher ratings require frames of much larger sizes than fan-cooled motors of corresponding rating.

Fig. 35.46. Totally-enclosed, fan-cooled 10-kW 440/400-V, 1000 r.p.m. 50-Hz induction motor (*Courtesy : Jyoti Limited*)

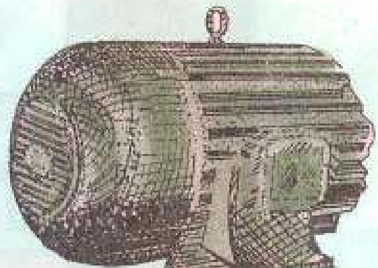


Fig. 35.47. Squirrel-cage motor, showing cowl over the external fan. (*Courtesy : General Electric Co. of India*)

(ii) Splash-proof Type

In the frames of such motors, the ventilating openings are so constructed that the liquid drops or dust particles falling on the motor are coming towards it in a straight line at any angle not greater than 100° from the vertical are not able to enter the motor either directly or by striking and running along the surface.

(iii) Totally-enclosed, Fan-cooled Type

In such motors (Fig. 35.46), cooling air is drawn into the motor by a fan mounted on the shaft. This air is forced through the motor between the inner fully-enclosed frame and an outer shell, over the end balls and the stator laminations and is then discharged through openings in the opposite side. An internal fan carries the generated heat to the totally enclosing frame, from where it is conducted to the outside. Because of totally enclosing frame, all working parts are protected against corrosive or abrasive effects of fumes, dust, and moisture.

(iv) Cowl-covered Motor

These motors are simplified form of fan-cooled motors (Fig. 35.47). These consist of totally-enclosed frame with a fan and cowl mounted at the end opposite to the driving end. The air is drawn into the cowl with the help of fan and is then forced over the frame. The contours of the cowl guide the cooling air in proper directions. These motors are superior to the usual fan-cooled motors for operation in extremely dusty atmosphere *i.e.* gas works, chemical works, collieries and quarries etc. because there are no air passages which will become clogged with dust.

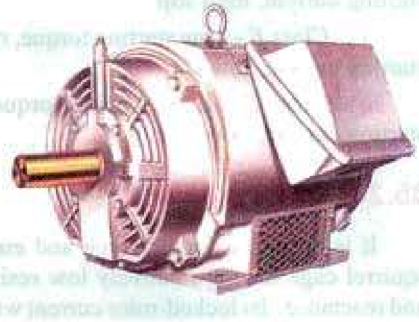


Fig. 35.48. Squirrel cage motor

(v) Protected Type

This construction consists of perforated covers for the openings in both end shields

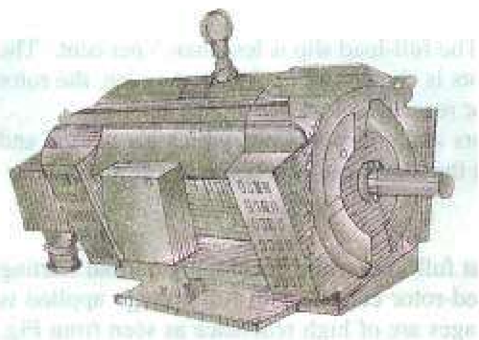


Fig. 35.49. Protected slip-ring motor with totally enclose slip -rings. (*Courtesy : General Electric Co. of India*)

(vi) Drip-proof Motors

The frames of such motors are so constructed that liquid drops or dust particles, falling on the machine at any angle greater than 15° from the vertical, cannot enter the motor, either directly or by striking and running along a horizontal or inwardly inclined smooth surface (Fig. 35.51).

(vii) Self (Pipe) Ventilated Type

The construction of such motors consists of enclosed shields with provision for pipe connection on both the shields. The motor fan circulates sufficient air through pipes which are of ample section.

(viii) Separately (Forced) Ventilated Type

These motors are similar to the self-ventilated type except that ventilation is provided by a separated blower.

35.22. Standard Types of Squirrel-cage Motors

Different types of 3-phase squirrel-cage motors have been standardized, according to their electric characteristics, into six types, designated as design *A*, *B*, *C*, *D*, *E* and *F* respectively. The original commercial squirrel-cage induction motors which were of shallow-slot type are designated as class *A*. For this reason, Class *A* motors are used as a reference and are referred to as 'normal starting-torque, normal starting-current, normal slip' motors.

(*i*) Class *A* - *N*ormal starting torque, normal starting current, normal slip

(*ii*) Class *B* - *N*ormal starting torque, low starting current, normal slip

(*iii*) Class *C* - High starting torque, low starting current, normal slip

Fig. 35.50. Squirrel cage A C induction motor

(*iv*) Class *D* - High starting torque, low starting current, high slip

(*v*) Class *E* - Low starting torque, normal starting current, low slip

(*vi*) Class *F* - Low starting torque, low starting current, normal slip

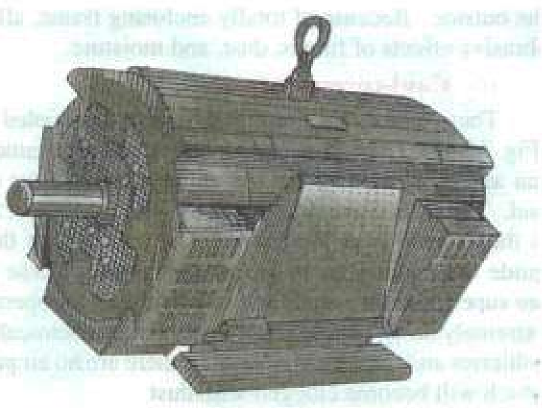


Fig. 35.51. Drip-proof slip-ring 50 up, 440/400-V, 50HZ, 1000 r.p.m. motor (courtesy : Jyoti Limited)

35.23. Class A Motors

It is the most popular type and employs squirrel cage having relatively low resistance and reactance. Its locked-rotor current with full voltage is generally more than 6 times the rated full-load current. For smaller sizes and number of poles, the starting torque with full voltage is nearly twice the full-load torque whereas for larger sizes and number of poles, the corresponding figure is 1.1 times the full-load torque. The full-load slip is less than 5 per cent. The general configuration of slot construction of such motors is shown in Fig. 35.52. As seen, the rotor bars are placed close to the surface so as to reduce rotor reactance.

Such motors are used for fans, pumps, compressors and conveyors etc. which are started and stopped in frequently and have low inertia loads so that the motor can accelerate in a few seconds.

35.24. Class B Motors

These motors are so built that they can be started at full-load while developing normal starting torque with relatively low starting current. Their locked-rotor current with full voltage applied is generally 5 to 5 1/2 times the full-load current. Their cages are of high reactance as seen from Fig. 35.53. The rotor is constructed with deep and narrow bars so as to obtain high reactance during starting.

Fig. 35.52

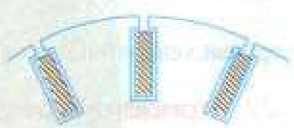


Fig. 35.53

Such motors are well-suited for those applications where there is limitation on the starting current or if the starting current is still in excess of what can be permitted, then reduced voltage starting is employed. One of the common applications of such motors is large fans most of which have high moment of inertia. It also finds wide use in many machine tool applications, for pumps of centrifugal type and for driving electric generators.

35.25. Class C Motors

Such motors are usually of double squirrel-cage type (Fig. 35.54) and combine high starting torque with low starting current. Their locked-rotor currents and slip with full voltage applied are nearly the same as for class *B* motors. Their starting torque with full voltage applied is usually 2.75 times the full-load torque.

For those applications where reduced voltage starting does not give sufficient torque to start the load with either class *A* or *B* motor, class C motor, with its high inherent starting torque along with reduced starting current supplied by reduced-voltage starting may be used. Hence, it is frequently used for crushers, compression pumps, large refrigerators, coveyor equipment, textile machinery, boring mills and wood-working equipment etc.

35.26. Class D Motors

Such motors are provided with a high-resistance squirrel cage giving the motor a high starting torque with low starting current. Their locked-rotor currents with full voltage applied are of the same order as for class *C* motors. Their full-load slip varies from 5% to 20 per cent depending on the application. Their slot structure is shown in Fig. 35.55. For obtaining high starting torque with low starting current, thin rotor bars are used which make the leakage flux of the rotor low and the useful flux high.

Since these motors are used where extremely high starting torque is essential, they are usually used for bulldozers, shearing machines, punch presses, foundry equipment, stamping machines, hoists, laundry equipment and metal drawing equipment etc.

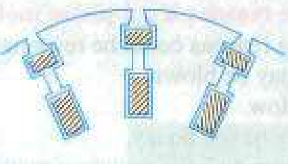


Fig. 35.54

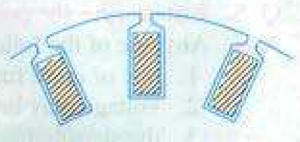
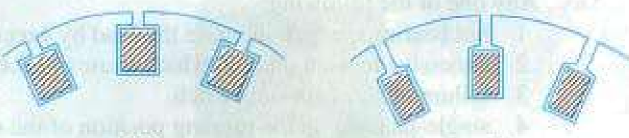


Fig. 35.55

35.27. Class E Motors

These motors have a relatively low slip at rated load. For motors above 5 kW rating, the starting current may be sufficiently high as to require a compensator or resistance starter. Their slot structure is shown in Fig. 35.56 (*a*).



(*a*) (*b*)

Fig. 35.56

35.28. Class F Motors

Such motors combine a low starting current with a low starting torque and may be started on full voltage. Their low starting current is due to the design of rotor which has high reactance during starting [Fig. 35.56 (*b*)]. The locked rotor currents with full voltage applied and the full-load slip are in the same range as those for class *B* and *C* motors. The starting torque with full voltage applied is nearly 1.25 times the full-torque.

QUESTIONS AND ANSWERS ON THREE-PHASE INDUCTION MOTORS

Q. 1. How do changes in supply voltage and frequency affect the performance of an induction motor ?

Ans. High voltage decreases both power factor and slip, but increases torque. Low voltage does just the opposite. Increase in frequency increases power factor but decreases the torque. However, per cent slip remains unchanged. Decrease in frequency decreases power factor but increases torque leaving per cent slip unaffected as before.

Q. 2. What is, in brief, the basis of operation of a 3-phase induction motor ?

Ans. The revolving magnetic field which is produced when a 3-phase stator winding is fed from a 3-phase supply.

Q. 3. What factors determine the direction of rotation of the motor ?

Ans. The phase sequence of the supply lines and the order in which these lines are connected to the stator winding.

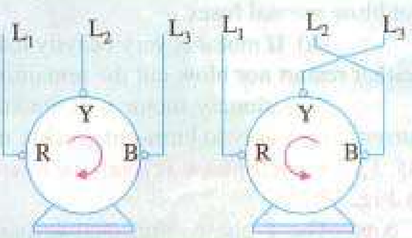


Fig. 35.57

Q. 4. How can the direction of rotation of the motor be reversed ?

Ans. By transposing or changing over any two line leads, as shown in Fig. 35.57.

Q. 5. Why are induction motors called asynchronous ?

Ans. Because their rotors can never run with the synchronous speed.

Q. 6. How does the slip vary with load ?

Ans. The greater the load, greater is the slip or slower is the rotor speed.

Q. 7. What modifications would be necessary if a motor is required to operate on voltage different from that for which it was originally designed ?

Ans. The number of conductors per slot will have to be changed in the same ratio as the change in voltage. If the voltage is doubled, the number of conductors per slot will have to be doubled.

Q. 8. Enumerate the possible reasons if a 3-phase motor fails to start.

Ans. Any one of the following reasons could be responsible :

1. one or more fuses may be blown.
2. voltage may be too low.
3. the starting load may be too heavy.
4. worn bearings due to which the armature may be touching field laminae, thus introducing excessive friction.

Q. 9. A motor stops after starting *i.e.* it fails to carry load. What could be the causes ?

Ans. Any one of the following:

1. hot bearings, which increase the load by excessive friction.
2. excessive tension on belt, which causes the bearings to heat.
3. failure of short cut-out switch.
4. single-phasing on the running position of the starter.

Q. 10. Which is the usual cause of blow-outs in induction motors ?

Ans. The commonest cause is single-phasing.

Q. 11. What is meant by 'single-phasing' and what are its causes ?

Ans. By single-phasing is meant the opening of one wire (or leg) of a three-phase circuit whereupon the remaining leg at once becomes single-phase. When a three-phase circuit functions normally, there are three distinct currents flowing in the circuit. As is known, any two of these currents use the third wire as the return path *i.e.* one of the three phases acts as a return path for the other two. Obviously, an open circuit in one leg kills two of the phases and there will be only one current or phase working, even though two wires are left intact. The remaining phase attempts to carry all the load. The usual cause of single-phasing is, what is generally referred to as ***running fuse***, which is a fuse whose current-carrying capacity is equal to the full-load current of the motor connected in the circuit. This fuse will blow-out whenever there is overload (either momentary or sustained) on the motor.

Q. 12. What happens if single-phasing occurs when the motor is running ? And when it is stationary ?

Ans. (*i*) If already running and carrying half load or less, the motor will continue running as a single-phase motor on the remaining single-phase supply, without damage because half loads do not blow normal fuses.

(*ii*) If motor is very heavily loaded, then it will stop under single-phasing and since it can neither restart nor blow out the remaining fuses, the burn-out is very prompt.

A stationary motor will not start with one line broken. In fact, due to heavy standstill current, it is likely to burn-out quickly unless immediately disconnected.

Q. 13. Which phase is likely to burn-out in a single-phasing delta-connected motor, shown in Fig. 35.58.

Ans. The Y-phase connected across the live or operative lines carries nearly three times its normal current and is the one most likely to burn-out.

The other two phases R and B, which are in series across L_2 and L_3 carry more than their full-load currents.

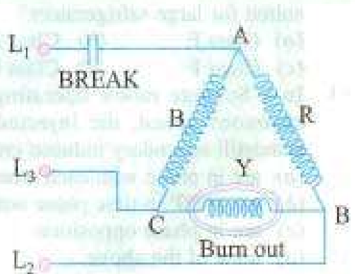


Fig. 35.58

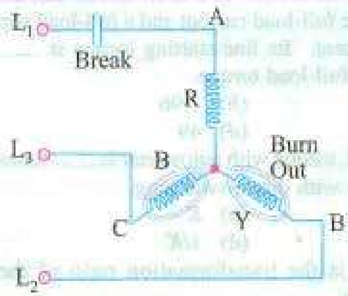


Fig. 35.59

Q. 14. What currents flow in single-phasing star-connected motor of Fig. 35.59.

Ans. With L_1 disabled, the currents flowing in L_2 and L_3 and through phases Y and B in series will be of the order of 250 per cent of the normal full-load current, 160 per cent on 3/4 load and 100 per cent on 1/2 load.

Q. 15. How can the motors be protected against single-phasing ?

Ans. (*i*) By incorporating a combined overload and single-phasing relay in the control gear.

(*ii*) By incorporating a phase-failure relay in the control gear. The relay may be either voltage or current-operated.

Q. 16. Can a 3-phase motor be run on a single-phase line ?

Ans. Yes, it can be. But a phase-splitter is essential.

Q. 17. What is a meant by a phase-splitter ?

Ans. It is a device consisting of a number of capacitors so connected in the motor circuit that it produces, from a single input wave, three output waves which differ in phase from each other.

Q. 18. What is the standard direction of rotation of an induction motor ?

Ans. Counterclockwise, when looking from the front end *i.e.* non-driving end of the motor.

Q. 19. Can a wound-motor be reversed by transposing any two leads from the slip-rings ?

Ans. No. There is only one way of doing so *i.e.* by transposing any two line leads.

Q. 20. What is jogging ?

Ans. It means inching a motor i.e. make it move a little at a time by constant starting and stopping.

Q. 21. What is meant by plugging ?

Ans. It means stopping a motor by instantaneously reversing it till it stops.

Q. 22. What are the indications of winding faults in an induction motor ?

Ans. Some of the indications are as under:

(*i*) excessive and unbalanced starting currents

(*ii*) some peculiar noises and (*iii*) overheating.

OBJECTIVE TESTS – 35

1. In the circle diagram for a 3-ϕ induction motor, the diameter of the circle is determined by
   (*a*) rotor current
   (*b*) exciting current
   (*c*) total stator current
   (*d*) rotor current referred to stator.
2. Point out the WRONG statement.
   Blocked rotor test on a 3-ϕ induction motor helps to find
   (*a*) short-circuit current with normal voltage
   (*b*) short-circuit power factor
   (*c*) fixed losses
   (*d*) motor resistance as referred to stator.
3. In the circle diagram of an induction motor, point of maximum input lies on the tangent drawn parallel to
   (*a*) output line
   (*b*) torque line
   (*c*) vertical axis
   (*d*) horizontal axis.

4. An induction motor has a short-circuit current 7 times the full-load current and a full-load slip of 4 per cent. Its line-starting torque is .... times the full-load torque.
(*a*) 7 (*b*) 1.96
(*c*) 4 (*d*) 49

5. In a SCIM, torque with autostarter is .... times the torque with direct-switching.
(*a*) K^2 (b) K
(c) $1/K^2$ (d) $1/K$
where K is the transformation ratio of the autostarter.

6. If stator voltage of a SCIM is reduced to 50 per cent of its rated value, torque developed is reduced by .... per cent of its full-load value.
(*a*) 50 (*b*) 25
(c) 75 (*d*) 57.7

7. For the purpose of starting an induction motor, a Y-Δ switch is equivalent to an auto-starter of ratio.......per cent.
(*a*) 33.3 (*b*) 57.7
(*c*) 73.2 (*d*) 60.

8. A double squirrel-cage motor (DSCM) scores over SCIM in the matter of
(*a*) starting torque
(b) high efficiency under running conditions
(c) speed regulation under normal operating conditions
(*d*) all of the above.

9. In a DSCM, outer cage is made of high resistance metal bars primarily for the purpose of increasing its
(*a*) speed regulation
(*b*) starting torque
(*c*) efficiency
(*d*) starting current.

10. A SCIM with 36-slot stator has two separate windings : one with 3 coil groups/ phase/pole and the other with 2 coil groups/phase/pole. The obtainable two motor speeds would be in the ratio of
(*a*) 3 : 2 (*b*) 2 : 3
(c) 2 : 1 (*d*) 1 : 2

11. A 6-pole 3-ϕ induction motor taking 25 kW from a 50-Hz supply is cumulatively-cascaded to a 4-pole motor. Neglecting all losses, speed of the 4-pole motor would be .... r.p.m.
(*a*) 1500 (*b*) 1000
(c) 600 (*d*) 3000.
and its output would be .... kW.
(*e*) 15 (*f*) 10
(g) 50/3 (*h*) 2.5.

12. Which class of induction motor will be well suited for large refrigerators?
(*a*) Class E (*b*) Class B
(*c*) Class F (*d*) Class C

13. In a Schrage motor operating at supersynchronous speed, the injected emf and the standstill secondary induced emf
(*a*) are in phase with each other
(*b*) are at 90° in time phase with each other
(*c*) are in phase opposition
(*d*) none of the above.
(Power App.-III, Delhi Univ. July 1987)

14. For starting a Schrage motor, 3-ϕ supply is connected to
(*a*) stator
(*b*) rotor via slip-rings
(*c*) regulating winding
(*d*) secondary winding via brushes.

15. Two separate induction motors, having 6 poles and 5 poles respectively and their cascade combination from 60 Hz, 3-phase supply can give the following synchronous speeds in rpm
(*a*) 720, 1200, 1500 and 3600
(*b*) 720, 1200 1800
(*c*) 600, 1000, 15000
(*d*) 720 and 3000
(Power App.-II, Delhi Univ.Jan 1987)

16. Mark the WRONG statement.
A Schrage motor is capable of behaving as a/an
(*a*) inverted induction motor
(*b*) slip-ring induction motor
(*c*) shunt motor
(*d*) series motor
(*e*) synchronous motor.

17. When a stationary 3-phase induction motor is switched on with one phase disconnected
(*a*) it is likely to burn out quickly unless immediately disconnected
(b) it will start but very slowly
(c) it will make jerky start with loud growing noise
(d) remaining intact fuses will be blown out due to heavy inrush of current

18. If single-phasing of a 3-phase induction motor occurs under running conditions, it
(*a*) will stall immediately
(*b*) will keep running though with slightly increased slip
(*c*) may either stall or keep running depending on the load carried by it
(*d*) will become noisy while it still keeps running.

ANSWERS

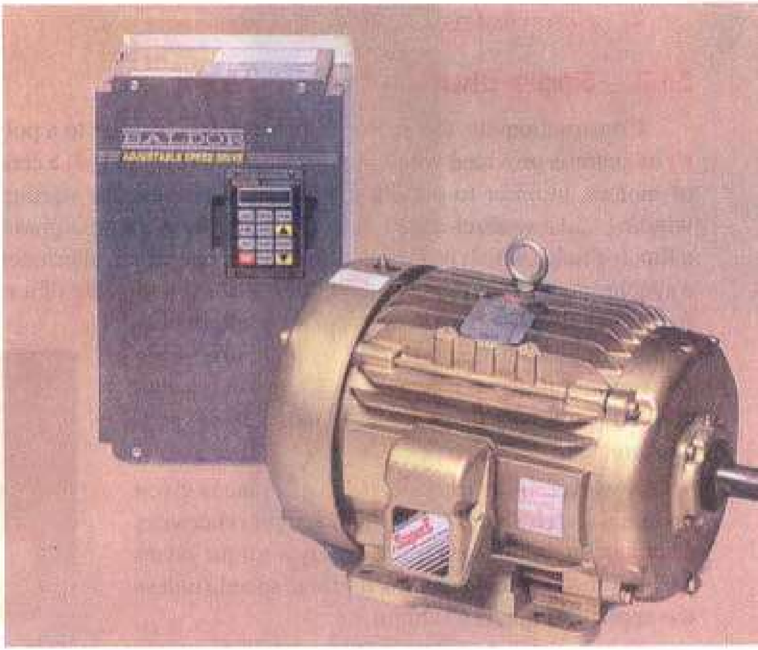
1. *c* 2. *c* 3. *d* 4. *b* 5. *a* 6. *c* 7. *b* 8. *d* 9. *b* 10. *a* 11. *c, f* 12. *d* 13. *a* 14. *b* 15. *a* 16. *d* 17. *a* 18. *c*

CHAPTER 36

SINGLE-PHASE MOTORS

Learning Objectives

- Types of Single-phase Motors
- Single-phase Induction Motor
- Double-field Revoling Theory
- Making Single-phase Induction Motor Self-starting
- Equivalent Circuit of Single-phase Induction Motor without Core Loss
- Equivalent Cuicuit With Core Loss
- Types of Capacitors Start Motors
- Capacitor Start-and-Run Motor
- Shaded-pole Single-phase Motor
- Repulsion Type Motors
- Repulsion Motor
- Repulsion Principle
- Compensated Repulsion Motor
- Repulsion-start Induction-Run Motor
- Repulsion Induction Motor
- A.C. Series Motors
- Universal Motor
- Speed control of Universal Motors
- Reluctance Motor
- Hysteresis Motor



Single phase motor

36.1. Types of Single-Phase Motors

Such motors, which are designed to operate from a single-phase supply, are manufactured in a large number of types to perform a wide variety of useful services in home, offices, factories, workshops and in business establishments etc. Small motors, particularly in the fractional kilo watt sizes are better known than any other. In fact, most of the new products of the manufacturers of space vehicles, aircrafts, business machines and power tools etc. have been possible due to the advances made in the design of fractional-kilowatt motors. Since the performance requirements of the various applications differ so widely, the motor-manufacturing industry has developed many different types of such motors, each being designed to meet specific demands.

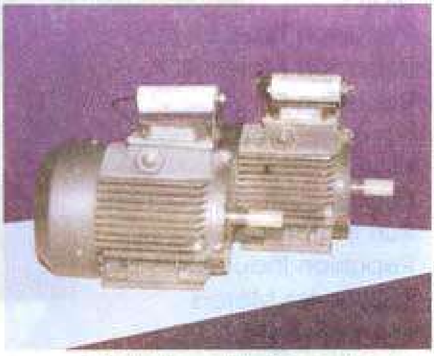
Split-phase motor. Switch mechanism disconnects start winding when motor reached three-fourths of rated speed

Single-phase motors may be classified as under, depending on their construction and method of starting :

1. *Induction Motors* (split-phase, capacitor and shaded-pole etc.)
2. *Repulsion Motors* (sometime called Inductive-Series Motors)
3. *A.C. Series Motor*
4. *Un-excited Synchronous Motors*

36.2. Single-phase Induction Motor

Constructionally, this motor is, more or less, similar to a polyphase induction motor, except that (*i*) its stator is provided with a single-phase winding and (*ii*) a centrifugal switch is used in some types of motors, in order to cut out a winding, used only for starting purposes. It has distributed stator winding and a squirrel-cage rotor. When fed from a single-phase supply, its stator winding produces a flux (or field) which is only *alternating i.e.* one which alternates along one space axis only. It is not a synchronously revolving (or rotating) flux, as in the case of a two- or a three-phase stator winding, fed from a 2-or 3-phase supply. Now, an alternating or pulsating flux acting on a *stationary* squirrel-cage rotor cannot produce rotation (only a revolving flux can). That is why a single-phase motor is *not self-starting*.



Single-phase induction motor

However, if the rotor of such a machine is given an initial start by hand (or small motor) or otherwise, *in either* direction, then immediately a torque arises and the motor accelerates to its final speed (unless the applied torque is too high).

This peculiar behaviour of the motor has been explained in two ways : (*i*) by two -field or double-field revolving theory and (*ii*) by cross-field theory. Only the first theory will be discussed briefly.

36.3. Double-field Revolving Theory

This theory makes use of the idea that an alternating uni-axial quantity can be represented by two oppositely-rotating vectors of half magnitude. Accordingly, an *alternating* sinusoidal flux can be represented by two ***revolving*** fluxes, each equal to half the value of the alternating flux and each rotating synchronously ($N_s = 120f/P$) in opposite direction*.

As shown in Fig. 36.1 (a), let the alternating flux have a maximum value of Φ_m. Its component fluxes A and B will each be equal to $\Phi_m/2$ revolving in anticlockwise and clockwise directions respectively.

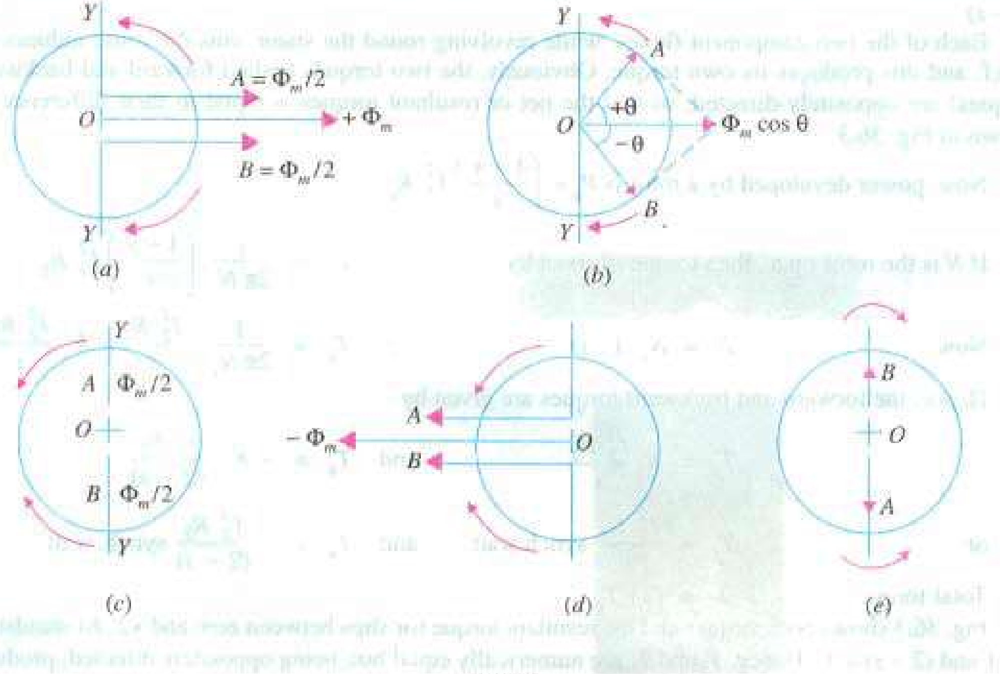


Fig. 36.1

After some time, when A and B would have rotated through angle $+\theta$ and $-\theta$, as in Fig. 36.1 (b), the resultant flux would be

$$= 2 \times \frac{\Phi_m}{2} \cos \frac{2\theta}{2} = \Phi_m \cos \theta$$

After a quarter cycle of rotation, fluxes A and B will be oppositely-directed as shown in Fig. 36.1 (c) so that the resultant flux would be zero.

After half a cycle, fluxes A and B will have a resultant of $-2 \times \Phi_m/2 = -\Phi_m$. After three-quarters of a cycle, again the resultant is

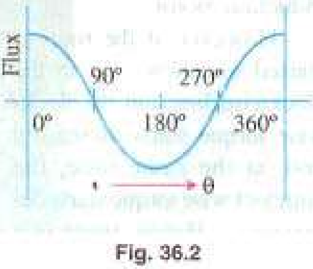


Fig. 36.2

* For example, a flux given by $\Phi = \Phi_m \cos 2\pi ft$ is equivalent to two fluxes revolving in opposite directions, each with a magnitude of $1/2\Phi$ and an angular velocity of $2\pi f$. It may be noted that Euler's expressions for $\cos\theta$ provides interesting justification for the decomposition of a pulsating flux. His expression is

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

The term $e^{j\theta}$ represents a vector rotated clockwise through an angle θ whereas $e^{-j\theta}$ represents rotation in anticlockwise direction. Now, the above given flux can be expressed as

$$\phi_m \cos 2\pi ft = \frac{\phi_m}{2}\left(e^{j2\pi ft} + e^{-j2\pi ft}\right)$$

The right-hand expression represents two oppositely-rotating vectors of half magnitude.

zero, as shown in Fig. 36.1 (*e*) and so on. If we plot the values of resultant flux against θ between limits $\theta = 0°$ to $\theta = 360°$, then a curve similar to the one shown in Fig. 36.2 is obtained. That is why an alternating flux can be looked upon as composed of two revolving fluxes, each of half the value and revolving synchronously in opposite directions.

It may be noted that if the slip of the rotor is s with respect to the forward rotating flux (*i.e.* one which rotates in the same direction as rotor) then its slip with respect to the backward rotating flux is $(2 - s)$*.

Each of the two component fluxes, while revolving round the stator, cuts the rotor, induces an e.m.f. and this produces its own torque. Obviously, the two torques (called forward and backward torques) are oppositely-directed, so that the net or resultant torques is equal to their difference as shown in Fig. 36.3.

Now, power developed by a rotor is $P_g = \left(\dfrac{1-s}{s}\right) I_2^2\, R_2$

If N is the rotor r.p.s., then torque is given by $\quad T_g = \dfrac{1}{2\pi N} \cdot \left(\dfrac{1-s}{s}\right) I_2^2\, R_2$

Now, $\quad N = N_s(1 - s) \quad \therefore \quad T_g = \dfrac{1}{2\pi N_s} \cdot \dfrac{I_2^2\, R_2}{s} = k \cdot \dfrac{I_2^2\, R_2}{s}$

Hence, the forward and backward torques are given by

$$T_f = K\,\frac{I_2^2\, R_2}{s} \quad \text{and} \quad T_b = -K \cdot \frac{I_2^2\, R_2}{(2-s)}$$

or

$$T_f = \frac{I_2^2\, R_2}{s} \text{ synch.watt} \quad \text{and} \quad T_b = -\frac{I_2^2\, R_2}{(2-s)} \text{ synch. watt}$$

Total torque $\quad T = T_f + T_b$

Fig. 36.3 shows both torques and the resultant torque for slips between zero and +2. At standstill, $s = 1$ and $(2 - s) = 1$. Hence, T_f and T_b are numerically equal but, being oppositely directed, produce no resultant torque. That explains why there is no ***starting*** torque in a single-phase induction motor.

However, if the rotor is started somehow, say, in the clockwise direction, the clockwise torque starts increasing and, at the same time, the anticlockwise torque starts decreasing. Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

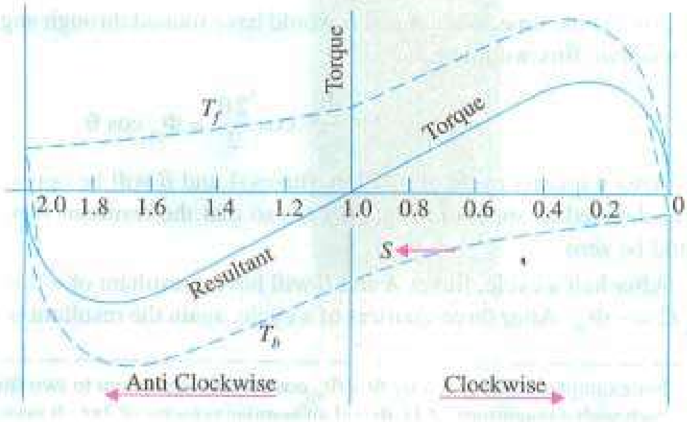


Fig. 36.3

* It may be proved thus : If N is the r.p.m. of the rotor, then its slip with respect to forward rotating flux is

$$s = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s} \quad \text{or} \quad \frac{N}{N_s} = 1 - s$$

Keeping in mind the fact that the backward rotating flux rotates opposite to the rotor, the rotor slip with respect to this flux is

$$s_b = \frac{N_s - (-N)}{N_s} = 1 + \frac{N}{N_s} = 1 + (1 - s) = (2 - s)$$

36.4. Making Single-phase Induction Motor Self-starting

As discussed above, a single-phase induction motor is not self-starting. To overcome this drawback and make the motor self-starting, it is temporarily converted into a two-phase motor during starting period. For this purpose, the stator of a single-phase motor is provided with an extra winding, known as *starting* (or auxiliary) winding, in addition to the *main* or *running winding*. The two windings are spaced 90° electrically apart and are connected in *parallel* across the single-phase supply as shown in Fig. 36.4.

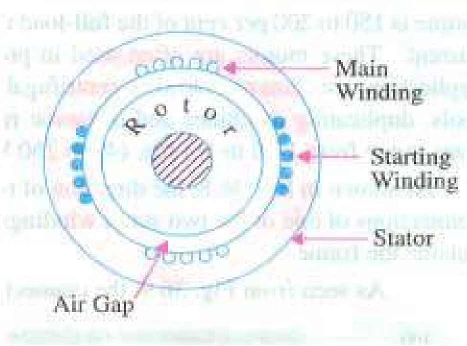


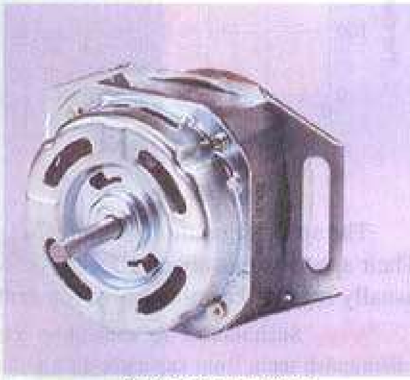
Fig. 36.4

It is so arranged that the phase-difference between the currents in the two stator windings is very large (ideal value being 90°). Hence, the motor behaves like a two-phase motor. These two currents produce a revolving flux and hence make the motor self-starting.

There are many methods by which the necessary phase-difference between the two currents can be created.

(*i*) In *split*-phase machine, shown in Fig. 36.5 (*a*), the main winding has low resistance but high reactance whereas the starting winding has a high resistance, but low reactance. The resistance of the starting winding may be increased either by connecting a high resistance R in series with it or by choosing a high-resistance fine copper wire for winding purposes.

Hence, as shown in Fig. 36.5 (*b*), the current I_s drawn by the starting winding lags behind the applied voltage V by a small angle whereas current I_m taken by the main winding lags behind V by a very large angle. Phase angle between I_s and I_m is made as large as possible because the starting torque of a split-phase motor is proportional to sin α. A centrifugul switch S is connected in series with the starting winding and is located inside the motor. Its function is to automatically disconnect the starting winding from the supply when the motor has reached 70 to 80 per cent of its full-load speed.


Single-phase motor.

In the case of split-phase motors that are hermetically sealed in refrigeration units, instead of internally-mounted centrifugal switch, an electromagnetic type of relay is used. As shown in Fig. 36.6, the relay coil is connected in series with main winding and the pair of contacts which are normally open, is included in the starting winding.

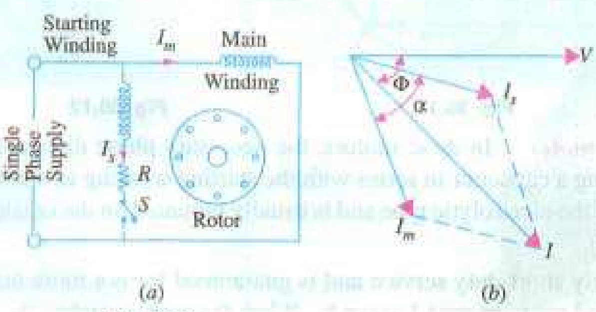


Fig. 36.5

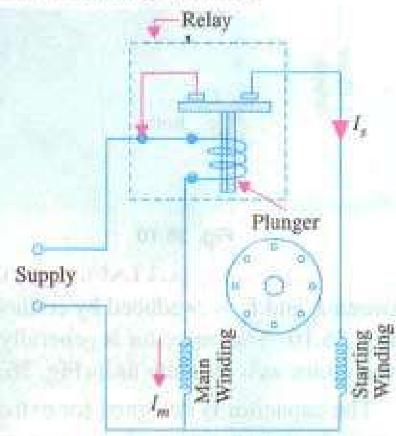


Fig. 36.6

During starting period, when I_m is large, relay contacts close thereby allowing I_s to flow and the motor starts as usual. After motor speeds up to 75 per cent of full-load speed, I_m drops to a value that is low enough to cause the contacts to open.

A typical torque/speed characteristic of such a motor is shown in Fig. 36.7. As seen, the starting torque is 150 to 200 per cent of the full-load torque with a starting current of 6 to 8 times the full-load current. These motors are often used in preference to the costlier capacitor-start motors. Typical applications are : fans and blowers, centrifugal pumps and separators, washing machines, small machine tools, duplicating machines and domestic refrigerators and oil burners etc. Commonly available sizes range from 1/20 to 1/3 h.p. (40 to 250 W) with speeds ranging from 3,450 to 865 r.p.m.

As shown in Fig. 36.8, the direction of rotation of such motors can be reversed by reversing the connections of one of the two stator windings (not both). For this purpose, the four leads are brought outside the frame.

As seen from Fig. 36.9, the connections of the starting winding have been reversed.

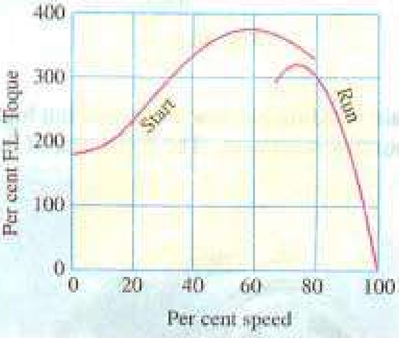


Fig. 36.7

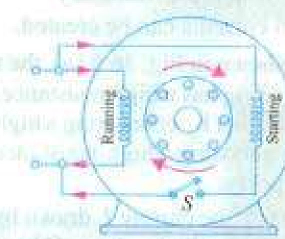


Fig. 36.8

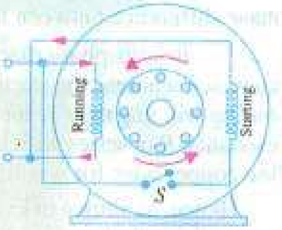


Fig. 36.9

The speed regulation of standard split-phase motors is nearly the same as of the 3-phase motors. Their speed varies about 2 to 5% between no load and full-load. For this reason such motors are usually regarded as practically constant-speed motors.

Note. Such motors are sometimes referred to as resistance-start split-phase induction motors in order to distinguish them from capacitor-start induction run and capacitor start-and-run motors described later.

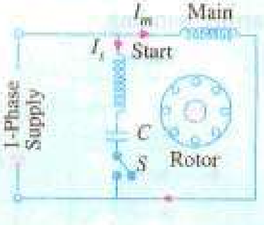


Fig. 36.10

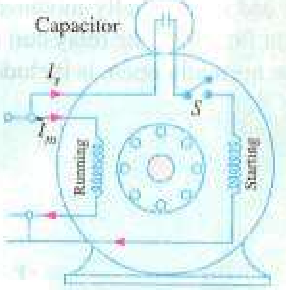


Fig. 36.11

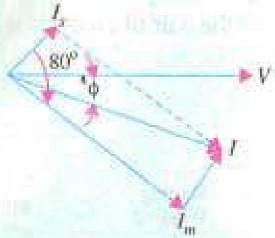


Fig. 36.12

(ii) Capacitor-start Induction-run motors. In these motors, the necessary phase difference between I_s and I_m is produced by connecting a capacitor in series with the starting winding as shown in Fig. 36.10. The capacitor is generally of the electrolytic type and is usually mounted on the outside of the motor as a separate unit (Fig. 36.11).

The capacitor is designed for extremely short-duty service and is guaranteed for not more than 20 periods of operation per hour, each period not to exceed 3 seconds. When the motor reaches about

75 per cent of full speed, the centrifugal switch S opens and cuts out both the starting winding and the capacitor from the supply, thus leaving only the running winding across the lines. As shown in Fig. 36.12, current I_m drawn by the main winding lags the supply voltage V by a large angle whereas I_s leads V by a certain angle. The two currents are out of phase with each other by about 80° (for a 200-W 50-Hz motor) as compared to nearly 30° for a split-phase motor. Their resultant current I is small and is almost in phase with V as shown in Fig. 36.12.

Capacitor start/capacitor-run motor.

Since the torque developed by a split-phase motor is proportional to the sine of the angle between I_s and I_m, it is obvious that the increase in the angle (from 30° to 80°) alone increases the starting torque to nearly twice the value developed by a standard split-phase induction motor. Other improvements in motor design have made it possible to increase the starting torque to a value as high as 350 to 450 per cent.

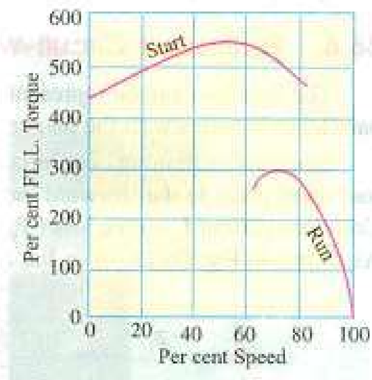


Fig. 36.13

Typical performance curve of such a motor is shown in Fig. 36.13.

36.5. Equivalent Circuit of a Single-phase Induction Motor–Without Core Loss

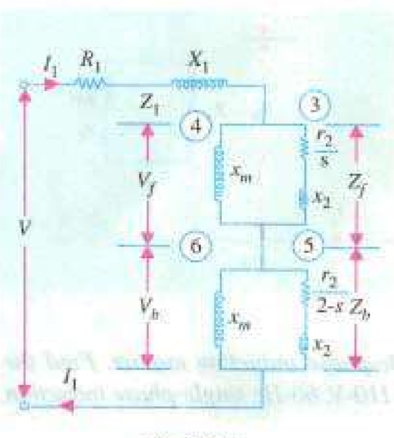


Fig. 36.14

A single-phase motor may be looked upon as consisting of two motors, having a common stator winding, but with their respective rotors revolving in opposite directions. The equivalent circuit of such a motor based on double-field revolving theory is shown in Fig. 36.14. Here, the single-phase motor has been imagined to be made-up of (*i*) one stator winding and (*ii*) two imaginary rotors. The stator impedance is $Z = R_1 + j\,X_1$. The impedance of each rotor is $(r_2 + jx_2)$ where r_2 and x_2 represent half the actual rotor values in stator terms (*i.e.* x_2 stands for half the standstill reactance of the rotor, as referred to stator). Since iron loss has been neglected, the exciting branch is shown consisting of exciting reactance only. Each rotor has been assigned half the magnetising reactance* (*i.e* x_m represents half the actual reactance). The impedance of 'forward running' rotor is

$$Z_f = \frac{j\,x_m\left(\dfrac{r_2}{s} + j\,x_2\right)}{\dfrac{r_2}{s} + j\,(x_m + x_2)}$$

and it runs with a slip of s. The impedance of 'backward' running rotor is

* In fact, full values are shown by capital letters and half values by small letters.

$$Z_b = \frac{j\,x_m\left(\dfrac{r_2}{2-s} + j\,x_2\right)}{\dfrac{r_2}{2-s} + j\,(x_m + x_2)}$$

and it runs with a slip of $(2 - s)$. Under standstill conditions, $V_f = V_b$, but under running conditions V_f is almost 90 to 95% of the applied voltage.

The forward torque in synchronous watts is $T_f = I_3^2\, r_2 / s$. Similarly, backward torque is $T_b = I_5^2\, r_2 / (2 - s)$

The total torque is $\quad T = T_f - T_b$.

36.6. Equivalent Circuit–With Core Loss

The core loss can be represented by an equivalent resistance which may be connected either in parallel or in series with the magnetising reactance as shown in Fig. 36.15.

Since under running conditions V_f is very high (and V_b is correspondingly, low) most of the iron loss takes place in the 'forward motor' consisting of the common stator and forward-running rotor. Core-loss current I_w = core loss / V_f. Hence, half value of core-loss equivalent resistance is $r_c = V_f / I_w$. As shown in Fig. 36.15 (a), r_c has been connected in parallel with x_m in each rotor.

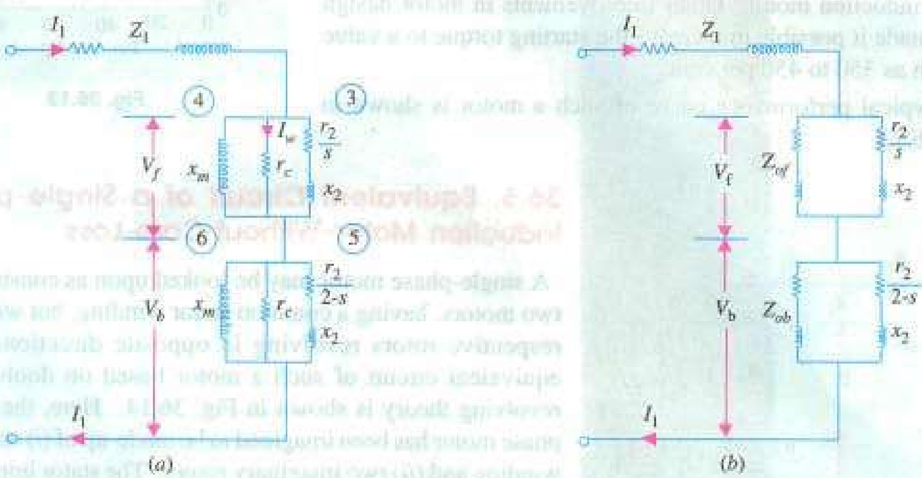


Fig. 36.15

Example 36.1. *Discuss the revolving field theory of single-phase induction motors. Find the mechanical power output at a slip of 0.05 of the 185-W, 4-pole, 110-V, 60-Hz single-phase induction motor, whose constants are given below:*

Resistance of the stator main winding	R_1	= 1.86 ohm
Reactance of the stator main winding	X_1	= 2.56 ohm
Magnetizing reactance of the stator main winding	X_m	= 53.5 ohm
Rotor resistance at standstill	R_2	= 3.56 ohm
Rotor reactance at standstill	X_2	= 2.56 ohm

(Elect. Machines, Nagpur Univ. 1991)

Solution. Here, $X_m = 53.5\ \Omega$, hence $x_m = 53.5/2 = 26.7\ \Omega$

Similarly, $\quad r_2 = R_2 / 2 = 3.56 / 2 = 1.78\ \Omega$ and $x_2 = X_2 / 2 = 2.56 / 2 = 1.28\ \Omega$

$$\therefore \quad Z_f = \frac{j x_m \left(\frac{r_2}{s} + j x_2\right)}{\frac{r_2}{s} + j (x_2 + x_m)} = x_m \frac{\frac{r_2}{s} \cdot x_m + j\left[(r_2/s)^2 + x_2 x_0\right]}{(r_2/s)^2 + x_0^2} \quad \text{where } x_0 = (x_m + x_2)$$

$$\therefore \quad Z_f = 26.7 \frac{(1.78/0.05) \times 26.7 + j\,[(1.78/0.05)^2 + 1.28 \times 27.98]}{(1.78/0.05)^2 + (27.98)^2}$$

$$= 12.4 + j\,17.15 = 21.15 \angle 54.2°$$

Similarly,
$$Z_b = \frac{j x_m \left(\frac{r_2}{2-s} + j x_2\right)}{\frac{r_2}{2-s} + j (x_2 + x_m)} = x_m \frac{\left(\frac{r_2}{2-s}\right) x_m + j\left[\left(\frac{r_2}{2-s}\right)^2 + x_0 x_2\right]}{\left(\frac{r_2}{2-s}\right)^2 + x_0^2}$$

$$= 26.7 \frac{(1.78/1.95) \times 26.7 + j\,[(1.78/1.95)^2 + 1.28 \times 27.98]}{(1.78/1.95)^2 + (27.98)^2}$$

$$= 0.84 + j\,1.26 = 1.51 \angle 56.3°$$

$$\mathbf{Z_1} = R_1 + j X_1 = 1.86 + j\,2.56 = 3.16 \angle 54°$$

Total circuit impedance is

$$\mathbf{Z_{01}} = \mathbf{Z_1} + \mathbf{Z_f} + \mathbf{Z_b} = (1.86 + j\,2.56) + (12.4 + j\,17.15) + (0.84 + j\,1.26)$$

$$= 15.1 + j\,20.97 = 25.85 \angle 54.3°$$

Motor current $I_1 = 110/25.85 = 4.27$ A

$$V_f = I_1 Z_f = 4.27 \times 21.15 = 90.4 \text{ V};\ V_b = I_1 Z_b = 4.27 \times 1.51 = 6.44 \text{ V}$$

$$Z_3 = \sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2} = 35.7\ \Omega,\ Z_5 = \sqrt{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} = 1.57\ \Omega$$

$$I_3 = V_f / Z_3 = 90.4/35.7 = 2.53 \text{ A},\ I_5 = V_b / Z_5 = 6.44/1.57 = 4.1 \text{ A}$$

$$T_f = I_3^2 R_2 / s = 228 \text{ synch. watts},\ T_5 = I_5^2\, r_2/(2-s) = 15.3 \text{ synch. watts.}$$

$$T = T_f - T_b = 228 - 15.3 = 212.7 \text{ synch. watts}$$

Output = synch. watt × $(1 - s) = 212.7 \times 0.95 =$ **202 W**

Since friction and windage losses are not given, this also represents the net output.

Example 36.2. *Find the mechanical power output of 185-W, 4 pole, 110-V, 50-Hz single-phase induction motor, whose constants are given below at a slip of 0.05.*

$R_1 = 1.86\ \Omega$ $X_1 = 2.56\ \Omega$ $X_\phi = 53.5\ \Omega$ $R_2 = 3.56\ \Omega$ $X_2 = 2.56\ \Omega$

Core loss = 3.5 W, Friction and windage loss = 13.5 W.

(Electrical Machines-III, Indore Univ. 1987)

Solution. It would be seen that major part of this problem has already been solved in Example 36.1. Let us, now, assume that V_f = 82.5% of 110 V = 90.7 V. Then the core-loss current $I_c = 35/90.7 = 0.386$ A ; $r_c = 90.7/0.386 = 235\ \Omega$.

Motor I

conductance of core-loss branch $= 1/r_c = 1/235 = 0.00426$ S

susceptance of magnetising branch $= -j/x_m = -j/26.7 = -j\,0.0374$ S

admittance of branch 3 $= \dfrac{(r_2/s) - j x_2}{(r_2/s)^2 + x_2^2} = 0.028 - j\,0.00101$ S

admittance of 'motor' *I* is $\mathbf{Y}_f$ = $0.00426 - j\,0.0374 + 0.028 - j\,0.00101$
$= 0.03226 - j\,0.03841$ S

impedance $\mathbf{Z}_f$ = $1/\mathbf{Y}_f = 12.96 + j\,15.2$ or $19.9\ \Omega$

Motor II

$$\text{admittance of branch 5} = \frac{\dfrac{r_2}{2-s} - j\,x_2}{\left(\dfrac{r_2}{2-s}\right)^2 + x_2^2} = \frac{0.91 - j\,1.28}{2.469} = 0.369 - j\,0.517$$

admittance of 'motor' **II**, $\mathbf{Y}_b$ = $0.00426 - j\,0.0374 + 0.369 - j\,0.517$
$= 0.3733 - j\,0.555$ S

Impedance of 'motor' II, $\mathbf{Z}_b = 1/\mathbf{Y}_b = 0.836 + j\,1.242$ or $1.5\ \Omega$

Impedance of entire motor (Fig. 36.16) $Z_{01} = \mathbf{Z}_1 + \mathbf{Z}_f + \mathbf{Z}_b = 15.66 + j\,19$ or $24.7\ \Omega$

$$I_1 = V/Z_{01} = 110/24.7 = 4.46 \text{ A}$$
$$V_f = I_1 Z_f = 4.46 \times 19.9 = 88.8 \text{ V}$$
$$V_b = 4.46 \times 1.5 = 6.69 \text{ V}$$

$$I_3 = 88.8/35.62 = 2.5 \text{ A}$$
$$I_5 = 6.69/1.57 = 4.25 \text{ A}$$
$$T_f = I_3^2(r_2/s) = 222 \text{ synch. watt}$$
$$T_b = I_5^2\left(\frac{r_2}{2-s}\right) = 16.5 \text{ synch. watt}$$
$$T = T_f - T_b = 205.5 \text{ synch. watt}$$

Watts converted = synch. watt $(1 - s)$
$= 205.5 \times 0.95 = 195$ W

Net output = $195 - 13.5 =$ **181.5 W.**

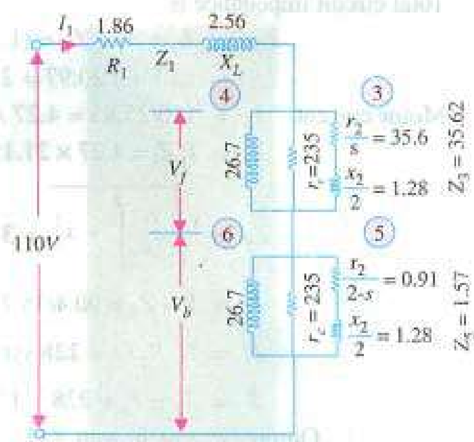


Fig. 36.16

Example 36.3. *A 250-W, 230-V, 50-Hz capacitor-start motor has the following constants for the main and auxiliary windings: Main winding, $Z_m = (4.5 + j\,3.7)$ ohm. Auxiliary winding $Z_a = (9.5 + j\,3.5)$ ohm. Determine the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.*

(Electrical Machines-III, South Gujarat Univ. 1985)

Solution. Let X_C be the reactance of the capacitor connected in the auxiliary winding.

Then $\mathbf{Z}_a = 9.5 + j\,3.5 - j\,X_C = (9.5 + jX)$
where X is the net reactance.

Now, $\mathbf{Z}_m = 4.5 + j\,3.5 = 5.82 \angle 39.4°$ ohm

Obviously, I_m lags behind V by 39.4°

Since, time phase angle between I_m and I_a has to be 90°, I_a must lead V by

$$(90° - 39.4°) = 50.6°.$$

For auxiliary winding, $\tan\phi_a = X/R$ or $\tan 50.6° = X/9.5$

or $X = 9.5 \times 1.217 = 11.56\ \Omega$ (capacitive)

$\therefore$ $X_C = 11.56 + 3.5 = 15.06\ \Omega$ $\quad\therefore\quad 15.06 = 1/314\,C;\ C =$ **211 μF.**

Tutorial Problem No. 36.1.

1. A 1-ϕ, induction motor has stator windings in space quadrature and is supplied with a single-phase voltage of 200 V at 50 Hz. The standstill impedance of the main winding is (5.2 + j 10.1) and of the auxiliary winding is (19.7 + j 14.2). Find the value of capitance to be inserted in the auxiliary winding for maximum starting torque. *(Electrical Machines-III, Indore Univ. July, 1977)*
2. A 230-V, 50-Hz, 6-pole, single-phase induction motor has the following constants.
$$r_1 = 0.12\ \Omega,\ r_2 = 0.14\ \Omega,\ x_1 = x_2 = 0.25\ \Omega,\ x_m = 15\ \Omega.$$
If the core loss is 250 W and friction and windage losses are 500 W, determine the efficiency and torque at $s = 0.05$. *(Electrical Machines-IV, Bangalore Univ. Aug. 1978)*
3. Explain how the pulsating mmf of a single-phase induction motor may be considered equivalent to two oppositely-rotating fields. Develop an expression for the torque of the motor.
A 125-W, 4-pole, 110-V, 50-Hz single-phase induction motor has the no-load rotational loss of 25 watts and total rotor copper loss at rated load of 25 watts at a slip of 0.06. The rotor I^2R loss may be neglected.
At a slip s = 0.06, what is the power input to the machine ?
(Electrical Machines-III, Indore Univ. July 1977)

36.7. Types of Capacitor-start Motors

Some of the important types of such motors are given below :

1. Single-voltage, externally-reversible type

In this motor, four leads are brought outside its housing; two from the main winding and two from the starting-winding circuit. These four leads are necessary for external reversing. As usual, internally, the starting winding is connected in series with the electrolytic capacitor and a centrifugal switch. The direction of rotation of the motor can be easily reversed externally by reversing the starting winding leads with respect to the running winding leads.

2. Single-voltage, non-reversible type

In this case, the starting winding leads are connected internally to the leads of the running winding. Consequently, there are only two external leads in such motors. Obviously, direction of rotation cannot be reversed unless the motor is taken apart and leads of the starting winding reversed.

3. Single-voltage reversible and with thermostat type

Many motors are fitted with a device called thermostat which provides protection against overload, overheating, and short-circuits etc. The thermostat usually consists of a bimetallic element that is connected in series with the motor and is often mounted on the outside of the motor.

The wiring diagram of a capacitor-start motor fitted with this protective device is shown in Fig. 36.17. When due to some reasons, excessive current flows through the motor, it produces abnormal heating of the bimetallic strip with the result that it bends and opens the contact points thus disconnecting the motor from the supply lines. When the thermostat element cools, it automatically closes the contacts*.

In the case of capacitor-start motors used for refrigerators, generally a terminal block is attached to the motor. Three out of the four block terminals are marked T, TL and L as shown in Fig. 36.18. Thermostat is connected to T and TL, capacitor between L and the unmarked terminal and the supply lines to TL and L.

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* However, in some thermal units, a reset button has to be operated manually to restore the motor to operation. In certain types of thermal units, a heating element is used for heating the bimetallic strip. In that case, the heating element is connected in the line and the element or bimetallic strip is placed either inside the heating unit or besides it.

4. Single-voltage, non-reversible with magnetic switch type

Such motors are commonly used in refrigerators where it is not possible to use a centrifugal switch. The circuit diagram is similar to that shown in Fig. 36.6. Since their application requires just one direction of rotation, these motors are not connected for reversing.

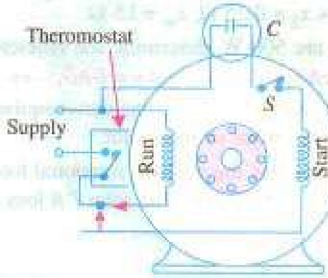


Fig. 36.17

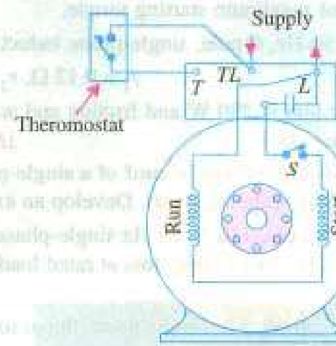


Fig. 36.18

One disadvantage of a capacitor-start motor having magnetic switch lies in the possibility that slight overloads may operate the plunger thereby connecting the starting winding circuit to the supply. Since this winding is designed to operate for very short periods (3 seconds or less) it is likely to be burnt out.

5. Two-voltage, non-reversible Type

These motors can be operated from two a.c. voltage either 110 V and 220 V or 220 V and 440 V. Such motors have two main windings (or one main winding in two sections) and one starting winding with suitable number of leads brought out to permit changeover from one voltage to another.

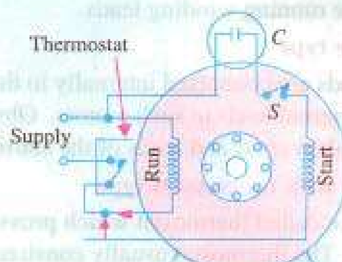


Fig. 36.19

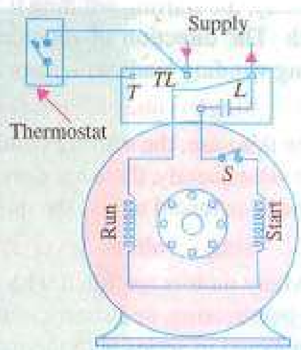


Fig. 36.20

When the motor is to operate from lower voltage, the two main windings are connected in parallel (Fig. 36.19). Whereas for higher voltage, they are connected in series (Fig. 34.20). As will be seen from the above circuit diagrams, the starting winding is always operated on the low-voltage for which purpose it is connected across one of the main windings.

6. Two-voltage, reversible type

External reversing is made possible by means of two additional leads that are brought out from the starting winding.

Fig. 36.21 and 36.22 show connections for clockwise and anticlockwise rotations respectively when motor is operated from lower voltage. Similar wiring diagram can be drawn for higher voltage supply.

7. Single-voltage, three-lead reversible type

In such motors, a two-section running winding is used. The two sections R_1 and R_2 are internally connected in series and one lead of the starting winding is connected to the mid-point of R_1 and R_2.

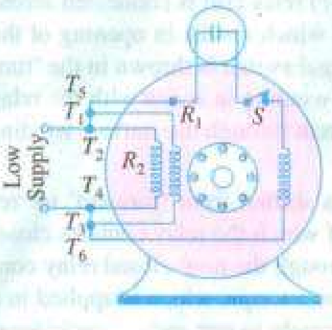


Fig. 36.21

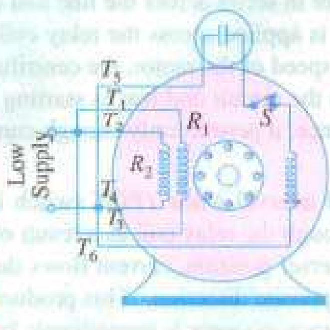


Fig. 36.22

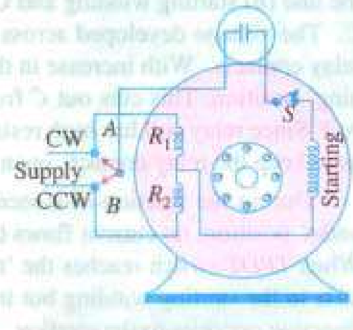


Fig. 36.23

The second lead of the starting winding and both leads of the running winding are brought outside as shown in Fig. 36.23. When the external lead of the starting winding is connected to point A, the winding is connected across R_1 and the motor runs clockwise. When the lead of starter winding is connected to point B, it is connected across R_2. Since current flowing through starting winding is reversed, the motor runs in counter-clockwise direction.

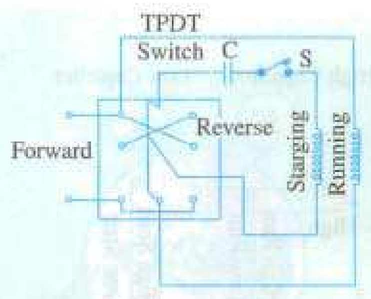


Fig. 36.24

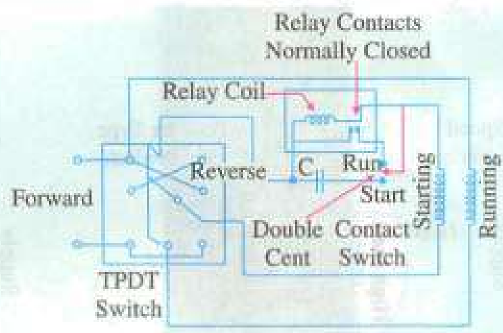


Fig. 36.25

8. Single-voltage, instantly-reversible type

Normally, a motor must be brought to complete rest before it can be started in the reverse direction. It is so because the centrifugal switch cannot close unless the motor has practically stopped. Since starting winding is disconnected from supply when the motor is running, reversal of starting winding leads will not affect the operation of the motor. This reversal is achieved by a triple-pole, double-throw (*TPDT*) switch as shown in Fig. 36.24. The switch consists of three blades or poles which move together as one unit in either of the two positions. In one position of the switch (shown in one figure) motor runs clockwise and in the other, in counter-clockwise direction. Obviously, in this type of arrangement, it is necessary to wait till motor stops.

In certain applications where instant reversal is necessary while the motor is operating at full speed, a relay is fitted in the circuit to short-circuit the centrifugal switch and connect the starting winding in the circuit in the reversed direction (Fig. 36.25).

It will be seen that when at rest, the double-contact centrifugal switch is in the 'start' position. In this position, two connections are made :

(*i*) the starting winding and capacitor *C* are placed in series across the supply line and

(*ii*) the coil of the normally-closed relay is connected across *C*

With the manual *TPDT* switch in the 'forward' position (*a*) running winding is connected across the line (*b*) starting winding and *C* are in series across the line and (*c*) relay coil is connected across *C*. The voltage developed across *C* is applied across the relay coil which results in opening of the relay contacts. With increase in the speed of the motor, the centrifugal switch is thrown in the 'running' position. This cuts out *C* from the circuit and leaves starting winding in series with the relay coil. Since relay coil has high resistance, it permits only enough current through the starting winding as to keep the relay contacts open.

During the fraction-of-a-second interval while *TPDT* switch is shifted from 'forward' to 'reverse' position, no current flows through the relay coil as a result of which the relay contacts close. When *TPDT* switch reaches the 'reverse' position, current flows through the now-closed relay contacts to the starting winding but in opposite direction. This produces a torque which is applied in a direction opposite to the rotation. Hence, (*i*) rotor is immediately brought to rest and (*ii*) centrifugal switch falls to the 'start' position. As before, *C* is put in series with the starter winding and the motor starts rotating in the opposite direction.

9. Two-speed type

Speed can be changed by changing the number of poles in the winding for which purpose two separate running windings are placed in the slots of the stator, one being 6-pole winding and the other, 8-pole winding. Only one starting winding is used which always acts in conjunction with the higher-speed running winding. The double-action or transfer type centrifugal switch *S* has two contact

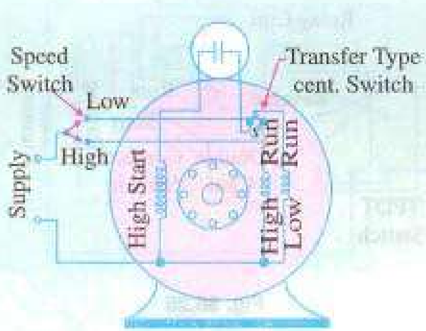


Fig. 36.26

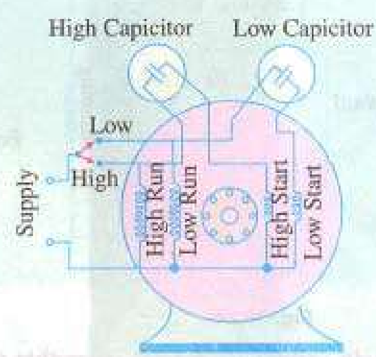


Fig. 36.27

points on the 'start' side and one on the 'run' side. As shown in Fig. 36.26, an external speed switch is used for changing the motor speed. The motor will always start on high speed irrespective of whether the speed switch is on the 'high' or 'low' contact. If speed switch is set on 'low', then as soon as the motor comes up to speed, the centrifugal switch

(*a*) cuts out the starting winding and high-speed running winding and

(*b*) cuts in the low-speed running winding.

10. Two-speed with two-capacitor type

As shown in Fig. 36.27, this motor has two running windings, two starting windings and two capacitors. One capacitor is used for high-speed operation and the other for low-speed operation. A double centrifugal switch *S* is employed for cutting out the starting winding after start.

36.8. Capacitor Start-and-Run Motor

This motor is similar to the capacitor-start motor [Art.36.4 (*ii*)] except that the starting winding and capacitor are connected in the circuit at ***all times***. The advantages of leaving the capacitor permanently in circuit are (*i*) improvement of over-load capacity of the motor (*ii*) a higher power factor (*iii*) higher efficiency and (*iv*) quieter running of the motor which is so much desirable for small power drives in offices and laboratories. Some of these motors which start and run with one value of capacitance in the circuit are called ***single-value*** capacitor-run motors. Other which start with high value of capacitance but run with a low value of capacitance are known as ***two-value*** capacitor-run motors.

Capacitor starts and run motor

(*i*) Single-value capacitor-Run Motor

It has one running winding and one starting winding in series with a capacitor as shown in Fig. 36.28. Since capacitor remains in the circuit permanently, this motor is often referred to as permanent-

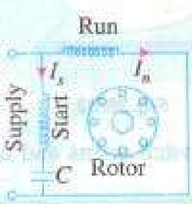


Fig. 36.28

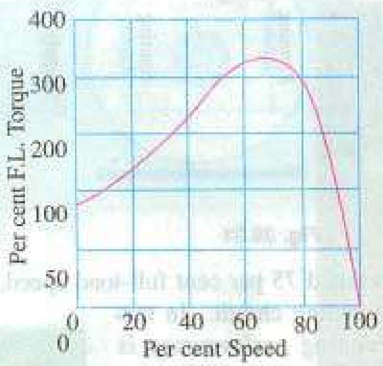


Fig. 36.29

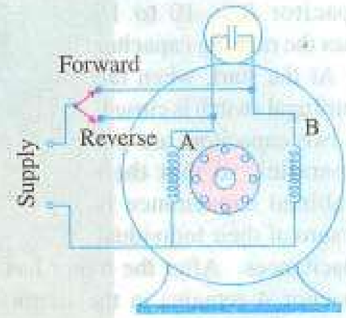


Fig. 36.30

split capacitor-run motor and behaves practically like an unbalanced 2-phase motor. Obviously, there is no need to use a centrifugal switch which was necessary in the case of capacitor-start motors. Since the same capacitor is used for starting and running, it is obvious that neither optimum starting nor optimum running performance can be obtained because value of capacitance used must be a compromise between the best value for starting and that for running. Generally, capacitors of 2 to 20 μF capacitance are employed and are more expensive oil or pyranol-insulated foil-paper capacitors because of continuous-duty rating. The low value of the capacitor results in small starting torque which is about 50 to 100 per cent of the rated torque (Fig. 36.29). Consequently, these motors are used where the required starting torque is low such as air - moving equipment *i.e.* fans, blowers and voltage regulators and also oil burners where quiet operation is particularly desirable.

One unique feature of this type of motor is that it can be easily reversed by an external switch provided its ***running and starting windings are identical***. One serves as the running winding and the other as a starting winding for one direction of rotation. For reverse rotation, the one that previously served as a running winding becomes the starting winding while the former starting winding serves as the running winding. As seen from Fig. 36.30 when the switch is in the forward position, winding *B* serves as running winding and *A* as starting winding. When switch is in 'reverse' position, winding *A* becomes the running winding and *B* the starting winding.

Such reversible motors are often used for operating devices that must be moved back and forth very frequently such as rheostats, induction regulations, furnace controls, valves and arc-welding controls.

(ii) Two-value capacitor-Run Motor

This motor starts with a high capacitor in series with the starting winding so that the starting torque is high. For running, a lower capacitor is substituted by the centrifugal switch. Both the running and starting windings remain in circuit.

The two values of capacitance can be obtained as follows:

1. by using two capacitors in parallel at the start and then switching out one for low-value run. (Fig. 36.31) or
2. by using a step-up auto-transformer in conjunction with one capacitor so that effective capacitance value is increased for starting purposes.

In Fig. 36.31, B is an electrolytic capacitor of high capacity (short duty) and A is an oil capacitor of low value (continuous duty). Generally, starting capacitor B is 10 to 15 times the running capacitor A. At the start, when the centrifugal switch is closed, the two capacitors are put in parallel, so that their combined capacitance is the sum of their individual capacitances. After the motor has reached 75 per cent full-load speed, the switch opens and only capacitor A remains in the starting winding circuit. In this way, both optimum starting and running performance is achieved in such motors. If properly designed, such motors have operating characteristics very closely resembling those displayed by two-phase motors. Their performance is characterised by

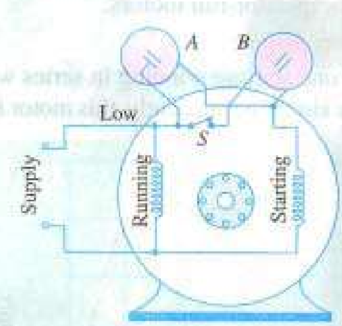


Fig. 36.31

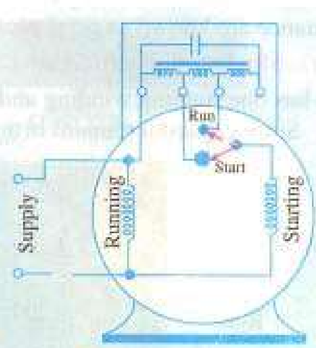


Fig.36.32

1. ability to start heavy loads
2. extremely quiet operation
3. higher efficiency and power factor
4. ability to develop 25 per cent overload capacity

Hence, such motors are ideally suited where load requirements are severe as in the case of compressors and fire strokers etc.

Auto-transformer

The use of an auto-transformer and single oil-type capacitor is illustrated in Fig.36.32. The transformer and capacitor are sealed in a rectangular iron box and mounted on top of the motor. The idea behind using this combination is that a capacitor of value C connected to the secondary of a step-up transformer, appears to the primary as though it had a value of K^2C where K is voltage transformation ratio. For example, if actual value of $C = 4$ μF and $K = 6$, then low-voltage primary acts as if it had a 144 μF ($= 6^2 \times 4$) capacitor connected across its terminals. Obviously, effective value of capacitance has increased 36 times. In the 'start' position of the switch, the connection is made to the mid-tap of the auto-transformer so that $K = 2$. Hence, effective value of capacitance at start is 4 times the running value and is sufficient to give a high starting torque. As the motor speeds up, the centrifugal switch shifts the capacitor from one voltage tap to another so that the voltage transformation ratio changes from higher value at starting to a lower value for running. The capacitor which is actually of the paper-tinfoil construction is immersed in a high grade insulation like wax or mineral oil.

36.9. Shaded-pole Single-phase Motor

Shaded pole single phase motor

In such motors, the necessary phase-splitting is produced by induction. These motors have salient poles on the stator and a squirrel-cage type rotor Fig. 36.33 shows a four-pole motor with the field poles connected in series for alternate polarity. One pole of such a motor is shown separately in Fig. 36.34. The laminated pole has a slot cut across the laminations approximately one-third distance from one edge. Around the small part of the pole is placed a short-circuited Cu coil known as ***shading coil***. This part of the pole is known as ***shaded*** part and the other as ***unshaded*** part. When an alternating current is passed through the exciting (or field) winding surrounding the whole pole, the axis of the pole shifts from the unshaded part a to the shaded part *b*. This shifting of the magnetic axis is, in effect, equivalent to the actual physical movement of the pole. Hence, the rotor starts rotating in the direction of this shift *i.e.* from unshaded part to the shaded part.

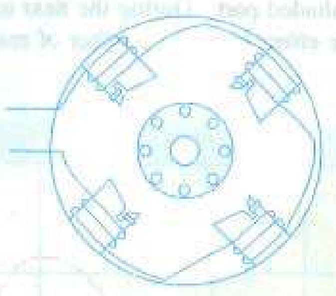


Fig. 36.33

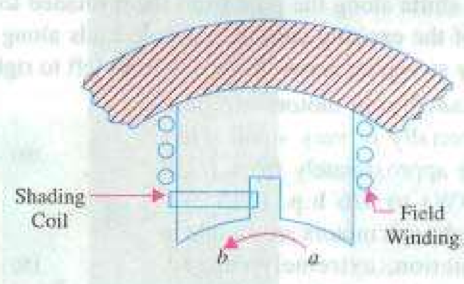


Fig. 36.34

Let us now discuss why shifting of the magnetic axis takes place. It is helpful to remember that the shading coil is highly inductive. When the alternating current through exciting coil tends to increase, it induces a current in the shading coil by transformer action in such a direction as to oppose its growth. Hence, flux density decreases in the shaded part when exciting current increases. However, flux density increases in the shaded part when exciting current starts decreasing (it being assumed that exciting current is sinusoidal).

In Fig. 36.35 (*a*) exciting current is rapidly increasing along *OA* (shown by dots). This will produce an e.m.f. in the shading coil. As shading coil is of low resistance, a large current will be set up in such a direction (according to Lenz's law) as to oppose the rise of exciting current (which is responsible for its production). Hence, the flux mostly shifts to the unshaded part and the magnetic axis lies along the middle of this part *i.e*. along *NC*.

Next, consider the moment when exciting current is near its peak value *i.e.* from point *A* to *B* [Fig. 36.35 (*b*)]. Here, the change in exciting current is very slow. Hence, practically no voltage and, therefore, no current is induced in the shading coil. The flux produced by exciting current is at its maximum value and is uniformly distributed over the pole face. So the magnetic axis shifts to the centre of the pole *i.e.* along positions *ND*.

Fig. 36.35 (*c*) represents the condition when the exciting current is rapidly decreasing from *B* to *C*. This again sets up induced current in the shading coil by transformer action. This current will flow in such a direction as to oppose this decrease in exciting current, with the result that the flux is strengthened in the shaded part of the pole. Consequently, the magnetic axis shifts to the middle part of the shaded pole *i.e.* along *NE*.

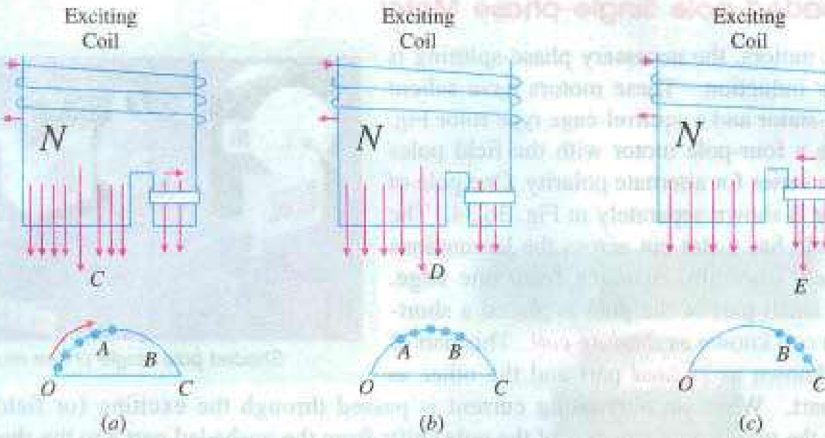


Fig. 36.35

From the above discussion we find that during the positive half-cycle of the exciting current, a *N*-pole shifts along the pole from the unshaded to the shaded part. During the next negative half-cycle of the exciting current, a *S*-pole trails along. The effect is as if a number of real poles were actually sweeping across the space from left to right.

Shaded pole motors are built commercially in very small sizes, varying approximately from 1/250 h.p. (3W) to 1/6 h.p. (125 W). Although such motors are simple in construction, extremely rugged, reliable and cheap, they suffer from the disadvantages of (*i*) low starting torque (*ii*) very little overload capacity and (*iii*) low efficiency. Efficiencies vary from 5% (for tiny sizes) to 35 (for higher ratings). Because of its low starting torque, the shaded-pole motor is generally used for small fans, toys, instruments, hair dryers, ventilators, circulators and electric clocks. It is also frequently used for such devices as churns, phonograph turntables and advertising displays etc. The direction of rotation of this motor cannot be changed, because it is fixed by the position of copper rings.

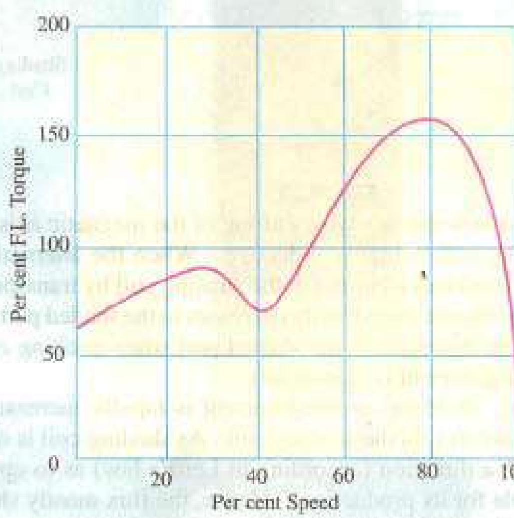
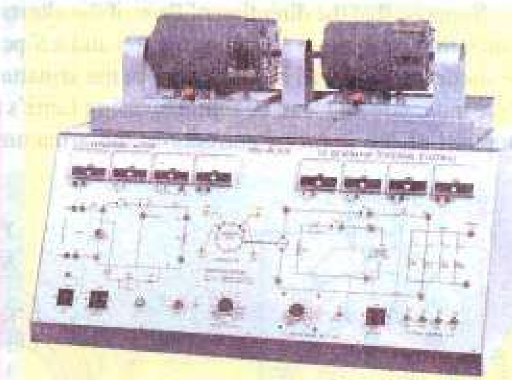


Fig. 36.36

A typical torque / speed curve for such a motor is shown in Fig. 36.36.

36.10. Repulsion Type Motors



Repulsion induction motor

These can be divided into the following four distinct categories :

1. **Repulsion Motor.** It consists of (*a*) one stator winding (*b*) one rotor which is wound like a d.c. armature (*c*) commutator and (*d*) a set of brushes, which are short-circuited and remain in contact with the commutator at all times. It operates continuously on the 'repulsion' principle. No short-circuiting mechanism is required for this type.
2. **Compensated Repulsion Motor.** It is identical with repulsion motor in all respects, except that (*a*) it carries an additional stator winding, called compensating winding (*b*) there is another set of two brushes which are placed midway between the usual short-circuited brush set. The compensating winding and this added set are connected in series.
3. **Repulsion-start Induction-run Motor.** This motor starts as a repulsion motor, but normally runs as an induction motor, with constant speed characteristics. It consists of (*a*) one stator winding (*b*) one rotor which is similar to the wire-wound d.c. armature (*c*) a commutator and (*d*) a centrifugal mechanism which short-circuits the commutator bars all the way round (with the help of a short-circuiting necklace) when the motor has reached nearly 75 per cent of full speed.
4. **Repulsion Induction Motor.** It works on the combined principle of repulsion and induction. It consists of (*a*) stator winding (*b*) two rotor windings : one squirrel cage and the other usual d.c. winding connected to the commutator and (*c*) a short-circuited set of two brushes.

It may be noted that repulsion motors have excellent characterstics, but are expensive and require more attention and maintenance than single-phase motors. Hence, they are being replaced by two-value capacitor motors for nearly all applications.

36.11. Repulsion Motor

Constructionally, it consists of the following :

1. Stator winding of the distributed non-salient pole type housed in the slots of a smooth-cored stator (just as in the case of split-phase motors). The stator is generally wound for four, six or eight poles.
2. A rotor (slotted core type) carrying a distributed winding (either lap or wave) which is connected to the commutator. The rotor is identical in construction to the d.c. armature.
3. A commutator, which may be one of the two types : an axial commutator with bars parallel to the shaft or a radial or vertical commutator having radial bars on which brushes press horizontally.
4. Carbon brushes (fitted in brush holders) which ride against the commutator and are used for conducting current through the armature (*i.e.* rotor) winding.

36.12. Repulsion Principle

To understand how torque is developed by the repulsion principle, consider Fig. 36.37 which shows a 2-pole salient pole motor with the magnetic axis vertical. For easy understanding, the stator winding has been shown with concentrated salient-pole construction (actually it is of distributed non-salient type). The basic functioning of the machine will be the same with either type of construction. As mentioned before, the armature is of standard d.c. construction with commutator and brushes (which are short-circuited with a low-resistance jumper).

Suppose that the direction of flow of the alternating current in the exciting or field (stator) winding is such that it creates a *N*-pole at the top and a *S*-pole at the bottom. The alternating flux produced by the stator winding will induce e.m.f. in the armature conductors by transformer action. The direction of the induced e.m.f. can be found by using Lenz's law and is as shown in Fig. 36.37 (*a*). However, the direction of the ***induced*** currents in the armature conductors will depend on the ***positions of the short-circuited brushes***. If brush axis is colinear with magnetic axis of the main poles, the directions of the induced currents (shown by dots and arrows) will be as indicated in Fig. 36.37 (*a*)*. As a result, the armature will become an electromagnet with a *N*-pole on its top, directly under the main *N*-pole and with a *S*-pole at the bottom, directly over the main *S*-pole. Because of this face-to-face positioning of the main and induced magnetic poles, no torque will be developed. The two forces of repulsion on top and bottom act along *YY′* in direct opposition to each other.**

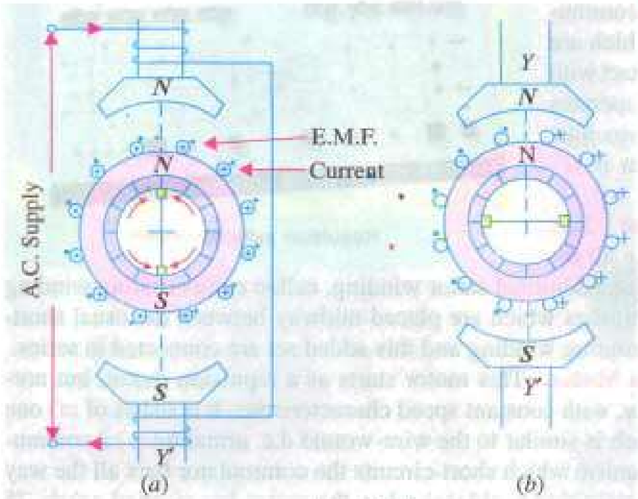


Fig. 36.37

If brushes are shifted through 90° to the position shown in Fig. 36.37 (*b*) so that the brush axis is at right angles to the magnetic axis of the main poles, the directions of the induced voltages at any time in the respective armature conductors are exactly the same as they were for the brush position of Fig. 36.37 (*a*). However, with brush positions of Fig. 36.37 (*b*), the voltages induced in the armature conductors in each path between the brush terminals will neutralize each other, hence there will be no net voltage across brushes to produce armature current. If there is no armature current, obviously, no torque will be developed.

If the brushes are set in position shown in Fig. 36.38 (*a*) so that the brush axis is neither in line with nor 90° from the magnetic axis *YY′* of the main poles, a net voltage*** will be induced between the brush terminals which will produce armature current. The armature will again act as an elec-

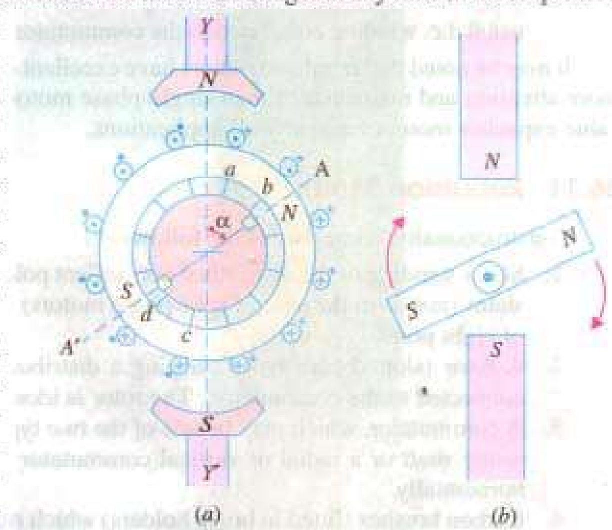


Fig. 36.38

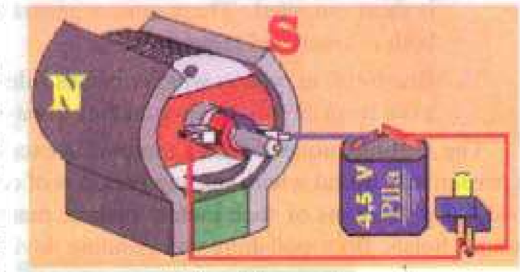
* It should be noted that during the next half-cycle of the supply current, the directions of the respective voltages will be in the opposite directions.

** Alternatively, the absence of the torque may be explained by arguing that the torques developed in the four quadrants neutralize each other.

*** It will be seen from Fig. 36.38 (*a*) that the induced voltages in conductors *a* and *b* oppose the voltages in other conductors lying above brush-axis. Similarly, induced voltages in conductors *c* and *d* oppose the voltages in other conductors, lying below the brush-axis. Yet the net voltage across brush terminals will be sufficient to produce current which will make the armature a powerful magnet.

tromagnet and develop its own N-and S-poles which, in this case, will not directly face the respective main poles. As shown in Fig. 36.38 (a), the armature poles lie along AA' making an angle of α with YY'.

Hence, *rotor* N-pole will be repelled by the main N-pole and the rotor S-pole will, similarly, be repelled by the ***main*** S-pole. Consequently, the rotor will rotate in clockwise direction [Fig.36.38 (b)]. Since the forces are those of ***repulsion***, it is appropriate to call the motor as repulsion motor.



Repulsion motor

It should be noted that if the brushes are shifted counter-clockwise from YY', rotation will also be counter-clockwise. Obviously, direction of rotation of the motor is determined by the position of brushes with respect to the main magnetic axis.

It is worth noting that the value of starting torque developed by such a motor will depends on the ***amount*** of brush-shift whereas direction of rotation will depend on the ***direction*** of shift [Fig. 36.39 (a)]. Maximum starting torque is developed at some position where brush axis makes, an angle lying between 0° and 45° with the magnetic axis of main poles. Motor speed can also be controlled by means of brush shift. Variation of starting torque of a repulsion motor with brush-shift is shown in Fig. 36.39 (b).

A straight repulsion type motor has high starting torque (about 350 per cent) and moderate starting current (about 3 to 4 times full-load value).

Principal shortcomings of such a motor are :

1. speed varies with changing load, becoming dangerously high at no load.
2. low power factor, except at high speeds.
3. tendency to spark at brushes.

36.13. Compensated Repulsion Motor

It is a modified form of the straight repulsion motor discussed above. It has an additional stator winding, called compensating winding whose purpose is (i) to improve power-factor and (ii) to provide better speed regulation. This winding is much smaller than the stator winding and is usually wound in the inner slots of each main pole and is connected in series with the armature (Fig. 36.40) through an additional set of brushes placed mid-way between the usual short-circuited brushes.

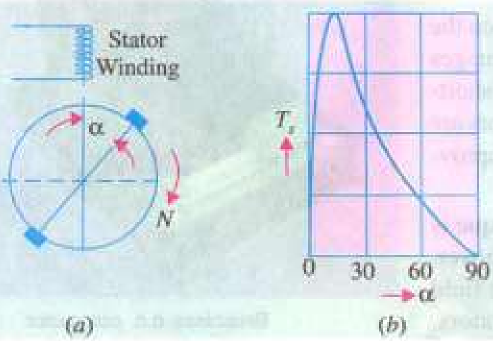


Fig. 36. 39

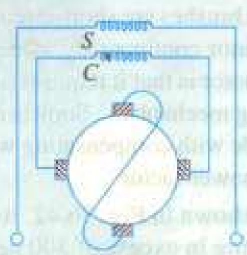


Fig. 36. 40

36.14. Repulsion-start Induction-Run Motor

As mentioned earlier, this motor starts as an ordinary repulsion motor, but after it reaches about 75 per cent of its full speed, centrifugal short-circuiting device short-circuits its commutator. From then on, it runs as an induction motor, with a short-circuited squirrel-cage rotor. After the commutator is short-circuited, brushes do not carry any current, hence they may also be lifted from the commutator, in order to avoid unnecessary wear and tear and friction losses.

Repulsion-start motors are of two different designs :

1. Brush-lifting type in which the brushes are automatically lifted from the commutator when it is short-circuited. These motors generally employ radial form of commutator and are built both in small and large sizes.
2. Brush-riding type in which brushes ride on the commutator at all times. These motors use axial form of commutator and are always built in small sizes.

The starting torque of such a motor is in excess of 350 per cent with moderate starting current. It is particularly useful where starting period is of comparatively long duration, because of high inertia loads. Applications of such motors include machine tools, commercial refrigerators, compressors, pumps, hoists, floor-polishing and grinding devices etc.

36.15. Repulsion Induction Motor

In the field of repulsion motor, this type is becoming very popular, because of its good all-round characteristics which are comparable to those of a compound d.c. motor. It is particularly suitable for those applications where the load can be removed entirely by de-clutching or by a loose pulley.

This motor is a combination of the repulsion and induction types and is sometimes referred to as ***squirrel-cage repulsion motor***. It possesses the desirable characteristics of a repulsion motor and the constant-speed characteristics of an induction motor.

It has the usual stator winding as in all repulsion motors. But there are two separate and independent windings in the rotor (Fig. 36.41).

(i) a squirrel-cage winding and

(ii) commutated winding similar to that of a d.c. armature.

Both these windings function during the *entire period* of operation of the motor. The commutated winding lies in the outer slots while squirrel-cage winding is located in the inner slots*. At start, the commutated winding supplies most of the torque, the squirrel-cage winding being practically inactive because of its high reactance. When the rotor accelerates, the squirrel-cage winding takes up a larger portion of the load.

The brushes are short-circuited and ride on the commutator continuously. One of the advantages of this motor is that it requires no centrifugal short-circuiting mechanism. Sometimes such motors are also made with compensating winding for improving the power factor.

As shown in Fig. 36.42, its starting torque is high, being in excess of 300 per cent. Moreover, it has a fairly constant speed regulation. Its field of application includes house-hold refrigerators, garage air pumps, petrol pumps, compressors, machine tools, mixing machines, lifts and hoists etc.

Brushless d.c. seromotor

* Hence, commutated winding has low resistance whereas the squirrel-cage winding has inherently a high reactance.

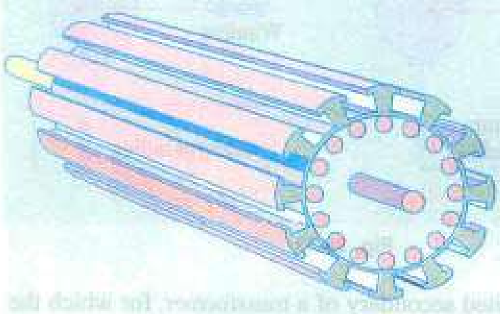


Fig. 36.41

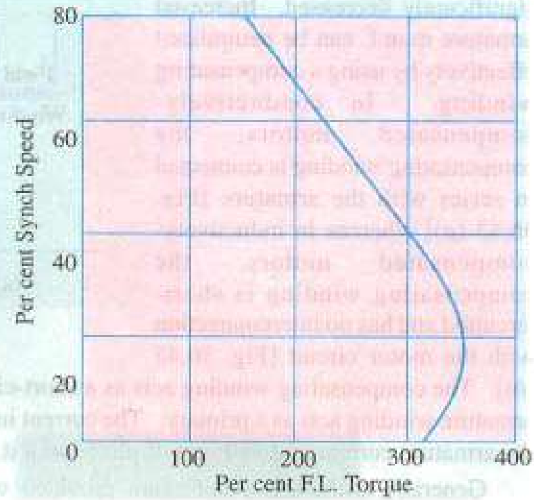


Fig. 36.42

These motors can be reversed by the usual brush-shifting arrangement.

36.16. A.C. Series Motors

If an ordinary d.c. series motor were connected to an a.c. supply, it will rotate and exert unidirectional torque* because the current flowing both in the armature and field reverses at the same time. But the performance of such a motor will not be satisfactory for the following reasons :

1. the alternating flux would cause excessive eddy current loss in the yoke and field cores which will become extremely heated.
2. vicious sparking will occur at brushes because of the huge voltage and current induced in the short-circuited armature coils during their commutation period.
3. power factor is low because of high inductance of the field and armature circuits.

Induction type AC sevro motor and control box

However, by proper modification of design and other refinements, a satisfactory single-phase motor has been produced.

The eddy current loss has been reduced by laminating the entire iron structure of the field cores and yoke.

Power factor improvement is possible only by reducing the magnitudes of the reactances of the field and armature windings. Field reactance is reduced by reducing the number of turns on the field windings. For a given current, it will reduce the field m.m.f. which will result in reduced air- gap flux. This will tend to increase the speed but reduce motor torque. To obtain the same torque, it will now be necessary to increase the number of armature turns proportionately. This will, however, result in increased inductive reactance of the armature, so that the overall reactance of the motor will not be

* However, torque developed is not of constant magnitude (as in d.c. series motors) but pulsates between zero and maximum value each half-cycle.

significantly decreased. Increased armature m.m.f. can be neutralized effectively by using a compensating winding. In conductively-compensated motors, the compensating winding is connected in series with the armature [Fig. 36.43 (*a*)] whereas in inductively-compensated motors, the compensating winding is short-circuited and has no interconnection with the motor circuit [Fig. 36.43 (*b*)]. The compensating winding acts as a short-circuited secondary of a transformer, for which the armature winding acts as a primary. The current in the compensating winding will be proportional to the armature current and 180° out of phase with it.

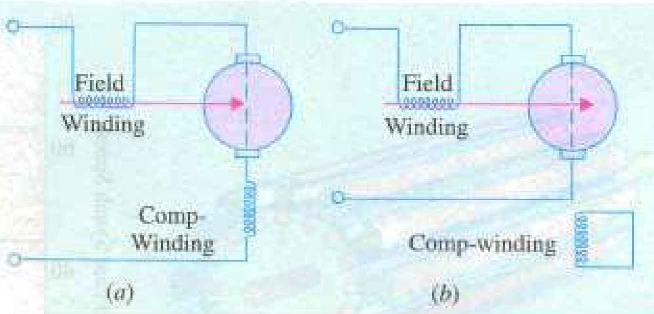


Fig. 36.43

Generally, all d.c. series motors are 'provided' with commutating poles for improving commutation (as in d.c. motors). But commutating poles alone will not produce satisfactory commutation, unless something is done to neutralize the huge voltage induced in the short-circuited armature coil by transformer action (this voltage is not there in d.c. series motor). It should be noted that in an a.c. series motor, the flux produced by the field winding is alternating and it induces voltage (by transformer action) in the short-circuited armature coil during its commutating period. The field winding, associated with the armature coil undergoing commutation, acts as primary and the armature coil during its commutating period acts as a short-circuited secondary. This transformer action produces heavy current in the armature coil as it passes through its commutating period and results in vicious sparking, unless the transformer voltage is neutralized. One method, which is often used for large motors, consists of shunting the winding of each commutating pole with a non-inductive resistance, as shown in Fig. 36.44 (*a*).

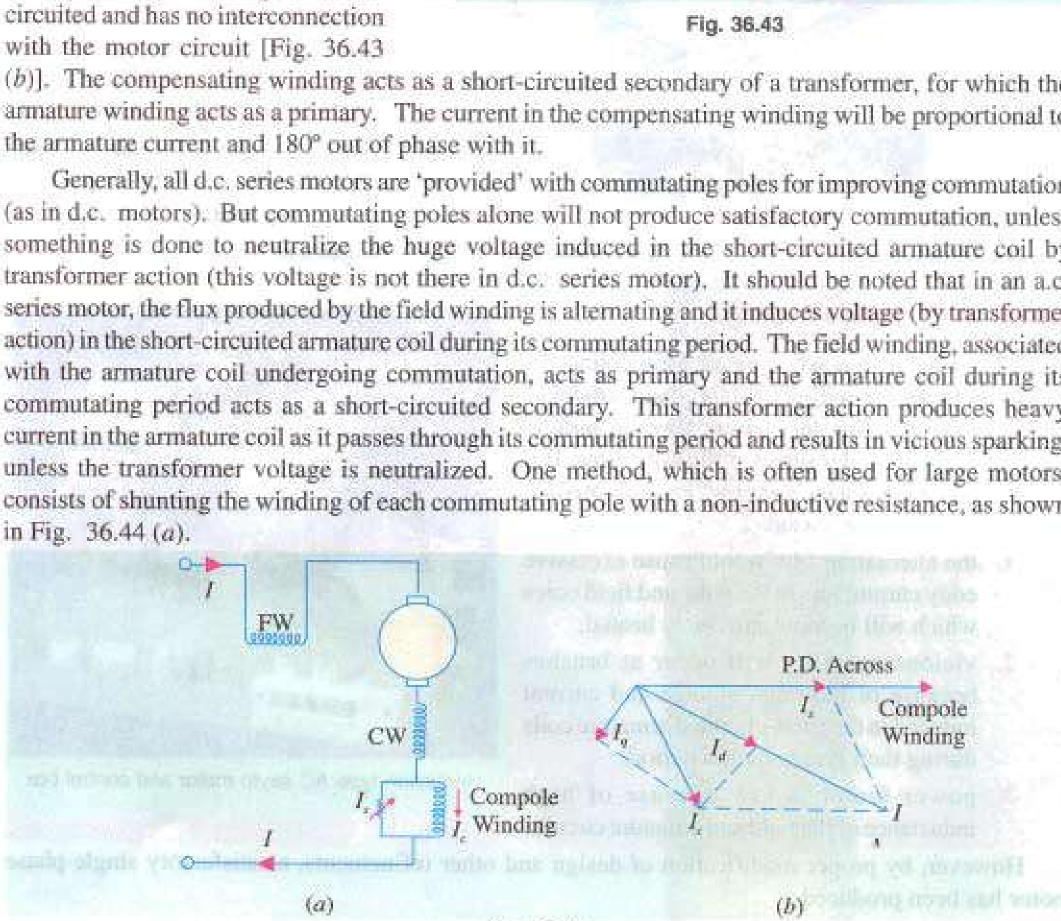


Fig. 36.44

Fig. 36.44. (*b*) shows the vector diagram of a shunted commutator pole. The current I_c through the commutating pole (which lags the total motor current) can be resolved into two rectangular components I_d and I_q as shown. I_d produces a flux which is in phase with total motor current I whereas flux produced by I_q lags I by 90°. By proper adjustment of shunt resistance (and hence I_s), the speed voltage generated in a short-circuited coil by the cutting of the 90° lagging component of the commutating pole flux may be made to neutralize the voltage induced by transformer action.

36.17. Universal Motor

A universal motor is defined as a motor which may be operated either on direct or single-phase a.c. supply at approximately the same speed and output.

In fact, it is a smaller version (5 to 150 W) of the a.c. series motor described in Art. 36.16. Being a series-wound motor, it has high starting torque and a variable speed characteristic. It runs at dangerously high speed on no-load. That is why such motors are usually built into the device they drive.

Generally, universal motors are manufactured in two types:

1. ***concentrated-pole, non-compensated type*** (low power rating)
2. ***distributed-field compensated type*** (high power rating)

Universal motor.

The non-compensated motor has two salient poles and is just like a 2-pole series d.c. motor except that whole of its magnetic path is laminated (Fig. 36.45). The laminated stator is necessary because the flux is alternating when motor is operated from a.c. supply. The armature is of wound type and similar to that of a small d.c. motor. It consists essentially of a laminated core having either straight or skewed slots and a commutator to which the leads of the armature winding are connected. The distributed-field compensated type motor has a stator core similar to that of a split-phase motor and a wound armature similar to that of a small d.c. motor. The compensating winding is used to reduce the reactance voltage present in the armature when motor runs on a.c. supply. This voltage is caused by the alternating flux by transformer action (Art. 36.16).

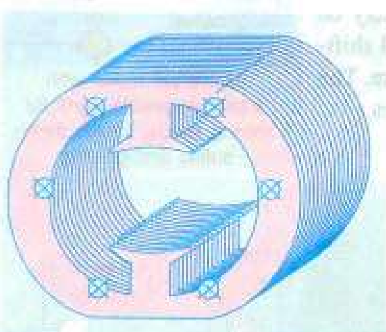


Fig. 36.45

In a 2-pole non-compensated motor, the voltage induced by transformer action in a coil during its commutation period is not sufficient to cause any serious commutation trouble. Moreover, high-resistance brushes are used to aid commutation.

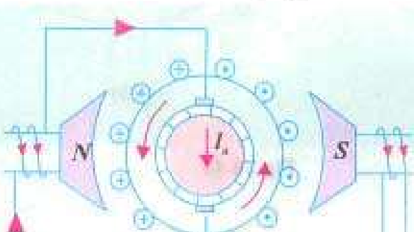


Fig. 36.46

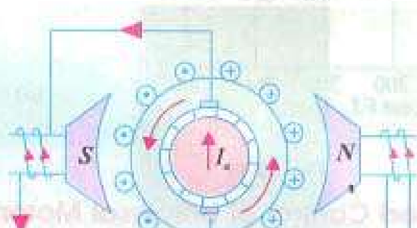


Fig.36.47

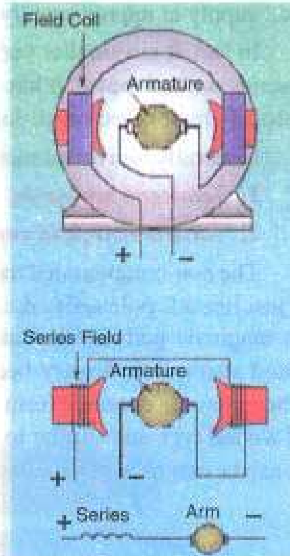
(*a*) Operation. As explained in Art. 36.16, such motors develop unidirectional torque, regardless of whether they operate on d.c. or a.c. supply. The production of unidirectional torque, when the motor runs on a.c. supply can be easily understood from Fig. 36.46. The motor works on the same principle as a d.c. motor *i.e.* force between the main pole flux and the current-carrying armature conductors. This is true regardless of whether the current is alternating or direct (Fig. 36.47).

(*b*) **Speed/Load Characteristic.** The speed of a universal motor varies just like that of a d.c. series motor *i.e.* low at full-load and high on no-load (about 20,000 r.p.m. in some cases). In fact, on no-load the speed is limited only by its own friction and windage load. Fig. 36.48 shows typical torque characteristics of a universal motor both for d.c. and a.c. supply. Usually, gear trains are used to reduce the actual load speeds to proper values.

(*c*) **Applications.** Universal motors are used in vacuum cleaners where actual motor speed is the load speed. Other applications where motor speed is reduced by a gear train are : drink and food mixers, portable drills and domestic sewing machine etc.

(*d*) **Reversal of Rotation.** The concentrated-pole (or salient-pole) type universal motor may be reversed by reversing the flow of current through either the armature or field windings. The usual method is to interchange the leads on the brush holders (Fig.36.49).

The distributed-field compensated type universal motor may be reversed by interchanging either the armature or field leads and shifting the brushes against the direction in which the motor will rotate. The extent of brush shift usually amounts to several commutator bars.



Universal motor with interchangeable parts for mixing solids, liquids semi-solids and coating.

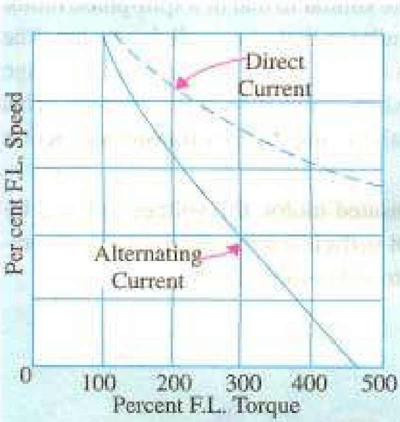


Fig. 36.48

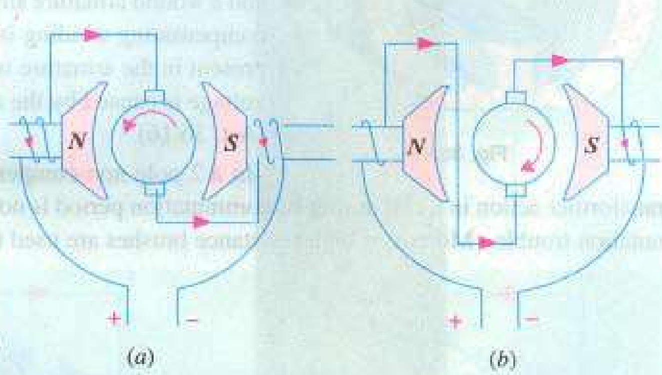


Fig.36.49

36.18. Speed Control of Universal Motors

The following methods are usually employed for speed-control purposes :

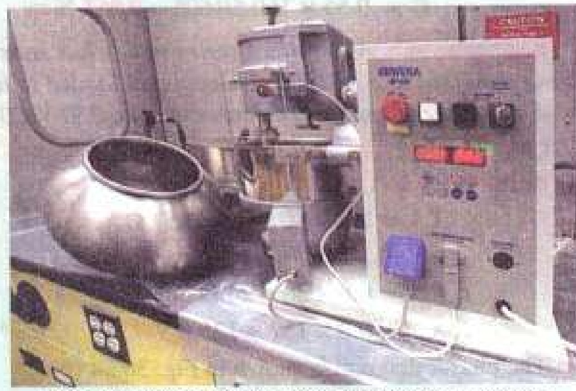
(*i*) **Resistance Method.** As shown in Fig. 36.50, the motor speed is controlled by connecting a variable resistance R in series with the motor. This method is employed for motors used in sewing machines. The amount of resistance in the circuit is changed by means of a foot-pedal.

(*ii*) **Tapping-field Method.** In this method, a field pole is tapped at various points and speed is controlled by varying the field strength (Fig. 36.51). For this purpose, either of the following two arrangements may be used :

(*a*) The field pole is wound in various sections with different sizes of wire and taps are brought out from each section.

(*b*) Nichrome resistance wire is wound over one field pole and taps are brought out from this wire.

(iii) **Centrifugal Mechanism.** Universal motors, particularly those used for home food and drink mixers, have a number of speeds. Selection is made by a centrifugal device located inside the motor and connected, as shown in Fig. 36.52. The switch is adjustable by means of an external lever. If the motor speed rises above that set by the lever, the centrifugal device opens two contacts and inserts resistance R in the circuit, which causes the motor speed to decrease. When motor runs slow, the two contacts close and short-circuit the resistance, so that the motor speed rises. This process is repeated so rapidly that variations in speed are not noticeable.



Universal motor used for home food and drink mixers

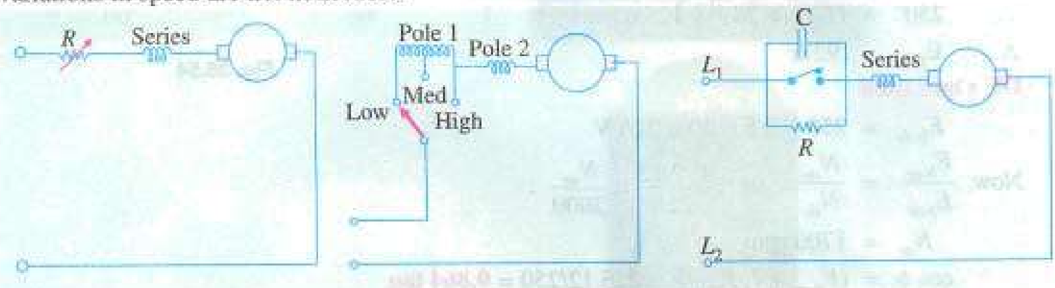


Fig. 36.50 Fig. 36.51 Fig. 36.52

The resistance R is connected across the governor points as shown in Fig. 36.52. A capacitor C is used across the contact points in order to reduce sparking produced due to the opening and closing of these points. Moreover, it prevents the pitting of contacts.

Example 36.4. *A 250-W, single-phase, 50-Hz, 220-V universal motor runs at 2000 rpm and takes 1.0 A when supplied from a 220-V dc. supply. If the motor is connected to 220-V ac supply and takes 1.0 A (r.m.s), calculate the speed, torque and power factor. Assume R_a = 20 Ω and L_a = 0.4 H.*

Solution. DC Operation : $E_{b.dc} = V - I_a R_a$

$$= 220 - 20 \times 1 = 200 \text{ V}$$

AC Operation

$$X_a = 2\pi \times 50 \times 0.4 = 125.7\ \Omega.$$

As seen from Fig.36.53.

$$V^2 = (E_{b.ac} + I_a R_a)^2 + (I_a \times X_a)^2$$

$$\therefore \quad E_{b.ac} = -I_a R_a + \sqrt{V^2 - (I_a X_a)^2}$$

$$= -1 \times 20 + \sqrt{220^2 - (125.7 \times 1)^2} = 160.5 \text{ V}$$

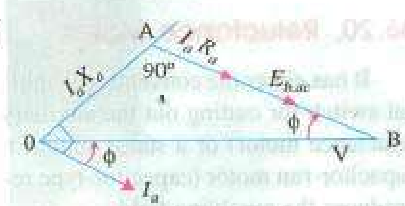


Fig. 36.53

Since armature current is the same for both dc and ac excitations, hence

$$\frac{E_{b.dc}}{E_{b.ac}} = \frac{N_{dc}}{N_{ac}}; \qquad \therefore \quad N_{ac} = 2000 \times \frac{160.5}{200} = 1605 \text{ rpm}$$

$$\cos\phi = AB/OB = (E_{b,ac} + I_a R_a)/V = (160.5 + 20)/220 = 0.82 \text{ lag}$$

$$P_{mech} = E_{b.ac} \cdot I_a = 160.5 \times 1 = 160.5 \text{ W}$$
$$T = 9.55 \times 160.5/1605 = \mathbf{0.955 \text{ N-m}}$$

Example 36.5. *A universal series motor has resistance of 30 Ω and an inductance of 0.5 H. When connected to a 250 V d.c. supply and loaded to take 0.8 A, it runs at 2000 r.p.m. Estimate its speed and power factor, when connected to a 250-V, 50-Hz a.c. supply and loaded to take the same current.* **(Elect. Machine, A.M.I.E. Sec. B, 1992)**

Solution. A.C. Operation

$$X_a = 2\pi \times 50 \times 0.5 = 157\ \Omega,$$
$$R_a = 30\ \Omega$$
$$I_a R_a = 0.8 \times 30 = 24\ \Omega$$
$$I_a X_a = 0.8 \times 157 = 125.6 \text{ V}$$

The phasor diagram is shown in Fig. 36.54 (*b*)

$$V^2 = (E_{b.ac} + I_a R_a)^2 + (I_a X_a)^2$$
$$250^2 = (E_{b.ac} + 24)^2 + 125.6^2$$
$$\therefore \quad E_{b.as} = 192.6$$

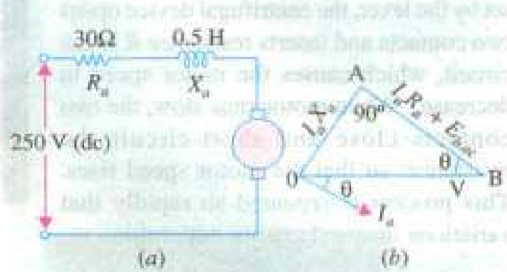


Fig. 36.54

DC Operation

$$E_{b.dc} = 250 - 0.8 \times 30 = 226 \text{ V}$$

$$\text{Now, } \frac{E_{b.ac}}{E_{b.dc}} = \frac{N_{ac}}{N_{dc}} \quad \text{or} \quad \frac{192.12}{226} = \frac{N_{ac}}{2000} \ ;$$
$$N_{ac} = 1700 \text{ rpm}$$
$$\cos\phi = (E_{b.ac} + I_a R_a)/V = 236.12/250 = \mathbf{0.864 \text{ lag}}$$

36.19. Unexcited Single-phase Synchronous Motors

These motors

1. operate from a single-phase a.c. supply
2. run at a constant speed – the synchronous speed of the revolving flux
3. need no d.c. excitation for their rotors (that is why they are called *unexcited*)
4. are self-starting.

These are of two types (*a*) reluctance motor and (*b*) hysteresis motor.

36.20. Reluctance Motor

It has either the conventional split-phase stator and a centrifugal switch for cutting out the auxiliary winding (split-phase type reluctance motor) or a stator similar to that of a permanent-split capacitor-run motor (capacitor-type reluctance motor). The stator produces the revolving field.

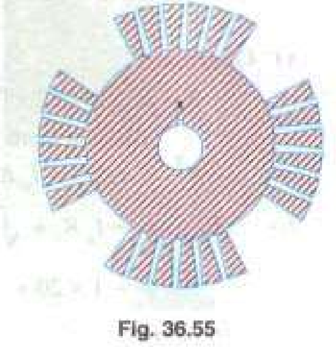


Fig. 36.55

The squirrel-cage rotor is of unsymmetrical magnetic construction. This type of unsymmetrical construction can be achieved by removing some of the teeth of a symmetrical squirrel-cage rotor punching. For example, in a 48-teeth, four-pole rotor following teeth may be cut away :

1, 2, 3, 4, 5, 6–13, 14, 15, 16, 17, 18–25, 26, 27, 28, 29, 30–37, 38, 39, 40, 41, 42.

This would leave four projecting or salient poles (Fig. 36.55) consisting of the following sets of teeth : 7–12; 19–24; 31–36 and 43–48. In this way, the rotor offers variable magnetic reluctance to the stator flux, the reluctance varying with the position of the rotor.

Working

Reluctance brushless motor

For understanding the working of such a motor one basic fact must be kept in mind. And it is that when a piece of magnetic material is located in a magnetic field, a force acts on the material, tending to bring it into the most dense portion of the field. The force tends to align the specimen of material in such a way that the reluctance of the magnetic path that lies through the material will be minimum.

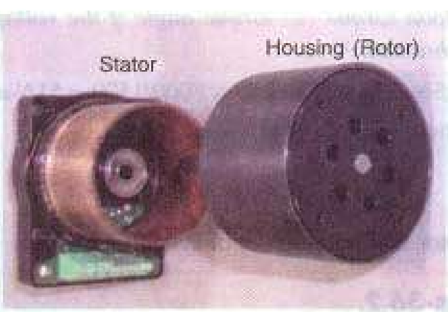
When the stator winding is energised, the revolving magnetic field exerts reluctance torque on the unsymmetrical rotor tending to align the salient pole axis of the rotor with the axis of the revolving magnetic field (because in this position, the reluctance of the magnetic path is minimum). If the reluctance torque is sufficient to start the motor and its load, the rotor will pull into step with the revolving field and continue to run at the speed of the revolving field.*

However, even though the rotor revolves synchronously, its poles lag behind the stator poles by a certain angle known as torque angle, (something similar to that in a synchronous motor). The reluctance torque increases with increase in torque angle, attaining maximum value when $\alpha = 45°$. If α increases beyond 45°, the rotor falls out of synchronism. The average value of the reluctance torque is given by $T = K\,(V/f)^2 \sin 2\,\delta$ where K is a motor constant.

It may be noted that the amount of load which a reluctance motor could carry at its ***constant speed*** would only be a fraction of the load that the motor could normally carry when functioning as an induction motor. If the load is increased beyond a value under which the reluctance torque cannot maintain synchronous speed, the rotor drops out of step with the field. The speed, then, drops to some value at which the slip is sufficient to develop necessary torque to drive the load by induction-motor action.

The constant-speed characteristic of a reluctance motor makes it very suitable for such applications as signalling devices, recording instruments, many kinds of timers and phonographs etc.

36.21. Hysteresis Motor



Hysteresis motor

The operation of this motor depends on the presence of a ***continuously-revolving*** magnetic flux. Hence, for the split-phase operation, its stator has two windings which remain connected to the single-phase supply continuously both at starting as well as during the running of the motor. Usually, shaded-pole principle is employed for this purpose giving shaded-pole hysteresis motor. Alternatively, stator winding of the type used in capacitor-type motor may be used giving capacitor-type shaded-pole motor. Obviously, in either type, no centrifugal device is used.

The rotor is a smooth chrome-steel cylinder** having

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* Actually, the motor starts as an induction motor and after it has reached its maximum speed as an induction, motor, the reluctance torque pulls its rotor into step with the revolving field so that the motor now runs as a synchronous motor by virtue of its saliency.

** Rotors of ceramic permanent magnet material are used whose resistivity approaches that of an insulator. Consequently, it is impossible to set up eddy currents in such a rotor. Hence, there is no eddy current loss but only hysteresis loss.

high retentivity so that the hysteresis loss is high. It has no winding. Because of high retentivity of the rotor material, it is very difficult to change the magnetic polarities once they are induced in the rotor by the revolving flux. The rotor revolves synchronously because the rotor poles magnetically lock up with the revolving stator poles of opposite polarity. However, the rotor poles always lag behind the stator poles by an angle α. Mechanical power developed by rotor is given by $P_m = P_h\left(\frac{1-s}{s}\right)$ where P_h is hysteres loss in rotor. Also $T_h = 9.55\ P_m/N_s$. It is seen that hysteresis torque depends solely on the area of rotor's hysteresis loop.

The fact that the rotor has no teeth or winding of any sort, results in making the motor extremely quiet in operation and free from mechanical and magnetic vibrations. This makes the motor particularly useful for driving tape-decks, tape-decks, turn-tables and other precision audio equipment. Since, commercial motors usually have two poles, they run at 3,000 r.p.m. at 50-Hz single-phase supply. In order to adopt such a motor for driving an electric clock and other indicating devices, gear train is connected to the motor shaft for reducing the load speed. The unit accelerates rapidly, changing from rest to full speed almost instantaneously. It must do so because it cannot accelerate gradually as an ordinary motor it is either operating at synchronous speed or not at all.

Fig. 36.56

Some unique features of a hysteresis motor are as under :

(i) since its hysteresis torque remains practically constant from locked rotor to synchronous speed, a hysteresis motor is able to synchronise any load it can accelerate–something no other motor does.

(ii) due to its smooth rotor, the motor operates quietly and does not suffer from magnetic pulsations caused by slots/salient-poles that are present in the rotors of other motors.

In Fig. 36.56. is shown a two-pole shaded-pole type hysteresis motor used for driving ordinary household electric clocks. The rotor is a thin metal cylinder and the shaft drives a gear train.

Example 36.6. *A 8-kW, 4-pole, 220-V, 50-Hz reluctance motor has a torque angle of 30° when operating under rated load conditions. Calculate (i) load torque (ii) torque angle if the voltage drops to 205 V and (iii) will the rotor pulled out of synchronism ?*

Solution. (i) $N_s = 120 \times 50/4 = 1500$ rpm; $T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 8000/1500 = 51\ N\text{-}m$

(ii) With the same load torque and constant frequency,

$$V_1 \sin 2\alpha_1 = V_2^2 \sin 2\,\alpha_2$$

$$\therefore \quad 220^2 \times \sin(2 \times 30°) = 205^2 \times \sin 2\alpha; \qquad \therefore \quad \alpha = 42.9°$$

(iii) since the new load angle is less than 45°, the rotor will not pull out of synchronous.

Tutorial Problems-36.2.

1. A 230-V, 50-Hz, 4-pole, class-A, single-phase induction motor has the following parameters at an operating temperature 63°C :

   $r_{1m} = 2.51$ ohms, $r_2' = 7.81$ ohm, $X_m = 150.88$ ohm, $X_{im} = 4.62$ ohm, $X_2' = 4.62$ ohms

   Determine stator main winding current and power factor when the motor is running at a slip of 0.05 at the specified temperature of 63°C.

   [3.74 ∠48.24°, 0.666] *(AMIE Sec. B Elect. Machines (E-B) Summer 1991)*

2. A fractional horse-power universal motor has armature circuit resistance of 20 ohm and inductance of 0.4 H. On being connected to a 220-V d.c. supply, it draws 1.0 A from the mains and runs at 2000 r.p.m. Estimate the speed and power factor of the motor, when connected to a 230-V, 50-Hz supply drawing the same armature current. Draw relevant phasor diagram.

[1726 rpm, 0.84] *(AMIE Sec. B Elect. Machines 1991)*

3. A universal series motor, when operating on 220 V d.c. draws 10 A and runs at 1400 r.p.m. Find the new speed and power factor, when connected to 220 V, 25 Hz supply, the motor current remaining the same. The motor has total resistance of 1 ohm and total inductance of 0.1 H.

[961 rpm; 0.7] *(AMIE Sec. B Elect. Machines 1990)*

QUESTIONS AND ANSWERS ON SINGLE-PHASE MOTORS

Q.1. How would you reverse the direction of rotation of a capacitor start-induction-run motor ?

Ans. By reversing either the running or starting-winding leads where they are connected to the lines. Both must not be reversed.

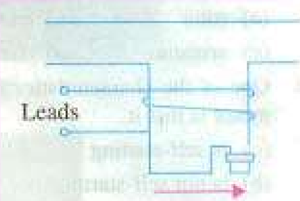


Fig. 36.57

Q.2. In which direction does a shaded-pole motor run ?

Ans. It runs from the unshaded to the shaded pole (Fig.36.57)

Q.3. Can such a motor be reversed ?

Ans. Normally, such motors are not reversible because that would involve mechanical dismantling and re-assembly. However, special motors are made having two rotors on a common shaft, each having one stator assembly for rotation in opposite direction.

Q.4. What is a universal motor ?

Ans. It is built like a series d.c. motor with the difference that both its stator and armature are laminated. They can be used either on d.c. or a.c. supply although the speed and power are greater on direct current. They cannot be satisfactorly made to run at less than about 2000 r.p.m.

Q.5. How can a universal motor be reversed ?

Ans. By reversing either the field leads or armature leads but not both.

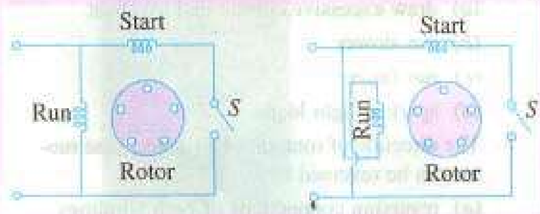


Fig. 36.58

Q.6. How can we reverse the direction of rotation of repulsion, repulsion-induction and repulsion-induction and repulsion- start-induction-run motors ?

Ans. By shifting the brush positions by about 15° electrical.

Q.7. How can we reverse the rotation of a 1-phase, split-phase motor ?

Ans. By reversing the leads to either the running or starter winding (Fig. 36.58) but not both.

Q.8. What could be the reasons if a repulsion-induction motor fails to start ?

Ans. Any one of the following :

1. no supply voltage
2. low voltage
3. excessive overload
4. the bearing lining may be stuck or 'frozen' to the shaft
5. armature may be rubbing
6. brush yoke may be incorrectly located
7. brush spacing may be wrong.

Q.9. What could be the reasons if a split-phase motor fails to start and hums loudly ?

Ans. It could be due to the starting winding being open or grounded or burnt out.

Q.10. What could be the reasons if a split-phase motor runs too slow ?

Ans. Any one of the following factors could be responsible :

1. wrong supply voltage and frequency
2. overload
3. grounded starting and running windings
4. short-circuited or open winding in field circuit.

OBJECTIVE TESTS – 36

1. The starting winding of a single-phase motor is placed in the
   (*a*) rotor (*b*) stator
   (*c*) armature (*d*) field.
2. One of the characteristics of a single- phase motor is that it
   (*a*) is self-starting
   (*b*) is not self-starting
   (*c*) requires only one winding
   (*d*) can rotate in one direction only.
3. After the starting winding of a single- phase induction motor is disconnected from supply, it continues to run only on ....winding.
   (*a*) rotor (*b*) compensating
   (*c*) field (*d*) running
4. If starting winding of a single-phase induction motor is left in the circuit, it will
   (*a*) draw excessive current and overheat
   (*b*) run slower
   (*c*) run faster
   (*d*) spark at light loads.
5. The direction of rotation of a single-phase motor can be reversed by
   (*a*) reversing connections of both windings
   (*b*) reversing connections of starting winding
   (*c*) using a reversing switch
   (*d*) reversing supply connections.
6. If a single-phase induction motor runs slower than normal, the more likely defect is
   (*a*) improper fuses
   (*b*) shorted running winding
   (*c*) open starting winding
   (*d*) worn bearings.
7. The capacitor in a capacitor-start induction- run ac motor is connected in series with ...... winding.
   (*a*) starting (*b*) running
   (*c*) squirrel-cage (*d*) compensating
8. A permanent-split single-phase capacitor motor does not have
   (*a*) centrifugal switch
   (*b*) starting winding
   (*c*) squirrel-cage rotor
   (*d*) high power factor.
9. The starting torque of a capacitor-start induction-run motor is directly related to the angle α between its two winding currents by the relation
   (*a*) $\cos \alpha$ (*b*) $\sin \alpha$
   (*c*) $\tan \alpha$ (*d*) $\sin \alpha/2$.
10. In a two-value capacitor motor, the capacitor used for running purposes is a/an
    (*a*) dry-type ac electrolytic capacitor
    (*b*) paper-spaced oil-filled type
    (*c*) air-capacitor
    (*d*) ceramic type.
11. If the centrifugal switch of a two-value capacitor motor using two capacitors fails to open, then
    (*a*) electrolytic capacitor will, in all probability, suffer breakdown
    (*b*) motor will not carry the load
    (*c*) motor will draw excessively high current
    (*d*) motor will not come upto the rated speed.
12. Each of the following statements regarding a shaded-pole motor is true **except**
    (*a*) its direction of rotation is from un- shaded to shaded portion of the poles
    (*b*) it has very poor efficiency
    (*c*) it has very poor p.f.
    (*d*) it has high starting torque.
13. Compensating winding is employed in an ac series motor in order to

(a) compensate for decrease in field flux
(b) increase the total torque
(c) reduce the sparking at brushes
(d) reduce effects of armature reaction.

14. A universal motor is one which
(a) is available universally
(b) can be marketed internationally
(c) can be operated either on dc or ac supply
(d) runs at dangerously high speed on no-load.

15. In a single-phase series motor the main purpose of inductively-wound compensating winding is to reduce the
(a) reactance emf of commutation
(b) rotational emf of commutation
(c) transformer emf of commutation
(d) none of the above.
(Power App.-II, Delhi Univ. Jan. 1987)

16. A repulsion motor is equipped with
(a) a commutator (b) slip-rings
(c) a repeller
(d) neither (a) nor (b).

17. A repulsion-start induction-run single- phase motor runs as an induction motor only when
(a) brushes are shifted to neutral plane
(b) short-circuiter is disconnected
(c) commutator segments are short- circuited
(d) stator winding is reversed.

18. If a dc series motor is operated on ac supply, it will
(a) have poor efficiency
(b) have poor power factor
(c) spark excessively
(d) all of the above
(e) none of the above.

19. An outstanding feature of a universal motor is its
(a) best performance at 50 Hz supply
(b) slow speed at all loads
(c) excellent performance on dc. supply
(d) highest output kW/kg ratio.

20. The direction of rotation of a hysteresis motor is determined by the
(a) retentivity of the rotor material
(b) amount of hysteresis loss
(c) permeability of rotor material
(d) position of shaded pole with respect to the main pole.

21. Speed of the universal motor is
(a) dependent on frequency of supply
(b) proportional to frequency of supply
(c) independent of frequency of supply
(d) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

22. In the shaded pole squirrel cage induction motor the flux in the shaded part always
(a) leads the flux in the unshaded pole segment
(b) is in phase with the flux in the unshaded pole segment
(c) lags the flux in the unshaded pole segment
(d) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

23. Which of the following motor is an interesting example of beneficially utilizing a phenomenon that is often considered undesirable ?
(a) hysteresis motor
(b) reluctance motor
(c) stepper motor
(d) shaded-pole motor.

24. Usually, large motors are more efficient than small ones. The efficiency of the tiny motor used in a wrist watch is approximately.......... per cent.
(a) 1
(b) 10
(c) 50
(d) 80

ANSWERS

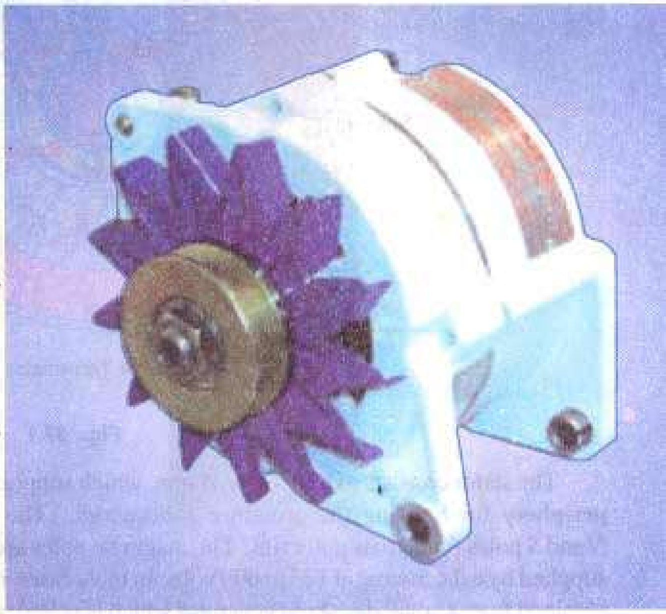
1. b **2.** b **3.** d **4.** a **5.** b **6.** d **7.** a **8.** a **9.** b **10.** b **11.** a **12.** d **13.** d **14.** c **15.** d **16.** a **17.** c **18.** d **19.** d **20.** d **21.** a **22.** c **23.** a **24.** a

C H A P T E R 37

ALTERNATORS

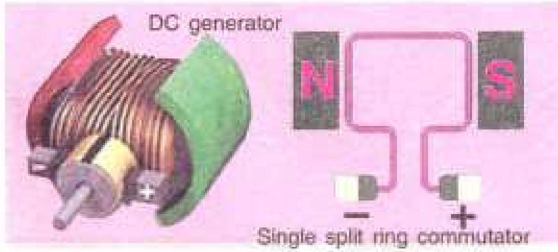
Learning Objectives

- Basic Principle
- Stationary Armature
- Rotor
- Armature Windings
- Wye and Delta Connections
- Distribution or Breadth Factor or Winding Factor or Spread Factor
- Equation of Induced E.M.F.
- Factors Affecting Alternator Size
- Alternator on Load
- Synchronous Reactance
- Vector Diagrams of Loaded Alternator
- Voltage Regulation
- Rothert's M.M.F. or Ampere-turn Method
- Zero Power Factor Method or Potier Method
- Operation of Salient Pole Synchronous Machine
- Power Developed by a Synchonous Generator
- Parallel Operation of Alternators
- Synchronizing of Alternators
- Alternators Connected to Infinite Bus-bars
- Synchronizing Torque Tsy
- Alternative Expression for Synchronizing Power
- Effect of Unequal Voltages
- Distribution of Load
- Maximum Power Output
- Questions and Answers on Alternators



Alternator

37.1. Basic Principle



A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the *armature rotates* and the field system is *stationary*, the arrangement in alternators is just the reverse of it. In their case, standard construction consists of armature winding mounted on a stationary element called *stator* and field windings on a rotating element called rotor. The details of construction are shown in Fig. 37.1.

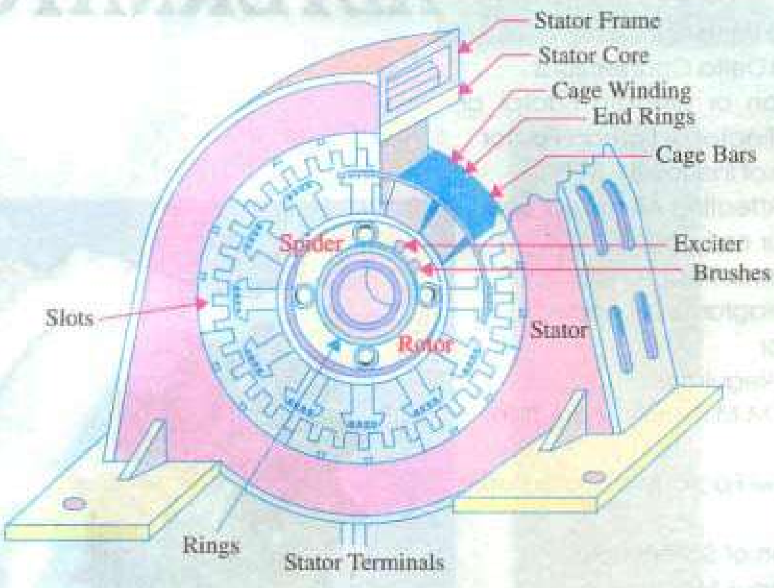


Fig. 37.1

The stator consists of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors. The rotor is like a flywheel having alternate *N* and *S* poles fixed to its outer rim. The magnetic poles are excited (or magnetised) from direct current supplied by a d.c. source at 125 to 600 volts. In most cases, necessary exciting (or magnetising) current is obtained from a small d.c. shunt generator which is belted or mounted on the shaft of the alternator itself. Because the field magnets are rotating, this current is supplied through two sliprings. As the exciting voltage is relatively small, the slip-rings and brush gear are of light construction. Recently, brushless excitation systems have been developed in which a 3-phase a.c. exciter and a group of rectifiers supply d.c. to the alternator. Hence, brushes, slip-rings and commutator are eliminated.

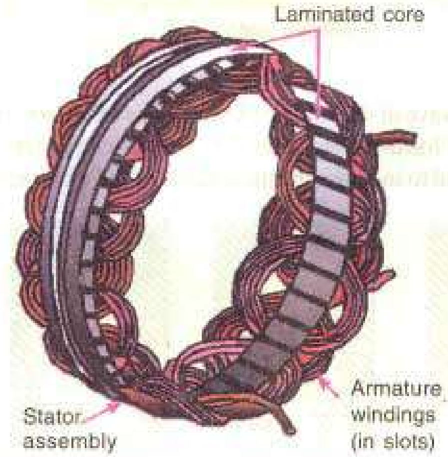
When the rotor rotates, the stator conductors (being stationary) are cut by the magnetic flux, hence they have induced e.m.f. produced in them. Because the magnetic poles are alternately *N* and *S*, they induce an e.m.f. and hence current in armature conductors, which first flows in one direction and then in the other. Hence, an alternating e.m.f. is produced in the stator conductors (*i*) whose frequency depends on the number of N and S poles moving past a conductor in one second and *(ii)* whose direction is given by Fleming's Right-hand rule.

37.2. Stationary Armature

Advantages of having stationary armature (and a rotating field system) are :

1. The output current can be led directly from fixed terminals on the stator (or armature windings) to the load circuit, without having to pass it through brush-contacts.

Alternators



Stationary armature windings

2. It is easier to insulate stationary armature winding for high a.c. voltages, which may have as high a value as 30 kV or more.
3. The sliding contacts *i.e.* slip-rings are transferred to the low-voltage, low-power d.c. field circuit which can, therefore, be easily insulated.
4. The armature windings can be more easily braced to prevent any deformation, which could be produced by the mechanical stresses set up as a result of short-circuit current and the high centrifugal forces brought into play.

37.3. Details of Construction

1. Stator Frame

In d.c. machines, the outer frame (or yoke) serves to carry the magnetic flux but in alternators, it is not meant for that purpose. Here, it is used for holding the armature stampings and windings in position. Low-speed large-diameter alternators have frames which because of ease of manufacture, are cast in sections. Ventilation is maintained with the help of holes cast in the frame itself. The provision of radial ventilating spaces in the stampings assists in cooling the machine.

But, these days, instead of using castings, frames are generally fabricated from mild steel plates welded together in such a way as to form a frame having a box type section.

In Fig. 37.2 is shown the section through the top of a typical stator.

2. Stator Core

The armature core is supported by the stator frame and is built up of laminations of

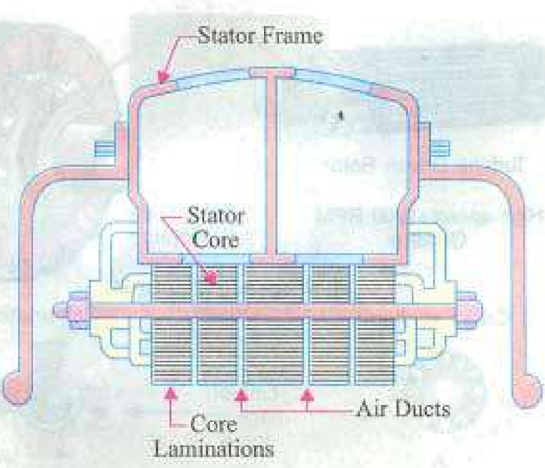


Fig. 37.2

special magnetic iron or steel alloy. The core is laminated to minimise loss due to eddy currents. The laminations are stamped out in complete rings (for smaller machine) or in segments (for larger machines). The laminations are insulated from each other and have spaces between them for allowing the cooling air to pass through. The slots for housing the armature conductors lie along the inner periphery of the core and are stamped out at the same time when laminations are formed. Different shapes of the armature slots are shown in Fig. 37.3.

Streetbike stator

The wide-open type slot (also used in d.c. machines) has the advantage of permitting easy installation of form-wound coils and their easy removal in case of repair. But it has the disadvantage of distributing the air-gap flux into bunches or tufts, that produce ripples in the wave of the generated e.m.f. The semi-closed type slots are better in this respect, but do not allow the use of form-wound coils. The wholly-closed type slots or tunnels do not disturb the air-gap flux but (*i*) they tend to increase the inductance of the windings (*ii*) the armature conductors have to be threaded through, thereby increasing initial labour and cost of winding and (*iii*) they present a complicated problem of end-connections. Hence, they are rarely used.

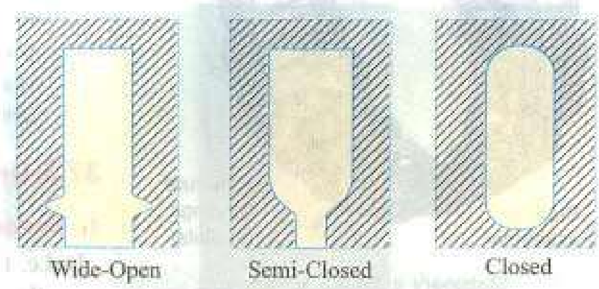
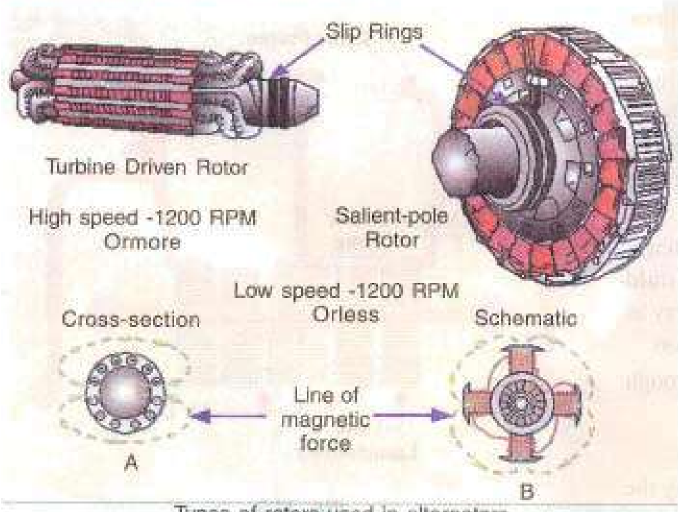


Fig. 37.3

37.4. Rotor

Two types of rotors are used in alternators (*i*) salient-pole type and (*ii*) smooth-cylindrical type.

(*i*) Salient (or projecting) Pole Type



Types of rotors used in alternators

It is used in low-and medium-speed (engine driven) alternators. It has a large number of projecting (salient) poles, having their cores bolted or dovetailed onto a heavy magnetic wheel of cast-iron, or steel of good magnetic quality (Fig. 37.4). Such generators are characterised by their large diameters and short axial lengths. The poles and pole-shoes (which cover 2/3 of pole-pitch) are laminated to minimize heating due to eddy currents. In large machines, field windings consist of rectangular copper strip wound on edge.

(ii) Smooth Cylindrical Type

It is used for steam turbine-driven alternators *i.e.* turbo-alternators, which run at very high speeds. The rotor consists of a smooth solid forged steel cylinder, having a number of slots milled out at intervals along the outer periphery (and parallel to the shaft) for accommodating field coils. Such rotors are designed mostly for 2-pole (or 4-pole) turbo-generators running at 3600 r.p.m. (or 1800 r.p.m.). Two (or four) regions corresponding to the central polar areas are left unslotted, as shown in Fig. 37.5 (*a*) and (*b*).

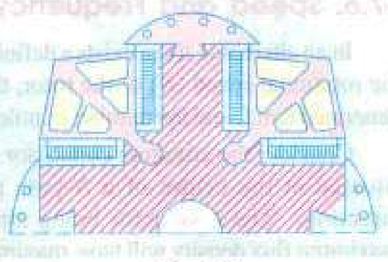


Fig. 37.4

Turbine alternator

The central polar areas are surrounded by the field windings placed in slots. The field coils are so arranged around these polar areas that flux density is maximum on the polar central line and gradually falls away on either side. It should be noted that in this case, poles are non-salient *i.e.* they do not project out from the surface of the rotor. To avoid excessive peripheral velocity, such rotors have very small diameters (about 1 metre or so). Hence, turbo-generators are characterised by small diameters and very long axial (or rotor) length. The cylindrical construction of the rotor gives better balance and quieter-operation and also less windage losses.

37.5. Damper Windings

Most of the alternators have their pole-shoes slotted for receiving copper bars of a grid or damper winding (also known as squirrel-cage winding). The copper bars are short-circuited at both ends by heavy copper rings (Fig. 37.6). These dampers are useful in preventing the hunting (momentary speed fluctuations) in generators and are needed in synchronous motors to provide the starting torque. Turbo-generators usually do not have these damper windings (except in special case to assist in synchronizing) because the solid field-poles themselves act as efficient dampers. It should be clearly understood that under normal running conditions, damper winding does not carry any current because rotor runs at synchronous speed.

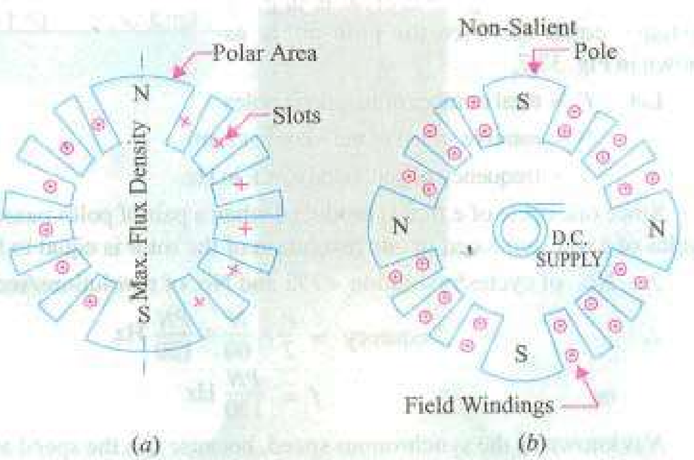


Fig. 37.5

The damper winding also tends to maintain balanced 3-ϕ voltage under unbalanced load conditions.

37.6. Speed and Frequency

Fig. 37.6

In an alternator, there exists a definite relationship between the rotational speed (N) of the rotor, the frequency (f) of the generated e.m.f. and the number of poles P.

Consider the armature conductor marked X in Fig. 37.7 situated at the centre of a N-pole rotating in clockwise direction. The conductor being situated at the place of maximum flux density will have maximum e.m.f. induced in it.

The direction of the induced e.m.f. is given by Fleming's right hand rule. But while applying this rule, one should be careful to note that the thumb indicates the direction of the motion of the conductor relative to the field. To an observer stationed on the clockwise revolving poles, the conductor would seem to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced e.m.f. is downwards, in a direction at right angles to the plane of the paper.

When the conductor is in the interpolar gap, as at A in Fig. 37.7, it has minimum e.m.f. induced in it, because flux density is minimum there. Again, when it is at the centre of a S-pole, it has maximum e.m.f. induced in it, because flux density at B is maximum. But the direction of the e.m.f. when conductor is over a N-pole is opposite to that when it is over a S-pole.

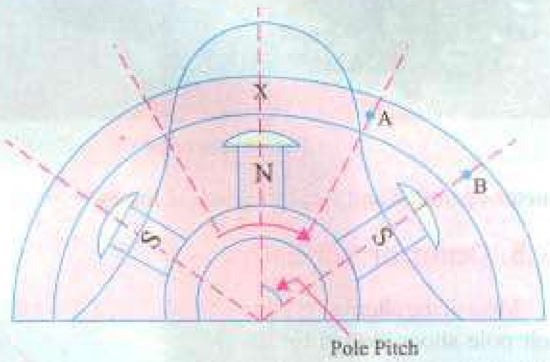


Fig. 37.7

Obviously, one cycle of e.m.f. is induced in a conductor when one pair of poles passes over it. In other words, the e.m.f. in an armature conductor goes through one cycle in angular distance equal to twice the pole-pitch, as shown in Fig. 37.7.

Let P = total number of magnetic poles

N = rotative speed of the rotor in r.p.m.

f = frequency of generated e.m.f. in Hz.

Since one cycle of e.m.f. is produced when a pair of poles passes past a conductor, the number of cycles of e.m.f. produced in one revolution of the rotor is equal to the number of pair of poles.

∴ No. of cycles/revolution = $P/2$ and No. of revolutions/second = $N/60$

$$\therefore \quad \text{frequency} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ Hz}$$

or

$$f = \frac{PN}{120} \text{ Hz}$$

N is known as the synchronous speed, because it is the speed at which an alternator must run, in order to generate an e.m.f. of the required frequency. In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, the alternator will have to run at the following speeds :

No. of poles	2	4	6	12	24	36
Speed (r.p.m.)	3600	1800	1200	600	300	200

Referring to the above equation, we get $P = 120f/N$

It is clear from the above that because of slow rotative speeds of engine-driven alternators, their number of poles is much greater as compared to that of the turbo-generators which run at very high speeds.

37.7. Armature Windings

The armature windings in alternators are different from those used in d.c. machines. The d.c. machines have closed circuit windings but alternator windings are open, in the sense that there is no closed path for the armature currents in the winding itself. One end of the winding is joined to the neutral point and the other is brought out (for a star-connected armature).

D.C. Armature

The two types of armature windings most commonly used for 3-phase alternators are :

(i) single-layer winding

(ii) double-layer winding

Single-layer Winding

It is variously referred to as concentric or chain winding. Sometimes, it is of simple bar type or wave winding.

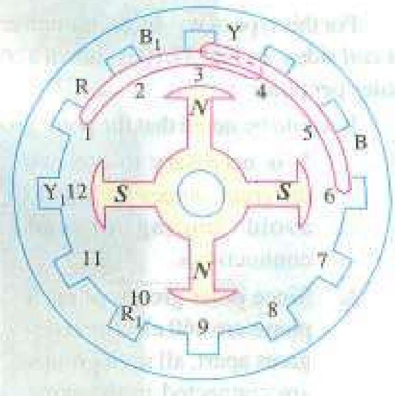
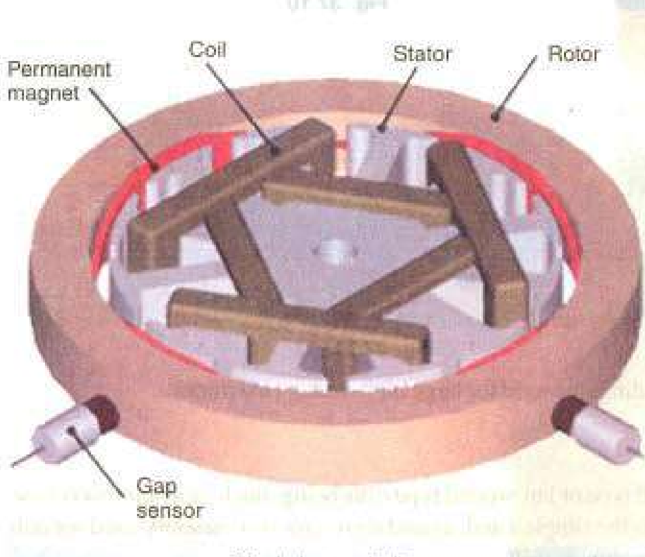


Fig. 37.8

The fundamental principle of such a winding is illustrated in Fig. 37.8 which shows a single-layer, one-turn, full-pitch winding for a four-pole generator. There are 12 slots in all, giving 3 slots per pole or 1 slot/phase/pole. The pole pitch is obviously 3. To get maximum e.m.f., two sides of a coil should be one pole-pitch apart *i.e.* coil span should be equal to one pole pitch. In other words, if one side of the coil is under the centre of a *N*-pole, then the, other side of the same *coil* should be under the centre of *S*-pole *i.e.* 180° (electrical) apart. In that case, the e.m.fs. induced in the two sides of the coil are added together. It is seen from the above figure, that *R* phase starts at slot No. 1, passes through slots 4, 7 and finishes at 10. The *Y*-phase starts 120° afterwards *i.e.* from slot No. 3 which is two slots away from the start of *R*-phase (because when 3 slots correspond to 180° electrical degrees, two slots correspond to an angular displacement of 120° electrical). It passes through slots 6, 9 and finishes at 12. Similarly, *B*-phase starts from slot No.5 *i.e.* two slots away from the start of *Y*-phase. It passes through slots 8, 11 and finishes at slot No. 2, The developed diagram is shown in Fig. 37.9. The ends of the windings are joined to form a star point for a *Y*-connection.



Single layer winding

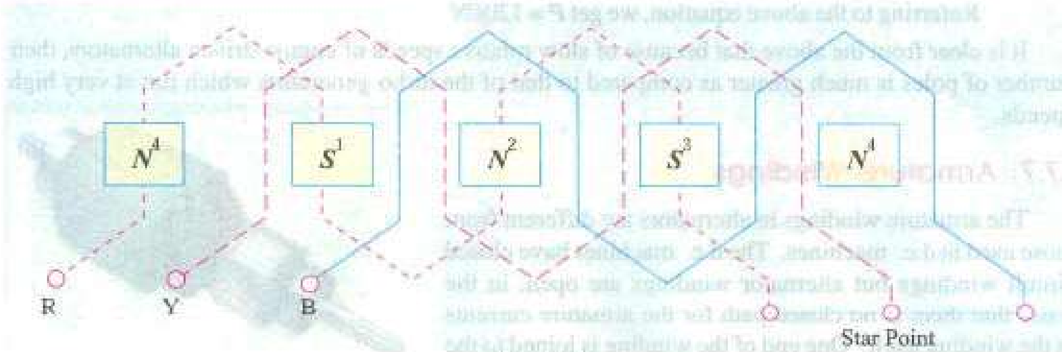


Fig. 37.9

37.8. Concentric or Chain Windings

For this type of winding, the number of slots is equal to twice the number of coils or equal to the number of *coil* sides. In Fig. 37.10 is shown a concentric winding for 3-phase alternator. It has one coil per pair of poles per phase.

It would be noted that the polar group of each phase is 360° (electrical) apart in this type of winding

1. It is necessary to use two different shapes of coils to avoid fouling of end connections.
2. Since polar groups of each phase are 360 electrical degrees apart, all such groups are connected in the same direction.
3. The disadvantage is that short-pitched coils cannot be used.

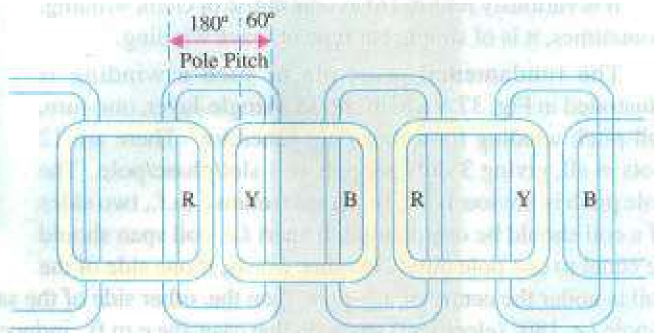


Fig. 37.10

In Fig. 37.11 is shown a concentric winding with two coils per group per pole. Different shapes of coils are required for this winding.

All coil groups of phase *R* are connected in the same direction. It is seen that in each group, one coil has a pitch of 5/6 and the other has a pitch of 7/6 so that pitch factor (explained later) is 0.966. Such windings are used for large high-voltage machines.

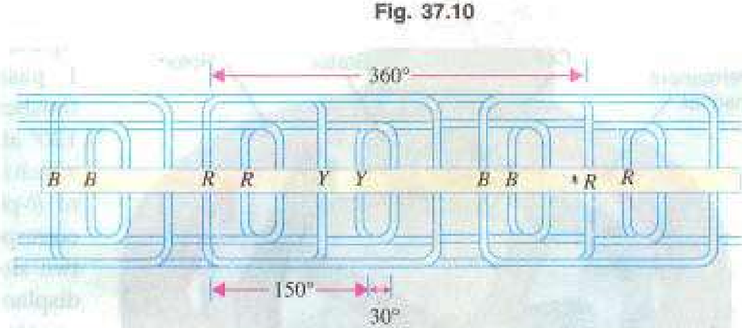


Fig. 37.11

37.9. Two-Layer Winding

This winding is either of wave-wound type or lap-wound type (this being much more common especially for high-speed turbo-generators). It is the simplest and, as said above, most commonly-used not only in synchronous machines but in induction motors as well.

Two important points regarding this winding should be noted :

(*a*) Ordinarily, the number of slots in stator (armature) is a multiple of the number of poles and the number of phases. Thus, the stator of a 4-pole, 3-phase alternator may have 12, 24, 36, 48 etc. slots all of which are seen to be multiple of 12 (*i.e.* 4×3).

(*b*) The number of stator slots is equal to the number of coils (which are all of the same shape). In other words, each slot contains two coil sides, one at the bottom of the slot and the other at the top. The coils overlap each other, just like shingles on a roof top.

For the 4-pole, 24-slot stator machine shown in Fig. 37.12, the pole-pitch is $24/4 = 6$. For maximum voltage, the coils should be full-pitched. It means that if one side of the coil is in slot No.1, the other side should be in slot No.7, the two slots 1 and 7 being one pole-pitch or 180° (electrical) apart. To make matters simple, coils have been labelled as 1, 2, 3 and 4 etc. In the developed diagram of Fig. 37.14, the coil number is the number of the slot in which the left-hand side of the coil is placed.

Each of the three phases has $24/3 = 8$ coils, these being so selected as to give maximum voltage when connected in series. When connected properly, coils 1, 7, 13 and 19 will add directly in phase. Hence, we get 4 coils for this phase. To complete eight coils for this phase, the other four selected are 2, 8, 14 and 20 each of which is at an angular displacement of 30° (elect.) from the adjacent coils of the first. The coils 1 and 2 of this phase are said to constitute a polar group (which is defined as the group of coils/phase/pole). Other polar groups for this phase are 7 and 8, 13 and 14, 19 and 20 etc. After the coils are placed in slots, the polar groups are joined. These groups are connected together with alternate poles reversed (Fig. 37.13) which shows winding for one phase only.

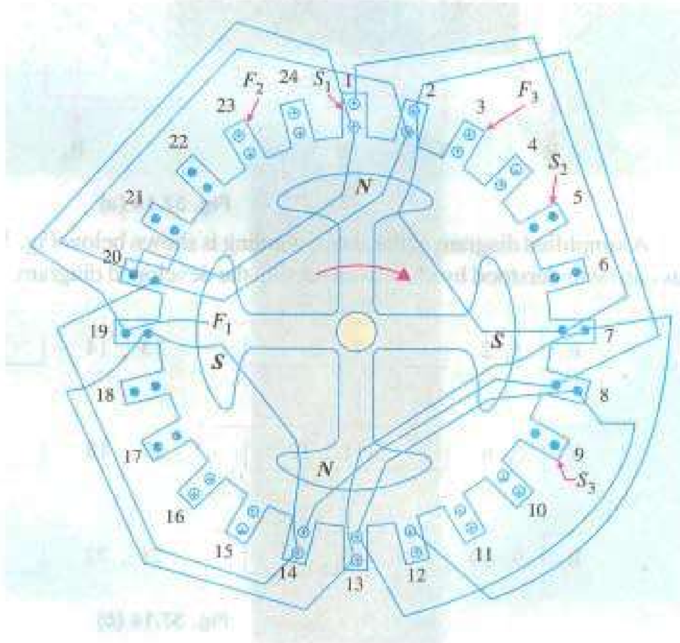


Fig. 37.12

Now, phase Y is to be so placed as to be 120° (elect.) away from phase R. Hence, it is started from slot 5 *i.e.* 4 slots away (Fig. 37.14). It should be noted that angular displacement between slot No. 1 and 5 is $4 \times 30 = 120°$ (elect). Starting from coil 5, each of the other eight coils of phase Y will be placed 4 slots to the right of corresponding coils for phase R. In the same way, B phase will start from coil 9. The complete wiring diagram for three phases is shown in Fig. 37.14. The terminals R_2, Y_2 and B_2 may be connected together to form a neutral for Y-connection.

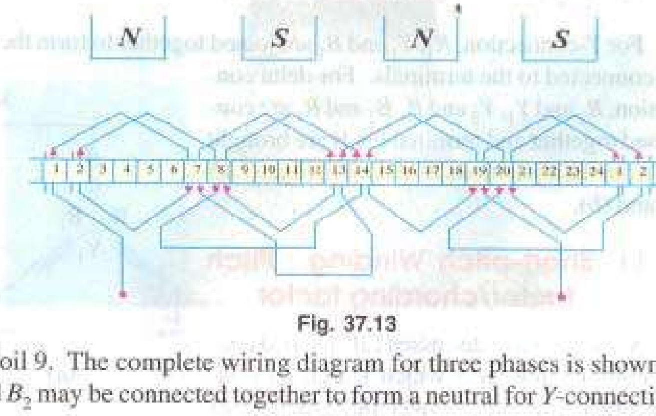


Fig. 37.13

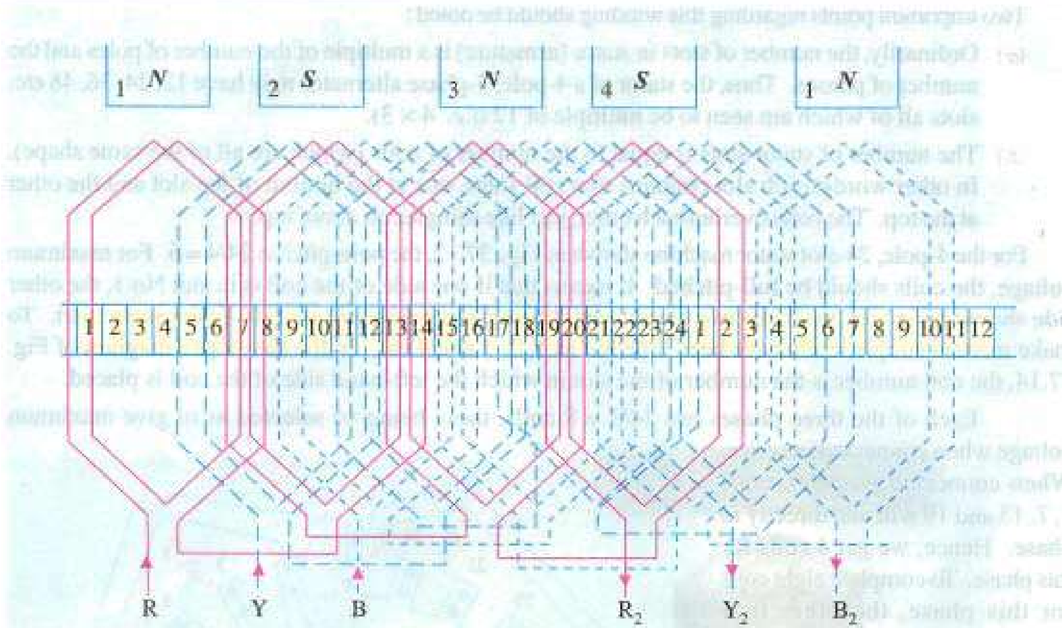


Fig. 37.14 (*a*)

A simplified diagram of the above winding is shown below Fig. 37.14. The method of construction for this can be understood by closely inspecting the developed diagram.

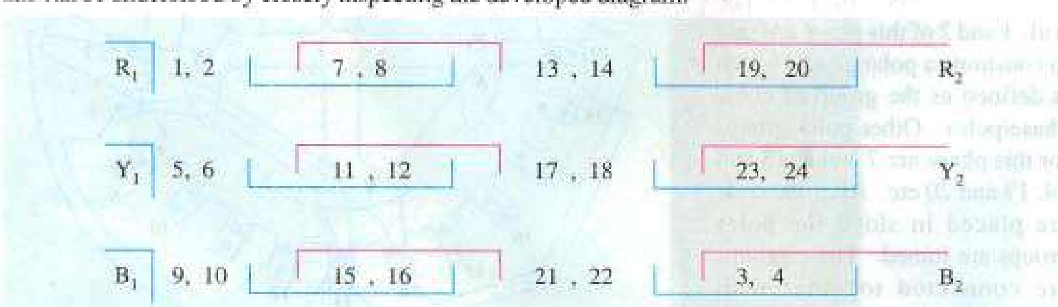


Fig. 37.14 (*b*)

37.10. Wye and Delta Connections

For Y-connection, R_1, Y_1 and B_1 are joined together to form the star-point. Then, ends R_2, Y_2 and B_2 are connected to the terminals. For delta connection, R_2 and Y_1, Y_2 and B_1 B_2 and R_1 are connected together and terminal leads are brought out from their junctions as shown in Fig. 37.15 (*a*) and (*b*).

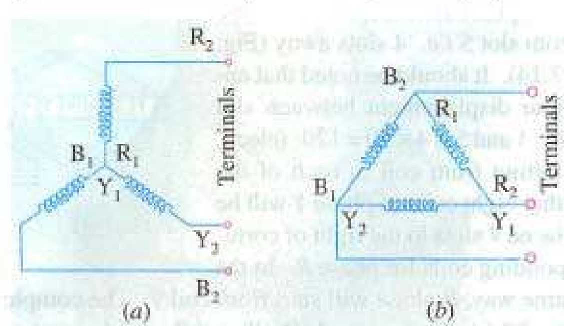


Fig. 37.15

37.11. Short-pitch Winding : Pitch factor/chording factor

So far we have discussed full-pitched coils *i.e.* coils having span which is equal to one pole-pitch *i.e.* spanning over 180° (electrical).

As shown in Fig. 37.16, if the coil sides are placed in slots 1 and 7, then it is full-pitched. If the coil sides are placed in slots 1 and 6, then it is short-pitched or fractional-pitched because coil span is equal to 5/6 of a pole-pitch. It falls short by 1/6 pole-pitch or by $180°/6 = 30°$. Short-pitched coils are deliberately used because of the following advantages:

1. They save copper of end connections.
2. They improve the wave-form of the generated e.m.f. *i.e.* the generated e.m.f. can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated.
3. Due to elimination of high frequency harmonics, eddy current and hysteresis losses are reduced thereby increasing the efficiency.

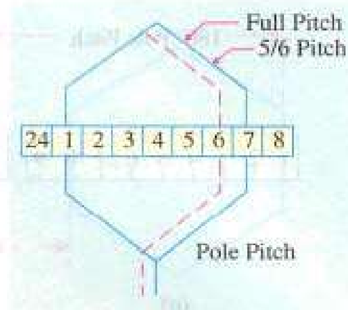


Fig. 37.16

But the disadvantage of using short-pitched coils is that the total voltage around the coils is somewhat reduced. Because the voltages induced in the two sides of the short-pitched coil are slightly out of phase, their resultant vectorial sum is less than their arithmetical sum.

The pitch factor or coil-span factor k_p or k_c is defined as

$$= \frac{\text{vector sum of the induced e.m.fs. per coil}}{\text{arithmetic sum of the induced e.m.fs. per coil}}$$

It is always less than unity.

Let E_S be the induced e.m.f. in each side of the coil. If the coil were full-pitched *i.e.* if its two sides were one pole-pitch apart, then total induced e.m.f. in the coil would have been $= 2E_S$ [Fig. 37.17 (*a*).

If it is short-pitched by 30° (elect.) then as shown in Fig. 37.17 (*b*), their resultant is E which is the vector sum of two voltage 30° (electrical) apart.

$$\therefore \quad E = 2\,E_S \cos 30°/2 = 2E_S \cos 15°$$

$$k_c = \frac{\text{vector sum}}{\text{arithmetic sum}} = \frac{E}{2\,E_S} = \frac{2\,E_S \cos 15°}{2\,E_S} = \cos 15° = 0.966$$

Hence, pitch factor, $k_c = 0.966$.

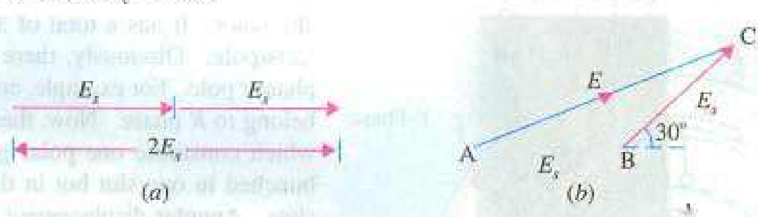


Fig. 37.17

In general, if the coil span falls short of full-pitch by an angle α (electrical)*,
then $k_c = \cos \alpha/2$.

Similarly, for a coil having a span of 2/3 pole-pitch, $k_c = \cos 60°/2 = \cos 30° = 0.866$.

It is lesser than the value in the first case.

Note. The value of α will usually be given in the question, if not, then assume $k_c = 1$.

---

* This angle is known as chording angle and the winding employing short-pitched coils is called chorded winding.

Example 37.1. *Calculate the pitch factor for the under-given windings : (a) 36 stator slots, 4-poles, coil-span, 1 to 8 (b) 72 stator slots, 6 poles, coils span 1 to 10 and (c) 96 stator slots, 6 poles, coil span 1 to 12. Sketch the three coil spans.*

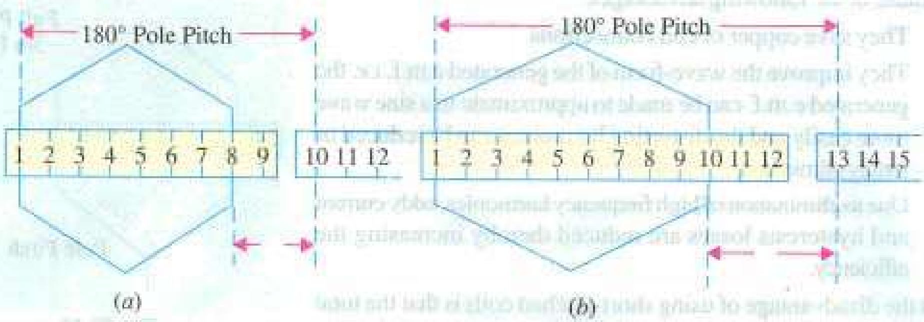


Fig. 37.18

Solution. (a) Here, the coil span falls short by (2/9) × 180° = 40°

$$\alpha = 40°$$

$\therefore\ k_c = \cos 40°/2 = \cos 20° = \mathbf{0.94}$

(b) Here $\alpha = (3/12) \times 180° = 45°$ $\quad\therefore\ k_c = \cos 45°/2 = \cos 22.5° = \mathbf{0.924}$

(c) Here $\alpha = (5/16) \times 180° = 56°\ 16'$ $\quad\therefore\ k_c = \cos 28°8' = \mathbf{0.882}$

The coil spans have been shown in Fig. 37.18.

37.12. Distribution or Breadth Factor or Winding Factor or Spread Factor

It will be seen that in each phase, coils are not concentrated or bunched in one slot, but are distributed in a number of slots to form polar groups under each pole. These coils/phase are displaced from each other by a certain angle. The result is that the e.m.fs. induced in coil sides constituting a polar group are not in phase with each other but differ by an angle equal to angular displacement of the slots.

In Fig. 37.19 are shown the end connections of a 3-phase single-layer winding for a 4-pole alternator. It has a total of 36 slots *i.e.* 9 slots/pole. Obviously, there are 3 slots / phase / pole. For example, coils 1, 2 and 3 belong to *R* phase. Now, these three coils which constitute one polar group are not bunched in one slot but in three different slots. Angular displacement between any two adjacent slots = 180°/9 = 20° (elect.)

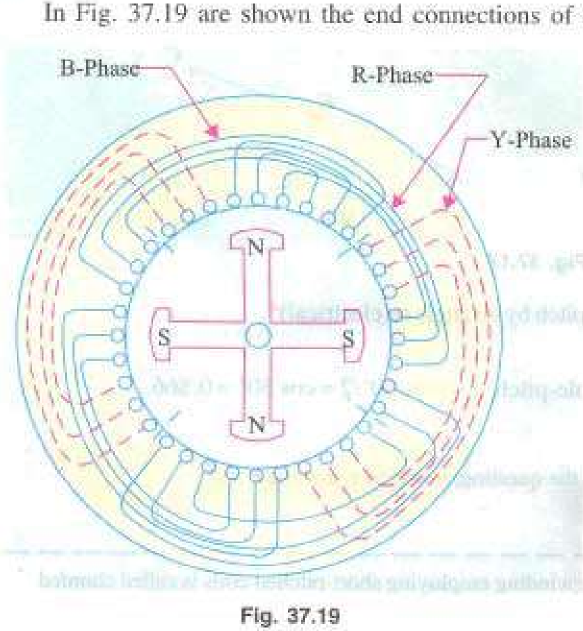


Fig. 37.19

If the three coils were bunched in one slot, then total e.m.f. induced in the three sides of the coil would be the arithmetic sum of the three e.m.f.s. *i.e.* = 3 E_S, where E_S is the e.m.f. induced in one coil *side* [Fig.37.20 (*a*)].

Since the coils are distributed, the individual e.m.fs. have a phase difference of 20° with each other. Their vector sum as seen from Fig. 35.20 (*b*) is

$$E = E_S \cos 20^\circ + E_S + E_S \cos 20^\circ$$
$$= 2\,E_S \cos 20^\circ + E_S$$
$$= 2\,E_S \times 0.9397 + E_S = 2.88\,E_S$$

The distribution factor (k_d) is defined as

$$= \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

In the present case

$$k_d = \frac{\text{e.m.f. with winding in 3 slots/pole/phase}}{\text{e.m.f. with winding in 1 slots/pole/phase}} = \frac{E}{3\,E_S} = \frac{2.88\,E_S}{3\,E_S} = 0.96$$

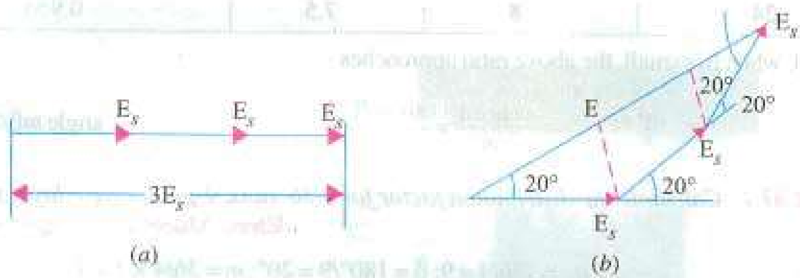


Fig. 37.20

General Case

Let β be the value of angular displacement between the slots. Its value is

$$\beta = \frac{180^\circ}{\text{No. of slots/pole}} = \frac{180^\circ}{n}$$

Let m = No. of slots/phase/pole

$m\beta$ = phase spread angle

Then, the resultant voltage induced in one polar group would be mE_S

where E_S is the voltage induced in one coil side. Fig. 37.21 illustrates the method for finding the vector sum of m voltages each of value E_S and having a mutual phase difference of β (if m is large, then the curve $ABCDE$ will become part of a circle of radius r).

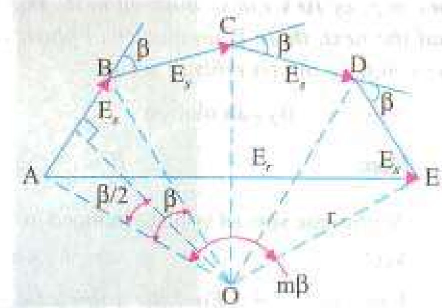


Fig. 37.21

$$AB = E_S = 2r \sin \beta/2$$

Arithmetic sum is $= mE_S = m \times 2r \sin \beta/2$

Their vector sum $= AE = E_r = 2r \sin m\beta/2$

$$k_d = \frac{\text{vector sum of coils e.m.fs.}}{\text{arithmetic sum of coil e.m.fs.}}$$

$$= \frac{2r \sin m\beta/2}{m \times 2r \sin \beta/2} = \frac{\sin m\,\beta/2}{m \sin \beta/2}$$

The value of distribution factor of a 3-phase alternator for different number of slots/pole/phase is given in table No. 37.1.

Table 37.1

Slots per pole	*m*	β°	*Distribution factor* k_d
3	1	60	1.000
6	2	30	0.966
9	3	20	0.960
12	4	15	0.958
15	5	12	0.957
18	6	10	0.956
24	8	7.5	0.955

In general, when β is small, the above ratio approaches

$$= \frac{\text{chord}}{\text{arc}} = \frac{\sin m\beta/2}{m\,\beta/2} \qquad \text{— angle } m\beta/2 \text{ in radians.}$$

Example 37.2. *Calculate the distribution factor for a 36-slots, 4-pole, single-layer three-phase winding.* **(Elect. Machine-I Nagpur Univ. 1993)**

Solution.

$$n = 36/4 = 9;\ \beta = 180°/9 = 20°;\ m = 36/4 \times 3 = 3$$

$$kd = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3\times 20°/2}{3 \sin 20°/2} = \mathbf{0.96}$$

Example. 37.3. *A part of an alternator winding consists of six coils in series, each coil having an e.m.f. of 10 V r.m.s. induced in it. The coils are placed in successive slots and between each slot and the next, there is an electrical phase displacement of 30°. Find graphically or by calculation, the e.m.f. of the six coils in series.*

Solution. By calculation

Here $\beta = 30°$; $m = 6$ $\therefore$ $k_d = \dfrac{\sin m\beta/2}{m \sin \beta/2} = \dfrac{\sin 90°}{6 \times \sin 15°} = \dfrac{1}{6 \times 0.2588}$

Arithmetic sum of voltage induced in 6 coils $= 6 \times 10 = 60$ V

Vector sum $= k_d \times$ arithmetic sum $= 60 \times 1/6 \times 0.2588 =$ **38.64V**

Example 37.4. *Find the value of k_d for an alternator with 9 slots per pole for the following cases :*

(i) One winding in all the slots (ii) one winding using only the first 2/3 of the slots/pole (iii) three equal windings placed sequentially in 60° group.

Solution. Here, $\beta = 180°/9 = 20°$ and values of m *i.e.* number of slots in a group are 9, 6 and 3 respectively.

(i) $m = 9$, $\beta = 20°$, $k_d = \dfrac{\sin 9\times 20°/2}{9 \sin 20°/2} = \mathbf{0.64}$ $\left[\text{or } k_d = \dfrac{\sin \pi/2}{\pi/2} = 0.637\right]$

(ii) $m = 6$, $\beta = 20°$, $k_d = \dfrac{\sin 6\times 20°/2}{6 \sin 20°/2} = \mathbf{0.83}$ $\left[\text{or } k_d = \dfrac{\sin \pi/3}{\pi/3} = 0.827\right]$

(iii) $m = 3$, $\beta = 20°$, $k_d = \dfrac{\sin 3\times 20°/2}{3 \sin 20°/2} = \mathbf{0.96}$ $\left[\text{or } k_d = \dfrac{\sin \pi/6}{\pi/6} = 0.955\right]$

37.13. Equation of Induced E.M.F.

Let Z = No. of conductors or coil sides in series/phase

$= 2T$ — where T is the No. of coils or turns per phase (remember one turn or coil has two sides)

P = No. of poles

f = frequency of induced e.m.f. in Hz

Φ = flux/pole in webers

k_d = distribution factor $= \dfrac{\sin m\beta/2}{m \sin \beta/2}$

k_c or k_p = pitch or coil span factor = $\cos \alpha/2$

k_f = from factor = **1.11** —if e.m.f. is assumed sinusoidal

N = rotor r.p.m.

In one revolution of the rotor (*i.e.* in $60/N$ second) each stator conductor is cut by a flux of ΦP webers.

$\therefore \quad d\Phi = \Phi P$ and $dt = 60/N$ second

$$\therefore \quad \text{Average e.m.f. induced per conductor} = \frac{d\Phi}{dt} = \frac{\Phi P}{60/N} = \frac{\Phi NP}{60}$$

Now, we know that $f = PN/120$ or $N = 120 f/P$

Substituting this value of N above, we get

$$\text{Average e.m.f. per conductor} = \frac{\Phi P}{60} \times \frac{120 f}{P} = 2f\,\Phi \text{ volt}$$

If there are Z conductors in series/phase, then Average e.m.f./phase $= 2f\,\Phi Z$ volt $= 4f\,\Phi T$ volt

R.M.S. value of e.m.f./phase $= 1.11 \times 4f\,\Phi T = 4.44 f\,\Phi T$ volt*.

This would have been the actual value of the induced voltage if all the coils in a phase were (*i*) full-pitched and (*ii*) concentrated or bunched in one slot (instead of being distributed in several slots under poles). But this not being so, the actually available voltage is reduced in the ratio of these two factors.

$\therefore$ Actually available voltage/phase $= 4.44\, k_c k_d f\,\Phi\, T = 4\, k_f k_c k_d f\,\Phi\, T$ volt.

If the alternator is star-connected (as is usually the case) then the line voltage is $\sqrt{3}$ times the phase voltage (as found from the above formula).

37.14. Effect of Harmonics on Pitch and Distribution Factors

(*a*) If the short-pitch angle or chording angle is α degrees (electrical) for the fundamental flux wave, then its values for different harmonics are

for 3rd harmonic $= 3\alpha$; for 5th harmonic $= 5\alpha$ and so on.

$\therefore$ pitch-factor, $k_c = \cos \alpha/2$ —for fundamental

$= \cos 3\alpha/2$ —for 3rd harmonic

$= \cos 5\alpha/2$ —for 5th harmonic etc.

(*b*) Similarly, the distribution factor is also different for different harmonics. Its value becomes

$$k_d = \frac{\sin m\beta/2}{m \sin \beta/2} \quad \text{where } n \text{ is the order of the harmonic}$$

* It is exactly the same equation as the e.m.f. equation of a transformer. (Art 32.6)

for fundamental, $n = 1$ $k_{d1} = \dfrac{\sin m\beta/2}{m\sin\beta/2}$

for 3rd harmonic, $n = 3$ $k_{d3} = \dfrac{\sin 3m\beta/2}{m\sin 3\beta/2}$

for 5th harmonic, $n = 5$ $k_{d5} = \dfrac{\sin 5m\beta/2}{m\sin 5\beta/2}$

(c) Frequency is also changed. If fundamental frequency is 50 Hz *i.e.* $f_1 = 50$ Hz then other frequencies are :

3rd harmonic, $f_3 = 3 \times 50 = 150$ Hz, 5th harmonic, $f_5 = 5 \times 50 = 250$ Hz etc.

Example 37.5. *An alternator has 18 slots/pole and the first coil lies in slots 1 and 16. Calculate the pitch factor for (i) fundamental (ii) 3rd harmonic (iii) 5th harmonic and (iv) 7th harmonic.*

Solution. Here, coil span is $= (16 - 1) = 15$ slots, which falls short by 3 slots.

Hence, $\alpha = 180° \times 3/18 = 30°$

(i) $k_{c1} = \cos 30°/2 = \cos 15° = \mathbf{0.966}$ (ii) $k_{c3} = \cos 3 \times 30°/2 = \mathbf{0.707}$

(iii) $k_{c5} = \cos 5 \times 30°/2 = \cos 75° = \mathbf{0.259}$ (iv) $k_{c7} = \cos 7 \times 30°/2 = \cos 105° = \cos 75° = \mathbf{0.259}$.

Example 37.6. *A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency rpm and the phase and line e.m.f. Assume full-pitched coil.*

(Elect. Machines, AMIE Sec. B, 1991)

Solution. $f = PN/120 = 16 \times 375/120 = \mathbf{50\ Hz}$

Since k_c is not given, it would be taken as unity.

$$n = 144/16 = 9;\ \beta = 180°/9 = 20°;\ m = 144/16 \times 3 = 3$$

$$k_d = \sin 3 \times (20°/2)/3 \sin (20°/2) = 0.96$$

$$Z = 144 \times 10/3 = 480;\ T = 480/2 = 240\ /\ \text{phase}$$

$$E_{ph} = 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 240 = \mathbf{15.34\ V}$$

Line voltage, $E_L = \sqrt{3}\,E_{ph} = \sqrt{3} \times 1534 = \mathbf{2658\ V}$

Example 37.7. *Find the no-load phase and line voltage of a star-connected 3-phase, 6-pole alternator which runs at 1200 rpm, having flux per pole of 0.1 Wb sinusoidally distributed. Its stator has 54 slots having double layer winding. Each coil has 8 turns and the coil is chorded by 1 slot.*

(Elect. Machines-I, Nagput Univ. 1993)

Solution. Since winding is chorded by one slot, it is short-pitched by 1/9 or 180°/9 = 20°

$\therefore$ $k_c = \cos 20°/2 = 0.98$; $f = 6 \times 1200/120 = 60$ Hz ,

$$n = 54/6 = 9;\ \beta = 180°/9 = 20°,\ m = 54/6 \times 3 = 3$$

$$k_d = \sin 3 \times (20°/2)/3 \sin (20°/2) = 0.96$$

$$Z = 54 \times 8/3 = 144;\ T = 144/2 = 72,\ f = 6 \times 1200/120 = 60\ \text{Hz}$$

$$E_{ph} = 4.44 \times 0.98 \times 0.96 \times 60 \times 0.1 \times 72 = 1805\ \text{V}$$

Line voltage, $E_L = \sqrt{3} \times 1805 = \mathbf{3125\ V.}$

Example 37.8. *The stator of a 3-phase, 16-pole alternator has 144 slots and there are 4 conductors per slot connected in two layers and the conductors of each phase are connected in series. If the speed of the alternator is 375 r.p.m., calculate the e.m.f. inducted per phase. Resultant flux in the air-gap is 5×10^{-2} webers per pole sinusoidally distributed. Assume the coil span as 150° electrical.*

(Elect. Machine, Nagpur Univ. 1993)

Solution. For sinusoidal flux distribution, $k_f = 1.11$; $\alpha = (180° - 150°) = 30°$ (elect)

$$k_c = \cos 30°/2 = 0.966^*$$

No. of slots / pole, $n = 144/16 = 9$;

$$\beta = 180°/9 = 20°$$

$$m = \text{No. of slots/pole/phase} = 144/16 \times 3 = 3$$

$$\therefore \quad k_d = \frac{\sin m\,\beta/2}{m \sin \beta/2} = \frac{\sin 3\times 20°/2}{3 \sin 20°/2} = 0.96; \; f = 16 \times 375/120 = 50 \text{ Hz}$$

No. of slots / phase $= 144/3 = 48$; No of conductors / slot = 4

$\therefore$ No. of conductors in series/phase $= 48 \times 4 = 192$

$\therefore$ turns / phase = conductors per phase/2 = 192/2 = 96

$$E_{ph} = 4\, k_f\, k_c\, k_d\, f\, \Phi T$$

$$= 4 \times 1.11 \times 0.966 \times 0.96 \times 50 \times 5 \times 10^{-2} \times 96 = \mathbf{988\ V}$$

Example 37.9. *A 10-pole, 50-Hz, 600 r.p.m. alternator has flux density distribution given by the following expression*

$$B = \sin\theta + 0.4 \sin 3\theta + 0.2 \sin 5\theta$$

The alternator has 180 slots wound with 2-layer 3-turn coils having a span of 15 slots. The coils are connected in 60° groups. If armature diameter is = 1.2 m and core length = 0.4 m, calculate

(i) *the expression for instantaneous e.m.f. / conductor*

(ii) *the expression for instantaneous e.m.f./coil*

(iii) *the r.m.s. phase and line voltages, if the machine is star-connected.*

Solution. For finding voltage/conductor, we may either use the relation *Blv* or use the relation of Art. 35-13.

Area of pole pitch $= (1.2\,\pi/10) \times 0.4 = 0.1508\ \text{m}^2$

Fundamental flux/pole, ϕ_1 = av. flux density × area $= 0.637 \times 1 \times 0.1508 = 0.096$ Wb

(a) RMS value of fundamental voltage per conductor,

$$= 1.1 \times 2 f \phi_1 = 1.1 \times 2 \times 50 \times 0.096 = \mathbf{10.56\ V}$$

Peak value $= \sqrt{2} \times 10.56 = 14.93$ V

Since harmonic conductor voltages are in proportion to their flux densities,

3rd harmonic voltage $= 0.4 \times 14.93 = 5.97$ V

5th harmonic voltage $= 0.2 \times 14.93 = 2.98$ V

Hence, equation of the instantaneous e.m.f./conductor is

$$\mathbf{e = 14.93 \sin\theta + 5.97 \sin 3\theta + 2.98 \sin 5\theta}$$

(b) Obviously, there are 6 conductors in a 3-turn coil. Using the values of k_c found in solved Ex. 37.5, we get

fundamental coil voltage $= 6 \times 14.93 \times 0.966 = 86.5$ V

3rd harmonic coil voltage $= 6 \times 5.97 \times 0.707 = 25.3$ V

5th harmonic coil voltage $= 6 \times 2.98 \times 0.259 = 4.63$ V

---

* Since they are not of much interest, the relative phase angles of the voltages have not been included in the expression.

Hence, coil voltage expression is*

$$e = 86.5 \sin \theta + 25.3 \sin 3\theta + 4.63 \sin 5\theta$$

(c) Here, $m = 6, \beta = 180°/18 = 10°$; $k_{d1} = \dfrac{\sin 6 \times 10°/2}{6 \sin 10°/2} = 0.956$

$$k_{d3} = \frac{\sin 3 \times 6 \times 10°/2}{6 \sin 3 \times 10°/2} = 0.644 \qquad k_{d5} = \frac{\sin 5 \times 6 \times 10°/2}{6 \sin 5 \times 10°/2} = 0.197$$

It should be noted that number of coils per phase = 180/3 = 60

Fundamental phase e.m.f.= $(86.5A\sqrt{2}) \times 60 \times 0.956 = 3510$ V

3rd harmonic phase e.m.f.= $(25.3A\sqrt{2}) \times 60 \times 0.644 = 691$ V

5th harmonic phase e.m.f.= $(4.63A\sqrt{2}) \times 60 \times 0.197 = 39$ V

RMS value of phase voltage = $(3510^2 + 691^2 + 39^2)^{1/2}$ = **3577 V**

RMS value of line voltage = $\sqrt{3} \times (3510^2 + 39^2)^{1/2}$ = **6080 V**

Example 37.10. *A 4-pole, 3-phase, 50-Hz, star-connected alternator has 60 slots, with 4 conductors per slot. Coils are short-pitched by 3 slots. If the phase spread is 60°, find the line voltage induced for a flux per pole of 0.943 Wb distributed sinusoidally in space. All the turns per phase are in series.* **(Electrical Machinery, Mysore Univ. 1987)**

Solution. As explained in Art. 37.12, phase spread = $m\beta = 60°$ —given

Now, $m = 60/4 \times 3 = 5 \quad \therefore \quad 5\beta = 60°, \beta = 12°$

$$k_d = \frac{\sin 5 \times 12°/2}{5 \sin 12°/2} = 0.957; \alpha = (3/15) \times 180° = 36°; k_c = \cos 18° = 0.95$$

$$Z = 60 \times 4/3 = 80; T = 80/2 = 40; \Phi = 0.943 \text{ Wb}; k_f = 1.11$$

$$\therefore \quad E_{ph} = 4 \times 1.11 \times 0.95 \times 0.975 \times 50 \times 0.943 \times 40 = 7613 \text{ V}$$

$$E_L = \sqrt{3} \times 7613 = \mathbf{13{,}185\ V}$$

Example 37.11. *A 4-pole, 50-Hz, star-connected alternator has 15 slots per pole and each slot has 10 conductors. All the conductors of each phase are connected in series' the winding factor being 0.95. When running on no-load for a certain flux per pole, the terminal e.m.f. was 1825 volt. If the windings are lap-connected as in a d.c. machine, what would be the e.m.f. between the brushes for the same speed and the same flux/pole. Assume sinusoidal distribution of flux.*

Solution. Here $k_f = 1.11, k_d = 0.95, k_c = 1$ (assumed)

$f = 50$ Hz; e.m.f./phase = $1825/\sqrt{3}$ V

Total No. of slots = $15 \times 4 = 60$

$\therefore$ No. of slots/phase = $60/3 = 20$; No. of turns/phase = $20 \times 10/2 = 100$

$\therefore \quad 1825/\sqrt{3} = 4 \times 1.11 \times 1 \times 0.95 \times \Phi \times 50 \times 100 \quad \therefore \Phi = 49.97$ mWb

When connected as a d.c. generator

$$E_g = (\Phi ZN/60) \times (P/A) \text{ volt}$$

$$Z = 60 \times 10 = 600, \qquad N = 120 f/P = 120 \times 50/4 = 1500 \text{ r.p.m.}$$

$$\therefore \quad E_g = \frac{49.97 \times 10^{-3} \times 600 \times 1500}{60} \times \frac{4}{4} = \mathbf{750\ V}$$

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* Also $k_d = \sin 150°/2 = \sin 75° = 0.966$

Example 37.12. *An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field current is 2.4 A.*

Solution. As seen from the e.m.f. equation of an alternator,

$$E \propto \Phi f \quad \therefore \frac{E_1}{E_2} = \frac{\Phi_1 f_1}{\Phi_2 f_2}$$

Since saturation is neglected, $\Phi \propto I_f$ where I_f is the field current

$$\therefore \quad \frac{E_1}{E_2} = \frac{I_{f1} \cdot f_1}{I_{f2} \cdot f_2} \quad \text{or} \quad \frac{360}{E_2} = \frac{3.6 \times 60}{2.4 \times 40}; E_2 = \textbf{160 V}$$

Example 37.13. *Calculate the R.M.S. value of the induced e.m.f. per phase of a 10-pole, 3-phase, 50-Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is 150°. The flux per pole has a fundamental component of 0.12 Wb and a 20% third component.* **(Elect. Machines-III, Punjab Univ. 1991)**

Solution. Fundamental E.M.F.

$$\alpha = (180° - 150°) = 30°; \; k_{c1} = \cos \alpha/2 = \cos 15° = 0.966$$

$$m = 2 \text{ ; No. of slots/pole} = 6 \text{ ; } \beta = 180°/6 = 30°$$

$$\therefore \quad k_{d1} = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 2 \times 30°/2}{2 \sin 30°/2} = 0.966$$

$$Z = 10 \times 2 \times 4 = 80 \text{ ; turn/phase, } T = 80/2 = 40$$

$\therefore$ Fundamental E.M.F./phase $= 4.44\, k_c\, k_d\, f\, \Phi\, T$

$$\therefore \quad E_1 = 4.44 \times 0.966 \times 0.966 \times 50 \times 0.12 \times 40 = 995\text{V}$$

Hormonic E.M.F.

$$K_{c3} = \cos 3\,\alpha/2 = \cos 3 \times 30°/2 = \cos 45° = 0.707$$

$$k_{d3} = \frac{\sin mn\,\beta/2}{m \sin n\,\beta/2} \text{ where } n \text{ is the order of the harmonic } i.e. \; n = 3$$

$$\therefore \quad k_{d3} = \frac{\sin 2 \times 3 \times 30°/2}{2 \sin 3 \times 30°/2} = \frac{\sin 90°}{2 \sin 45°} = 0.707, f_2 = 50 \times 3 = 150 \text{ Hz}$$

$$\Phi_3 = (1/3) \times 20\% \text{ of fundamental flux} = (1/3) \times 0.02 \times 0.12 = 0.008 \text{ Wb}$$

$$\therefore \quad E_3 = 4.44 \times 0.707 \times 0.707 \times 150 \times 0.008 \times 40 = 106 \text{ V}$$

$$\therefore \quad E \text{ per phase} = \sqrt{E_1^2 + E_3^2} = \sqrt{995^2 + 106^2} = \textbf{1000 V}$$

Note. Since phase e.m.fs. induced by the 3rd, 9th and 15th harmonics etc. are eliminated from the line voltages, the line voltage for a *Y*-connection would be $= 995 \times \sqrt{3}$ volt.

Example 37.14. *A 3-phase alternator has generated e.m.f. per phase of 230 V with 10 per cent third harmonic and 6 per cent fifth harmonic content. Calculate the r.m.s. line voltage for (a) star connection (b) delta-connection. Find also the circulating current in delta connection if the reactance per phase of the machine at 50-Hz is 10 Ω.* **(Elect. Machines-III, Osmania Univ. 1988)**

Solution. It should be noted that in both star and delta-connections, the third harmonic components of the three phases cancel out at the line terminals because they are co-phased. Hence, the line e.m.f. is composed of the fundamental and the fifth harmonic only.

(a) Star-connection

$$E_1 = 230 \text{ V} \; ; E_5 = 0.06 \times 230 = 13.8 \text{ V}$$

$$\text{E.M.F./phase} = \sqrt{E_1^2 + E_3^2} = \sqrt{230^2 + 13.8^2} = 230.2\ V$$

$$\text{R.M.S. value of line e.m.f.} = \sqrt{3} \times 230.2 = \mathbf{3.99\ V}$$

(b) Delta-connection

Since for delta-connection, line e.m.f. is the same as the phase e.m.f.

R.M.S. value of line e.m.f. = **230.2 V**

In delta-connection, third harmonic components are additive round the mesh, hence a circulating current is set up whose magnitude depends on the reactance per phase at the third harmonic frequency.

R.M.S. value of third harmonic e.m.f. per phase = $0.1 \times 230 = 23$ V

$$\text{Reactance at triple frequency} = 10 \times 3 = 30\ \Omega$$

$$\text{Circulating current} = 23/30 = \mathbf{0.77\ A}$$

Example 37.15 (a). *A motor generator set used for providing variable frequency a.c. supply consists of a three-phase, 10-pole synchronous motor and a 24-pole, three- phase synchronous generator. The motor-generator set is fed from a 25 Hz, three-phase a.c. supply. A 6-pole, three-phase induction motor is electrically connected to the terminals of the synchronous generator and runs at a slip of 5%. Determine :*

(i) *the frequency of the generated voltage of the synchronous generator.*

(ii) *the speed at which the induction motor is running.* **(U.P. Technical University 2001)**

Solution. Speed of synchronous motor = $(120 \times 25)/10 = 300$ rpm.

(i) At 300 rpm, frequency of the voltage generated by 24-pole synchronous generator

$$= \frac{24 \times 300}{120} = 60 \text{ Hz}$$

Synchronous speed of the 6-pole induction motor fed from a 60 Hz supply

$$= \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

(ii) With 5 % slip, the speed of this induction motor = $0.95 \times 1200 = 1140$ rpm.

Further, the frequency of the rotor-currents = $s\,f = 0.05 \times 60 = 3$ Hz.

Example 37.15 (b). *Find the no load line voltage of a star connected 4-pole alternator from the following :*

Flux per pole = 0.12 Weber, Slots per pole per phase = 4

Conductors/ slot = 4, Two layer winding, with coil span = 150°

[Bharthithasan University, April 1997]

Solution. Total number of slots = $4 \times 3 \times 4 = 48$, Slot pitch = 15° electrical

Total number of conductors = $48 \times 4 = 192$, Total number of turns = 96

No. of turns in series per phase = 32

For a 60° phase spread,

$$k_b = \frac{\sin\,(60°/2)}{4 \times \sin 7.5°} = 0.958$$

For 150° coil-span, pitch factor $k_p = \cos 15° = 0.966$, and for 50 Hz frequency,

$$E_{ph} = 4.44 \times 50 \times 0.12 \times 0.958 \times 0.966 \times 32 = 789 \text{ volts}$$

$$E_{line} = 789 \times 1.732 = 1366.6 \text{ volts}$$

Tutorial Problems 37.1

1. Find the no-load phase and line voltage of a star-connected, 4-pole alternator having flux per pole of 0.1 Wb sinusoidally distributed; 4 slots per pole per phase, 4 conductors per slot, double-layer winding with a coil span of 150°.

   [Assuming f = 50 Hz; 789 V; 1366 V] *(Elect. Technology-I, Bombay Univ. 1978)*

2. A 3-ϕ, 10-pole, *Y*-connected alternator runs at 600 r.p.m. It has 120 stator slots with 8 conductors per slot and the conductors of each phase are connected in series. Determine the phase and line e.m.fs. if the flux per pole is 56 mWb. Assume full-pitch coils.

   [1910 V; 3300 V] *(Electrical Technology-II, Madras Univ. April 1977)*

3. Calculate the speed and open-circuit line and phase voltages of a 4-pole, 3-phase, 50-Hz, star-connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.0496 Wb and is sinusoidally distributed. **[1500 r.p.m.; 3,300 V; 1,905 V]** *(Elect. Engg-II, Bombay Univ. 1979)*

4. A 4-pole, 3-phase, star-connected alternator armature has 12 slots with 24 conductors per slot and the flux per pole is 0.1 Wb sinusoidally distributed.

   Calculate the line e.m.f. generated at 50 Hz. **[1850 V]**

5. A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 30 mWb sinusoidally distributed. Find the frequency, the phase and line voltage if the speed is 375 rpm. **[50 Hz; 1530 V; 2650 V]** *(Electrical Machines-I, Indore Univ. April 1977)*

6. A synchronous generator has 9 slots per pole. If each coil spans 8 slot pitches, what is the value of the pitch factor ? **[0.985]** *(Elect. Machines, A.M.I.E. Sec. B. 1989)*

7. A 3-phase, *Y*-connected, 2-pole alternator runs at 3,600 r.p.m. If there are 500 conductors per phase in series on the armature winding and the sinusoidal flux per pole is 0.1 Wb, calculate the magnitude and frequency of the generated voltage from first principles. **[60 Hz; 11.5 kV]**

8. One phase of a 3-phase alternator consists of twelve coils in series. Each coil has an r.m.s. voltage of 10 V induced in it and the coils are arranged in slots so that there is a successive phase displacement of 10 electrical degrees between the e.m.f. in each coil and the next. Find graphically or by calculation, the r.m.s. value of the total phase voltage developed by the winding. If the alternator has six pole and is driven at 100 r.p.m., calculate the frequency of the e.m.f. generated. **[108 V; 50 Hz]**

9. A 4-pole, 50-Hz, 3-phase, Y-connected alternator has a single-layer, full-pitch winding with 21 slots per pole and two conductors per slot. The fundamental flux is 0.6 Wb and air-gap flux contains a third harmonic of 5% amplitude. Find the r.m.s. values of the phase e.m.f. due to the fundamental and the 3rd harmonic flux and the total induced e.m.f.

   [3,550 V; 119.5 V; 3,553 V] *(Elect. Machines-III, Osmania Univ. 1977)*

10. A 3-phase, 10-pole alternator has 90 slots, each containing 12 conductors. If the speed is 600 r.p.m. and the flux per pole is 0.1 Wb, calculate the line e.m.f. when the phases are (*i*) star connected (*ii*) delta connected. Assume the winding factor to be 0.96 and the flux sinusoidally distributed.

    [(i) 6.93 kV (ii) 4 kV] *(Elect. Engg-II, Kerala Univ. 1979)*

11. A star-connected 3-phase, 6-pole synchronous generator has a stator with 90 slots and 8 conductors per slot. The rotor revolves at 1000 r.p.m. The flux per pole is 4×10^{-2} weber. Calculate the e.m.f. generated, if all the conductors in each phase are in series. Assume sinusoidal flux distribution and full-pitched coils. **[Eph = 1,066 V]** *(Elect. Machines, A.M.I.E. Summer, 1979)*

12. A six-pole machine has an armature of 90 slots and 8 conductors per slot and revolves at 1000 r.p.m. the flux per pole being 50 milli weber. Calculate the e.m.f. generated as a three-phase star-connected machine if the winding factor is 0.96 and all the conductors in each phase are in series.

    [1280 V] *(Elect. Machines, AMIE, Sec. B, (E-3), Summer 1992)*

13. A 3-phase, 16 pole alternator has a star connected winding with 144 slots and 10 conductors per slot. The flux/pole is 0.04 wb (sinusoidal) and the speed is 375 rpm. Find the frequency and phase and line e.m.f. The total turns/phase may be assumed to series connected.

    [50 Hz, 2035 Hz, 3525 V] *(Rajiv Gandhi Technical University, Bhopal, 2000)*

37.15. Factors Affecting Alternator Size

The efficiency of an alternator always increases as its power increases. For example, if an alternator of 1 kW has an efficiency of 50%, then one of 10 MW will inevitably have an efficiency of about 90%. It is because of this improvement in efficiency with size that alternators of 1000 MW and above possess efficiencies of the order of 99%.


Light weight alternator

Another advantage of large machines is that power output per kilogram increases as the alternator power increases. If 1 kW alternator weighs 20 kg (*i.e.* 50 W/kg), then 10MW alternator weighing 20,000 kg yields 500 W/kg. In other words, larger alternators weigh relatively less than smaller ones and are, consequently, cheaper.

However, as alternator size increases, cooling problem becomes more serious. Since large machines inherently produce high power loss per unit surface area (W/m^2), they tend to overheat. To keep the temperature rise within acceptable limits, we have to design efficient cooling system which becomes ever more elaborate as the power increases. For cooling alternators of rating upto 50 MW, circulating cold-air system is adequate but for those of rating between 50 and 300 MW, we have to resort to hydrogen cooling. Very big machines in 1000 MW range have to be equipped with hollow water-cooled conductors. Ultimately, a point is reached where increased cost of cooling exceeds the saving made elsewhere and this fixes the upper limit of the alternator size.


Large weight alternator

So for as the speed is concerned, low-speed alternators are always bigger than high speed alternators of the same power. Bigness always simplifies the cooling problem. For example, the large 200-rpm, 500-MVA alternators installed in a typical hydropower plant are air-cooled whereas much smaller 1800-r.p.m., 500-MVA alternators installed in a steam plant are hydrogen cooled.

37.16. Alternator on Load

As the load on an alternator is varied, its terminal voltage is also found to vary as in d.c. generators. This variation in terminal voltage V is due to the following reasons:

1. voltage drop due to armature resistance R_a
2. voltage drop due to armature leakage reactance X_L
3. voltage drop due to armature reaction

(a) Armature Resistance

The armature resistance/phase R_a causes a voltage drop/phase of IR_a which is in phase with the armature current I. However, this voltage drop is practically negligible.

(b) Armature Leakage Reactance

When current flows through the armature conductors, fluxes are set up which do not cross the air-gap, but take different paths. Such fluxes are known as *leakage fluxes*. Various types of leakage fluxes are shown in Fig. 37.22.

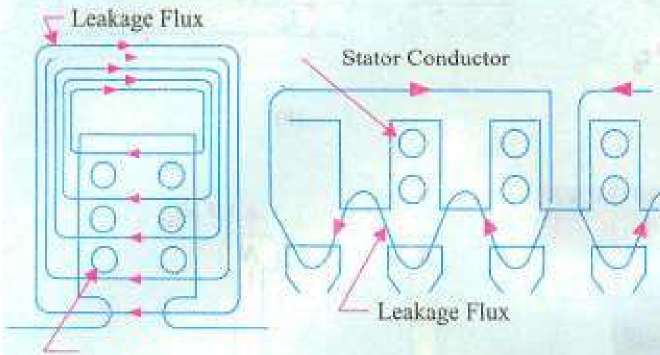

Fig. 37.22

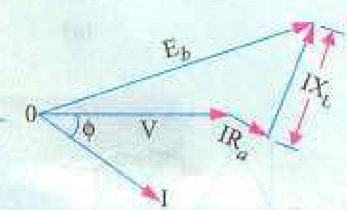

Fig. 37.23

The leakage flux is practically independent of saturation, but is dependent on I and its phase angle with terminal voltage V. This leakage flux sets up an e.m.f. of self-inductance which is known as *reactance e.m.f.* and which is ahead of I by 90°. Hence, armature winding is assumed to possess leakage reactance X_L (also known as Potier rectance X_P) such that voltage drop due to this equals IX_L. A part of the generated e.m.f. is used up in overcoming this reactance e.m.f.

$$\therefore \qquad E = V + I\,(R + jX_L)$$

This fact is illustrated in the vector diagram of Fig. 37.23.

(c) Armature Reaction

As in d.c. generators, armature reaction is the effect of armature flux on the main field flux. ***In the case of alternators, the power factor of the load has a considerable effect on the armature reaction***. We will consider three cases : *(i)* when load of p.f. is unity *(ii)* when p.f. is zero lagging and *(iii)* when p.f. is zero leading.

Before discussing this, it should be noted that in a 3-phase machine the combined ampere-turn wave (or m.m.f. wave) is sinusoidal which moves synchronously. This amp-turn or m.m.f. wave is fixed relative to the poles, its amplitude is proportional to the load current, but its position depends on the p.f. of the load.

Consider a 3-phase, 2-pole alternator having a single-layer winding, as shown in Fig. 37.24 (*a*). For the sake of simplicity, assume that winding of each phase is concentrated (instead of being distributed) and that the number of turns per phase is N. Further suppose that the alternator is loaded with a resistive load of unity power factor, so that phase currents I_a, I_b and I_c are in phase with their respective phase voltages. Maximum current I_a will flow when the poles are in position shown in Fig. 37.24 (*a*) or at a time t_1 in Fig. 37.24 (*c*). When I_a has a maximum value, I_b and I_c have one-half their maximum values (the arrows attached to I_a, I_b and I_c are only polarity marks and are not meant to give the instantaneous directions of these currents at time t_1). The instantaneous directions of currents are shown in Fig. 37.24 (*a*). At the instant t_1, I_a flows in conductor α whereas I_b and I_c flow out.

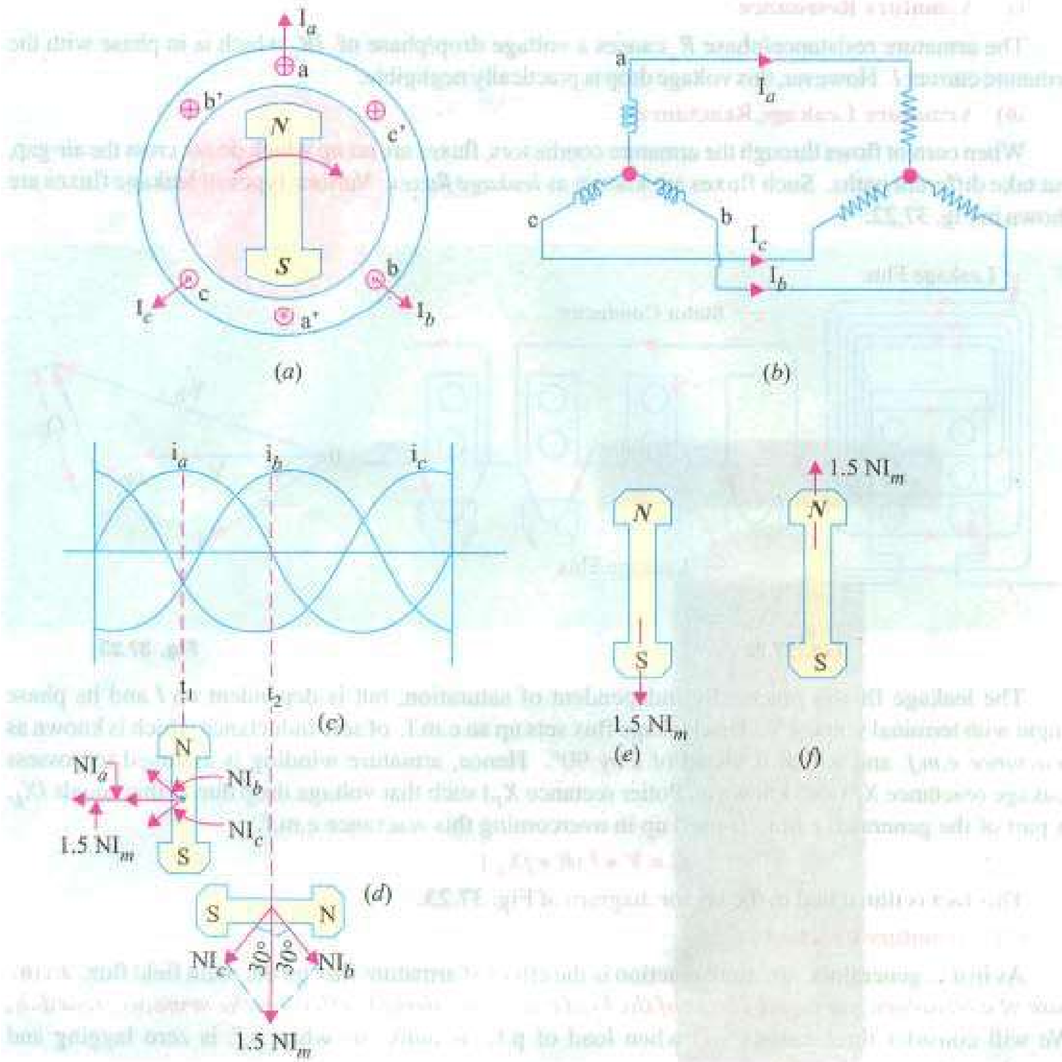

Fig. 37.24

As seen from Fig. 37.24 (*d*), the m.m.f. ($= NI_m$) produced by phase *a-a′* is horizontal, whereas that produced by other two phases is $(I_m/2)\ N$ each at 60° to the horizontal. The total armature m.m.f. is equal to the vector sum of these three m.m.fs.

$\therefore$ Armature m.m.f. $= NI_m + 2.(1/2\ NI_m) \cos 60° = 1.5\ NI_m$

As seen, at this instant t_1, the m.m.f. of the main field is upwards and the armature m.m.f. is behind it by 90 electrical degrees.

Next, let us investigate the armature m.m.f. at instant t_2. At this instant, the poles are in the horizontal position. Also $I_a = 0$, but I_b and I_c are each equal to 0.866 of their maximum values. Since I_c has not changed in direction during the interval t_1 to t_2, the direction of its m.m.f. vector remains unchanged. But I_b has changed direction, hence, its m.m.f. vector will now be in the position shown in Fig. 37.24 (*d*). Total armature m.m.f. is again the vector sum of these two m.m.fs.

$\therefore$ Armature m.m.f. $= 2 \times (0.866\ NI_m) \times \cos 30° = 1.5\ NI_m$.

If further investigations are made, it will be found that.

1. armature m.m.f. remains constant with time.

2. it is 90 space degrees behind the main field m.m.f., so that it is only distortional in nature.
3. it rotates synchronously round the armature *i.e.* stator.

For a lagging load of zero power factor, all currents would be delayed in time 90° and armature m.m.f. would be shifted 90° with respect to the poles as shown in Fig. 37.24 (*e*). Obviously, armature m.m.f. would demagnetise the poles and cause a reduction in the induced e.m.f. and hence the terminal voltage.

For leading loads of zero power factor, the armature m.m.f. is advanced 90° with respect to the position shown in Fig. 37.24 (*d*). As shown in Fig. 37.24 (*f*), the armature m.m.f. strengthens the main m.m.f. In this case, armature reaction is wholly magnetising and causes an increase in the terminal voltage.

The above facts have been summarized briefly in the following paragraphs where the matter is discussed in terms of 'flux' rather than m.m.f. waves.

1. Unity Power Factor

In this case [Fig. 37.25 (*a*)] the armature flux is cross-magnetising. The result is that the flux at the leading tips of the poles is reduced while it is increased at the trailing tips. However, these two effects nearly offset each other leaving the average field strength constant. In other words, armature reaction for unity p.f. is distortional.

2. Zero P.F. lagging

As seen from Fig. 37.25 (*b*), here the armature flux (whose wave has moved backward by 90°) is in direct opposition to the main flux.

Hence, the main flux is decreased. Therefore, it is found that armature reaction, in this case, is wholly ***demagnetising***, with the result, that due to weakening of the main flux, less e.m.f. is generated. To keep the value of generated e.m.f. the same, field excitation will have to be increased to compensate for this weakening.

3. Zero P.F. leading

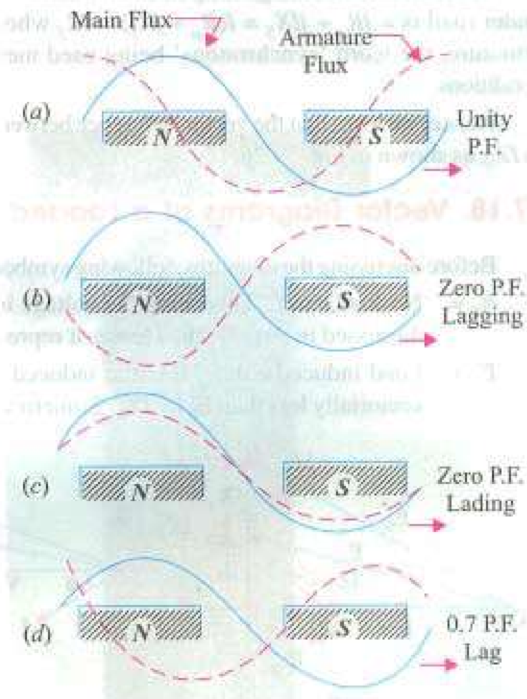

Fig. 37.25

In this case, shown in Fig. 37.25 (*c*) armature flux wave has moved forward by 90° so that it is in phase with the main flux wave. This results in added main flux. Hence, in this case, armature reaction is wholly *magnetising*, which results in greater induced e.m.f. To keep the value of generated e.m.f. the same, field excitation will have to be reduced somewhat.

4. For intermediate power factor [Fig. 37.25 (*d*)], the effect is partly distortional and partly demagnetising (because p.f. is lagging).

37.17. Synchronous Reactance

From the above discussion, it is clear that for the same field excitation, terminal voltage is decreased from its no-load value E_0 to V (for a lagging power factor). This is because of

1. drop due to armature resistance, IR_a
2. drop due to leakage reactance, IX_L
3. drop due to armature reaction.

The drop in voltage due to armature reaction may be accounted for by assumiung the presence of a fictitious reactance X_a in the armature winding. The value of X_a is such that IX_a represents the voltage drop due to armature reaction.

The leakage reactance X_L (or X_P) and the armature reactance X_a may be combined to give synchronous reactance X_S.

Hence $\quad X_S = X_L + X_a$*

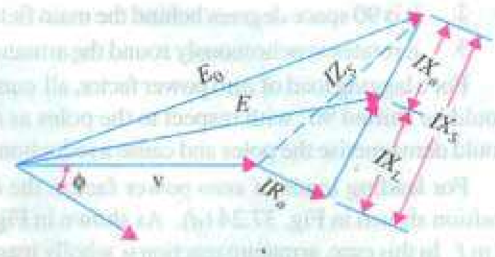

Fig. 37.26

Therefore, total voltage drop in an alternator under load is $= IR_a + jIX_S = I(R_a + jX_S) = IZ_S$ where Z_S is known as synchronous impedance of the armature, the word 'synchronous' being used merely as an indication that it refers to the working conditions.

Hence, we learn that the vector difference between no-load voltage E_0 and terminal voltage V is equal to IZ_S, as shown in Fig. 37.26.

37.18. Vector Diagrams of a Loaded Alternator

Before discussing the diagrams, following symbols should be clearly kept in mind.

E_0 = No-load e.m.f. This being the voltage induced in armature in the absence of three factors discussed in Art. 37.16. Hence, it represents the maximum value of the induced e.m.f.

E = Load induced e.m.f. It is the induced e.m.f. after allowing for armature reaction. E is vectorially less than E_0 by IX_a. Sometimes, it is written as E_a (Ex. 37.16).

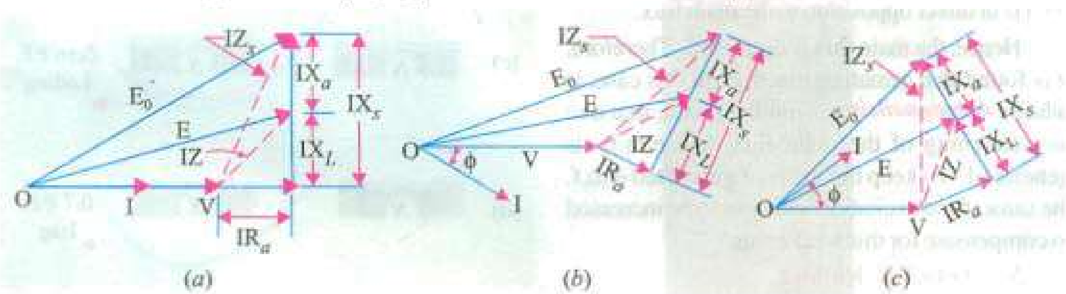

Fig. 37.27

V = Terminal voltage. It is vectorially less than E_0 by IZ_S or it is vectorially less than E by I_Z where

$Z = \sqrt{(R_a^2 + X_L^2)}$. It may also be written as Z_a.

I = armature current/phase and ϕ = load p.f. angle.

In Fig. 37.27 (*a*) is shown the case for unity p.f., in Fig. 37.27 (*b*) for lagging p.f. and in Fig. 37.27 (*c*) for leading p.f. All these diagrams apply to one phase of a 3-phase machine. Diagrams for the other phases can also be drawn similary.

Example 37.16. *A 3-phase, star-connected alternator supplies a load of 10 MW at p.f. 0.85 lagging and at 11 kV (terminal voltage). Its resistance is 0.1 ohm per phase and synchronous reactance 0.66 ohm per phase. Calculate the line value of e.m.f. generated.*

(Electrical Technology, Aligarh Muslim Univ. 1988)

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* The ohmic value of X_a varies with the p.f. of the load because armature reaction depends on load p.f.

Solution. F.L. output current $= \dfrac{10\times10^6}{\sqrt{3}\times11,000\times0.85} = 618\ A$

IR_a drop $= 618\times0.1 = 61.8$ V

IX_S drop $= 618\times0.66 = 408$ V

Terminal voltage/phase $= 11,000/\sqrt{3} = 6,350$ V

$\phi = \cos^{-1}(0.85) = 31.8°;\ \sin\phi = 0.527$

As seen from the vector diagram of Fig. 37.28 where I instead of V has been taken along reference vector,

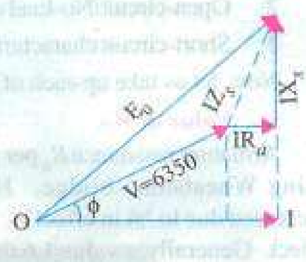


Fig. 37.28

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2}$$

$$= \sqrt{(6350\times0.85+61.8)^2 + (6350\times0.527+408)^2}$$

$$= 6,625\ \text{V}$$

Line e.m.f. $= \sqrt{3}\times6,625 =$ **11,486 volt**

37.19. Voltage Regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends ***not only on the load but also on the load power factor.***

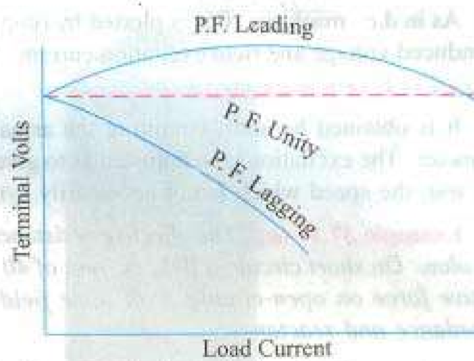


Fig. 37.29

The voltage regulation of an alternator is defined as "the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage."

$\therefore$ % regulation 'up' $= \dfrac{E_0 - V}{V}\times100$

Note. *(i)* $E_0 - V$ is the arithmetical difference and not the vectorial one.

(ii) In the case of *leading* load p.f., terminal voltage will fall on removing the full-load. Hence, regulation is negative in that case.

(iii) The rise in voltage when full-load is thrown off is not the same as the fall in voltage when full-load is applied.

Voltage characteristics of an alternator are shown in Fig. 37.29.

37.20. Determination of Voltage Regulation

In the case of small machines, the regulation may be found by direct loading. The procedure is as follows :

The alternator is driven at synchronous speed and the terminal voltage is adjusted to its rated value V. The load is varied until the wattmeter and ammeter (connected for the purpose) indicate the rated values at desired p.f. Then the entire load is thrown off while the speed and field excitation are kept constant. The open-circuit or no-load voltage E_0 is read. Hence, regulation can be found from

$$\%\ \text{regn} = \frac{E_0 - V}{V}\times100$$

In the case of large machines, the cost of finding the regulation by direct loading becomes prohibitive. Hence, other indirect methods are used as discussed below. It will be found that all these methods differ chiefly in the way the no-load voltage E_0 is found in each case.

1. ***Synchronous Impedance or E.M.F. Method.*** It is due to Behn Eschenberg.
2. ***The Ampere-turn or M.M.F. Method.*** This method is due to Rothert.
3. ***Zero Power Factor or Potier Method.*** As the name indicates, it is due to Potier.

All these methods require—

1. Armature (or stator) resistance R_a
2. Open-circuit/No-load characteristic.
3. Short-circuit characteristic (but zero power factor lagging characteristic for Potier method).

Now, let us take up each of these methods one by one.

(*i*) Value of Ra

Armature resistance R_a per phase can be measured directly by voltmeter and ammeter method or by using Wheatstone bridge. However, under working conditions, the effective value of R_a is increased due to 'skin effect'*. The value of R_a so obtained is increased by 60% or so to allow for this effect. Generally, a value 1.6 times the d.c. value is taken.

(*ii*) O.C. Characteristic

As in d.c. machines, this is plotted by running the machine on no-load and by noting the values of induced voltage and field excitation current. It is just like the *B-H* curve.

(*iii*) S.C. Characteristic

It is obtained by short-circuiting the armature (*i.e.* stator) windings through a low-resistance ammeter. The excitation is so adjusted as to give 1.5 to 2 times the value of full-load current. During this test, the speed which is not necessarily synchronous, is kept constant.

Example 37.17 (*a*). *The effective resistance of a 2200V, 50Hz, 440 KVA, 1-phase, alternator is 0.5 ohm. On short circuit, a field current of 40 A gives the full load current of 200 A. The electromotive force on open-circuits with same field excitation is 1160 V. Calculate the synchronous impedance and reactance.* **(Madras University, 1997)**

Solution. For the 1-ph alternator, since the field current is same for O.C. and S.C. conditions

$$Z_S = \frac{1160}{200} = 5.8 \text{ ohms}$$

$$X_S = \sqrt{5.8^2 - 0.5^2} = 5.7784 \text{ ohms}$$

Example 37.17 (*b*). *A 60-KVA, 220 V, 50-Hz, 1-φ alternator has effective armature resistance of 0.016 ohm and an armature leakage reactance of 0.07 ohm. Compute the voltage induced in the armature when the alternator is delivering rated current at a load power factor of (a) unity (b) 0.7 lagging and (c) 0.7 leading.* **(Elect. Machines-I, Indore Univ. 1981)**

Solution. Full load rated current $I = 60{,}000/220 = 272.2$ A

$$IR_a = 272.2 \times 0.016 = 4.3 \text{ V};$$
$$IX_L = 272.2 \times 0.07 = 19 \text{ V}$$

(*a*) Unity p.f. — Fig. 37.30 (*a*)

$$E = \sqrt{(V + IR_a)^2 + (I\,X_L)^2} = \sqrt{(220 + 4.3)^2 + 19^2} = \mathbf{225\ V}$$

* The 'skin effect' may sometimes increase the effective resistance of armature conductors as high as 6 times its d.c. value.

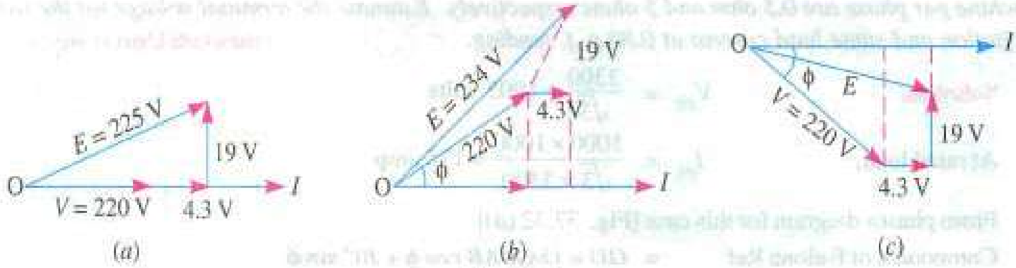


Fig. 37.30

(b) p.f. 0.7 (lag) —Fig. 37.30 (b)

$$E = [V\cos\phi + IR_a)^2 + (V\sin\phi + IX_L)^2]^{1/2}$$
$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 + 19)^2]^{1/2} = \mathbf{234\ V}$$

(c) p.f. = 0.7 (lead) —Fig. 37.30 (c)

$$E = [(V\cos\phi + IR_a)^2 + (V\sin\phi - IX_L)^2]^{1/2}$$
$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 - 19)^2]^{1/2} = \mathbf{208\ V}$$

Example 37.18 (a). *In a 50-kVA, star-connected, 440-V, 3-phase, 50-Hz alternator, the effective armature resistance is 0.25 ohm per phase. The synchronous reactance is 3.2 ohm per phase and leakage reactance is 0.5 ohm per phase. Determine at rated load and unity power factor :*

(a) Internal e.m.f. E_a (b) no-load e.m.f. E_0 (c) percentage regulation on full-load (d) value of synchronous reactance which replaces armature reaction.

(Electrical Engg. Bombay Univ. 1987)

Solution. (a) The e.m.f. E_a is the vector sum of (i) terminal voltage V (ii) IR_a and (iii) IX_L as detailed in Art. 37.17. Here,

$$V = 440/\sqrt{3} = 254\ \text{V}$$

F.L. output current at u.p.f. is

$$= 50{,}000/\sqrt{3} \times 440 = 65.6\ \text{A}$$

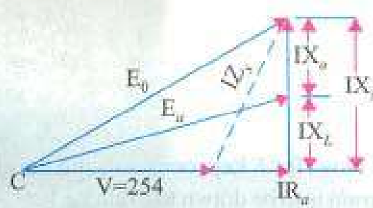


Fig. 37.31

Resistive drop = 65.6 × 0.25 = 16.4 V

Leakage reactance drop IX_L = 65.6 × 0.5 = 32.8 V

$$\therefore \quad E_a = \sqrt{(V + IR_a)^2 + (IX_L)^2}$$
$$= \sqrt{(254 + 16.4)^2 + 32.8^2} = 272\ \text{volt}$$

Line value $= \sqrt{3} \times 272 = \mathbf{471\ volt.}$

(b) The no-load e.m.f. E_0 is the vector sum of (i) V (ii) IR_a and (iii) IX_S or is the vector sum of V and IZ_S (Fig. 37.31).

$$\therefore \quad E_0 = \sqrt{(V + IR_a)^2 + (IX_S)^2} = \sqrt{(254 + 16.4)^2 + (65.6 \times 3.2)^2} = 342\ \text{volt}$$

Line value $= \sqrt{3} \times 342 = \mathbf{592\ volt}$

(c) % age regulation 'up' $= \dfrac{E_0 - V}{V} \times 100 = \dfrac{342 - 254}{254} \times 100 = \mathbf{34.65\ per\ cent}$

(d) $X_a = X_S - X_L = 3.2 - 0.5 = \mathbf{2.7\ \Omega}$ ...Art. 37.17

Example 37.18 (b). *A 1000 kVA, 3300-V, 3-phase, star-connected alternator delivers full-load current at rated voltage at 0.80 p. f. Lagging. The resistance and synchronous reactance of the* *machine per phase are 0.5 ohm and 5 ohms respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p. f. leading.* **(Amravati University, 1999)**

Solution. $V_{ph} = \dfrac{3300}{\sqrt{3}} = 1905$ volts

At rated load, $I_{ph} = \dfrac{1000 \times 1000}{\sqrt{3} \times 3300} = 175$ amp

From phasor diagram for this case [Fig. 37.32 (*a*)]

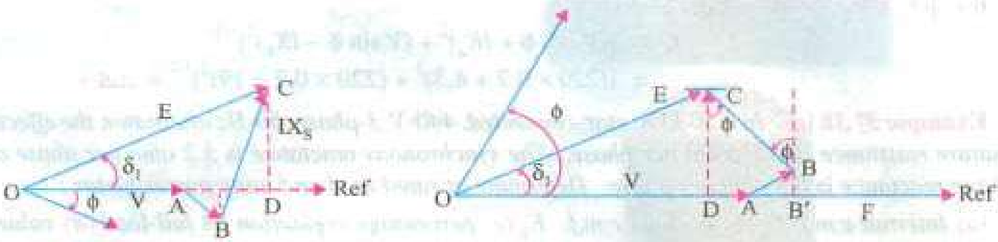
Component of E along Ref $= OD = OA + AB \cos\phi + BC \sin\phi$

$= 1905 + (87.5 \times 0.80) + (875 \times 0.60) = 2500$

Component of E along perpendicular direction

$= CD = -AB \sin\phi + BC \cos\phi$

$= 87.5 \times 0.6 + 875 \times 0.80 = 647.5$ volts



(*a*) Phasor diagram at lagging P.f. (*b*) Phasor diagram for leading P.F.

Fig. 37.32

$$OA = 1950, AB = I_r = 87.5, BC = IX_S = 875$$

$$OC = E = \sqrt{OD^2 + DC^2} = \sqrt{2500^2 + 647.5^2} = \mathbf{2582.5\ volts}$$

$$\delta_1 = \sin^{-1}\frac{CD}{OC} = \sin^{-1}(647.5/2582.5) = 14.52°$$

Now, for *E* kept constant, and the alternator delivering rated current at 0.80 leading p.f., the phasor diagram is to be drawn to evaluate *V*.

Construction of the phasor diagram starts with marking the reference. Take a point *A* which is the terminating point of phasor *V* which starts from *O*. *O* is the point yet to be marked, for which the other phasors have to be drawn.

$$AB = 87.5, BC = 875$$

$$BAF = 36.8°$$

BC perpendicular to *AB*. From *C*, draw an arc of length *E*, *i.e.* 2582.5 volts to locate *O*.

Note. Construction of Phasor diagram starts from known *AE*, *V* is to be found.

Along the direction of the current, $AB = 87.5$, $\angle BAF = 36.8°$, since the current is leading. $BC = 875$ which must be perpendicular to *AB*. Having located *C*, draw a line *CD* which is perpendicular to the reference, with point *D*, on it, as shown.

Either proceed graphically drawing to scale or calculate geometrically :

$CD = AB \sin\phi + BC \cos\phi = (87.5 \times 0.60) + (875 \times 0.80) = 752.5$ volts

Since $CD = E \sin\delta_2$, $\sin\delta_2 = 752.5/2582.5$ giving $\delta_2 = 17°$

$OD = E \cos\delta = 2470$ volts

$DA = DB' - AB'$

$= BC \sin\phi - AB \cos\phi - 875 \times 0.6 - 87.5 \times 0.8 = 455$ volts

Terminal voltage, $V = OA = OD + DA = 2470 + 455 = 2925$ volts/phase

Since the alternator is star connected, line voltage $\sqrt{3} \times 2925 = 5066$ volts

Check : While delivering lagging p.f. current,

Total power delivered= (100 kVA) × 0.80 = 800 kW

In terms of E and δ_1 referring to the impedance-triangle in Fig. 37.32 (c)

total power delivered

$$= 3\left[\frac{VE}{Z_S}\cos(\theta - \delta_1) - \frac{V^2}{Z_S}\cos\theta\right]$$

$$= 3\left[\frac{1905 \times 2582.5}{5.025}\cos(84.3° - 14.52°) - 1905^2 \times \cos 84.3°\right]$$

$$= 800 \text{ kW} \quad \text{...checked}$$

$$OA = 0.5, \quad AB = 5$$

$$OB = \sqrt{0.5^2 + 5^2} = 5.025\ \Omega$$

$$\theta = \angle BOA = \tan^{-1}(X_S/R) = \tan^{-1} 10 = 84.3°$$

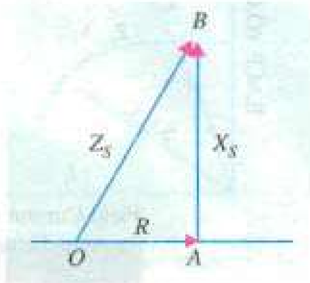


Fig. 37.32 (c) Impedance triangle for the Alternator

While delivering leading p.f. current the terminal voltage is 5.066 kV line to line.

Total power delivered in terms of V and I

$$= \sqrt{3} \times 5.066 \times 175 \times 0.8 \text{ kW} = 1228.4 \text{ kW}$$

In terms of E and with voltages expressed in volts,

total power output

$$= 3\left[\frac{VE}{Z_S}\cos(\theta - \delta_2) - \frac{V^2}{Z_S}\cos\theta\right] \times 10^{-3} \text{ kW}$$

$$= 3\left[\frac{2925 \times 2582.5}{5.025}\cos(84.3 - 17°) - \frac{2925^2}{5.025} \times \cos 84.3°\right] \times 10^{-3} \text{ kW}$$

$$= 3\left[\frac{2925 \times 2582.5}{5.025} \times \cos(67.3°) - \frac{2925 \times 2925}{5.025}\cos 84.3°\right] \text{kW}$$

$= 3\,(580.11 - 169.10) = 1233$ kW, which agrees fairly closely to the previous figure and hence checks our answer.

37.21. Synchronous Impedance Method

Following procedural steps are involved in this method:

1. O.C.C is plotted from the given data as shown in Fig. 37.33 (a).

2. Similarly, S.C.C. is drawn from the data given by the short-circuit test. It is a straight line passing through the origin. Both these curves are drawn on a common field-current base.

Consider a field current I_f. The O.C. voltage corresponding to this field current is E_1. When winding is short-circuited, the terminal voltage is zero. Hence, it may be assumed that the whole of this voltage E_1 is being used to circulate the armature short-circuit current I_1 against the synchronous impedance Z_S.

$$\therefore \qquad E_1 = I_1 Z_S \qquad \therefore Z_S = \frac{E_1 \text{ (open-circuit)}}{I_1 \text{ (short-circuit)}}$$

3. Since R_a can be found as discussed earlier, $X_S = \sqrt{(Z_S^2 - R_a^2)}$

4. Knowing R_a and X_S, vector diagram as shown in Fig. 37.33 (*b*) can be drawn for any load and any power factor.

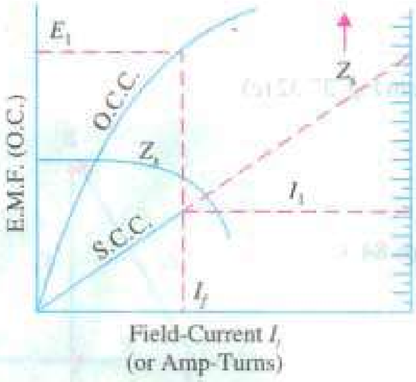


Fig. 37.33 (a)

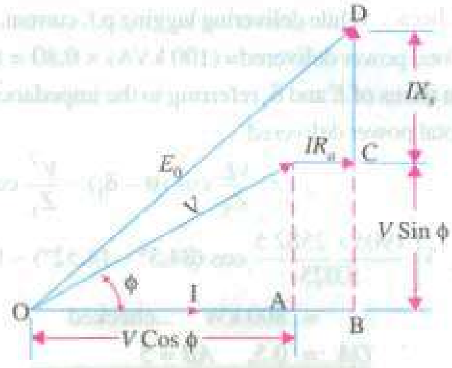


Fig. 37.33 (b)

Here $\quad OD = E_0 \quad \therefore E_0 = \sqrt{(OB^2 + BD^2)}$

or $$E_0 = \sqrt{[(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]}$$

$\therefore$ $$\%\text{ regn. 'up'} = \frac{E_0 - V}{V} \times 100$$

Note. (*i*) Value of regulation for unity power factor or leading p.f. can also be found in a similar way.

(*ii*) This method is not accurate because the value of Z_S so found is always more than its value under normal voltage conditions and saturation. Hence, the value of regulation so obtained is always more than that found from an actual test. That is why it is called pessimistic method. The value of Z_S is not constant but varies with saturation. At low saturation, its value is larger because then the effect of a given armature ampere-turns is much more than at high saturation. Now, under short-circuit conditions, saturation is very low, because armature m.m.f. is directly demagnetising. Different values of Z_S corresponding to different values of field current are also plotted in Fig. 37.33 (*a*).

(*iii*) The value of Z_S usually taken is that obtained from full-load current in the short-circuit test.

(*iv*) Here, armature reactance X_a has not been treated separately but along with leakage reactance X_L.

Example 37.19. *Find the synchronous impedance and reactance of an alternator in which a given field current produces an armature current of 200 A on short-circuit and a generated e.m.f. of 50 V on open-circuit. The armature resistance is 0.1 ohm. To what induced voltage must the alternator be excited if it is to deliver a load of 100 A at a p.f. of 0.8 lagging, with a terminal voltage of 200V.* **(Elect. Machinery, Banglore Univ. 1991)**

Solution. It will be assumed that alternator is a single phase one. Now, for same field current,

$$Z_S = \frac{\text{O.C. volts}}{\text{S.C. current}} = \frac{50}{200} = \mathbf{0.25\ \Omega.}$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{0.25^2 - 0.1^2} = \mathbf{0.23\ \Omega.}$$

Now, $\quad IR_a = 100 \times 0.1 = 10\text{ V}, IX_S = 100 \times 0.23 = 23\text{ V};$

$\cos\phi = 0.8, \sin\phi = 0.6.$ As seen from Fig. 37.34.

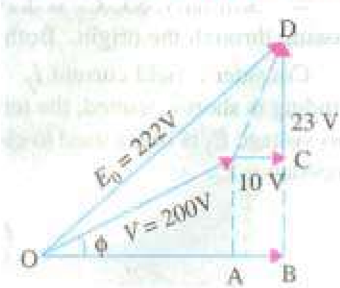


Fig. 37.34

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2}$$

$$= [(200 \times 0.8 + 10)^2 + (200 \times 0.6 + 23)^2]^{1/2} = \mathbf{222\ V}$$

Example 37.20. *From the following test results, determine the voltage regulation of a 2000-V, 1-phase alternator delivering a current of 100 A at (i) unity p.f. (ii) 0.8 leading p.f. and (iii) 0.71 lagging p.f.*

Test results : Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8Ω. **(Elect. Engg.-II, M.S. Univ. 1987)**

Solution. $Z_S = \dfrac{\text{O.C. volts}}{\text{S.C. current}}$ —for same excitation

for same excitation

$$= 500/100 = 5\ \Omega$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.936\ \Omega$$

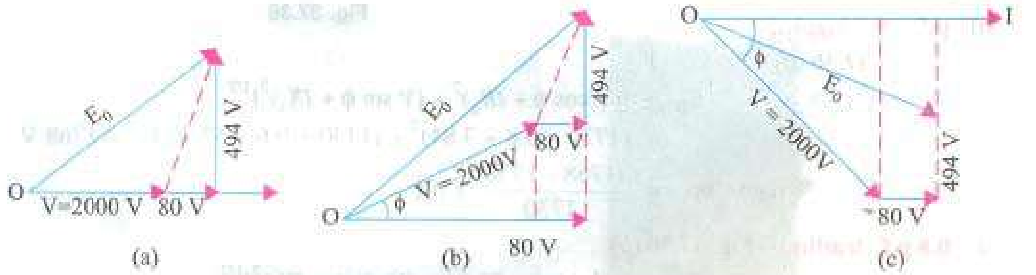


Fig. 37.35

(i) **Unity p.f.** (Fig. 37.35 (*a*)]

$$IR_a = 100 \times 0.8 = 80\ \text{V};\quad IX_S = 100 \times 4.936 = 494\ \text{V}$$

$$\therefore \quad E_0 = \sqrt{(2000 + 80)^2 + 494^2} = 2140\ V$$

$$\%\ \text{regn} = \frac{2140 - 2000}{2000} \times 100 = \mathbf{7\%}$$

(ii) **p.f. = 0.8 (lead)** [Fig. 37.35 (*c*)]

$$E_0 = [(2000 \times 0.8 + 80)^2 + (2000 \times 0.6 - 494)^2]^{1/2} = 1820\ \text{V}$$

$$\%\ \text{regn} = \frac{1820 - 2000}{2000} \times 100 = \mathbf{-9\%}$$

(iii) **p.f. = 0.71(lag)** [Fig.37.35 (*b*)]

$$E_0 = [(2000 \times 0.71 + 80)^2 + (2000 \times 0.71 + 494)^2]^{1/2} = 2432\ \text{V}$$

$$\%\ \text{regn} = \frac{2432 - 2000}{2000} \times 100 = \mathbf{21.6\%}$$

Example 37.21. *A 100-kVA, 3000-V, 50-Hz 3-phase star-connected alternator has effective armature resistance of 0.2 ohm. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V (line value). Calculate the full-load voltage regulation at 0.8 p.f. lagging and 0.8 p.f. leading. Draw phasor diagrams.*

(Basic Elect. Machines, Nagpur Univ. 1993)

Solution.
$$Z_S = \frac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}} \quad \text{— for same excitation}$$
$$= \frac{1040/\sqrt{3}}{200} = 3\ \Omega$$
$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{3^2 - 0.2^2}$$
$$= 2.99\ \Omega$$

F.L. current,
$$I = 100{,}000/\sqrt{3} \times 3000$$
$$= 19.2\ \text{A}$$
$$IR_a = 19.2 \times 0.2 = 3.84\ \text{V}$$
$$IX_S = 19.2 \times 2.99 = 57.4\ \text{V}$$

Voltage/phase
$$= 3000/\sqrt{3} = 1730\text{V}$$

$\cos\phi = 0.8$; $\sin\phi = 0.6$

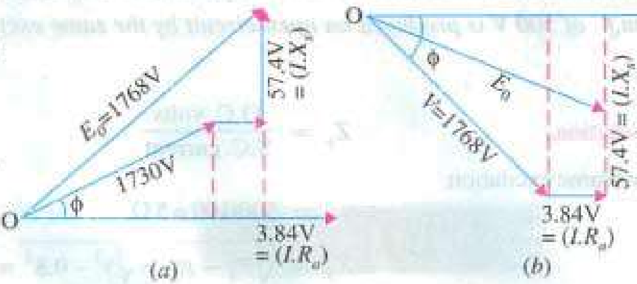


Fig. 37.36

(i) p.f. = 0.8 lagging —Fig. 37.36 (*a*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$
$$= (1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 + 57.4)^2]^{1/2} = 1768\ \text{V}$$
$$\%\ \text{regn. 'up'} = \frac{(1768 - 1730)}{1730} \times 100 = \mathbf{2.2\%}$$

(ii) 0.8 p.f. leading—Fig. 37.36 (*b*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi - IX_S)^2]^{1/2}$$
$$= [(1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 - 57.4)^2]^{1/2}$$
$$= 1699\ \text{V}$$
$$\%\ \text{regn.} = \frac{1699 - 1730}{1730} \times 100 = \mathbf{-1.8\%}$$

Example 37.22. *A 3-phase, star-connected alternator is rated at 1600 kVA, 13,500 V. The armature resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at 0.8 leading power factor.*

(Advanced Elect. Machines AMIE Sec. B, 1991)

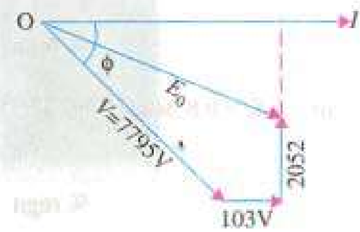


Fig. 37.37

Solution.
$$1280{,}000 = \sqrt{3} \times 13{,}500 \times I \times 0.8;$$
$$\therefore \quad I = 68.4\ \text{A}$$
$$IR_a = 68.4 \times 1.5 = 103\ \text{V} \; ; \; IX_S = 68.4 \times 30 = 2052$$
$$\text{Voltage/phase} = 13{,}500/\sqrt{3} = 7795\ \text{V}$$

As seen from Fig. 37.37.
$$E_0 = [(7795 \times 0.8 + 103)^2 + (7795 \times 0.6 - 2052)]^{1/2} = 6663\ \text{V}$$
$$\%\ \text{regn.} = (6663 - 7795)/7795$$
$$= -0.1411 \text{ or } - \mathbf{14.11\%}$$

Example 37.23. *A 3-phase, 10-kVA, 400-V, 50-Hz, Y-connected alternator supplies the rated load at 0.8 p.f. lag. If arm. resistance is 0.5 ohm and syn. reactance is 10 ohms, find the power angle and voltage regulation.* **(Elect. Machines-I Nagpur Univ. 1993)**

Solution. F.L. current, $I = 10{,}000/\sqrt{3} \times 400 = 14.4\ A$

$$IR_a = 14.4 \times 0.5 = 7.2\ \text{V}$$

$$IX_S = 14.4 \times 10 = 144\ \text{V}$$

Voltage/phase = $400/\sqrt{3} = 231$ V

$\phi = \cos^{-1} 0.8 = 36.87°$, as shown in Fig. 37.38.

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$

$$= (231 \times 0.8 + 7.2)^2 + (231 \times 0.6 + 144)^2]^{1/2}$$

$$= 342\ \text{V}$$

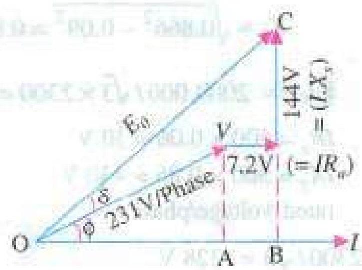


Fig. 37.38

$$\%\ \text{regn.} = \frac{342 - 231}{231} \times 100 = 0.48 \text{ or } 48\%$$

The power angle of the machine is defined as the angle between V and E_0 *i.e.* angle δ

As seen from Fig. 37.38, $\tan(\phi + \delta) = \dfrac{BC}{OB} = \dfrac{231 \times 0.6 + 144}{231 \times 0.8 + 7.2} = \dfrac{282.6}{192} = 1.4419$;

$\therefore \qquad (\phi + \delta) = 55.26°$

$\therefore$ power angle $\qquad \delta = 55.26° - 36.87° = \mathbf{18.39°}$

Example 37.24. *The following test results are obtained from a 3-phase, 6,000-kVA, 6,600 V, star-connected, 2-pole, 50-Hz turbo-alternator:*

With a field current of 125 A, the open-circuit voltage is 8,000 V at the rated speed; with the same field current and rated speed, the short-circuit current is 800 A. At the rated full-load, the resistance drop is 3 per cent. Find the regulation of the alternator on full-load and at a power factor of 0.8 lagging. **(Electrical Technology, Utkal Univ. 1987)**

Solution. $$Z_S = \frac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}} = \frac{8000/\sqrt{3}}{800} = 5.77\ \Omega$$

Voltage/phase $= 6{,}600\sqrt{3} = 3{,}810$ V

Resistive drop $= 3\%$ of $3{,}810\ \text{V} = 0.03 \times 3{,}810 = 114.3$ V

Full-load current $= 6{,}000 \times 10^3/\sqrt{3} \times 6{,}600 = 525$ A

Now $\qquad IR_a = 114.3\text{V}$

$\therefore \qquad R_a = 114.3/525 = 0.218\ \Omega$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{5.77^2 - 0.218^2} = 5.74\ \Omega \text{ (approx.)}$$

As seen from the vector diagram of Fig. 37.33, (*b*)

$$E_0 = \sqrt{[3{,}810 \times 0.8 + 114.3)^2 + (3{,}810 \times 0.6 + 525 \times 5.74)^2]} = 6{,}180\ \text{V}$$

$\therefore$ regulation $= (6{,}180 - 3{,}810) \times 100/3{,}810 = \mathbf{62.2\%}$

Example 37.25. *A 3-phase 50-Hz star-connected 2000-kVA, 2300 V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the open circuit voltage was 900 V. The resistance between a pair of terminals was 0.12 Ω. Find full-load regulation at (i) UPF (ii) 0.8 p.f. lagging.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. $$Z_S = \frac{\text{O.C. volts/phase}}{\text{S.C. current/phase}} = \frac{900/\sqrt{3}}{600} = 0.866\ \Omega$$

Resistance between the terminals is 0.12 Ω. It is the resistance of two phases connected in series.

∴ Resistance/phase = 0.12/2 = 0.06 Ω;

effective resistance/phase = 0.06 × 1.5 = 0.09 Ω;

$$X_S = \sqrt{0.866^2 - 0.09^2} = 0.86\ \Omega$$

F.L. $I = 2000{,}000/\sqrt{3} \times 2300 = 500$ A

$IR_a = 500 \times 0.06 = 30$ V ;

$IX_S = 500 \times 0.86 = 430$ V

rated voltage/phase

$= 2300/\sqrt{3} = 1328$ V

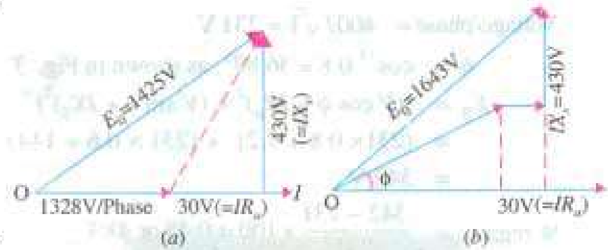


Fig. 37.39

(*i*) **U.P.F.** —Fig. 37.39 (*a*).

$$E_0 = [(V\cos\phi + IR_a)^2 + (IX_S)^2]^{1/2}$$

$$= \sqrt{(1328 + 30)^2 + 430^2} = 1425\text{ V}$$

% regn. = (1425 − 1328)/1328 = 0.073 or **7.3%**

(*ii*) **0.8 p.f. lagging** —Fig. 37.39 (*b*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2]^{1/2}$$

$$= [(1328 \times 0.8 + 30)^2 + (1328 \times 0.6 + 430)^2]^{1/2} = 1643\text{ V}$$

∴ % regn. = (1643 − 1328)/1328 = 0.237 or **23.7%**.

Example 37.26. *A 2000-kVA, 11-kV, 3-phase, star-connected alternator has a resistance of 0.3 ohm and reactance of 5 ohm per phase. It delivers full-load current at 0.8 lagging power factor at rated voltage. Compute the terminal voltage for the same excitation and load current at 0.8 power factor leading.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. (*i*) **At 0.8 p.f. lagging**

F.L. $I = 2000{,}000/\sqrt{3} \times 11{,}000 = 105$ A

Terminal voltage $= 11{,}000/\sqrt{3} = 6350$ V

$IR_a = 105 \times 0.3 = 31.5$ V;

$IX_S = 105 \times 5 = 525$ V

As seen from Fig. 37.40 (*a*)

$E_0 = [6350 \times 0.8 + 31.5)^2 + (6350 \times 0.6 + 525)^2]^{1/2} = 6700$ V

As seen from Fig. 37.40 (*b*), now, we are given $E_0 = 6700$ V and we are required to find the terminal voltage V at 0.8 p.f. leading.

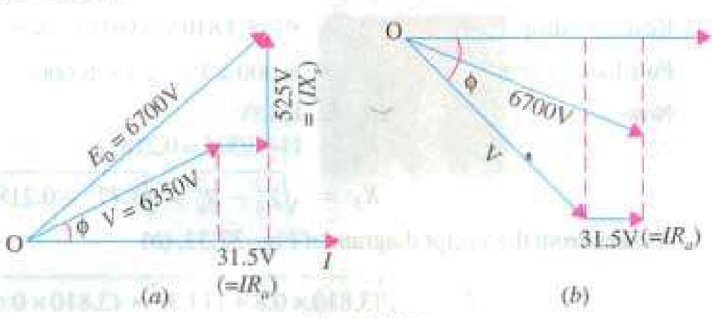


Fig. 37.40

$6700^2 = (0.8\text{V} + 31.5)^2 + (0.6\text{V} - 525)^2$; V = **6975 V**

Example 37.27. *The effective resistance of a 1200-kVA, 3.3-kV, 50-Hz, 3-phase, Y-connected alternator is 0.25 Ω/phase. A field current of 35 A produces a current of 200 A on short-circuit and 1.1 kV (line to line) on open circuit. Calculate the power angle and p.u. change in magnitude of the terminal voltage when the full load of 1200 kVA at 0.8 p.f. (lag) is thrown off. Draw the corresponding phasor diagram.* **(Elect. Machines, A.M.I.E. Sec. B, 1993)**

Solution. $Z_s = \dfrac{\text{O.C. voltage}}{\text{S.C. voltage}}$ —same excitation

$$= \frac{1.1\times10^3/\sqrt{3}}{200} = 3.175\,\Omega$$

$$X_S = \sqrt{3.175^2 - 0.25^2} = 3.165\,\Omega$$

$$V = 3.3\times10^3/\sqrt{3} = 1905\text{ V}$$

$$\tan\theta = X_s/R_a = 3.165/0.25,\ \theta = 85.48°$$

$$\therefore\quad Z_s = 3.175\angle 85.48°$$

Rated $I_a = 1200\times10^3/\sqrt{3}\times3.3\times10^3$

$$= 210\text{ A}$$

Let, $V = 1905\angle 0°,\quad I_a = 210\angle -36.87°$

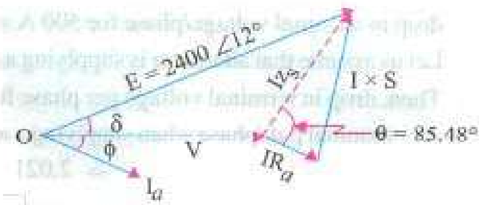


Fig. 37.41

As seen from Fig. 37.41,

$E = V + I_aZ_s = 1905 + 210\angle -36.87° \times 3.175\angle 85.48° = 2400\angle 12°$

Power angle $= \delta = 12°$

Per unit change in terminal voltage is

$$= (2400 - 1905)/1905 = \mathbf{0.26}$$

Example 37.28. *A given 3-MVA, 50-Hz, 11-kV, 3-φ, Y-connected alternator when supplying 100 A at zero p.f. leading has a line-to-line voltage of 12,370 V; when the load is removed, the terminal voltage falls down to 11,000 V. Predict the regulation of the alternator when supplying full-load at 0.8 p.f. lag. Assume an effective resistance of 0.4 Ω per phase.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. As seen from Fig. 37.42 (*a*), at zero p.f. leading

$$E_0^2 = (V\cos\phi + IR_a)^2 + V\sin\phi - IX_S)^2$$

Now, $E_0 = 11{,}000/\sqrt{3} = 6350\text{ V}$

$$V = 12370/\sqrt{3} = 7{,}142\text{ V},$$

$$\cos\phi = 0,\ \sin\phi = 1$$

$$\therefore\quad 6350^2 = (0 + 100\times0.4)^2 + (7142 - 100X_S)^2$$

$$\therefore\quad 100X_S = 790 \text{ or } X_S = 7.9\,\Omega$$

F.L. current

$$I = \frac{3\times10^6}{\sqrt{3}\times11{,}000} = 157\text{ A}$$

$IR_a = 0.4\times157 = 63\text{ V};\ IX_S$

$= 157\times7.9 = 1240\text{ V}$

$\therefore\quad E_0 = [(6350\times0.8+63)^2 + (6350\times0.6+1240)^2]^{1/2} = 7210\text{ V/phase}$

$$\therefore\quad \%\text{ regn} = \frac{7210-6350}{6350}\times100 = \mathbf{13.5\%}$$

Fig. 37.42

Example 37.29. *A straight line law connects terminal voltage and load of a 3-phase star-connected alternator delivering current at 0.8 power factor lagging. At no-load, the terminal voltage is 3,500 V and at full-load of 2,280 kW, it is 3,300 V. Calculate the terminal voltage when delivering current to a 3-φ, star-connected load having a resistance of 8 Ω and a reactance of 6 Ω per phase. Assume constant speed and field excitation.* **(London Univ.)**

Solution. No-load phase voltage $= 3{,}500/\sqrt{3} = 2{,}021$ V

Phase voltage on full-load and 0.8 power factor $= 3{,}300/\sqrt{3} = 1905$ V

Full-load current is given by

$$\sqrt{3}\,V_L I_L \cos\phi = 2{,}280 \times 1000 \qquad \therefore \quad I_L = \frac{2{,}280 \times 1000}{\sqrt{3} \times 3{,}300 \times 0.8} = 500 \text{ A}$$

drop in terminal voltage/phase for 500 A $= 2{,}021 - 1{,}905 = 116$ V

Let us assume that alternator is supplying a current of x ampere.

Then, drop in terminal voltage per phase for x ampere is $= 116\,x/500 = 0.232\,x$ volt

$\therefore$ terminal p.d./phase when supplying x amperes at a p.f. of 0.8 lagging is

$$= 2{,}021 - 0.232\,x \text{ volt}$$

Impedance of connected load/phase $= \sqrt{(8^2 + 6^2)} = 10\ \Omega$

load p.f. $= \cos\phi = 8/10 = 0.8$

When current is x, the applied p.d. is $= 10\,x$

$\therefore \quad 10\,x = 2021 - 0.232\,x$ or $x = 197.5$ A

$\therefore$ terminal voltage/phase $= 2021 - (0.232 \times 197.5) = 1975.2$ V

$\therefore$ terminal voltage of alternator $= 1975.2 \times \sqrt{3} =$ **3,421 V**

Tutorial Problem No. 37.2

1. If a field excitation of 10 A in a certain alternator gives a current of 150 A on short-circuit and a terminal voltage of 900 V on open-circuit, find the internal voltage drop with a load current of 60A. **[360 V]**
2. A 500-V, 50-kVA, 1-ϕ alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short-circuit and an e.m.f. of 450 V on open-circuit. Calculate the full-load regulation at p.f. 0.8 lag. **[34.4%]**
(Electrical Technology, Bombay Univ. 1978)
3. A 3-ϕ star-connected alternator is rated at 1600 kVA, 13,500 V. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of (*a*) 0.8 leading and (*b*) 0.8 lagging. **[(*a*) −11.8% (*b*) 18.6%]** *(Elect. Engg.-II, Bombay Univ. 1977)*
4. Determine the voltage regulation of a 2,000-V, 1-phase alternator giving a current of 100 A at 0.8 p.f. leading from the test results. Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5 A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8 Ω. Draw the vector diagram.
[− 8.9%] *(Electrical Machines-I, Gujarat Univ. Apr. 1976)*
5. In a single-phase alternator, a given field current produces an armature current of 250 A on short-circuit and a generated e.m.f. of 1500 V on open-circuit. Calculate the terminal p.d. when a load of 250 A at 6.6 kV and 0.8 p.f. lagging is switched off. Calculate also the regulation of the alternator at the given load.
[7.898 V; 19.7%] *(Elect. Machines-II, Indore Univ. Dec. 1977)*
6. A 500-V, 50-kVA, single-phase alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short-circuit and e.m.f. of 450 V on open circuit. Calculate (a) the synchronous impedance and reactance and (*b*) the full-load regulation with 0.8 p.f. lagging.
[(*a*) 2.25 Ω, 2.24 Ω, (b) 34.4%] *(Elect. Technology, Mysore Univ. 1979)*

7. A 100-kVA, 3,000-V, 50-Hz, 3-phase star-connected alternator has effective armature resistance of 0.2 Ω. A field current of 40 A produces short-circuit current of 200 A and an open-circuit e.m.f. of 1040 V (line value). Calculate the full-load percentage regulation at a power factor of 0.8 lagging. How will the regulation be affected if the alternator delivers its full-load output at a power factor of 0.8 leading?

[24.4% – 13.5%] *(Elect. Machines-II, Indore Univ. July 1977)*

8. A 3-φ, 50-Hz, star-connected, 2,000 kVA, 2,300-V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the O.C. voltage was 900 V. The resistance between a pair of terminals was 0.12 Ω. Find full-load regulation at (*a*) u.p.f. (*b*) 0.8 p.f. lagging (*c*) 0.8 p.f. leading.

[(*a*) 7.3% (b) 23.8% (*c*) –13.2%] *(Elect. Machinery-III, Bangalore Univ. Aug. 1979)*

9. A 3-phase star-connected alternator is excited to give 6600 V between lines on open circuit. It has a resistance of 0.5 Ω and synchronous reactance of 5 Ω per phase. Calculate the terminal voltage and regulation at full load current of 130 A when the P.F. is (*i*) 0.8 lagging, (*ii*) 0.6 leading. *[Rajive Gandhi Technical University, Bhopal, 2000]*

[(*i*) 3318 Volts/Ph, + 14.83% (*ii*) 4265 Volts/Ph, - 10.65%]

37.22. Rothert's M.M.F. or Ampere-turn Method

This method also utilizes O.C. and S.C. data, but is the converse of the E.M.F. method in the sense ***that armature leakage reactance is treated as an additional armature reaction***. In other words, it is assumed that the change in terminal p.d. on load is due entirely to armature reaction (and due to the ohmic resistance drop which, in most cases, is negligible). This fact is shown in Fig. 37.43.

Now, field A.T. required to produce a voltage of V on full-load is the vector sum of the following :

(*i*) Field A.T. required to produce V (or if R_a is to be taken into account, then $V + I R_a \cos \phi$) on *no-load*. This can be found from O.C.C. and

(*ii*) Field A.T. required to overcome the demagnetising effect of armature reaction on full-load. This value is found from short-circuit test. The field A.T. required to produce full-load current on short-circuit balances the armature reaction and the ***impedance drop***.

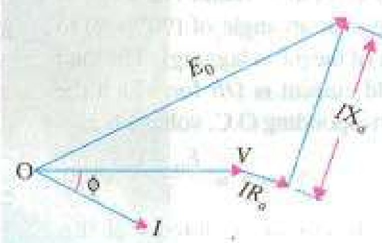


Fig. 37.43

The impedance drop can be neglected because R_a is usually very small and X_S is also small under short-circuit conditions. Hence, p.f. on short-circuit is almost zero lagging and the field A.T. are used entirely to overcome the armature reaction which is wholly demagnetising (Art. 37.15). ***In other words, the demagnetising armature A.T. on full-load are equal and opposite to the field A.T. required to produce full-load current on short-circuit.***

Now, if the alternator, instead of being on short-circuit, is supplying full-load current at its normal voltage and zero p.f. lagging, then total field A.T. required are the vector sum of

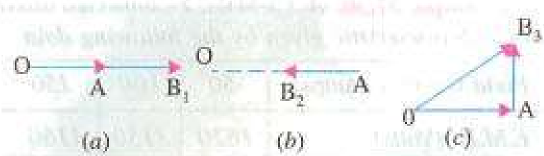


Fig. 37.44

(*i*) the field A.T. = OA necessary to produce normal voltage (as obtained from O.C.C.) and

(*ii*) the field A.T. necessary to neutralize the armature reaction AB_1. The total field A.T. are represented by OB_1 in Fig. 37.44 (*a*) and equals the vector sum of OA and AB_1

If the p.f. is zero leading, the armature reaction is *wholly magnetising*. Hence, in that case, the field A.T. required is OB_2 which is less than OA by the field A.T. $= AB_2$ required to produce full-load current on short-circuit [Fig. 37.44 (*b*)].

If p.f. is unity, the armature reaction is *cross-magnetising i.e.* its effect is distortional only. Hence, field A.T. required is OB_3 *i.e.* vector sum of OA and AB_3 which is drawn at right angles to OA as in Fig. 37.44 (*c*).

37.23. General Case

Let us consider the general case when the p.f. has any value between zero (lagging or leading) and unity. Field ampere-turns OA corresponding to V(or $V + IR_a \cos \phi$) is laid off horizontally. Then AB_1, representing full-load short-circuit field A.T. is drawn at an angle of $(90° + \phi)$ for a lagging p.f. The total field A.T. are given by OB_1 as in Fig. 37.45. (*a*). For a leading p.f., short-circuit A.T. $= AB_2$ is drawn at an angle of $(90° - \phi)$ as shown in Fig. 37.45 (*b*) and for unity p.f., AB_3 is drawn at right angles as shown in Fig. 37.45 (*c*).

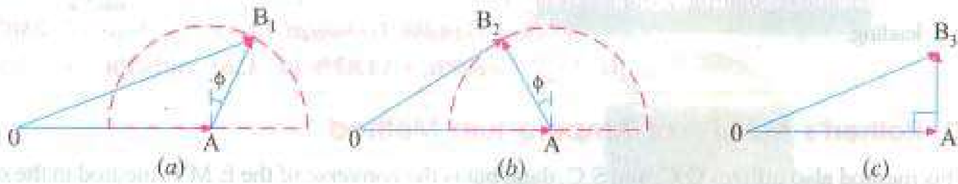


Fig. 37.45

In those cases where the number of turns on the field coils is not known, it is usual to work in terms of the field current as shown in Fig. 37.46.

In Fig. 37.47. is shown the complete diagram along with O.C. and S.C. characteristics. OA represents field current for normal voltage V. OC represents field current required for producing full-load current on *short-circuit*. Vector $AB = OC$ is drawn at an angle of $(90° + \phi)$ to OA (if the p.f. is lagging). The total field current is OB for which the corresponding O.C. voltage is E_0

$$\therefore \quad \% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

It should be noted that this method gives results which are less than the actual results, that is why it is sometimes referred to as optimistic method.

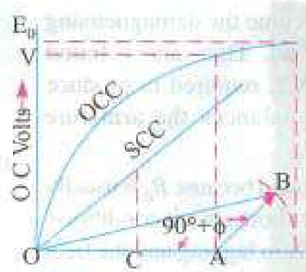


Fig. 37.46

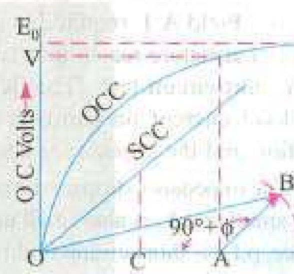


Fig. 37.47

Example 37.30. *A 3.5-MVA, Y-connected alternator rated at 4160 volts at 50-Hz has the open-circuit characteristic given by the following data :*

Field Current (Amps)	*50*	*100*	*150*	*200*	*250*	*300*	*350*	*400*	*450*
E.M.F. (Volts)	*1620*	*3150*	*4160*	*4750*	*5130*	*5370*	*5550*	*5650*	*5750*

A field current of 200 A is found necessary to circulate full-load current on short-circuit of the alternator. Calculate by (i) synchronous impedance method and (ii) ampere-turn method the full-load voltage regulation at 0.8 p.f. lagging. Neglect resistance. Comment on the results obtained.

(Electrical Machines-II, Indore Univ. 1984)

Solution. (*i*) As seen from the given data, a field current of 200 A produces O.C. voltage of 4750 (line value) and full-load current on short-circuit which is

$$= 3.5 \times 10^6 / \sqrt{3} \times 4160 = 486 \text{ A}$$

$$Z_S = \frac{\text{O.C. volt/phase}}{\text{S.C. current/phase}} = \frac{4750/\sqrt{3}}{486} = \frac{2740}{486} = 5.64\ \Omega/\text{phase}$$

Since $R_a = 0, X_S = Z_S \quad \therefore \quad IR_a = 0, IX_S = IZ_S = 486 \times 5.64 = 2740$ V

$$\text{F.L. Voltage/phase} = 4160/\sqrt{3} = 2400 \text{ V}, \cos\phi = 0.8, \sin\phi = 0.6$$

$$E_0 = (V\cos\phi + IR_a)^2 + (V\sin\phi + I.X_S)^2]^{1/2}$$

$$= [(2400 \times 0.8 + 0)^2 + (2400 \times 0.6 + 2740)^2]^{1/2} = 4600 \text{ V}$$

$$\%\text{ regn. up} = \frac{4600 - 2400}{2400} \times 100 = \mathbf{92.5\%}$$

(*ii*) It is seen from the given data that for normal voltage of 4160 V, field current needed is 150 A. Field current necessary to circulate F.L. current on short-circuit is 200 A.

In Fig. 37.48, *OA* represents 150 A. The vector *AB* which represents 200 A is vectorially added to *OA* at $(90° + \phi) = (90° + 36°52') = 126°52'$. Vector *OB* represents excitation necessary to produce a terminal p.d. of 4160 *V* at 0.8 p.f. lagging at *full-load*.

$$OB = [150^2 + 200^2 + 2 \times 150 \times 200 \times \cos(180° - 126°52')]^{1/2}$$

$$= 313.8 \text{ A}$$

The generated phase e.m.f. E_0, corresponding to this excitation as found from *OCC* (if drawn) is 3140 V. Line value is $3140 \times \sqrt{3} = 5440$ V.

$$\%\text{ regn.} = \frac{5440 - 4160}{4160} \times 100 = \mathbf{30.7\%}$$

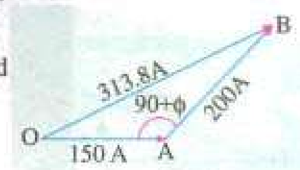


Fig. 37.48

Example 37.31. *The following test results are obtained on a 6,600-V alternator:*

Open-circuit voltage :	*3,100*	*4,900*	*6,600*	*7,500*	*8,300*
Field current (amps) :	*16*	*25*	*37.5*	*50*	*70*

A field current of 20 A is found necessary to circulate full-load current on short-circuit of the armature. Calculate by (i) the ampere-turn method and (ii) the synchronous impedance method the full-load regulation at 0.8 p.f. (lag). Neglect resistance and leakage reactance. State the drawbacks of each of these methods. **(Elect. Machinery-II, Bangalore Univ. 1992)**

Solution. (*i*) Ampere-turn Method

It is seen from the given data that for the normal voltage of 6,600 V, the field current needed is 37.5 A.

Field-current for full-load current, on short-circuit, is given as 20 A.

In Fig. 37.49, *OA* represents 37.5 A. The vector *AB*, which represents 20 A, is vectorially added to *OA* at $(90° + 36°52') = 126°52'$. Vector *OB* represents the excitation necessary to produce a terminal p.d. of 6,600 V at 0.8 p.f. lagging on ***full-load***

$$OB = \sqrt{37.5^2 + 20^2 + 2 \times 3.75 \times 20 \times \cos 53°8'} = 52 \text{ A}$$

The generated e.m.f. E_0 corresponding to this excitation, as found from O.C.C. of Fig. 37.49 is 7,600 V.

$$\text{Percentage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{7,600 - 6,600}{6,600} \times 100 = \mathbf{15.16\%}$$

(ii) Synchronous Impedance Method

Let the voltage of 6,600 V be taken as 100 per cent and also let 100 per cent excitation be that which is required to produce 6,600 V on open-circuit, that is, the excitation of 37.5 A.

Full-load or 100 per cent armature current is produced on short-circuit by a field current of 20 A. If 100 per cent field current were applied on short-circuit, then S.C. current would be $100 \times 37.5/20 =$ 187.5 per cent.

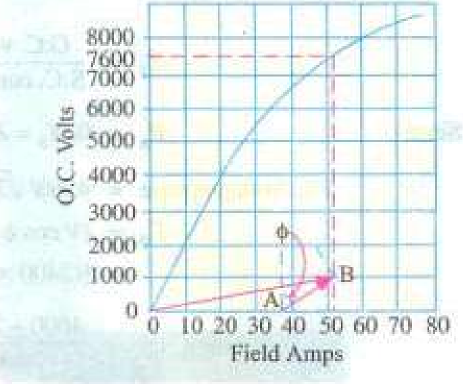


Fig. 37.49

$$\therefore \quad Z_S = \left. \frac{\text{O.C. voltage}}{\text{S.C. current}} \right| \text{same excitation}$$

$$= 100/187.5 \text{ or } 0.533 \text{ or } 53.3\%$$

The impedance drop IZ_S is equal to 53.3% of the normal voltage. When the two are added vectorially (Fig. 37.50), the value of voltage is

$$E_0 = \sqrt{[100 + 53.3 \cos (90 - \phi)]^2 + [53.3 \sin (90 - \phi)]^2}$$

$$= \sqrt{(100 + 53.3 \times 0.6)^2 + (53.3 \times 0.8)^2} = 138.7\%$$

$$\% \text{ reg} = \frac{138.7 - 100}{100} \times 100 = \mathbf{38.7\%}$$

Fig. 37.50

The two value of regulation, found by the two methods, are found to differ widely from each other. The first method gives somewhat lesser value, while the other method gives a little higher value as compared to the actual value. However, the first value is more likely to be nearer the actual value, because the second method employs Z_S, which does not have a constant value. Its value depends on the field excitation.

Example 37.32. *The open-and short-circuit test readings for a 3-ϕ, star-connected, 1000-kVA, 2000 V, 50-Hz, synchronous generator are :*

Field Amps ;	*10*	*20*	*25*	*30*	*40*	*50*
O.C. Terminal V	*800*	*1500*	*1760*	*2000*	*2350*	*2600*
S.C. armature current in A:	*—*	*200*	*250*	*300*	*—*	*—*

The armature effective resistance is 0.2 Ω per phase. Draw the characteristic curves and estimate the full-load percentage regulation at (a) 0.8 p.f. lagging (b) 0.8 p.f. leading.

Solution. The O.C.C. and S.C.C. are plotted in Fig. 37.51

The phase voltages are : 462, 866, 1016, 1155, 1357, 1502.

Full-load phase voltage $= 2000/\sqrt{3} = 1155$ V

Full-load current $= 1,000,000/2000 \times \sqrt{3} = 288.7$ A

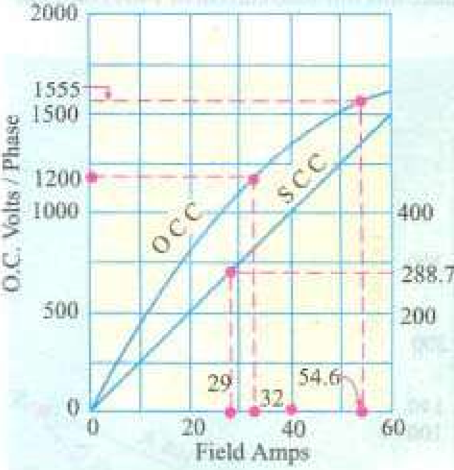


Fig. 37.51

Fig. 37.52

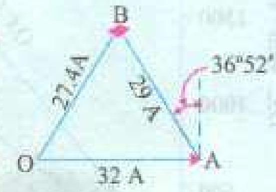


Fig. 37.53

Voltage/phase at full-load at 0.8 p.f. $= V + IR_a \cos\phi = 1155 + (288.7 \times 0.2 \times 0.8) = 1200$ volt

Form open-circuit curve, it is found that field current necessary to produce this voltage = 32 A.

From short-circuit characteristic, it is found that field current necessary to produce full-load current of 288.7 A is = 29 A.

(*a*) $\cos\phi = 0.8, \phi = 36°52'$ (lagging)

In Fig. 37.52, $OA = 32$ A, $AB = 29$ A and is at an angle of $(90° + 36°52') = 126°52'$ with OA. The total field current at full-load 0.8 p.f. lagging is $OB = 54.6$ A

O.C. volt corresponding to a field current of 54.6 A is = 1555 V

% regn. $= (1555 - 1155) \times 100/1155 = 34.6\%$

(*b*) In this case, as p.f. is leading, AB is drawn with OA (Fig. 37.53) at an angle of $90° - 36°52' = 53°8'$. $OB = 27.4$ A.

O.C. voltage corresponding to 27.4 A of field excitation is 1080 V.

$$\%\text{ regn.} = \frac{1080 - 1155}{1155} \times 100 = \mathbf{-6.4\%}$$

Example 37.33. *A 3-phase, 800-kV A, 3,300-V, 50-Hz alternator gave the following results:*

Exciting current (A)	*50*	*60*	*70*	*80*	*90*	*100*
O.C. volt (line)	*2560*	*3000*	*3300*	*3600*	*3800*	*3960*
S.C. current	*190*	—	—	—	—	—

The armature leakage reactance drop is 10% and the resistance drop is 2% of the normal voltage. Determine the excitation at full-load 0.8 power factor lagging by the m.m.f. method.

Solution. The phase voltages are : 1478, 1732, 1905, 2080, 2195, 2287

The O.C.C. is drawn in Fig. 37.54.

Normal phase voltage $= 3300/\sqrt{3} = 1905$ V; IR_a drop = 2% of 1905 = 38.1 volt

Leakage reactance drop = 10% of 1905 = 190.5 Volt

$$\therefore \quad E = \sqrt{[(1905 \times 0.8 \times + 38.1)^2 + (1905 \times 0.6 + 190.5)^2]} = 2{,}068 \text{ V}$$

The exciting current required to produce this voltage (as found from O.C.C.) is 82 A.

$$\text{Full load current} = 800.000/\sqrt{3} \times 3300 = 140\text{A}$$

As seen from S.C.C., the exciting current required to produce this full-load current of 140 A on short-circuit is 37 A.

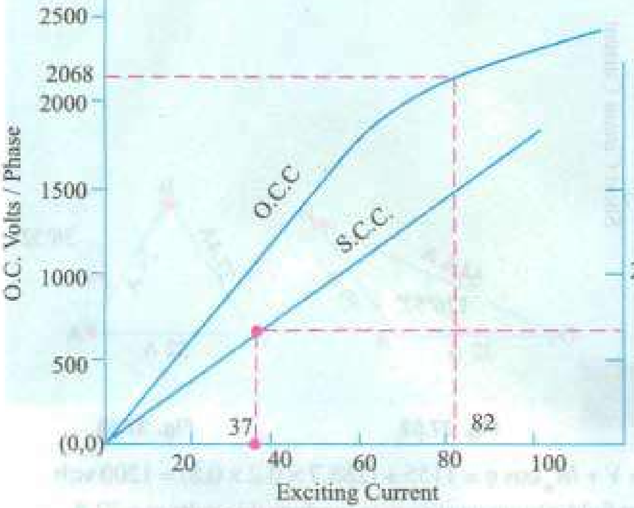


Fig. 37.54

Fig. 37.55

In Fig. 37.55, OB gives the excitation required on full-load to give a terminal phase voltage of 1905 V (or line voltage of 3300 V) at 0.8 p.f. lagging and its value is

$$= \sqrt{82^2 + 37^2 + 2 \times 82 \times 37 \times \cos 53°8'} = 108 \text{ A}$$

Tutorial Problem No. 37.3

1. A 30-kVA, 440-V, 50-Hz, 3-φ, star-connected synchronous generator gave the following test data :

Field current (A)	:	2	4	6	7	8	10	12	14
Terminal volts	:	155	287	395	440	475	530	570	592
S.C. current	:	11	22	34	40	46	57	69	80

Resistance between any two terminals is 0.3 Ω

Find regulation at full-load 0.8 p.f. lagging by (*a*) synchronous impedance method and (*b*) Rothert's ampere-turn method. Take Z_S corresponding to S.C. current of 80 A. **[(*a*) 51% (*b*) 29.9%]**

37.24. Zero Power Factor Method or Potier Method

This method is based on the separation of armature-leakage reactance drop and the armature reaction effects. Hence, it gives more accurate results. It makes use of the first two methods to some extent. The experimental data required is (*i*) no-load curve and (*ii*) full-load zero power factor curve (not the short-circuit characteristic) also called wattless load characteristic. It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero p.f.

The reduction in voltage due to armature reaction is found from above and (*ii*) voltage drop due to armature leakage reactance X_L (also called Potier reactance) is found from both. By combining these two, E_0 can be calculated.

It should be noted that if we vectorially add to V the drop due to resistance and leakage reactance X_L, we get E. If to E is further added the drop due to armature reaction (assuming lagging p.f.), then we get E_0 (Art. 37.18).

The zero p.f. lagging curve can be obtained.

(*a*) if a similar machine is available which may be driven at no-load as a synchronous motor at practically zero p.f. or

(*b*) by loading the alternator with pure reactors

(*c*) by connecting the alternator to a 3-ϕ line with ammeters and wattmeters connected for measuring current and power and by so adjusting the field current that we get full- load armature current with zero wattmeter reading.

Point B (Fig. 37.56) was obtained in this manner when wattmeter was reading zero. Point A is obtained from a short-circuit test with full-load armature current. Hence, OA represents field current which is equal and opposite to the demagnetising armature reaction and for balancing leakage reactance drop at full-load (please refer to A.T. method). Knowing these two points, full-load zero p.f. curve AB can be drawn as under.

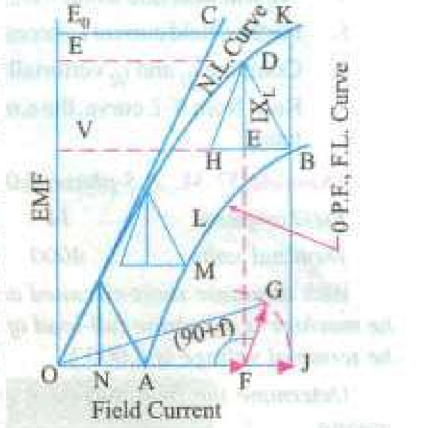


Fig. 37.56

From B, BH is drawn equal to and parallel to OA. From H, HD is drawn parallel to initial straight part of N-L curve *i.e.* parallel to OC, which is tangential to N-L curve. Hence, we get point D on no-load curve, which corresponds to point B on full-load zero p.f. curve. The triangle BHD is known as Potier triangle. This triangle is constant for a given armature current and hence can be transferred to give us other points like M, L etc. Draw DE perpendicular to BH. The length DE represents the drop in voltage due to armature leakage reactance X_L *i.e.* $I.X_L$. BE gives field current necessary to overcome demagnetising effect of armature reaction at full-load and EH for balancing the armature leakage reactance drop DE.

Let V be the terminal voltage on full-load, then if we add to it vectorially the voltage drop due to armature leakage reactance alone (neglecting R_a), then we get voltage $E = DF$ (and not E_0). Obviously, field excitation corresponding to E is given by OF. NA ($= BE$) represents the field current needed to overcome armature reaction. Hence, if we add NA vectorially to OF (as in Rothert's A.T. method) we get excitation for E_0 whose value can be read from N-L curve.

In Fig. 37.56, FG ($= NA$) is drawn at an angle of ($90° + \phi$) for a lagging p.f. (or it is drawn at an angle of $90° - \phi$ for a leading p.f.). The voltage corresponding to this excitation is $JK = E_0$

$$\therefore \quad \%\ \text{regn.} = \frac{E_0 - V}{V} \times 100$$

The vector diagram is also shown separately in Fig. 37.57.

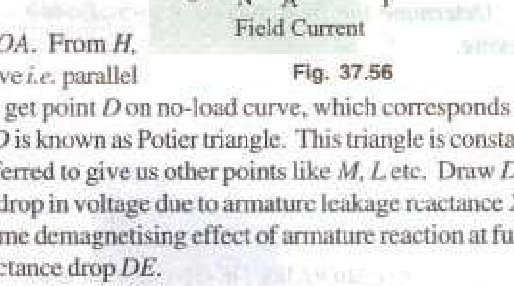


Fig. 37.57

Assuming a lagging p.f. with angle ϕ, vector for I is drawn at an angle of ϕ to V. IR_a is drawn parallel to current vector and IX_L is drawn perpendicular to it. OD represents voltage E. The excitation corresponding to it *i.e.*. OF is drawn at 90° ahead of it. FG ($= NA = BE$ in Fig. 37.56) representing field current equivalent of full-load armature reaction, is drawn parallel to current vector OI. The closing side OG gives field excitation for E_0. Vector for E_0 is 90° lagging behind OG. DL represents voltage drop due to armature reaction.

37.25. Procedural Steps for Potier Method 1.

1. Suppose we are given V-the terminal voltage/phase.
2. We will be given or else we can calculate armature leakage reactance X_L and hence can calculate IX_L.

3. Adding IX_L (and IR_a if given) vectorially to V, we get voltage E.
4. We will next find from *N-L* curve, field excitation for voltage E. Let it be i_{f1}.
5. Further, field current i_{f2} necessary for balancing armature reaction is found from Potier triangle.
6. Combine i_{f1} and i_{f2} vertorially (as in *A.T.* method) to get i_f.
7. Read from *N-L* curve, the e.m.f. corresponding to i_f. This gives us E_0. Hence, regulation can be found.

Example 37.34. *A 3-phase, 6,00-V alternator has the following O.C.C. at normal speed :*

Field amperes :	*14*	*18*	*23*	*30*	*43*
Terminal volts :	*4000*	*5000*	*6000*	*7000*	*8000*

With armature short-circuited and full-load current flowing the field current is 17 A and when the machine is supplying full-load of 2,000 kVA at zero power factor, the field current is 42.5 A and the terminal voltage is 6,000 V.

Determine the field current required when the machine is supplying the full-load at 0.8 p.f. lagging. **(A.C. Machines-I, Jadavpur Univ. 1988)**

Solution. The *O.C.C.* is drawn in Fig. 37.58 with phase voltages which are

2310, 2828, 3465 4042 4620

The full-load zero p.f. characteristic can be drawn because two points are known *i.e.* (17, 0) and (42.5, 3465).

In the Potier ΔBDH, line DE represents the leakage reactance drop ($= IX_L$) and is (by measurement) equal to 450 V. As seen from Fig. 37.59.

$$E = \sqrt{(V \cos \phi)^2 + (V \sin \phi + IX_L)^2}$$

$$= \sqrt{(3465 \times 0.8)^2 + (3465 \times 0.6 + 450)^2}$$

$$= 3750 \text{ V}$$

From *O.C.C.* of Fig. 37.58, it is found that field amperes required for this voltage = 26.5 *A*.

Field amperes required for balancing armature reaction = BE = 14.5 A (by measure-ment from Potier triangle BDH).

As seen from Fig. 37.60, the field currents are added vectorially at an angle of $(90° + \phi) = 126° 52'$.

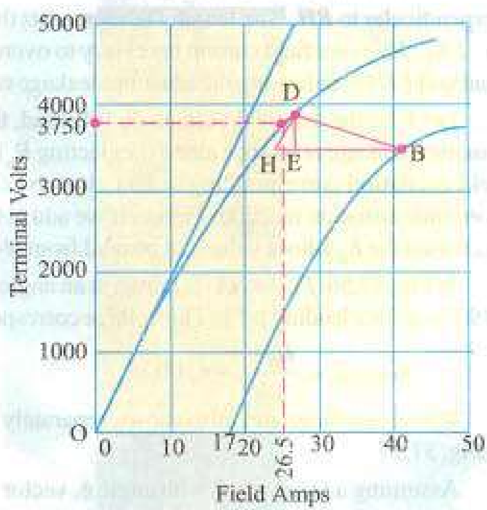


Fig. 37.58

Resultant field current is $OB = \sqrt{26.5^2 + 14.5^2 + 2 \times 26.5 \times 14.4 \cos 53°8'}$ = **37.2 A**

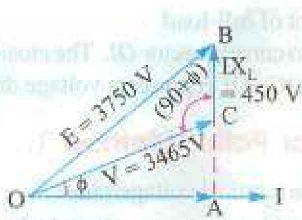


Fig. 37.59

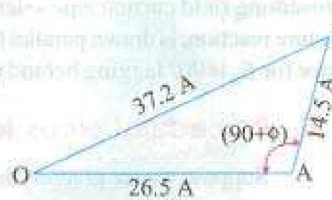


Fig. 37.60

Example 37.35. *An 11-kV, 1000-kVA, 3-phase, Y-connected alternator has a resistance of 2 Ω per phase. The open-circuit and full-load zero power factor characteristics are given below. Find the voltage regulation of the alternator for full load current at 0.8 p.f. lagging by Potier method.*

Field current (A)	:	*40*	*50*	*110*	*140*	*180*
O.C.C. line voltage	:	*5,800*	*7,000*	*12,500*	*13,750*	*15,000*
Line volts zero p.f.		*0*	*1500*	*8500*	*10,500*	*12,500*

(Calcutta Univ. 1987 and S. Ramanandtirtha Univ. Nanded, 2001)

Solution. The O.C.C. and full-load zero p.f. curve for phase voltage are drawn in Fig. 37.61. The corresponding phase voltages are :

O.C.C. phase voltage	3350	4040	7220	7940	8660
Phase voltage zero p.f.	0	866	4900	6060	7220

Full-load current $= 1000 \times 1000/\sqrt{3} \times 11,000 = 52.5$ A

Phase voltage $= 11,000/\sqrt{3} = 6,350$ A

In the Potier ΔABC, $AC = 40$ A, CB is parallel to the tangent to the initial portion of the *O.C.C.* and BD is $\perp$ to AC.

BD = leakage reactance drop $IX_L = 1000$ V – by measurement

$AD = 30$ A — field current required to overcome demagnetising effect of armature reaction on full-load.

As shown in Fig. 37.62,

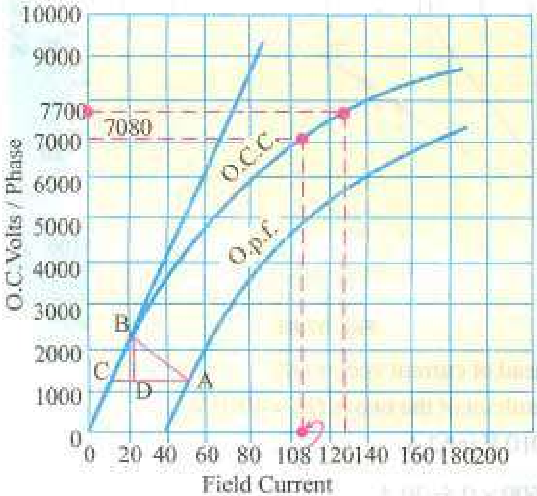


Fig. 37.61

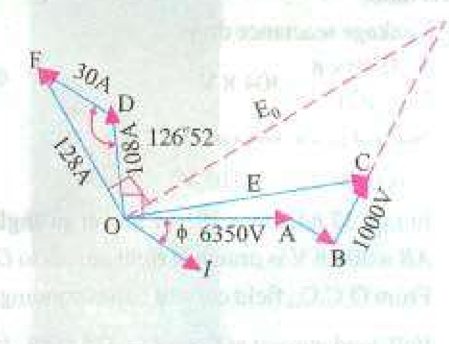


Fig. 37.62

$$OA = 6,350 \text{ V}; AB = IR_a = 52.5 \times 2 = 105 \text{ V}$$

$$IX_L = BC = 1000 \text{ V} \qquad \text{—by measurement}$$

$$OC = E = \sqrt{(V \cos\phi + IR_a)^2 + (V \sin\phi + IX_L)^2}$$

$$= \sqrt{(6350 \times 0.8 + 105)^2 + (6350 \times 0.6 + 1000)^2}\ ;\ E = 7,080 \text{ V}$$

As seen from O.C.C., field current required for 7,080 V is 108 A. Vector OD (Fig. 37.62) represents 108 A and is drawn $\perp$ to OC. DF represents 30 A and is drawn parallel to OI or at $(90° + 36° 52') = 126° 52'$ with OD. Total field current is OF.

$$OF = \sqrt{108^2 + 30^2 + 2 \times 108 \times 30 \cos 53°8'} = 128 \text{ A}$$

From O.C.C., it is found that the e.m.f. corresponding to this field current is 7,700V

$$\therefore \quad E_0 = 7,700 \text{ V}; \text{regulation} = \frac{7,700 - 6,350}{6,350} \times 100 = \textbf{21.3 per cent}$$

Example 37.36. *The following test results were obtained on a 275-kW, 3-ϕ, 6,600-V non-salient pole type generator.*

Open-circuit characteristic :

Volts	:	*5600*	*6600*	*7240*	*8100*
Exciting amperes	:	*46.5*	*58*	*67.5*	*96*

Short-circuit characteristic : Stator current 35 A with an exciting current of 50 A. Leakage reactance on full-load = 8%. Neglect armature resistance. Calculate as accurately as possible the exciting current (for full-load) at power factor 0.8 lagging and at unity. **(City & Guilds, London)**

Solution. First convert the O.C. line volts into phase volts by dividing the given terminal values by √3.

∴ O.C. volts (phase) : 3233, 3810, 4180, 4677.

O.C.C. is plotted in Fig. 37.63. For plotting S.C.C., we need two points. One is (0, 0) and the other is (50 A, 35 A). In fact, we can do without plotting the S.C.C. because it being a straight line, values of field currents corresponding to any armature current can be found by direct ratio.

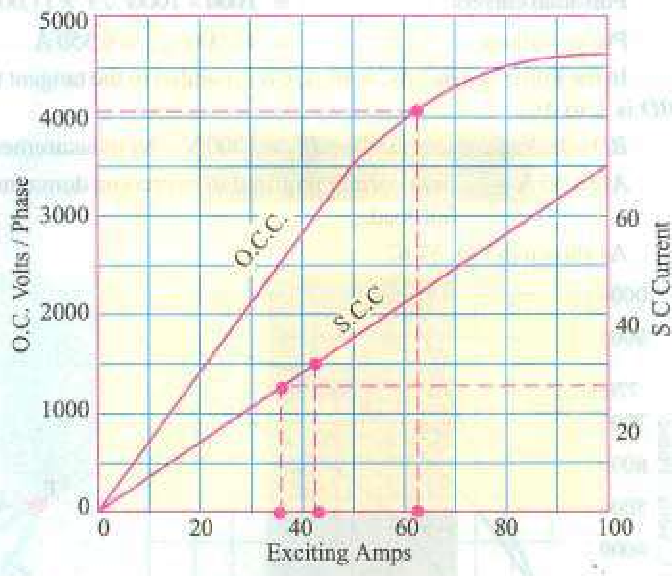


Fig. 37.63

Leakage reactance drop

$= \dfrac{3810 \times 8}{100} = 304.8$ V

Normal phase voltage

$= 6,600/\sqrt{3} = 3,810$ V

In Fig. 37.64, $OA = 3810$ V and at an angle ϕ ahead of current vector OI.

$AB = 304.8$ V is drawn at right angles to OI. Resultant of the two is $OB = 4010$ V.

From O.C.C., field current corresponding to 4,010 V is 62 A.

Full-load current at 0.8 p.f. = $275,000/\sqrt{3} \times 6600 \times 0.8 = 30$ A

35 A of armature current need 50 A of field current, hence 30 A of armature current need $30 \times 50/35 = 43$ A.

In Fig. 37.64, $OC = 62$ A is drawn at right angles to OB. Vector $CD = 43$ A is drawn parallel to OI. Then, OD = **94.3 A**

Note. Here. 43 A pf field excitation is assumed as having all been used for balancing armature reaction. In fact, a part of it is used for balancing armature leakage drop of 304.8 V. This fact has been clarified in the next example.

At Unity p.f.

In Fig. 37.65, OA again represents $V = 3810$ V, $AB = 304.8$ V and at right angles to OA.

The resultant $\quad OB = \sqrt{(3810^2 + 304.8^2)} = 3830$ V

Field current from O.C.C. corresponding to this voltage = 59.8 A.

Hence, $\quad OC$ = 59.8 A is drawn perpendicular to OB (as before)

Full-load current at u.p.f. $= 275,000/\sqrt{3} \times 6600 \times 1 = 24$ A

Now, 35 A armature current corresponds to a field current of 50 A, hence 24 A of armature current corresponds to $50 \times 24/35 = 34.3$ A.

Hence, $\qquad CD = 34.3$ A is drawn $\parallel$ to OA (and $\perp$ to OC approximately).*

Fig. 37.64

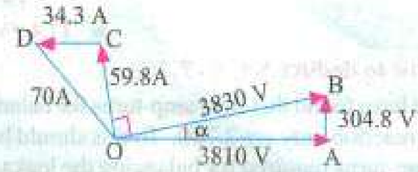


Fig. 37.65

$$\therefore \qquad OD = \sqrt{(59.8^2 + 34.3^2)} = \mathbf{70\ A}$$

Example 37.37. *A 600-kVA, 3,300-V, 8-pole, 3-phase, 50-Hz alternator has following characteristic :*

Amp-turns/pole :	*4000*	*5000*	*7000*	*10,000*
Terminal E.M.F. :	*2850*	*3400*	*3850*	*4400*

There are 200 conductor in series per phase.

Find the short-circuit characteristic, the field ampere-turns for full-load 0.8 p.f. (lagging) and the voltage regulation, having given that the inductive drop at full-load is 7% and that the equivalent armature reaction in amp-turns per pole = 1.06 × ampere-conductors per phase per pole.

(London Univ.)

Solution. O.C. terminal voltages are first converted into phase voltages and plotted against field amp-turns, as shown in Fig. 37.66.

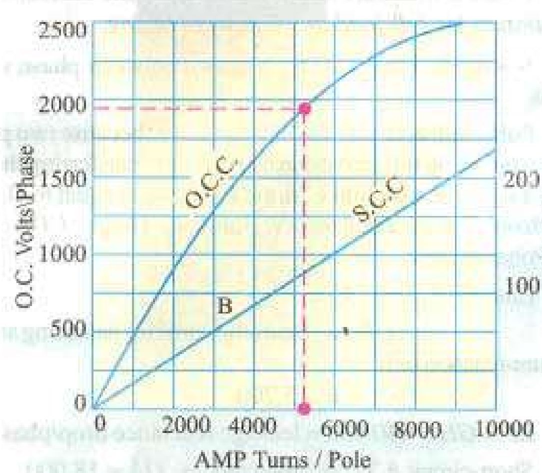


Fig. 37.66

Full-load current

$$= \frac{600{,}000}{\sqrt{3} \times 3300} = 105 \text{ A}$$

Demagnetising amp-turns per pole per phase for full-load at zero p.f.

$$= 1.06 \times 105 \times 200/8$$
$$= 2{,}780$$

Normal phase voltage

$$= 3300/\sqrt{3} = 1910 \text{ volt}$$

Leakage reactance drop

$$= \frac{3300 \times 7}{\sqrt{3} \times 100} = 133 \text{ V}$$

In Fig. 37.67, OA represents 1910 V.

$AB = 133$ V is drawn $\perp$ OI, OB is the resultant voltage E (not E_0).

$\therefore OB = E = 1987$ volt

From O.C.C., we find that 1987 V correspond to 5100 field amp-turns. Hence, $OC = 5100$ is drawn $\perp$ to OB. $CD = 2780$ is $\parallel$ to OI. Hence, $OD = 7240$ (approx). From O.C.C. it is found that this

* It is so because angle between OA and OB is negligibly small. If not, then CD should be drawn at an angle of $(90 + \alpha)$ where α is the angle between OA and OB.

corresponds to an O.C. voltage of 2242 volt. Hence, when load is thrown off, the voltage will rise to 2242 V.

$$\therefore \qquad \% \text{ regn.} = \frac{2242 - 1910}{1910} \times 100 = \mathbf{17.6\ \%}$$

How to deduce S.C.C. ?

We have found that field amp-turns for balancing armature reaction only are 2,780. To this should be added field amp-turns required for balancing the leakage reactance voltage drop of 133 V.

Field amp-turns corresponding to 133 volt on O.C. are 300 approximately. Hence, with reference to Fig. 37.56, $NA = 2780$, $ON = 300$

$\therefore$ Short-circuit field amp-turns

$= OA = 2780 + 300$

$= 3080$

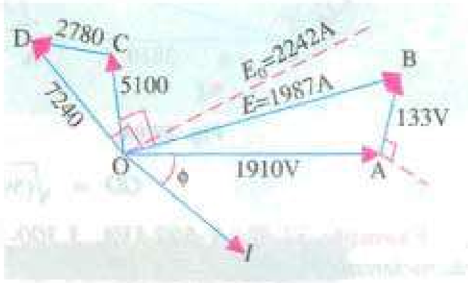


Fig. 37.67

Hence, we get a point B on S.C.C. *i.e.* (3080, 105) and the other point is the origin. So S.C.C. (which is a straight line) can be drawn as shown in Fig. 37.66.

Example 37.38. *The following figures give the open-circuit and full-load zero p.f saturation curves for a 15,000-kVA, 11,000 V, 3-φ, 50-Hz, star-connected turbo-alternator:*

Field AT in 10^3 :	*10*	*18*	*24*	*30*	*40*	*45*	*50*
O.C. line kV :	*4.9*	*8.4*	*10.1*	*11.5*	*12.8*	*13.3*	*13.65*
Zero p.f. full-load line kV :	—	*0*	—	—	—	*10.2*	—

Find the armature reaction, the armature reactance and the synchronous reactance. Deduce the regulation for full-load at 0.8 power lagging.

Solution. First, O.C.C. is drawn between phase voltages and field amp-turns, as shown in Fig. 37.68.

Full-load, zero p.f. line can be drawn, because two points are known *i.e.* A (18, 0) and C (45, 5890). Other points on this curve can be found by transferring the Potier triangle. At point C, draw CD ∥ to and equal to OA and from D draw DE ∥ to ON. Join EC. Hence, CDE is the Potier triangle.

Line EF is ⊥ to DC

CF = field amp-turns for balancing armature-reaction only

= 15,700

$EF = GH = 640$ volt = leakage reactance drop/phase

Short-circuit A.T. required = OA = 18,000

$$\text{Full-load current} = \frac{15{,}000 \times 1000}{\sqrt{3} \times 11{,}000} = 788 \text{ A}$$

$\therefore \quad 640 = I \times X_L \quad \therefore \quad X_L = 640/788 = 0.812\ \Omega$

From O.C.C., we find that 18,000 A.T. correspond to an O.C. voltage of $8{,}400/\sqrt{3} = 4{,}850$ V.

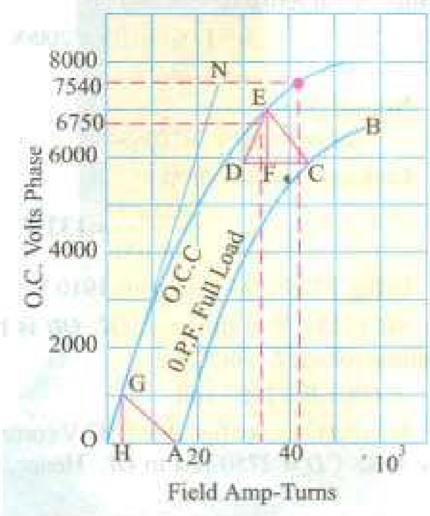


Fig. 37.68

$$\therefore \qquad Z_S = \frac{\text{O.C. volt}}{\text{S.C. cuerrent}} = \frac{4{,}850}{788} = 6.16\ \Omega \qquad \text{(Art. 37.21)}$$

As R_a is negligible, hence Z_S equals X_S.

Regulation

In Fig. 37.69, OA = phase voltage = $11,000/\sqrt{3}$

= 6,350 V

AB = 640 V and is drawn at right angles to OI or at $(90° + \phi)$ to OA.

Resultant is OB = 6,750 V

Field A.T. corresponding to O.C voltage of 6,750 V is = OC = 30,800 and is drawn ⊥ to OB.

CD = armature reaction at F.L. = 15,700 and is drawn ∥ to OI or at $(90° + \phi)$ to OC.

Hence, OD = 42,800.

From O.C.C., e.m.f. corresponding to 42,800 A.T. of rotor = 7,540 V

∴ % regn. up = (7,540 − 6,350)/6,350 = **0.187 or 18.7%**

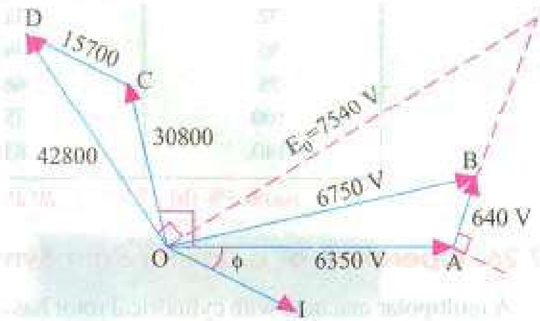


Fig. 37.69

Tutorial Problem No. 37.4

1. The following data relate to a 6,600-V, 10,000-kVA, 50-Hz, 3-φ, turbo- alternator:

O.C. kilovolt	4.25	5.45	6.6	7.3	8	9
Exciting A.T. in 10^3	60	80	100	120	145	220

Excitation needed to circulate full-load current on short circuit : 117,000 A.T. Inductive drop in stator winding at full-load = 15%. Find the voltage regulation at full-load 0.8 power factor.

[34.4%] ***(City & Guilds, London)***

2. Deduce the exciting current for a 3-φ, 3300-V generator when supplying 100 kW at 0.8 power factor lagging, given magnetisation curve on open-circuit :

Line voltage :	3300	3600	3900
Exciting current :	80	96	118

There are 16 poles, 144 slots, 5 conductors/slot, single-circuit, full-pitched winding, star- connected. The stator winding has a resistance per phase of 0.15 Ω and a leakage reactance of 1.2 Ω. The field coils have each 108 turns.

[124 A] ***(London Univ.)***

3. Estimate the percentage regulation at full-load and power factor 0.8 lagging of a 1000-kVA, 6,600-V, 3-φ, 50-Hz, star-connected salient-pole synchronous generator. The open-circuit characteristic is as follows :

Terminal volt	4000	6000	6600	7200	8000
Field A.T.	5200	8500	10,000	12,500	17,500

Leakage reactance 10%, resistance 2%. Short-circuit characteristic : full-load current with a field excitation of 5000 A.T. Take the permeance to cross armature reaction as 35% of that to direct reaction. **[20% up]**

4. A 1000-kVA, 11,000-V, 3-φ, 50-Hz, star-connected turbo-generator has an effective resistance of 2Ω /phase. The O.C.C. and zero p.f. full-load data is as follows :

O.C. volt	5,805	7,000	12,550	13,755	15,000
Field current A	40	50	110	140	180
Terminal volt at F.L. zero p.f.	0	1500	8,500	10,500	12,400

Estimate the % regulation for F.L. at 0.8 p.f. lagging. [22 %]

5. A 5-MVA, 6.6 kV, 3-ϕ, star-connected alternator has a resistance of 0.075 Ω per phase. Estimate the regulation for a load of 500 A at p.f. (*a*) unity and (*b*) 0.9 leading (*c*) 0.71 lagging from the following open-circuit and full-load zero power factor curve.

Field current (A)	Open-circuit terminal voltage (V)	Saturation curve zero p.f.
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

[(a) 6.3% (b) −7.9% (c) 20.2%] (*Electrical Machines-II, Indore Univ. Feb. 1978*)

37.26. Operation of a Salient Pole Synchronous Machine

A multipolar machine with cylindrical rotor has a uniform air-gap, because of which its reactance remains the same, irrespective of the spatial position of the rotor. However, a synchronous machine with salient or projecting poles has non-uniform air-gap due to which its reactance varies with the rotor position. Consequently, a cylindrical rotor machine possesses one axis of symmetry (pole axis or direct axis) whereas salient-pole machine possesses two axes of geometric symmetry (*i*) field poles axis, called direct axis or *d*-axis and (*ii*) axis passing through the centre of the interpolar space, called the quadrature axis or *q*-axis, as shown in Fig. 37.70.

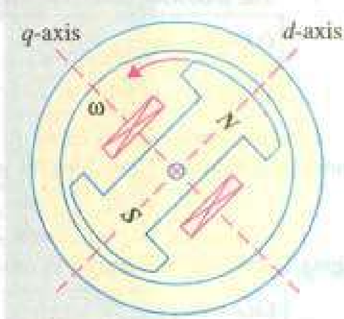


Fig. 37.70

Obviously, two mmfs act on the *d*-axis of a salient-pole synchronous machine *i.e.* field m.m.f. and armature m.m.f. whereas only one m.m.f., *i.e.* armature mmf acts on the *q*-axis, because field mmf has no component in the *q*-axis. The magnetic reluctance is low along the poles and high between the poles. The above facts form the basis of the two-reaction theory proposed by Blondel, according to which

(*i*) armature current I_a can be resolved into two components *i.e.* I_d perpendicular to E_0 and I_q along E_0 as shown in Fig. 37.71 (*b*).

(*ii*) armature reactance has two components *i.e.* *q*-axis armature reactance X_{ad} associated with I_d and *d*-axis armature reactance X_{aq} linked with I_q.

If we include the armature leakage reactance X_l which is the same on both axes, we get

$$X_d = X_{ad} + X_l \text{ and } X_q = X_{aq} + X_l$$

Since reluctance on the *q*-axis is higher, owing to the larger air-gap, hence,

$$X_{aq} < X_{ad} \text{ or } X_q < X_d \text{ or } X_d > X_q$$

37.27. Phasor Diagram for a Salient Pole Synchronous Machine

The equivalent circuit of a salient-pole synchronous generator is shown in Fig. 37.71 (*a*). The component currents I_d and I_q provide component voltage drops jI_dX_d and $j\,I_qX_q$ as shown in Fig. 37.71(*b*) for a lagging load power factor.

The armature current I_a has been resolved into its rectangular components with respect to the axis for excitation voltage E_0. The angle ψ between E_0 and I_a is known as the internal power factor angle. The vector for the armature resistance drop $I_a R_a$ is drawn parallel to I_a. Vector for the drop $I_d X_d$ is drawn perpendicular to I_d whereas that for $I_q \times X_q$ is drawn perpendicular to I_q. The angle δ between E_0 and V is called the power angle. Following phasor relationships are obvious from Fig. 37.71 (*b*)

$$E_0 = V + I_a R_a + jI_d X_d + jI_q X_q \text{ and } I_a = I_d + I_q$$

If R_a is neglected the phasor diagram becomes as shown in Fig. 37.72 (*a*). In this case,

$$E_0 = V + jI_d X_d + jI_q X_q$$

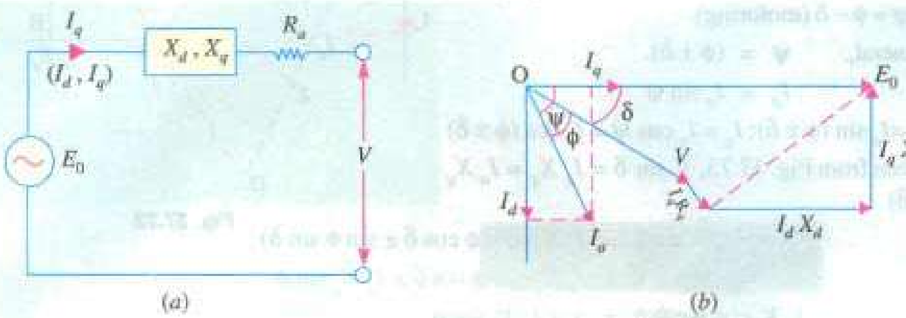


Fig. 37.71

Incidentally, we may also draw the phasor diagram with terminal voltage V lying in the horizontal direction as shown in Fig. 37-72 (*b*). Here, again drop $I_a R_a$ is $\parallel I_a$ and $I_d X_d$ is $\perp$ to I_d and drop $I_q X_q$ is $\perp$ to I_q as usual.

37.28. Calculations from Phasor Diagram

In Fig. 37.73, dotted line AC has been drawn perpendicular to I_a and CB is perpendicular to the phasor for E_0. The angle $ACB = \psi$ because angle between two lines is the same as between their perpendiculars. It is also seen that

$$I_d = I_a \sin \psi \,;\, I_q = I_a \cos \psi \,;\, \text{hence, } I_a = I_q / \cos \psi$$

In ΔABC, $\quad BC/AC = \cos \psi$ or $AC = BC/\cos \psi = I_q X_q / \cos \psi = I_a X_q$

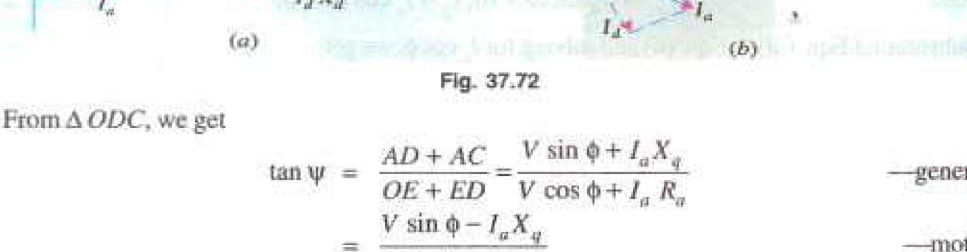


Fig. 37.72

From ΔODC, we get

$$\tan \psi = \frac{AD + AC}{OE + ED} = \frac{V \sin \phi + I_a X_q}{V \cos \phi + I_a R_a} \qquad \text{—generating}$$

$$= \frac{V \sin \phi - I_a X_q}{V \sin \phi - I_a R_a} \qquad \text{—motoring}$$

The angle ψ can be found from the above equation. Then, $\delta = \psi - \phi$ (generating) and $\delta = \phi - \psi$ (motoring)

As seen from Fig. 37.73, the excitation voltage is given by

$E_0 = V \cos \delta + I_q R_a + I_d X_d$ —generating

$= V \cos \delta - I_q R_a - I_d X_d$ —motoring

Note. Since angle ϕ is taken positive for lagging p.f., it will be taken negative for leading p.f.

If we neglect the armatrue resistance as shown in Fig. 37.72, then angle δ can be found directly as under :

$\psi = \phi + \delta$ (generating)

and $\psi = \phi - \delta$ (motoring).

In general, $\psi = (\phi \pm \delta)$,

$$I_d = I_a \sin \psi$$

$$= I_a \sin (\phi \pm \delta); \; I_q = I_a \cos \psi = I_a \cos (\phi \pm \delta)$$

As seen from Fig. 37.73, $V \sin \delta = I_q X_q = I_a X_q \cos (\phi \pm \delta)$

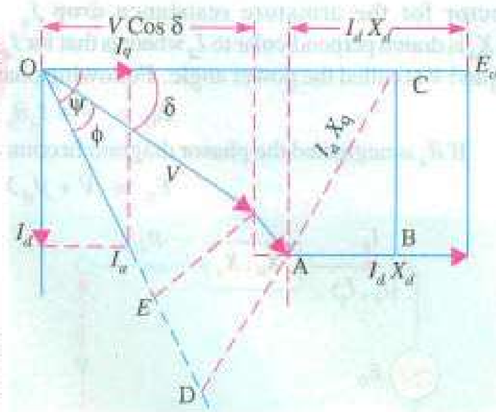


Fig. 37.73

$$\therefore \quad V \sin \delta = I_a X_q (\cos \phi \cos \delta \pm \sin \phi \sin \delta)$$

$$\text{or} \quad V = I_a X_q \cos \phi \cot \delta \pm I_a X_q \sin \phi$$

$$\therefore \quad I_a X_q \cos \phi \cot \delta = V \pm I_a X_q \sin \phi$$

$$\therefore \quad \tan \delta = \frac{I_a X_q \cos \phi}{V \pm I_a X_q \sin \phi}$$

In the above expression, plus sign is for synchronous generators and minus sign for synchronous motors.

Similarly, when R_a is neglected, then,

$$E_0 = V \cos \delta \pm I_d X_d$$

However, if R_a and hence $I_a R_a$ drop is not negligible then,

$$E_0 = V \cos \delta + I_q R_a + I_d X_d \quad \text{—generating}$$

$$= V \cos \delta - I_q R_a - I_d X_d \quad \text{—motoring}$$

37.29. Power Developed by a synchronous Generator

If we neglect R_a and hence Cu loss, then the power developed (P_d) by an alternator is equal to the power output (P_{out}). Hence, the per phase power output of an alternator is

$$P_{out} = VI_a \cos \phi = \text{power developed } (p_d) \quad \ldots(i)$$

Now, as seen from Fig., 37.72 (*a*), $I_q X_q = V \sin \delta \,;\, I_d X_d = E_0 - V \cos \delta$ ...(*ii*)

Also, $\quad I_d = I_a \sin (\phi + \delta); \; I_q = I_a \cos (\phi + \delta)$ ...(*iii*)

Substituting Eqn. (*iii*) in Eqn. (*ii*) and solving for $I_a \cos \phi$, we get

$$I_a \cos \phi = \frac{V}{X_d} \sin \delta + \frac{V}{2\,X_q} \sin 2\,\delta - \frac{V}{2\,X_d} \sin 2\,\delta$$

Finally, substituting the above in Eqn. (*i*), we get

$$P_d = \frac{E_0 V}{X_d} \sin \delta + \frac{1}{2} V^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\,\delta = \frac{E_0 V}{X_d} \sin \delta + \frac{V^2 (X_d - X_q)}{2\,X_d X_q} \sin 2\,\delta$$

The total power developed would be three times the above power.

As seen from the above expression, the power developed consists of two components, the first term represents power due to field excitation and the second term gives the reluctance power *i.e.*

power due to saliency. If $X_d = X_q$ *i.e.* the machine has a cylinderical rotor, then the second term becomes zero and the power is given by the first term only. If, on the other hand, there is no field excitation *i.e.* $E_0 = 0$, then the first term in the above expression becomes zero and the power developed is given by the second term. It may be noted that value of δ is positive for a generator and negative for a motor.

Example 37.39. *A 3-phase alternator has a direct-axis synchronous reactance of 0.7 p.u. and a quadrature axis synchronous reactance of 0.4 p.u. Draw the vector diagram for full-load 0.8 p.f. lagging and obtain therefrom (i) the load angle and (ii) the no-load per unit voltage.*

(Advanced Elect. Machines, AMIE Sec. B 1991)

Solution.

$$V = 1 \text{ p.u.}; X_d = 0.7 \text{ p.u.}; X_q = 0.4 \text{ p.u.};$$
$$\cos\phi = 0.8; \sin\phi = 0.6\,; \phi = \cos^{-1} 0.8 = 36.9°; I_a = 1 \text{ p.u.}$$

(i)
$$\tan\delta = \frac{I_a X_q \cos\phi}{V + I_q \sin\phi} = \frac{1\times0.4\times0.8}{1+0.4\times0.6} = 0.258, \delta = 16.5°$$

(ii)
$$I_d = I_a \sin(\phi+\delta) = 1 \sin(36.9° + 14.9°) = 0.78 \text{ A}$$
$$E_0 = V\cos\delta + I_d X_d = 1\times0.966 + 0.78\times0.75 = \mathbf{1.553}$$

Example 37.40. *A 3-phase, star-connected, 50-Hz synchronous generator has direct-axis synchronous reactance of 0.6 p.u. and quadrature-axis synchronous reactance of 0.45 p.u. The generator delivers rated kVA at rated voltage. Draw the phasor diagram at full-load 0.8 p.f. lagging and hence calculate the open-circuit voltage and voltage regulation. Resistive drop at full-load is 0.015 p.u.*

(Elect. Machines-II, Nagpur Univ. 1993)

Solution.

$$I_a = 1 \text{ p.u.}; V = 1 \text{ p.u.}; X_d = 0.6 \text{ p.u.}; X_q = 0.45 \text{ p.u.}\ ; R_a = 0.015 \text{ p.u.}$$
$$\tan\psi = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a} = \frac{1\times0.6+1\times0.45}{1\times0.8+1\times0.015} = 1.288; \quad \psi = 52.2°$$
$$\delta = \psi - \phi = 52.2° - 36.9° = 15.3°$$
$$I_d = I_a \sin\psi = 1\times0.79 = 0.79 \text{ A}\ ; I_q = I_a\cos\psi = 1\times0.61 = 0.61 \text{A}$$
$$E_0 = V\cos\delta + I_q R_a + I_d X_d$$
$$= 1\times0.965 + 0.61\times0.015 + 0.79\times0.6 = 1.448$$
$$\therefore \quad \%\text{ regn.} = \frac{1.448-1}{1}\times100 = \mathbf{44.8\%}$$

Example 37.41. *A 3-phase, Y-connected syn. generator supplies current of 10 A having phase angle of 20° lagging at 400 V. Find the load angle and the components of armature current I_d and I_q if X_d = 10 ohm and X_q = 6.5 ohm. Assume arm. resistance to be negligible.*

(Elect. Machines-I, Nagpur Univ. 1993)

Solution.

$$\cos\phi = \cos 20° = 0.94; \sin\phi = 0.342; I_a = 10 \text{ A}$$
$$\tan\delta = \frac{I_a X_q \cos\phi}{V + I_a X_q \sin\phi} = \frac{10\times6.5\times0.94}{400+10\times6.5\times0.342} = 0.1447$$
$$\delta = 8.23°$$
$$I_d = I_a\sin(\phi+\delta) = 10\sin(20° + 8.23°) = 4.73 \text{ A}$$
$$I_q = I_a\cos(\phi+\delta) = 10\cos(20° + 8.23°) = 8.81 \text{ A}$$

Incidentally, if required, voltage regulation of the above generator can be found as under:

$$I_d X_d = 4.73\times10 = 47.3 \text{ V}$$
$$E_0 = V\cos\delta + I_d X_d = 400\cos 8.23° + 47.3 = 443 \text{ V}$$

$$\% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

$$= \frac{443 - 400}{400} \times 100 = \mathbf{10.75\%}$$

Tutorial Problem No. 37.5.

1. A 20 MVA, 3-phase, star-connected, 50-Hz, salient-pole has $X_d = 1$ p.u.; $X_q = 0.65$ p.u. and $R_a = 0.01$ p.u. The generator delivers 15 MW at 0.8 p.f. lagging to an 11-kV, 50-Hz system. What is the load angle and excitation e.m.f. under these conditions? **[18°; 1.73 p.u]**
2. A salient-pole synchronous generator delivers rated kVA at 0.8 p.f. lagging at rated terminal voltage. It has $X_d = 1.0$ p.u. and $X_q = 0.6$ p.u. If its armature resistance is negligible, compute the excitation voltage under these conditions. **[1.77 p.u]**
3. A 20-kVA, 220-V, 50-Hz, star-connected, 3-phase salient-pole synchronous generator supplies load at a lagging power factor angle of 45°. The phase constants of the generator are $X_d = 4.0\,\Omega$; $X_q = 2\,\Omega$ and $R_a = 0.5\,\Omega$. Calculate (*i*) power angle and (*ii*) voltage regulation under the given load conditions. **[(*i*) 20.6° (*ii*) 142%]**
4. A 3-phase salient-pole synchronous generator has $X_d = 0.8$ p.u.; $X_q = 0.5$ p.u. and $R_a = 0$. Generator supplies full-load at 0.8 p.f. lagging at rated terminal voltage. Compute (*i*) power angle and (*ii*) no-load voltage if excitation remains constant. **[(*i*) 17.1° (*ii*) 1.6 p.u]**

37.30. Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as ***synchronizing***. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to ***infinite*** bus-bars.

It is never advisable to connect a stationary alternator to live bus-bars, because, stator induced e.m.f. being zero, a short-circuit will result. For proper synchronization of alternators, the following three conditions must be satisfied :

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.
2. The speed of the incoming machine must be such that its frequency (= $PN/120$) equals bus-bar frequency.
3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage. It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

Condition (1) is indicated by a voltmeter, conditions (2) and (3) are indicated by synchronizing lamps or a synchronoscope.

37.31. Synchronizing of Alternators

(*a*) Single-phase Alternators

Suppose machine 2 is to be synchronized with or 'put on' the bus-bars to which machine 1 is already connected. This is done with the help of two lamps L_1 and L_2 (known as synchronizing lamps) connected as shown in Fig. 37.74.

It should be noted that E_1 and E_2 are in-phase relative to the external circuit but are in direct phase opposition in the local circuit (shown dotted).

If the speed of the incoming machine 2 is not brought up to that of machine 1, then its frequency will also be different, hence there will be a phase-difference between their voltages (even when they are equal in magnitude, which is determined by field excitation). This phase-difference will be continously changing with the changes in their frequencies. The result is that their resultant voltage will undergo changes similar to the frequency changes of beats produced, when two sound sources of nearly equal frequency are sounded together, as shown in Fig. 37.75.

Sometimes the resultant voltage is maximum and some other times minimum. Hence, the current is alternatingly maximum and minimum. Due to this changing current through the lamps, a flicker will be produced, the frequency of flicker being $(f_2 - f_1)$. Lamps will dark out and glow up alternately. Darkness indicates that the two voltages E_1 and E_2 are in exact phase opposition relative to the local circuit and hence

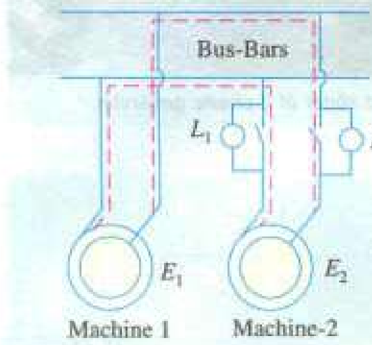


Fig. 37.74

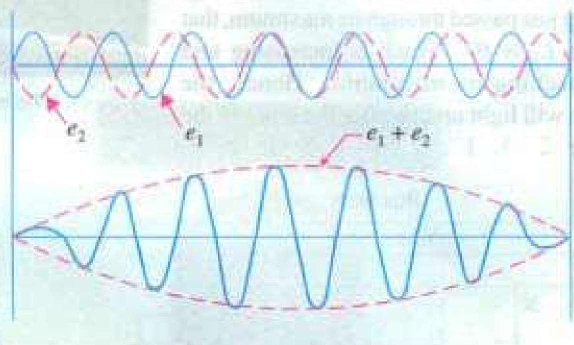


Fig. 37.75

there is no resultant current through the lamps. Synchronizing is done at the middle of the dark period. That is why, sometimes, it is known as 'lamps dark' synchronizing. Some engineers prefer 'lamps bright' synchronization because of the fact the lamps are much more sensitive to changes in voltage at their maximum brightness than when they are dark. Hence, a sharper and more accurate synchronization is obtained. In that case, the lamps are connected as shown in Fig. 37.76. Now, the lamps will glow brightest when the two voltages are in-phase with the bus-bar voltage because then voltage across them is twice the voltage of each machine.

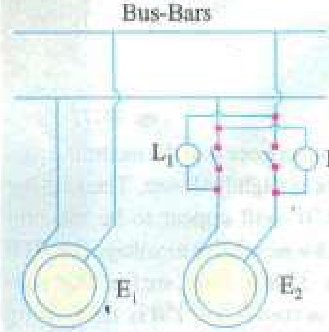


Fig. 37.76

(b) Three-phase Alternators

In 3-ϕ alternators, it is necessary to synchronize one phase only, the other two phases will then be synchronized automatically. However, first it is necessary that the incoming alternator is correctly 'phased out' *i.e.* the phases are connected in the proper order of *R, Y, B* and not *R, B, Y* etc.

In this case, three lamps are used. But they are deliberately connected asymmetrically, as shown in Fig. 37.77 and 37.78.

This transposition of two lamps, suggested by Siemens and Halske, helps to indicate whether the incoming machine is running too slow. If lamps were connected symmetrically, they would dark out or glow up simultaneously (if the phase rotation is the same as that of the bus-bars).

Lamp L_1 is connected between R and R', L_2 between Y and B' (not Y and Y') and L_3 between B and Y' (and not B and B'), as shown in Fig. 37.78.

Voltage stars of two machines are shown superimposed on each other in Fig. 37.79.

Two sets of star vectors will rotate at unequal speeds if the frequencies of the two machines are different. If the incoming alternator is running faster, then voltage star $R'Y'B'$ will appear to rotate anticlockwise with respect to the bus-bar voltage star RYB at a speed corresponding to the difference between their frequencies. With reference to Fig. 37.79, it is seen that voltage across L_1 is RR' and is seen to be increasing from zero, that across L_2 is YB' which is decreasing, having just passed through its maximum, that across L_3 is BY' which is increasing and approaching its maximum. Hence, the lamps will light up one after the other in the order 2, 3, 1 ; 2, 3, 1 or 1, 2, 3.

The rotor and stator of 3-phase generator

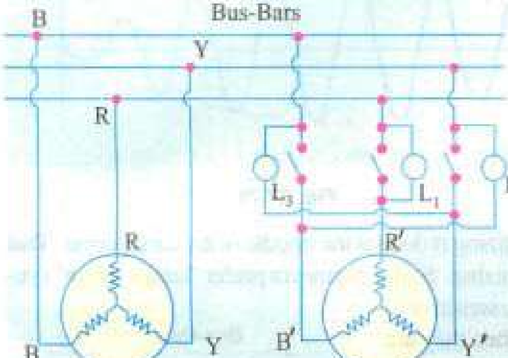


Fig. 37.77

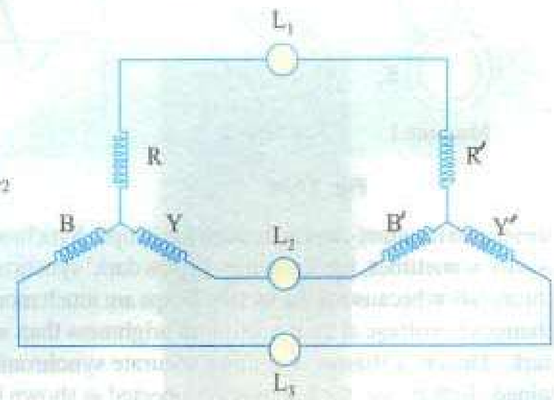


Fig. 37.78

Now, suppose that the incoming machine is slightly slower. Then the star R′Y′B′ will appear to be rotating clockwise relative to voltage star RYB (Fig. 37.80). Here, we find that voltage across L_3 *i.e.* $Y'B$ is decreasing having just passed through its maximum, that across L_2 *i.e.* YB' is increasing and approaching its maximum, that across L_1 is decreasing having passed through its maximum earlier. Hence, the lamps will light up one after the other in the order 3, 2, 1 ; 3, 2, 1, etc. which is just the reverse of the first order. Usually, the three lamps are mounted at the three corners of a triangle and the apparent direction of rotation of light

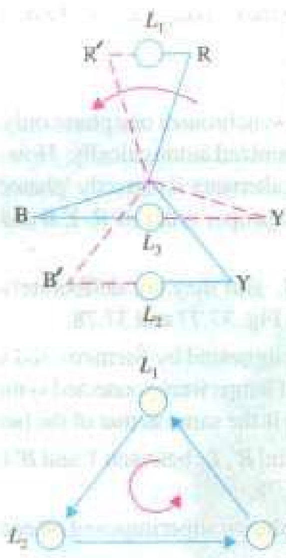


Fig. 37.79

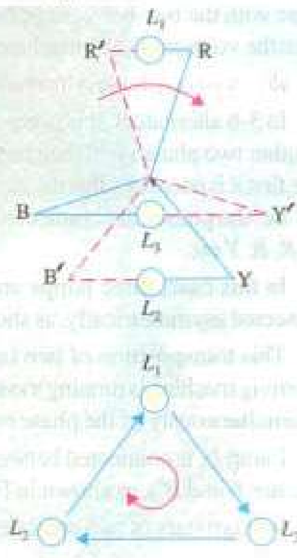


Fig. 37.80

indicates whether the incoming alternator is running too fast or too slow (Fig. 37.81). Synchronization is done at the moment the uncrossed lamp L_1 is in the middle of the dark period. When the alternator voltage is too high for the lamps to be used directly, then usually step-down transformers are used and the synchronizing lamps are connected to the secondaries.

It will be noted that when the uncrossed lamp L_1 is dark, the other two 'crossed' lamps L_2 and L_3 are dimly but equally bright. Hence, this method of synchronizing is also sometimes known as 'two bright and one dark' method.

It should be noted that synchronization by lamps is not quite accurate, because to a large extent, it depends on the sense of correct judgement of the operator. Hence, to eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.

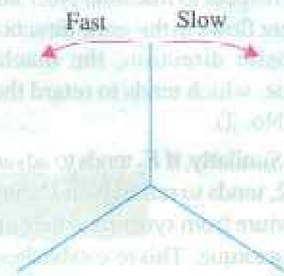


Fig. 37.81

Example 37.42. *In Fig. 37.74, E_1 = 220 V and f_1 = 60 Hz, whereas E_2 = 222 V and f_2 = 59 Hz. With the switch open; calculate*

(*i*) *maximum and minimum voltage across each lamp.*
(*ii*) *frequency of voltage across the lamps.*
(*iii*) *peak value of voltage across each lamp.*
(*iv*) *phase relations at the instants maximum and minimum voltages occur.*
(*v*) *the number of maximum light pulsations/minute.*

Solution. (*i*) E_{max}/lamp = (220 + 222)/2 = **221 V**

E_{min}/lamp = (222 − 220)/2 = **1.0 V**

(*ii*) $f = (f_1 - f_2) = (60 - 59) =$ **1.0 Hz**

(*iii*) E_{peak} = 221/0.707 = **313 V**

(*iv*) in-phase and anti-phase respectively in the local circuit.

(*v*) No. of pulsation/min = (60 − 59) × 60 = **60.**

37.32. Synchronizing Current

Once synchronized properly, two alternators continue to run in synchronism. Any tendency on the part of one to drop out of synchronism is immediately counteracted by the production of a synchronizing torque, which brings it back to synchronism.

When in exact synchronism, the two alternators have equal terminal p.d.'s and are in exact phase opposition, so far as the local circuit (consisting of their armatures) is concerned. Hence, there is no current circulating round the local circuit. As shown in Fig. 37.82 (*b*) e.m.f. E_1 of machine No. 1 is in exact phase opposition to the e.m.f. of machine No. 2 *i.e.* E_2. It should be clearly understood that the two e.m.f.s. are in opposition, so far as their local circuit is concerned but are in the same direction with respect to the external circuit. Hence, there is no resultant voltage (assuming $E_1 = E_2$ in magnitude) round the local circuit.

But now suppose that due to change in the speed of the governor of second machine, E_2 falls back* by a phase angle of α electrical degrees, as shown in Fig. 37.82 (*c*) (though still $E_1 = E_2$). Now, they have a resultant voltage E_r, which when acting on the local circuit, circulates a current known as synchronizing current. The value of this current is given by $I_{SY} = E_r/Z_S$ where Z_S is the synchronous impedance of the phase windings of both the machines (or of one machine only if it is connected to infinite bus-bars**). The current I_{SY} lags behind E_r by an angle θ given by $\tan\theta = X_S/R_a$ where X_S is the combined synchronous reactance of the two machines and R_a their armature resistance. Since R_a is negligibly small, θ is almost 90 degrees. So I_{SY} lags E_r by 90° and is almost in phase with E_1. It is seen that I_{SY} is generating current with respect to machine No.1 and motoring current with respect to machine No. 2 (remember when the current flows in the same direction as e.m.f., then the alternator acts as a generator, and when it flows in the opposite direction, the machine acts as a motor). This current I_{SY} sets up a synchronising torque, which tends to retard the generating machine (*i.e.* No. 1) and accelerate the motoring machine (*i.e.* No. 2).

Similarly, if E_2 tends to advance in phase [Fig. 37.82 (*d*)], then I_{SY}, being generating current for machine No. 2, tends to retard it and being motoring current for machine No. 1 tends to accelerate it. Hence, any departure from synchronism results in the production of a synchronizing current I_{SY} which sets up synchronizing torque. This re-establishes synchronism between the two machines by retarding the leading machine and by accelerating the lagging one. This current I_{SY}, it should be noted, is superimposed on the load currents in case the machines are loaded.

37.33. Synchronizing Power

Consider Fig. 37.82 (*c*) where machine No. 1 is generating and supplying the synchronizing power $= E_1 I_{SY} \cos\phi_1$ which is approximately equal to $E_1 I_{SY}$ (ϕ_1 is small). Since $\phi_1 = (90° - \theta)$, synchronizing power $= E_1 I_{SY} \cos\phi_1 = E_1 I_{SY} \cos(90° - \theta) = E_1 I_{SY} \sin\theta \cong E_1 I_{SY}$ because $\theta \cong 90°$ so that

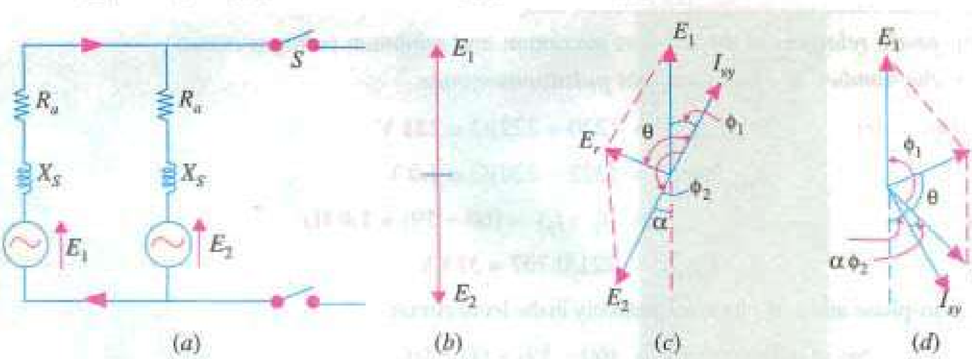


Fig. 37.82

$\sin\theta \cong 1$. This power output from machine No. 1 goes to supply (*a*) power input to machine No. 2 (which is motoring) and (*b*) the Cu losses in the local armature circuit of the two machines. Power input to machine No. 2 is $E_2 I_{SY} \cos\phi_2$ which is approximately equal to $E_2 I_{SY}$.

$\therefore \quad E_1 I_{SY} = E_2 I_{SY} + \text{Cu losses}$

Now, let $\quad E_1 = E_2 = E$ (say)

Then, $\quad E_r = 2E\cos[(180° - \alpha)/2]$*** $= 2E\cos[90° - (\alpha/2)]$

---

* Please remember that vectors are supposed to be rotating anticlockwise.

** Infinite bus-bars are those whose frequency and the phase of p.d.'s are not affected by changes in the conditions of any one machine connected in parallel to it. In other words, they are constant-frequency, constant-voltage bus-bars.

*** Strictly speaking, $E_r = 2E\sin\theta$. $\sin\alpha/2 \cong 2E\sin\alpha/2$.

$$= 2\,E \sin \alpha/2 = 2\,E \times \alpha/2 = \alpha E \qquad (\because \alpha \text{ is small})$$

Here, the angle α is in electrical radians.

Now, $$I_{SY} = \frac{E_r}{\text{synch. impedance } Z_S} \cong \frac{E_r}{2\,X_S} = \frac{\alpha\,E}{2\,X_S}$$

—if R_a of both machines is negligible

Here, X_S represents synchronous reactance of one machine and not of both as in Art. 37.31 Synchronizing power (supplied by machine No. 1) is

$$P_{SY} = E_1 I_{SY} \cos \phi_1 = E\,I_{SY} \cos (90° - \theta) = EI_{SY} \sin \theta \cong EI_{SY}$$

Substituting the value of I_{SY} from above,

$$P_{SY} = E.\alpha E/2\,Z_S = \alpha E^2/2Z_S \cong \alpha E^2/2X_S \qquad \text{—per phase}$$

(more accurately, $P_{SY} = \alpha\,E^2 \sin \theta/2X_S$)

Total synchronizing power for three phases

$$= 3P_{SY} = 3\,\alpha E^2/2X_S \text{ (or } 3\,\alpha E^2 \sin \theta/2X_S)$$

This is the value of the synchronizing power when two alternators are connected in parallel and are on no-load.

37.34. Alternators Connected to Infinite Bus-bars

Now, consider the case of an alternator which is connected to infinite bus-bars. The expression for P_{SY} given above is still applicable but with one important difference *i.e.* impedance (or reactance) of only that one alternator is considered (and not of two as done above). Hence, expression for synchronizing power in this case becomes

$$E_r = \alpha E \qquad \text{—as before}$$

$$I_{SY} = E/Z_S \cong E/X_S = \alpha E/X_S \qquad \text{—if } R_a \text{ is negligible}$$

$\therefore$ Synchronizing power $$P_{SY} = E\,I_{SY} = E.\alpha E/Z_S = \alpha E^2/Z_S \cong \alpha E^2/X_S \qquad \text{— per phase}$$

Now, $$E/Z_S \cong E/X_S = \text{S.C. current } I_{SC}$$

$\therefore$ $$P_{SY} = \alpha E^2/XS = \alpha E.E/X_S = \alpha E.I_{SY} \qquad \text{—per phase}$$

(more accurately, $P_{SY} = \alpha E^2 \sin \theta/X_S = \alpha E.I_{SC}.\sin \theta$)

Total synchronizing power for three phases = $3\,P_{SY}$

37.35. Synchronizing Torque T_{SY}

Let T_{SY} be the synchronizing torque per phase in newton-metre (N-m)

(*a*) When there are two alternators in parallel

$\therefore$ $$T_{SY} \times \frac{2\,\pi\,N_S}{60} = P_{SY} \quad \therefore \quad T_{SY} = \frac{P_{SY}}{2\,\pi\,N_S/60} = \frac{\alpha E^2/2X_S}{2\pi\,N_S/60} \text{ N-m}$$

Total torque due to three phases. $$= \frac{3P_{SY}}{2\pi N_S/60} = \frac{3\alpha E^2/2X_S}{2\pi N_S/60} \text{ N-m}$$

(*b*) Alternator connected to infinite bus-bars

$$T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \text{ or } T_{SY} = \frac{P_{SY}}{2\pi N_S/60} = \frac{\alpha E^2/X_S}{2\pi N_S/60} \text{ N-m}$$

Again, torque due to 3 phase $$= \frac{3P_{SY}}{2\pi N_S/60} = \frac{3\alpha E^2/X_S}{2\pi N_S/60} \text{ N-m}$$

where N_S = synchronous speed in r.p.m. = $120\,f/P$

37.36. Effect of Load on Synchronizing Power

In this case, instead of $P_{SY} = \alpha\, E^2/X_S$, the approximate value of synchronizing power would be $\cong \alpha EV/X_S$ where V is bus-bar voltage and E is the alternator induced e.m.f. per phase. The value of $E = V + IZ_S$

As seen from Fig. 37.83, for a lagging p.f.,

$E = [(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$

Fig. 37.83

Example 37.43. *Find the power angle when a 1500-kVA, 6.6 kV, 3-phase, Y-connected alternator having a resistance of 0.4 ohm and a reactance of 6 ohm per phase delivers full-load current at normal rated voltage and 0.8 p.f. lag. Draw the phasor diagram.*

(Electrical Machinery-II, Bangalore Univ. 1981)

Solution. It should be remembered that angle α between V and E is known as power angle (Fig. 37.84)

Full-load $\quad I = 15 \times 10^5/\sqrt{3} \times 6600 = 131$ A

$IR_a = 131 \times 0.4 = 52.4$ V, $IX_S = 131 \times 6 = 786$ V

V/phase $\quad = 6600/\sqrt{3} = 3810$ V;

$\phi = \cos^{-1}(0.8) = 36°50'$.

As seen from Fig. 37.84

$$\tan(\phi + \alpha) = \frac{AB}{OA} = \frac{V \sin \phi + IX_S}{V \cos \phi + I\, R_a}$$

$$= \frac{3810 \times 0.6 + 786}{3810 \times 0.8 + 52.4} = 0.991$$

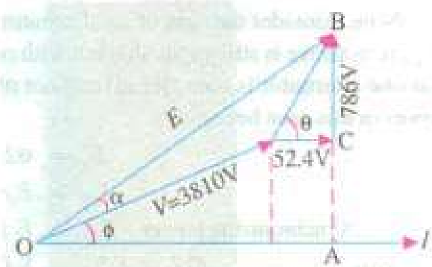


Fig. 37.84

$\therefore \quad (\phi + \alpha) = 44° \quad \therefore\ \alpha = 44° - 36°50' = 7°10'$

The angle α is also known as load angle or torque angle.

37.37. Alternative Expression for Synchronizing Power

As shown in Fig. 37.85, let V and E (or E_0) be the terminal voltage and induced e.m.f. per phase of the rotor. Then, taking $V = V\angle 0°$, the load current supplied by the alternator is

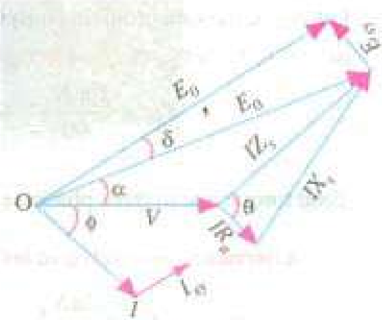


Fig. 37.85

$$I = \frac{E - V}{Z_S} = \frac{E\angle\alpha - V\angle 0°}{Z_S \angle\theta}$$

$$= \frac{E}{Z_S}\angle\alpha - \theta - \frac{V}{Z_S}\angle -\theta$$

$$= \frac{E}{Z_S}[\cos(\theta - \alpha) - j \sin(\theta - \alpha)]$$

$$= -\frac{V}{Z_S}(\cos\theta - j\sin\theta)$$

$$= \left[\frac{E}{Z_S}\cos(\theta - \alpha) - \frac{V}{Z_S}\cos\theta\right] - j\left[\frac{E}{Z_S}\sin(\theta - \alpha) - \frac{V}{Z_S}\sin\theta\right]$$

These components represent the $I \cos\phi$ and $I \sin\phi$ respectively. The power P converted internally is given be the sum of the product of corresponding components of the current with $E \cos\alpha$ and $E \sin\alpha$.

$$\therefore \quad P = E\cos\alpha\left[\frac{E}{Z_S}\cos(\theta-\alpha) - \frac{V}{Z_S}\cos\theta\right] - E\sin\alpha\left[\frac{E}{Z_S}\sin(\theta-\alpha) - \frac{V}{Z_S}\sin\theta\right]$$

$$= E\left[\frac{E}{Z_S}\cos\theta\right] - E\left[\frac{V}{Z_S}.\cos(\theta+\alpha)\right] = \frac{E}{Z_S}\,[E\cos\theta - V(\cos\theta+\alpha)] \qquad \text{—per phase*}$$

Now, let, for some reason, angle α be changed to $(\alpha \pm \delta)$. Since V is held rigidly constant, due to displacement $\pm\delta$, an additional e.m.f. of divergence *i.e.* $I_{SY} = 2E.\sin\alpha/2$ will be produced, which will set up an additional current I_{SY} given by $I_{SY} = E_{SY}/Z_S$. The internal power will become

$$P' = \frac{E}{Z_s}\,[E\cos\theta - V\cos(\theta+\alpha\pm\delta)]$$

The difference between P' and P gives the synchronizing power.

$$\therefore \quad P_{SY} = P' - P = \frac{EV}{Z_s}\,[\cos(\theta+\alpha) - \cos(\theta+\alpha\pm\delta)]$$

$$= \frac{EV}{Z_s}\,[\sin\delta\,.\,\sin(\theta+\alpha) \pm 2\cos(\theta+\alpha)\sin^2\delta/2]$$

If δ is very small, then $\sin^2(\delta/2)$ is zero, hence P_{SY} per phase is

$$P_{SY} = \frac{EV}{Z_S}.\sin(\theta+\alpha)\sin\delta \qquad ...(i)$$

(*i*) In large alternators, R_a is negligible, hence $\tan\theta = X_S/R_a = \infty$, so that $\theta \cong 90°$. Therefore, $\sin(\theta+\alpha) = \cos\alpha$.

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}.\cos\alpha\sin\delta \quad \text{— per phase} \qquad ...(ii)$$

$$= \frac{EV}{X_S}\cos\alpha\sin\delta \quad \text{—per phase} \qquad ...(iii)$$

(*ii*) Consider the case of synchronizing an unloaded machine on to a constant-voltage bus-bars. For proper operation, $\alpha = 0$ so that E coincides with V. In that case, $\sin(\theta+\alpha) = \sin\theta$.

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}\sin\theta\sin\delta \text{ —from } (i) \text{ above.}$$

Since δ is very small, $\sin\delta = \delta$,

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}\,\delta\sin\theta = \frac{EV}{X_S}\,\delta\sin\theta$$

Usually, $\sin\theta \cong 1$, hence

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}.\delta^{**} = V\left(\frac{E}{Z_S}\right)\delta = V\left(\frac{E}{X_S}\right)\delta = VI_{SC}\,.\,\delta \quad \text{—per phase}$$

37.38. Parallel Operation of Two Alternators

Consider two alternators with identical speed/load characteristics connected in parallel as shown in Fig. 37.86. The common terminal voltage **V** is given by

$$\mathbf{V} = \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 = \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2$$

$$\therefore \quad \mathbf{E}_1 - \mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2$$

Also $\quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$ and $\mathbf{V} = \mathbf{IZ}$

$$\therefore \quad \mathbf{E}_1 = \mathbf{I}_1\mathbf{Z}_1 + \mathbf{IZ} = \mathbf{I}_1(\mathbf{Z}+\mathbf{Z}_1) + \mathbf{I}_2\mathbf{Z}$$

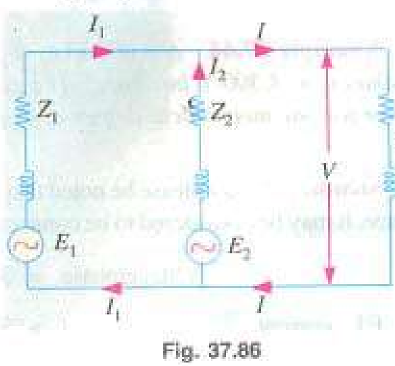


Fig. 37.86

* In large machines, R_a is very small so that $\theta = 90°$, hence $P = \frac{E}{Z_S}V\cos(90°\,\alpha) = \frac{E}{Z_S}V\sin\alpha = \alpha\,EV/Z_S$ —*if α is so small that $\sin\alpha = \alpha$*

** With $E = V$, the expression becomes $P_{SY} = \frac{V^2}{Z_S}\delta = \frac{\delta V^2}{X_S}$. It is the same as in Art. 37.33 —*per phase*

$$\mathbf{E}_2 = \mathbf{I}_2\mathbf{Z}_2 + \mathbf{IZ} = \mathbf{I}_2(\mathbf{Z} + \mathbf{Z}_2) + \mathbf{I}_1\mathbf{Z}$$

$$\therefore \qquad \mathbf{I}_1 = \frac{(\mathbf{E}_1 - \mathbf{E}_2)\,\mathbf{Z} + \mathbf{E}_1\mathbf{Z}_2}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{I}_2 = \frac{(\mathbf{E}_2 - \mathbf{E}_1)\,\mathbf{Z} + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2};$$

$$\mathbf{I} = \frac{\mathbf{E}_1\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{V} = \mathbf{IZ} = \frac{\mathbf{E}_1\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2 + (\mathbf{Z}_1\mathbf{Z}_2/\mathbf{Z})}; \mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{V}}{\mathbf{Z}_1}; \mathbf{I}_2 = \frac{\mathbf{E}_2 - \mathbf{V}}{\mathbf{Z}_2}$$

The circulating current under no-load condition is $\mathbf{I}_C = (\mathbf{E}_1 - \mathbf{E}_2)/(\mathbf{Z}_1 + \mathbf{Z}_2)$.

Using Admittances

The terminal Voltage may also be expressed in terms of admittances as shown below:

$$\mathbf{V} = \mathbf{IZ} = (\mathbf{I}_1 + \mathbf{I}_2)\mathbf{Z} \qquad \therefore \mathbf{I}_1 + \mathbf{I}_2 = \mathbf{V}/\mathbf{Z} = \mathbf{VY} \qquad \text{...}(i)$$

Also $\mathbf{I}_1 = (\mathbf{E}_1 - \mathbf{V})/\mathbf{Z}_1 = (\mathbf{E}_1 - \mathbf{V})\mathbf{Y}_1$; $\quad \mathbf{I}_2 = (\mathbf{E}_2 - \mathbf{V})/\mathbf{Z}_2 = (\mathbf{E}_2 - \mathbf{V})\mathbf{Y}_2$

$$\therefore \qquad \mathbf{I}_1 + \mathbf{I}_2 = (\mathbf{E}_1 - \mathbf{V})\,\mathbf{Y}_1 + (\mathbf{E}_2 - \mathbf{V})\mathbf{Y}_2 \qquad \text{...}(ii)$$

From Eq. (*i*) and (*ii*), we get

$$\mathbf{VY} = (\mathbf{E}_1 - \mathbf{V})\mathbf{Y}_1 + (\mathbf{E}_2 - \mathbf{V})\mathbf{Y}_2 \quad \text{or} \quad \mathbf{V} = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}}$$

Using Parallel Generator Theorem

$$\mathbf{V} = \mathbf{IZ} = (\mathbf{I}_1 + \mathbf{I}_2)\,\mathbf{Z} = \left(\frac{\mathbf{E}_1 - \mathbf{V}}{\mathbf{Z}_1} + \frac{\mathbf{E}_2 - \mathbf{V}}{\mathbf{Z}_2}\right)\mathbf{Z}$$

$$= \left(\frac{\mathbf{E}_1}{\mathbf{Z}_1} + \frac{\mathbf{E}_2}{\mathbf{Z}_2}\right)\mathbf{Z} - \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right)\mathbf{Z}$$

$$\therefore \qquad \mathbf{V}\left(\frac{1}{\mathbf{Z}} + \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right) = \frac{\mathbf{E}_1}{\mathbf{Z}_1} + \frac{\mathbf{E}_2}{\mathbf{Z}_2} = \mathbf{I}_{SC1} + \mathbf{I}_{SC2} = \mathbf{I}_{SC}$$

where $\mathbf{I}_{SC1}$ and $\mathbf{I}_{SC2}$ are the short-circuit currents of the two alternators.

If $$\frac{1}{\mathbf{Z}_0} = \left(\frac{1}{\mathbf{Z}} + \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right); \text{ then } \mathbf{V} \times \frac{1}{\mathbf{Z}_0} = \mathbf{I}_{SC} \text{ or } \mathbf{V} = \mathbf{Z}_0\,\mathbf{I}_{SC}$$

Example 37.44. *A 3,000-kVA, 6-pole alternator runs at 1000 r.p.m. in parallel with other machines on 3,300-V bus-bars. The synchronous reactance is 25%. Calculate the synchronizing power for one mechanical degree of displacement and the corresponding synchronizing torque.*

(Elect. Machines-I, Gwalior Univ. 1984)

Solution. It may please be noted that here the alternator is working in parallel with many alternators. Hence, it may be considered to be connected to infinite bus-bars.

$$\text{Voltage/phase} = 3{,}300/\sqrt{3} = 1905 \text{ V}$$

F.L. current $\quad I = 3 \times 10^6/\sqrt{3} \times 3300 = 525$ A

Now, $\quad IX_S = 25\%$ of $1905 \quad \therefore X_S = 0.25 \times 1905/525 = 0.9075\ \Omega$

Also, $\quad P_{SY} = 3 \times \alpha E^2/X_S$

Here $\quad \alpha = 1°$ (mech.); α (elect.) $= 1 \times (6/2) = 3°$

$\therefore \quad \alpha = 3 \times \pi/180 = \pi/60$ elect. radian.

$$\therefore \qquad P_{SY} = \frac{3 \times \pi \times 1905^2}{60 \times 0.9075 \times 1000} = 628.4 \text{ kW}$$

$$T_{SY} = \frac{60.P_{SY}}{2\pi N_S} = 9.55\frac{P_{SY}}{N_S} = 9.55\frac{628.4 \times 10^3}{1000} = \textbf{6,000 N-m}$$

Example 37.45. *A 3-MVA, 6-pole alternator runs at 1000 r.p.m on 3.3-kV bus-bars. The synchronous reactance is 25 percent. Calculate the synchronising power and torque per mechanical degree of displacement when the alternator is supplying full-load at 0.8 lag.*

(Electrical Machines-1, Bombay Univ. 1987)

Solution. $V = 3{,}300/\sqrt{3} = 1905$ V/phase, F.L. $I = 3 \times 10^6/\sqrt{3} \times 3{,}300 = 525$ A

$IX_S = 25\%$ of $1905 = 476$ V; $X_S = 476/525 = 0.9075\ \Omega$

Let, $\mathbf{I} = 525 \angle 0°$, then, $\mathbf{V} = 1905\,(0.8 + j0.6) = 1524 + j1143$

$\mathbf{E_0} = \mathbf{V} + \mathbf{IX_S} = (1524 + j1143) + (0 + j476) = (1524 + j1619) = 2220 \angle 46°44'$

Obviously, E_0 leads I by $46°44'$. However, V leads I by $\cos^{-1}(0.8) = 36°50'$.

Hence, $\alpha = 46°44' - 36°50' = 9°54'$

$\alpha = 1°$ (mech.), No. of pair of poles $= 6/2 = 3 \quad \therefore\ \alpha = 1 \times 3 = 3°$ (elect.)

$$P_{SY} \text{ per phase} = \frac{EV}{X_S}\cos\alpha \sin\delta = \frac{2220 \times 1905}{0.9075} \times \cos 9°54' \sin 3° = 218 \text{ kW}$$

P_{SY} for three phases $= 3 \times 218 =$ **654 kW**

$T_{SY} = 9.55 \times P_{SY}/N_S = 9.55 \times 654 \times 10^2/1000 = 6245$ N-m

Example 37.46. *A 750-kVA, 11-kV, 4-pole, 3-φ, star-connected alternator has percentage resistance and reactance of 1 and 15 respectively. Calculate the synchronising power per mechanical degree of displacement at (a) no-load (b) at full-load 0.8 p.f. lag. The terminal voltage in each case is 11 kV.*

(Electrical Machines-II, Indore Univ. 1985)

Solution. F.L. Current $I = 75 \times 10^3/\sqrt{3} \times 11 \times 10^3 = 40$ A

$V_{ph} = 11{,}000/\sqrt{3} = 6{,}350$ V, $IR_a = 1\%$ of $6{,}350 = 63.5$

or $40\,R_a = 63.5,\ R_a = 1.6\ \Omega;\ 40 \times X_S = 15\%$ of $6{,}350 = 952.5$ V

$\therefore \quad X_S = 23.8\ \Omega;\ Z_S = \sqrt{1.6^2 + 23.8^2} \cong 23.8\ \Omega$

(a) No-load

α (mech) $= 1°$: α (elect) $= 1 \times (4/2) = 2°$

$= 2 \times \pi/180 = \pi/90$ elect. radian.

$$P_{SY} = \frac{\alpha E^2}{Z_S} \cong \frac{\alpha E^2}{X_S} = \frac{(\pi/90) \times 6350^2}{23.8}$$

$= 59{,}140$ W $=$ **59.14 kW/phase.**

On no-load, V has been taken to be equal to E.

(b) F.L. 0.8 p.f.

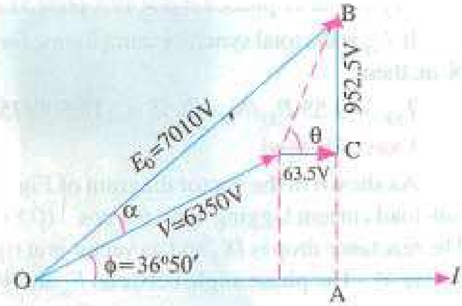


Fig. 37.87

As indicated in Art. 37.35, $P_{SY} = \alpha EV/X_S$. The value of E (or E_0) can be found from Fig. 37.87.

$$E = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$

$$= [(6350 \times 0.8 + 63.5)^2 + (6350 \times 0.6 + 952.5)^2]^{1/2} = 7010 \text{ V}$$

$$P_{SY} = \frac{\alpha EV}{X_S} = \frac{(\pi/90) \times 7010 \times 6350}{23.8} = 65{,}290 \text{ W}$$

$=$ **65.29 kW/phase**

More Accurate Method [Art. 37.35]

$$P_{SY} = \frac{EV}{X_S}\cos\alpha\sin\delta$$

Now, $E = 7010$ V, V = 6350 V, $\delta = 1° \times (4/2) = 2°$ (elect)

As seen from Fig. 37.87, $\sin(\phi+\alpha) = AB/OB = (6350 \times 0.6 + 952.5)/7010 = 0.6794$

$\therefore \quad (\phi + \alpha) = 42°30'; \; \alpha = 42°30' - 36°50' = 5°40'$

$$\therefore \quad P_{SY} = \frac{7010 - 6350}{23.8} \times \cos 5°40' \times \sin 2°$$

$$= 7010 \times 6350 \times 0.9953 \times 0.0349/23.8 = 64{,}970 \text{ W} = \mathbf{64.97 \text{ kW/phase}}$$

Note. It would be instructive to link this example with Ex. 38.1 since both are concerned with synchronous machines, one generating and the other motoring.

Example 37.47. *A 2,000-kVA, 3-phase, 8-pole alternator runs at 750 r.p.m. in parallel with other machines on 6,000 V bus-bars. Find synchronizing power on full-load 0.8 p.f. lagging per mechanical degree of displacement and the corresponding synchronizing torque. The synchronous reactance is 6 ohm per phase.* **(Elect. Machines-II, Bombay Univ. 1987)**

Solution. Approximate Method

As seen from Art. 37.37 and 38, $P_{SY} = \alpha EV/X_S$ —per phase

Now $\alpha = 1°$(mech); No. of pair of poles $= 8/2 = 4$;

$\alpha = 1 \times 4 = 4°$ (elect)

$= 4\pi/180 = \pi/45$ elect. radian

$V = 6000/\sqrt{3} = 3{,}465$ —assuming Y-connection

F.L. current $I = 2000 \times 10^3/\sqrt{3} \times 6000 = 192.4$ A

As seen from Fig. 37.88,

$E_0 = [(V\cos\phi)^2 + (V\sin\phi + IX_S)^2]^{1/2} = 4295$ V

$= [(3465 \times 0.8)^2 + (3465 \times 0.6 + 192.4 \times 6)^2]^{1/2}$

$= 4295$ V

$P_{SY} = (\pi/45) \times 4295 \times 3465/6 = 173{,}160$ W

$= \mathbf{173.16 \text{ kW/phase}}$

P_{SY} for three phases $= 3 \times 173.16 = 519.5$ kW

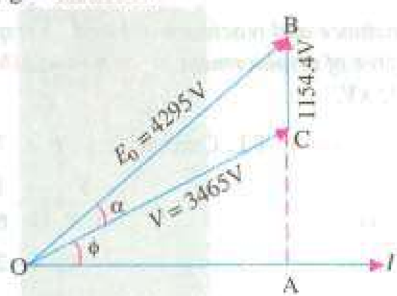


Fig. 37.88

If T_{SY} is the total synchronizing torque for three phases in N-m, then

$T_{SY} = 9.55\, P_{SY}/N_S = 9.55 \times 519{,}500/750 = \mathbf{6{,}614 \text{ N-m}}$

Exact Method

As shown in the vector diagram of Fig. 37.89, I is full-load current lagging V by $\phi = \cos^{-1}(0.8) = 36°50'$. The reactance drop is IX_S and its vector is at right angles to (lag.)*. The phase angle between E_0 and V is α.

F.L. current $I = 2{,}000{,}000/\sqrt{3} \times 6{,}000$

$= 192.4$ A

Let, $\quad I = 192.4 \angle 0°$

$\mathbf{V} = 3{,}465\,(0.8 + j\,0.6) = 2{,}772 + j\,2{,}079$

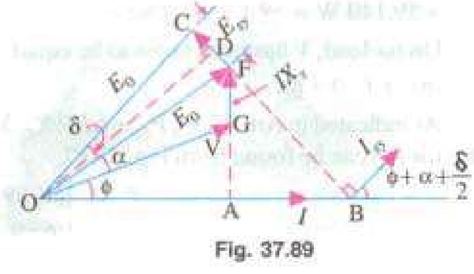


Fig. 37.89

* Earlier, we had called this e.m.f. as E when discussing regulation.

$IX_S = 192.4 \times 6 = 1154\ V = (0 + j\ 1154)\ V$

$\mathbf{E_0} = \mathbf{V + IX_S}$

$= (2{,}772 + j\ 2{,}079) + (0 + j\ 1154)$

$= 2{,}772 + j\ 3{,}233 = 4{,}259 \angle 49°24'$

$\alpha = 49°24' - 36°50' = 12°34'$

$E_{SY} = 2E_0 \sin \delta/2 = 2E_0 \sin (4°/2)$

$= 2 \times 4{,}259 \times 0.0349 = 297.3\ V$

$I_{SY} = 297.3/6 = 49.55\ A$

As seen, V leads I by ϕ and I_{SY} leads I by $(\phi + \alpha + \delta/2)$, hence I_{SY} leads V by $(\alpha + \delta/2) - 12°34' + (4°/2) = 14°34'$.

$\therefore \quad P_{SY}$/phase $= VI_{SY} \cos 14°34' = 3465 \times 49.55 \times \cos 14°34' = 166{,}200\ W = 166.2\ kW$

Synchronising power for three phases is $= 3 \times 166.2 =$ **498.6 kW**

If T_{SY} is the total synchronizing torque, then $T_{SY} \times 2\pi \times 750/60 = 498{,}600$

$\therefore \quad T_{SY} = 9.55 \times 498{,}600/750 =$ **6,348 N-m**

Alternative Method

We may use Eq. (*iii*) of Art. 37.36 to find the total synchronizing power.

$$P_{SY} = \frac{EV}{X_s} \cos \alpha \sin \delta \qquad \text{—per phase}$$

Here, $E = 4{,}259\ V$; $V = 3{,}465\ V$; $\alpha = 12°\ 34'$; $\delta = 4°$ (elect.)

$\therefore \quad P_{SY}$/phase $= 4{,}259 \times 3,465\ '\ \cos 12°\ 34' \times \sin 4°/6$

$= 4{,}259 \times 3{,}465 \times 0.976 \times 0.0698/6 = 167{,}500\ W = 167.5\ kW$

P_{SY} for 3 phases $= 3 \times 167.5 =$ **502.5 kW**

Next, T_{SY} may be found as above.

Example 37.48. *A 5,000-kV A, 10,000 V, 1500-r.p.m., 50-Hz alternator runs in parallel with other machines. Its synchronous reactance is 20%. Find for (a) no-load (b) full-load at power factor 0.8 lagging, synchronizing power per unit mechanical angle of phase displacement and calculate the synchronizing torque, if the mechanical displacement is 0.5°.*

(Elect. Engg. V, M.S. Univ. Baroda, 1986)

Solution. Voltage / phase $= 10{,}000 / \sqrt{3} = 5,775\ V$

Full-load current $= 5{,}000{,}000 / \sqrt{3} \times 10{,}000 = 288.7\ A$

$$X_S = \frac{20}{100} \times \frac{5,775}{288.7} = 4\ \Omega,\ P = \frac{120\ f}{N_S} = \frac{120 \times 50}{1500} = 4$$

$\alpha = 1°$ (mech.) ; No. of pair of poles $= 2 \qquad \therefore \alpha = 1 \times 2 = 2°$ (elect.) $= 2\ \pi/180 = \pi / 90$ radian

***(a)* At no-load**

$$P_{SY} = \frac{3\ \alpha\ E^2}{X_S} = 3 \times \frac{\pi}{90} \times \frac{5,775^2}{4 \times 1000} = \mathbf{873.4\ kW}$$

$\therefore \quad T_{SY} = 9.55 \times (873.4 \times 10^3)/1500 = 5{,}564$ N-m

$\therefore \quad T_{SY}$ for $0.5° = 5564/2 =$ **2,782 N-m**

***(b)* At F.L. p.f. 0.8 lagging**

Let $\mathbf{I} = 288.7 \angle 0°$. Then $\mathbf{V} = 5775\ (0.8 + j\ 0.6) = 4620 + j\ 3465$

$\mathbf{IX_S} = 288.7 \angle 0° \times 4 \angle 90° = (0 + j\ 1155)$

$\mathbf{E_0} = \mathbf{V + IX_S} = (4620 + j3465) + (0 + j1155)$

$$= 4620 + j4620 = 6533 \angle 45°$$

$$\cos\phi = 0.8,\ \phi = \cos^{-1}(0.8) = 36°50'$$

Now, E_0 leads I by 45° and V leads I by 36° 50′. Hence, E_0 leads V by (45° − 36°50′) =8° 10′ *i.e.* α = 8° 10′. As before, δ = 2° (elect).

As seen from Art. 37.36, $P_{SY} = \dfrac{EV}{X_s}\cos\alpha\sin\delta$ —per phase

$$= 6533 \times 5775 \times \cos 8°\ 10' \times \sin 2°/4 = 326 \text{ kW}$$

P_{SY} for three phases $= 3 \times 326 = 978$ kW

T_{SY}/ unit displacement $= 9.55 \times 978 \times 10^3/1500 = 6{,}237$ N-m

T_{SY} for 0.5° displacement $= 6{,}237/2 =$ **3118.5 N-m**

(c) We could also use the approximate expression of Art. 37.36

P_{SY} per phase $= \alpha\, E\, V/X_S = (\pi/90) \times 6533 \times 5775/4 =$ **329.3 kW**

Example 37.49. *Two 3-phase, 6.6-kW, star-connected alternators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of machine A is (0.5 + j10) Ω and of machine B is (0.4 + j 12) Ω. The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor and the governors are so set that load is shared equally between the machines. Determine the current, power factor, induced e.m.f. and load angle of each machine.*

(Electrical Machines-II, South Gujarat Univ. 1985)

Solution. It is given that each machine carries a load of 1500 kW. Also, $V = 6600/\sqrt{3} = 3810$ V. Let $V = 3810 \angle 0° = (3810 + j\,0)$.

For machine No. 1

$\sqrt{3}/6600 \times 150 \times \cos\phi_1 = 1500 \times 10^3$;

$\cos\phi_1 = 0.874,\ \phi_1 = 29°;\ \sin\phi_1 = 0.485$

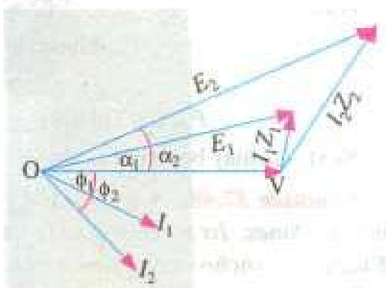


Fig. 37.90

Total current $I = 3000/\sqrt{3} \times 6.6 \times 0.8 = 328$ A

or $\quad I = 828\,(0.8 - j\,0.6) = 262 - j\,195$

Now, $\quad \mathbf{I_1} = 150\,(0.874 - j\,0.485) = 131 - j\,72.6$

$\therefore \quad \mathbf{I_2} = (262 - j\,195) - (131 - j\,72.6)$

$= (131 - j\,124.4)$

or $\quad I_2 =$ **181 A**, $\cos\phi_2 = 131/181 =$ **0.723 (lag).**

$\mathbf{E_A} = \mathbf{V + I_1Z_1} = 3810 + (131 - j\,72.6)\,(0.5 + j10)$

$= 4600 + j1270$

Line value of e.m.f. $= \sqrt{3}\sqrt{(4600^2 + 1270^2)} =$ **8,260 V**

Load angle $\quad \alpha_1 = (1270/4600) =$ **15.4°**

$\mathbf{E_B} = \mathbf{V + I_2Z_2} = 3810 + (131 - j124.4)\,(0.4 + j12)$

$= 5350 + j1520$

Line value of e.m.f $= \sqrt{3}\sqrt{5350^2 + 1520^2} =$ **9600 V**

Load angle $\quad \alpha_2 = \tan^{-1}(1520/5350) =$ **15.9°**

Example 37.50. *Two single-phase alternator operating in parallel have induced e.m.fs on open circuit of 230 ∠ 0° and 230 ∠ 10° volts and respective reactances of j2 Ω and j3 Ω. Calculate (i) terminal voltage (ii) currents and (iii) power delivered by each of the alternators to a load of impedance 6 Ω (resistive).* **(Electrical Machines-II, Indore Univ. 1987)**

Solution. Here, $\mathbf{Z_1} = j2$, $\mathbf{Z_2} = j3$, $\mathbf{Z} = 6$; $\mathbf{E_1} = 230 \angle 0°$ and

$$\mathbf{E}_2 = 230 \angle 10° = 230\,(0.985 + j\,0.174) = (226.5 + j\,39.9), \text{ as in Fig. 37.90}$$

$$(ii)\quad \mathbf{I}_1 = \frac{(E_1 - E_2)\,Z + E_1 Z_2}{Z\,(Z_1 + Z_2) + Z_1 Z_2} = \frac{[(230 + j0) - (226.5 + j39.9)] \times 6 + 230 \times j3}{6\,(j2 - j3) + j2 \times j3}$$

$$= 14.3 - j3.56 = 14.73 \angle -14° \qquad \text{—Art. 37.38}$$

$$\mathbf{I}_2 = \frac{(E_2 - E_1)\,Z + E_2 Z_1}{Z\,(Z_1 + Z_2) + Z_1 Z_2} = \frac{(-3.5 + j39.9) + (222.5 + j39.9) \times j2}{6\,(j2 + j3) + j2 \times j3}$$

$$= 22.6 - j\,1.15 = 22.63 \angle -3.4°$$

$$(i)\quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 36.9 - j\,4.71 = 37.2 \angle -7.3°$$

$$\mathbf{V} = \mathbf{IZ} = (36.9 - j4.71) \times 6 = 221.4 - j28.3 = 223.2 \angle -7.3°$$

$$(iii)\quad P_1 = V I_1 \cos\phi_1 = 223.2 \times 14.73 \times \cos 14° = \mathbf{3190\ W}$$

$$P_2 = V I_2 \cos\phi_1 = 223.2 \times 22.63 \times \cos 3.4° = \mathbf{5040\ W}$$

Tutorial Problem No. 37.6.

1. Calculate the synchronizing torque for unit mechanical angle of phase displacement for a 5,000-kVA, 3-ϕ alternator running at 1,500 r.p.m. when connected to 6,600-volt, 50-Hz bus-bars. The armature has a short-circuit reactance of 15%. **[43,370 kg-m]** *(City & Guilds, London)*
2. Calculate the synchronizing torque for one mechanical degree of phase displacement in a 6,000-kVA, 50-Hz, alternator when running at 1,500 r.p.m with a generated e.m.f. of 10,000 volt. The machine has a synchronous impedance of 25%.
[544 kg.m] *(Electrical Engineering-III, Madras Univ. April 1978; Osmania Univ. May 1976)*
3. A 10,000-kVA, 6,600-V, 16-pole, 50-Hz, 3-phase alternator has a synchronous reactance of 15%. Calculate the synchronous power per mechanical degree of phase displacement from the full load position at power factor 0.8 lagging. **[10 MW]** *(Elect.Machines-I, Gwalior Univ. 1977)*
4. A 6.6 kV, 3-phase, star-connected turbo-alternator of synchronous reactance 0.5 ohm/phase is applying 40 MVA at 0.8 lagging p.f. to a large system. If the steam supply is suddenly cut off, explain what takes place and determine the current the machine will then carry. Neglect losses.
[2100 A] *(Elect. Machines (E-3) AMIE Sec. B Summer 1990)*
5. A 3-phase 400 kVA, 6.6 kV, 1500 rpm., 50 Hz alternator is running in parallel with infinite bus bars. Its synchronous reactance is 25%. Calculate (*i*) for no load (*ii*) full load 0.8 p.f. lagging the synchronizing power and torque per unit mechanical angle of displacement.
[Rajive Gandhi Technical University, 2000] **[(*i*) 55.82 kW, 355 Nw-m (*ii*) 64.2 kW, 409 Nw-m]**

37.39. Effect of Unequal Voltages

Let us consider two alternators, which are running exactly in-phase (relative to the external circuit) but which have slightly unequal voltages, as shown in Fig. 37.91. If E_1 is greater than E_2, then their resultant is $E_r = (E_1 - E_2)$ and is in-phase with E_1. This E_r or E_{SY} set up a local synchronizing current I_{SY} which (as discussed earlier) is almost 90° behind E_{SY} and hence behind E_1 also. This lagging current produces demagnetising effect (Art. 37.16) on the first machine, hence E_1 is reduced. The other machine runs as a synchronous motor, taking almost 90° leading current. Hence, its field is strengthened due to magnetising effect of armature reaction (Art. 37.16). This tends to increase E_2. These two effects act together and hence lessen the inequalities between the two voltages and tend to establish stable conditions.

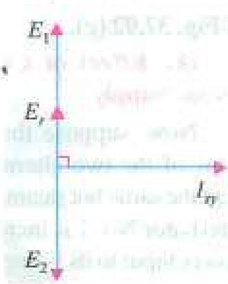


Fig. 37.91

37.40. Distribution of Load

It will, now be shown that the amount of load taken up by an alternator running, in parallel with other machines, is solely determined by its driving torque *i.e.* by the power input to its prime mover (by giving it more or less steam, in the case of steam drive). Any alternation in its excitation merely changes its kVA output, but not its kW output. In other words, it merely changes the power factor at which the load is delivered.

(a) Effect of Change in Excitation

Suppose the initial operating conditions of the two parallel alternators are identical *i.e.* each alternator supplies one half of the active load (kW) and one-half of the reactive load (kVAR), the operating power factors thus being equal to the load p.f. In other words, both active and reactive powers are divided equally thereby giving equal apparent power triangles for the two machines as shown in Fig. 37.92 (*b*). As shown in Fig. 37.92 (*a*), each alternator supplies a load current I so that total output current is $2I$.

Now, let excitation of alternator No. 1 be increased, so that E_1 becomes greater than E_2. The difference between the two e.m.fs. sets up a circulating current $I_C = I_{SY} = (E_1 - E_2)/2Z_S$ which is confined to the local path through the armatures and round the bus-bars. This current is superimposed on the original current distribution. As seen, I_C is vectorially added to the load current of alternator No. 1 and subtracted from that of No. 2. The two machines now deliver load currents I_1 and I_2 at respective power factors of cos ϕ_1 and cos ϕ_2. These changes in load currents lead to changes in power factors, such that cos ϕ_1 is reduced, whereas cos ϕ_2 is increased. However, effect on the

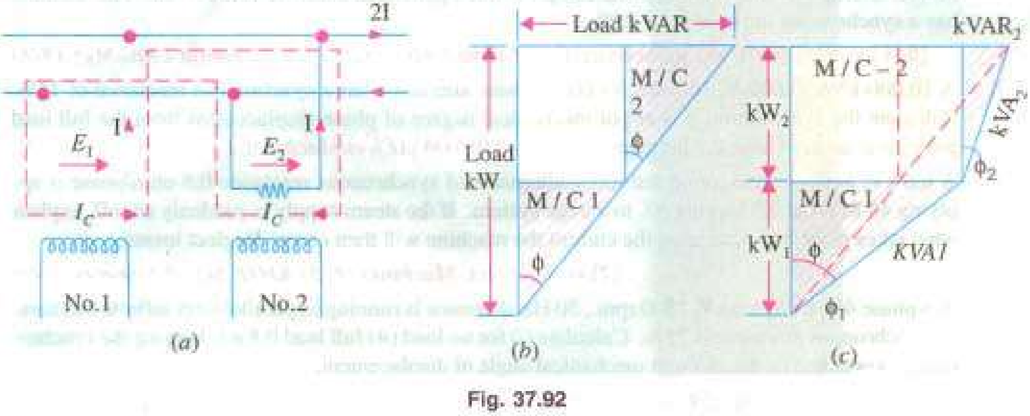


Fig. 37.92

kW loading of the two alternators is negligible, but kVAR$_1$ supplied by alternator No. 1 is increased, whereas kVAR$_2$ supplied by alternator No. 2 is correspondingly decreased, as shown by the kVA triangles of Fig. 37.92 (*c*).

(b) Effect of Change in Steam Supply

Now, suppose that excitations of the two alternators are kept the same but steam supply to alternator No. 1 is increased *i.e.* power input to its prime mover is increased. Since the speeds of the two machines are tied together by their synchronous bond, machine No. 1 cannot overrun machine No 2. Alternatively, it utilizes its increased power input for carrying

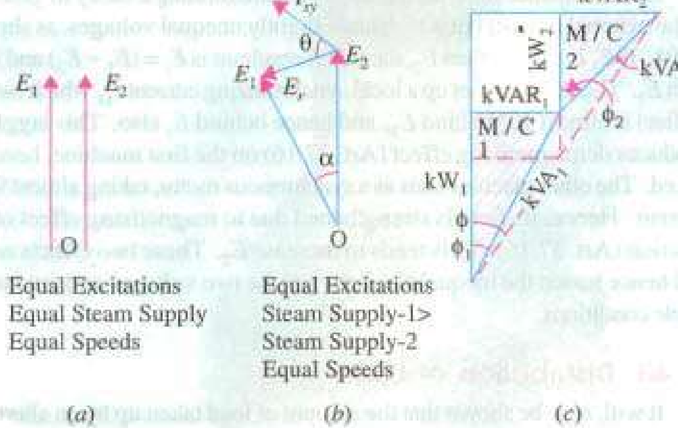


Fig. 37.93

more load than No. 2. This can be made possible only when rotor No. 1 advances its angular position with respect to No. 2 as shown in Fig. 37.93 (*b*) where E_1 is shown advanced ahead of E_2 by an angle α. Consequently, resultant voltage E_r (or E_{sy}) is produced which, acting on the local circuit, sets up a current I_{sy} which lags by almost 90° behind E_r but is almost in phase with E_1 (so long as angle α is small). Hence, power per phase of No. 1 is increased by an amount $= E_1 I_{sy}$ whereas that of No. 2 is decreased by the same amount (assuming total load power demand to remain unchanged). Since I_{sy} has no appreciable reactive (or quadrature) component, the increase in steam supply does not disturb the division of reactive powers, but it increases the active power output of alternator No. 1 and decreases that of No. 2. Load division, when steam supply to alternator No. 1 is increased, is shown in Fig. 37.93 (*c*).

So, it is found that by increasing the input to its prime mover, an alternator can be made to take a greater share of the load, though at a different power factor.

The points worth remembering are :

1. The load taken up by an alternators directly depends upon its driving torque or in other words, upon the angular advance of its rotor.

2. The excitation merely changes the p.f. at which the load is delivered without affecting the load so long as steam supply remains unchanged.

3. If input to the prime mover of an alternator is kept constant, but its excitation is changed, then kVA component of its output is changed, not kW.

Example 37.51. *Two identical 3-phase alternators work in parallel and supply a total load of 1, 500 kW at 11 kV at a power factor of 0.867 lagging. Each machine supplies half the total power. The synchronous reactance of each is 50 Ω per phase and the resistance is 4 Ω per phase. The field excitation of the first machine is so adjusted that its armature current is 50 A lagging. Determine the armature current of the second alternator and the generated voltage of the first machine.*

(Elect. Technology, Utkal Univ. 1983)

Solution. Load current at 0.867 p.f. lagging is

$$= \frac{1,500 \times 1,000}{\sqrt{3} \times 11,000 \times 0.887} = 90.4 \text{ A}; \cos\phi = 0.867; \sin\phi = 0.4985$$

Wattful component of the current = 90.4 × 0.867 = 78.5 A

Wattless component of the current = 90.4 × 0.4985 = 45.2 A

Each alternator supplies half of each of the above two component when conditions are identical (Fig. 37.94).

Current supplied by each machine = 90.4/2 = 45.2 A

Since the steam supply of first machine is not changed, the working components of both machines would remain the same at 78.5 / 2 = 39.25 A. But the wattless or reactive components would be redivided due to change in excitation. The armature current of the first machine is changed from 45.2 A to 50 A.

∴ Wattless component of the 1st machine $= \sqrt{50^2 - 39.25^2} = 31$ A

Wattless component of the 2nd machine = 45.2 − 31 = 14.1 A

The new current diagram is shown in Fig. 37.95 (*a*)

(*i*) Armative current of the 2nd alternator, $I_2 = \sqrt{39.25^2 + 14.1^2} = 41.75$ A

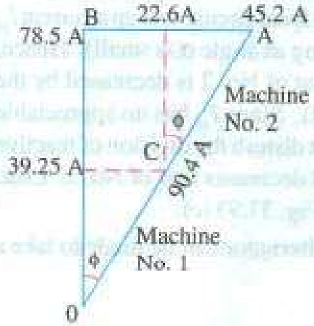


Fig. 37.94

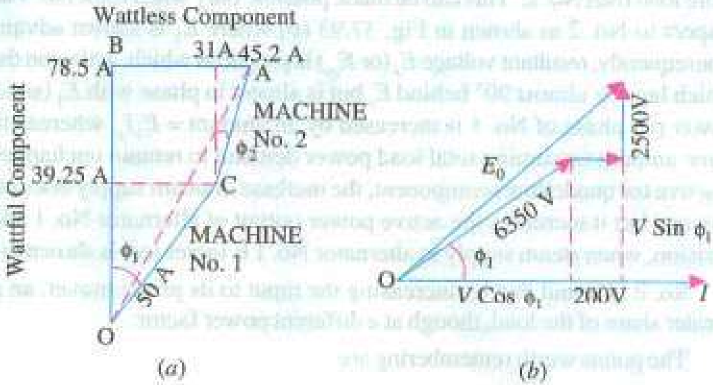


Fig. 37.95

(*ii*) Terminal voltage / phase = $11,000 / \sqrt{3} = 6350$ V

Considering the first alternator,

$$IR \text{ drop } = 4 \times 50 = 200 \text{ V}; \quad IX \text{ drop } = 50 \times 50 = 2,500 \text{ V}$$

$$\cos \phi_1 = 39.25/50 = 0.785; \quad \sin \phi_1 = 0.62$$

Then, as seen from Fig. 37.95 (*b*)

$$E = \sqrt{(V \cos \phi_1 + IR)^2 + (V \sin \phi_1 + IX)^2}$$

$$= \sqrt{(6,350 \times 0.785 + 200)^2 + (6,350 \times 0.62 + 2,500)^2} = 8,350 \text{ V}$$

Line voltage $= 8,350 \times \sqrt{3}$ = **14,450 V**

Example 37.52. *Two alternators A and B operate in parallel and supply a load of 10 MW at 0.8 p.f. lagging (a) By adjusting steam supply of A, its power output is adjusted to 6,000 kW and by changing its excitation, its p.f. is adjusted to 0.92 lag. Find the p.f. of alternator B.*

(b) If steam supply of both machines is left unchanged, but excitation of B is reduced so that its p.f. becomes 0.92 lead, find new p.f. of A.

Solution. (*a*) $\cos \phi = 0.8$, $\phi = 36.9°$, $\tan \phi = 0.7508$; $\cos \phi_A = 0.92$, $\phi_A = 23°$; $\tan \phi_A = 0.4245$

load kW = 10,000, load kVAR = 10,000 × 0.7508 = 7508 (lag)

kW of A = 6,000, kVAR of A = 6,000 × 0.4245 = 2547 (lag)

Keeping in mind the convention that lagging kVAR is taken as negative we have,

kW of B = (10,000 − 6,000) = 4,000 : kVAR of B = (7508 − 2547) = 4961,(lag)

∴ kVA of $B = 4,000 - j4961 = 6373 \angle -51.1°$; $\cos \phi_B = \cos 51.1° =$ **0.628**

(*b*) Since steam supply remains unchanged, load kW of each machine remains as before but due to change in excitation, kVARs of the two machines are changed.

kW of B = 4,000, new kVAR of B = 4000 × 0.4245 = 1698 (lead)

kW of A = 6,000, new kVAR of A = −7508 − (+1698) = − 9206 (lag.)

∴ new kVA of $A = 6,000 - j9206 = 10,988 \angle -56.9°$; $\cos \phi_A =$ **0.546 (lag)**

Example 37.53. *A 6,000-V, 1,000-kVA, 3-ϕ alternator is delivering full-load at 0.8 p.f. lagging. Its reactance is 20% and resistance negligible. By changing the excitation, the e.m.f. is increased by 25% at this load. Calculate the new current and the power factor. The machine is connected to infinite bus-bars.*

Solution. Full-load current $I = \dfrac{1,000,000}{\sqrt{3} \times 6,600} = 87.5$ A

Voltage/phase = $6,600/\sqrt{3} = 3,810$ V

Reactance $= \dfrac{3810 \times 20}{87.5 \times 100} = 8.7\ \Omega$

$IX = 20\%$ of $3810 = 762$ V

In Fig. 37.96, current vector is taken along X-axis. ON represents bus-bar or terminal voltage and is hence constant.

Current I has been split up into its active and reactive components I_R and I_X respectively.

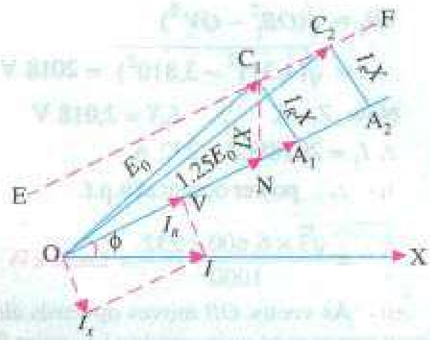


Fig. 37.96

$NA_1 = I_X X = 52.5 \times 7.8 = 457$ V

$A_1C_1 = I_R.X = 70 \times 8.7 = 609$ V

$E_0 = OC_1 = \sqrt{[(V + I_X X)^2 + (I_R X)^2]}$

$= \sqrt{[(3,810 + 457)^2 + 609^2]} = 4,311$ V

When e.m.f. is increased by 25%, then E_0 becomes equal to $4,311 \times 1.25 = 5,389$ V

The locus of the extremity of E_0 lies on the line EF which is parallel to ON. Since the kW is unchanged, I_R and hence $I_R.X$ will remain the same. It is only the $I_X.X$ component which will be changed. Let OC_2 be the new value of E_0. Then $A_2C_2 = A_1C_1 = I_R X$ as before. But the $I_X X$ component will change. Let I' be the new line current having active component I_R (the same as before) and the new reactive component I_X'. Then, $I_X' X = NA_2$

From right-angled triangle OC_2A_2

$$OC_2^{\,2} = OA_2^{\,2} + A_2C_2^{\,2};\ 5,389^2 = (3810 + VA_2)^2 + 609^2$$

$\therefore \quad VA_2 = 1546\ V$ or $I_X'\ X = 1546$

$\therefore \quad I_X' = 1546/8.7 = 177.7$ A

$\therefore$ New line current $\quad I' = \sqrt{(70^2 + 177.7^2)} =$ **191 A**

New angle of lag, $\quad \phi' = \tan^{-1}(177.7/70) = 68°\ 30'$; $\cos\phi' = \cos 68°30' =$ **0.3665**

As a check, the new power $= \sqrt{3} \times 6,600 \times 191 \times 0.3665 = 800$ kW

It is the same as before $= 1000 \times 0.8 = 800$ kW

Example 37.54. *A 6,600-V, 1000-kVA alternator has a reactance of 20% and is delivering full-load at 0.8 p.f. lagging. It is connected to constant-frequency bus-bars. If steam supply is gradually increased, calculate* **(i)** *at what output will the power factor become unity* **(ii)** *the maximum load which it can supply without dropping out of synchronism and the corresponding power factor.*

Solution. We have found in Example 37.52 that

$I = 87.5\ A$, $X = 8.7\ \Omega$, V/phase $= 3,810$ V

$E_0 = 4,311\ V$, $I_R = 70\ A$, $I_X = 52.5\ A$ and

$IX = 87.5 \times 8.7 = 762$ V

Using this data, vector diagram of Fig. 37.97 can be constructed.

Since excitation is constant, E_0 remains constant, the extremity of E_0 lies on the arc of a circle of radius E_0 and centre O. Constant power lines have been shown dotted and they are all parallel to OV. Zero power output line coincides with OV. When p.f. is unity, the current vector lies along OV, I_1Z is ⊥ to OV and cuts the arc at B_1. Obviously

$VB_1 = \sqrt{(OB_1^2 - OV^2)}$

$= \sqrt{(4{,}311^2 - 3{,}810^2)} = 2018$ V

Now $Z = X \quad \therefore \quad I_1X = 2{,}018$ V

$\therefore I_1 = 2018/8.7 = 232$ A

(*i*) $\therefore$ power output at u.p.f.

$$= \frac{\sqrt{3} \times 6{,}600 \times 232}{1000} = \mathbf{2{,}652\ kW}$$

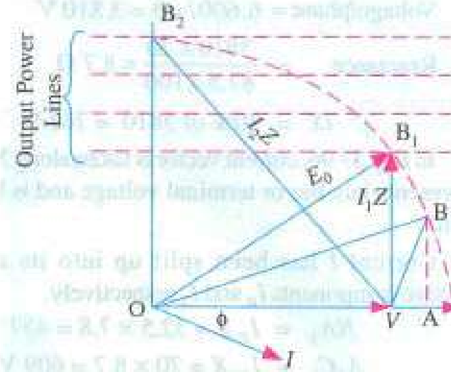


Fig. 37.97

(*ii*) As vector *OB* moves upwards along the arc, output power goes on increasing *i.e.*, point *B* shifts on to a higher output power line. Maximum output power is reached when *OB* reaches the position OB_2 where it is vertical to *OV*. The output power line passing through B_2 represents the maximum output for that excitation. If *OB* is further rotated, the point B_2 shifts down to a lower power line *i.e.* power is decreased. Hence, $B_2V = I_2Z$ where I_2 is the new current corresponding to maximum output.

From triangle OB_2V, it is seen that

$$B_2V = \sqrt{(OV^2 + OB_2^2)} = \sqrt{(3{,}810^2 + 4{,}311^2)} = 5{,}753 \text{ V}$$

$$\therefore \quad I_2Z = 5{,}753 \text{ V} \quad \therefore \quad I_2 = 5753/8.7 = 661 \text{ A}$$

Let I_{2R} and I_{2X} be the power and wattless components of I_2, then

$$I_{2R}\,X = OB_2 = 4311 \text{ and } I_{2R} = 4311/8.7 = 495.6 \text{ A}$$

Similarly $\quad I_{2X} = 3810/8.7 = 438$ A; $\tan\phi_2 = 438/495.6 = \mathbf{0.884}$

$\therefore \quad \phi_2 = 41°28'$; $\cos\phi_2 = 0.749$

$$\therefore \quad \text{Max. power output} = \frac{\sqrt{3} \times 6{,}600 \times 661 \times 0.749}{1000} = \mathbf{5{,}658\ kW}$$

Example 37.55. *A 3-phase, star-connected turbo-alternator, having a synchronous reactance of 10 Ω per phase and negligible armature resistance, has an armature current of 220 A at unity p.f. The supply voltage is constant at 11 kV at constant frequency. If the steam admission is unchanged and the e.m.f. raised by 25%, determine the current and power factor.*

If the higher value of excitation is maintained and the steam supply is slowly increased, at what power output will the alternator break away from synchronism ?

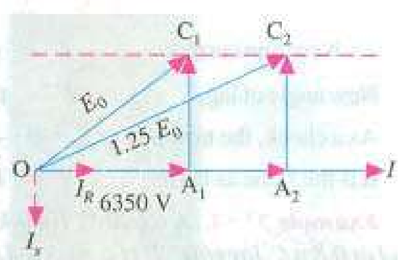


Fig. 37.98

Draw the vector diagram under maximum power condition.

(Elect.Machinery-III, Banglore Univ. 1992)

Solution. The vector diagram for unity power factor is shown in Fig. 37.98. Here, the current is wholly active.

$$OA_1 = 11{,}000/\sqrt{3} = 6{,}350 \text{ V}$$

$$A_1C_1 = 220 \times 10 = 2{,}200 \text{ V}$$

$$E_0 = \sqrt{(6350^2 + 2{,}200^2)} = 6{,}810 \text{ V}$$

When e.m.f. is increased by 25%, the e.m.f. becomes $1.25 \times 6{,}810 = 8{,}512$ V and is represented by OC_2. Since the kW remains unchanged, $A_1C_1 = A_2C_2$. If I' is the new current, then its active component

I_R would be the same as before and equal to 220 A. Let its reactive component be I_X. Then

$$A_1A_2 = I_X X_S = 10\, I_X$$

From right-angled $\Delta\, OA_2C_2$, we have

$$8{,}512^2 = (6350 + A_1A_2)^2 + 2{,}200^2$$

$\therefore \quad A_1A_2 = 1870 \text{ V} \quad \therefore \quad 10\, I_X = 1870 \quad I_X = 187 \text{ A}$

Hence, the new current has active component of 220 A and a reactive component of 187 A.

$$\text{New current} = \sqrt{220^2 + 187^2} = \mathbf{288.6\ A}$$

$$\text{New power factor} = \frac{\text{active component}}{\text{total current}} = \frac{220}{288.6} = \mathbf{0.762\ (lag)}$$

Since excitation remains constant, E_0 is constant. But as the steam supply is increased, the extremity of E_0 lies on a circle of radius E_0 and centre O as shown in Fig. 37.99.

The constant-power lines (shown dotted) are drawn parallel to OV and each represents the locus of the e.m.f. vector for a constant power output at varying excitation. Maximum power output condition is reached when the vector E_0 becomes perpendicular to OV. In other words, when the circular e.m.f. locus becomes tangential to the constant-power lines *i.e.* at point B. If the steam supply is increased further, the alternator will break away from synchronism.

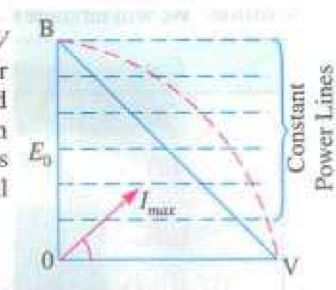


Fig. 37.99

$$B.V. = \sqrt{6350^2 + 8{,}512^2} = 10{,}620 \text{ V}$$

$\therefore \quad I_{max} \times 10 = 10{,}620$ or $I_{max} = 1{,}062$ A

If I_R and I_X are the active and reactive components of I_{max}, then

$10\, I_R = 8{,}512 \quad \therefore \quad I_R = 851.2$ A; $10\, I_X = 6{,}350 \quad \therefore \quad I_X = 635$ A

Power factor at maximum power output = 851.2/1062 = 0.8 (lead)

$$\text{Maximum power output} = \sqrt{3} \times 11{,}000 \times 1062 \times 0.8 \times 10^{-3} = \mathbf{16{,}200\ kW}$$

Example 37.56. *Two 20-MVA, 3-φ alternators operate in parallel to supply a load of 35MVA at 0.8 p.f. lagging. If the output of one machine is 25 MVA at 0.9 lagging, what is the output and p.f. of the other machine?* **(Elect. Machines, Punjab Univ. 1990)**

Solution. Load MW = 35 × 0.8 = 28; load MVAR = 35 × 0.6 = 21

First Machine $\cos \phi_1 = 0.9$, $\sin \phi_1 = 0.436$; $\text{MVA}_1 = 25$, $\text{MW}_1 = 25 \times 0.9 = 22.5$

$\text{MVAR}_1 = 25 \times 0.436 = 10.9$

Second Machine $\text{MW}_2 = \text{MW} - \text{MW}_1 = 28 - 22.5 = 5.5$

$\text{MVAR}_2 = \text{MVAR} - \text{MVAR}_1 = 21 - 10.9 = 10.1$

$$\therefore \quad \text{MVA}_2 = \sqrt{\text{MW}_2^2 + \text{MVAR}_2^2} = \sqrt{5.5^2 + 10.1^2} = \mathbf{11.5}$$

$$\cos \phi_2 = 5.5/11.5 = \mathbf{0.478\ (lag)}$$

Example 37.57. *A lighting load of 600 kW and a motor load of 707 kW at 0.707 p.f. are supplied by two alternators running in parallel. One of the machines supplies 900 kW at 0.9 p.f. lagging. Find the load and p.f. of the second machine.*

(Electrical Technology, Bombay Univ. 1988 & Bharatiar University, 1997)

Solution.

	Active Power	*kVA*	*Reactive Power*
(*a*) Lighting Load (unity P.f.)	600 kW	600	—
(*b*) Motor, 0.707 P.f.	707 kW	1000	707 k VAR
Total Load :	1307	By Phasor addition	707 k VAR

One machine supplies an active power of 900 kW, and due to 0.9 lagging p.f., kVA = 1000 kVA and its k VAR = $1000 \times \sqrt{(1-0.9^2)}$ = 436 kVAR. Remaining share will be catered to by the second machine.

Active power shared by second machine = 1307 – 900 = **407 kW**

Reactive power shared by second machine = 707 – 436 = **271 kVAR**

Example 37.58. *Two alternators, working in parallel, supply the following loads :*

(i) Lighting load of 500 kW *(ii) 1000 kW at p.f. 0.9 lagging*

(iii) 800 kW at p.f. 0.8 lagging *(iv) 500 kW at p.f. 0.9 leading*

One alternator is supplying 1500 kW at 0.95 p.f. lagging. Calculate the kW output and p.f of the other machine.

Solution. We will tabulate the kW and kVAR components of each load separately :

Load	*kW*	*kVAR*
(i)	500	
(ii)	1000	$\frac{1000}{0.9} \times 0.436 = 485$
(iii)	800	$\frac{800 \times 0.6}{0.8} = 600$
(iv)	500	$\frac{500 \times 0.436}{0.9} = -242$
Total	2800	+ 843

For 1st machine, it is given : kW = 1500, kVAR = (1500/0.95) × 0.3123 = 493

∴ kW supplied by other machine = 2800 – 1500 = **1300**

kVAR supplied = 843 – 493 = 350 ∴ tan φ 350/1300 = 0.27 ∴ cos φ = **0.966**

Example 37.59 *Two 3-φ synchronous mechanically-coupled generators operate in parallel on the same load. Determine the kW output and p.f. of each machine under the following conditions: synchronous impedance of each generator : 0.2 + j2 ohm/phase. Equivalent impedance of the load : 3 + j4 ohm/phase. Induced e.m.f. per phase, 2000 + j0 volt for machine I and 2,2000 + j 100 for II.* **[London Univ.]**

Solution. Current of 1st machine $= \mathbf{I}_1 = \dfrac{\mathbf{E}_1 - \mathbf{V}}{0.2 + j2}$ or $\mathbf{E}_1 - \mathbf{V} = \mathbf{I}_1\,(0.2 + j2)$

Similarly $\mathbf{E}_2 - \mathbf{V} = \mathbf{I}_2\,(0.2 + j2)$

Also $\mathbf{V} = (\mathbf{I}_1 + \mathbf{I}_2)\,(3 + j4)$ where $3 + j4$ = load impedance

Now $\mathbf{E}_1 = 2{,}000 + j\,0,\ \mathbf{E}_2 = 2{,}200 + j100$

Solving from above, we get $\mathbf{I}_1 = 68.2 - j\,102.5$

Similarly $\mathbf{I}_2 = 127 - j\,196.4\ ;\ \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 195.2 - j299$

Now $\mathbf{V} = \mathbf{IZ} = (192.2 - j\,299)\,(3 + j\,4) = 1781 - j\,115.9$

Using the method of conjugate for power calculating, we have for the first machine

$P_{VA1} = (1781 - j\,115.9)\,(68.2 + j\,102.5) = 133{,}344 + j\,174{,}648$

∴ kW_1 = 133.344 kW/phase = 3 × 133.344 = **400 kW** —for 3 phases

Now $\tan^{-1}(102.5/68.2) = 56°24'$; $\tan^{-1}(115.9/1781) = 3°43'$

∴ for 1st machine ; $\cos(56°24' - 3°43')$ = **0.6062**

$PVA_2 = (1781 - j115.9)(127 + j196.4) = 248{,}950 + j335{,}069$

$\therefore \quad kW_2 = 248.95$ kW/phase $= 746.85$ kW —for 3 phases

$\tan^{-1}(196.4/127) = 57°6'$; $\cos\phi = \cos(57°6' - 3°43') =$ **0.596**

Example 37.60. *The speed regulations of two 800-kW alternators A and B, running in parallel, are 100% to 104% and 100% to 105% from full-load to no-load respectively. How will the two alternators share a load of 1000 kW? Also, find the load at which one machine ceases to supply any portion of the load.* **(Power Systems-I, A.M.I.E. 1989)**

Solution. The speed / load characteristics (assumed straight) for the alternators are shown in Fig. 37.100. Out of the combined load $AB = 1000$ kW, A's share is AM and B's share is BM. Hence, $AM + BM = 1000$ kW. PQ is the horizontal line drawn through point C, which is the point of intersection.

From simiiar Δs GDA and CDP, we have

$$\frac{CP}{GA} = \frac{PD}{AD} \text{ or } CP = GA \cdot \frac{PD}{AD}$$

Since, $PD = (4 - h)$

$\therefore \quad CP = 800(4 - h)/4 = 200(4 - h)$

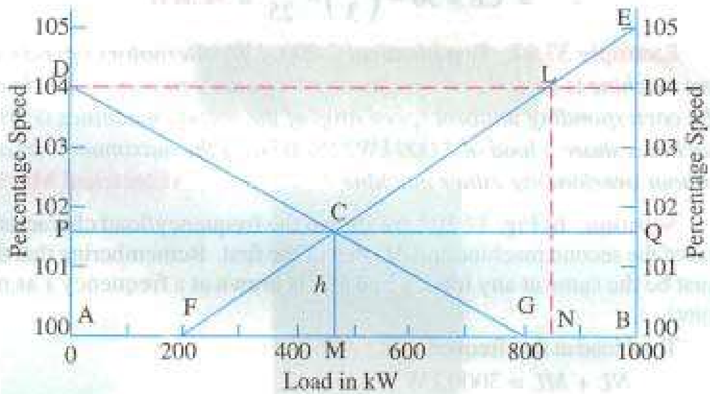


Fig. 37.100

Similarly, from similar Δs BEF and QEC, we get

$$\frac{QC}{BF} = \frac{QE}{BE} \text{ or}$$

$QC = 800(5 - h)/5 = 160(5 - h)$

$\therefore \quad CP + QC = 1000$ or $200(4 - h) + 160(5 - h) = 1000$ or $h = 5/3$

$\therefore \quad CP = 200(4 - 5/3) = 467$ kW, $QC = 160(5 - 5/3) = 533$ kW

Hence, alternator A supplies **467 kW** and B supplies **533 kW.**

Alternator A will cease supplying any load when line PQ is shifted to point D. Then, load supplied by alternator B (= BN) is such that the speed variation is from 105% to 104%.

Knowing that when its speed varies from 105% to 100%, alternator B supplies a load of 800 kW, hence load supplied for speed variation from 105% to 100% is (by proportion)

$$= 800 \times 1/5 = \mathbf{160\ kW\ (= BN)}$$

Hence, when load drops from 1000 kW to 160 kW, alternator A will cease supplying any portion of this load.

Example 37.61. *Two 50-MVA, 3-φ alternators operate in parallel. The settings of the governors are such that the rise in speed from full-load to no-load is 2 per cent in one machine and 3 per cent in the other, the characteristics being straight lines in both cases. If each machine is fully loaded when the total load is 100 MW, what would be the load on each machine when the total load is 60 MW?* **(Electrical Machines-II, Punjab Univ. 1991)**

Solution. Fig. 37.101 shows the speed/load characteristics of the two machines, NB is of the first machine and MA is that of the second. Base AB shows equal load division at full-load and speed. As the machines are running in parallel, their frequencies must be the same. Let CD be drawn through x% speed where total load is 60 MW.

$$CE = 50 - AP = 50 - \frac{50}{3}x$$

$$ED = 50 - QB = 50 - \frac{50}{2}x$$

$$\therefore \quad CD = 50 - (50/3)x + 50 - 25x$$

$$\therefore \quad 60 = 50 - (50/3)x + 50 - 25x$$

$$x = \frac{24}{25}; \therefore \; LE = 100\frac{24}{25}\,\%$$

Load supplied by 1st machine

$$= ED = 50 - 25 \times \frac{24}{25} = \textbf{26 MW}$$

Load supplied by 2nd machine.

$$= CE = 50 - \left(\frac{50}{3}\right) \times \frac{24}{25} = \textbf{34 MW}$$

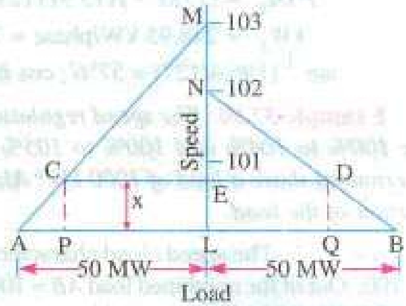


Fig. 37.101

Example 37.62. *Two identical 2,000 -kVA alternators operate in parallel. The governor of the first machine is such that the frequency drops uniformly from 50-Hz on no-load to 48-Hz on full-load. The corresponding uniform speed drop of the second machines is 50 to 47.5 Hz* **(a)** *How will the two machines share a load of 3,000 kW?* **(b)** *What is the maximum load at unity p.f. that can be delivered without overloading either machine ?* **(Electrical Machinery-II, Osmania Univ. 1989)**

Solution. In Fig. 37.102 are shown the frequency/load characteristics of the two machines, AB is that of the second machine and AD that of the first. Remembering that the frequency of the two machines must be the same at any load, a line MN is drawn at a frequency x as measured from point A (common point).

Total load at that frequency is

$NL + ML = 3000$ kW

From Δs ABC and ANL, $NL/2000 = x/2.5$

$\therefore \quad NL = 2000\,x/2.5 = 800\,x$

Similarly, $ML = 2000\,x/2 = 1000x$

$\therefore \quad 1800\,x = 3000$ or $x = 5/3$

Frequency $= 50 - 5/3 = 145/3$ Hz.

(a) $NL = 800 \times 5/3 =$ **1333 kW (assuming u.p.f.)**

$ML = 1000 \times 5/3 =$ **1667 kW (assuming u.p.f.)**

(b) For getting maximum load, DE is extended to cut AB at F. Max. load $= DF$.

Now, $EF = 2000 \times 2/3.5 = 1600$ kW

$\therefore$ Max. load $= DF = 2{,}000 + 1{,}600$

$=$ **3,600 kW.**

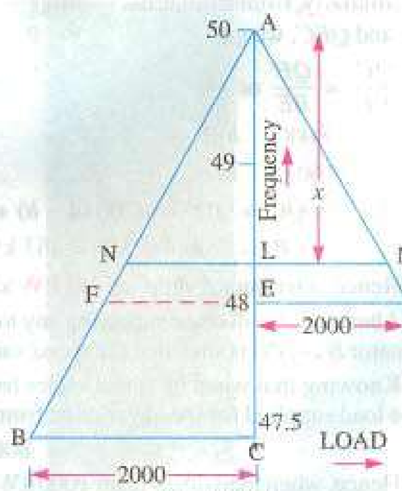


Fig. 37.102

Tutorial Problem No. 37.7

1. Two similar 6,600-V, 3-ϕ, generators are running in parallel on constant-voltage and frequency bus-bars. Each has an equivalent resistance and reactance of 0.05 Ω and 0.5 Ω respectively and supplies one half of a total load of 10,000 kW at a lagging p.f. of 0.8, the two machines being similarly excited. If the excitation of one machine be adjusted until the armature current is 438 A and the steam supply to the turbine remains unchanged, find the armature current, the e.m.f. and the p.f. of the other alternator. **[789 A, 7200 V, 0.556] (*City & Guilds, London*)**

2. A single-phase alternator connected to 6,600-V bus-bars has a synchronous impedance of 10Ω and a resistance of 1 Ω. If its excitation is such that on open circuit the p.d. would be 5000 V, calculate the maximum load the machine can supply to the external circuit before dropping out of step and the corresponding armature current and p.f. **[2864 kW, 787 A, 0.551]** *(London Univ.)*

3. A turbo-alternator having a reactance of 10 Ω has an armature current of 220 A at unity power factor when running on 11,000 V, constant-frequency bus-bars. If the steam admission is unchanged and the e.m.f. raised by 25%, determine graphically or otherwise the new value of the machine current and power factor. If this higher value of excitation were kept constant and the steam supply gradually increased, at what power output would the alternator break from synchronism? Find also the current and power factor to which this maximum load corresponds. State whether this p.f. is lagging or leading.

[360 A at 0.611 p.f. ; 15.427 kW ; 1785 A at 0.7865 leading] *(City & Guilds, London)*

4. Two single-phase alternators are connected to a 50-Hz bus-bars having a constant voltage of $10 \angle 0°$ kV. Generator *A* has an induced e.m.f. of $13 \angle 22.6°$ kV and a reactance of 2 Ω; generator *B* has an e.m.f. of $12.5 \angle 36.9°$ kV and a reactance of 3 Ω. Find the current, kW and kVAR supplied by each generator. *(Electrical Machine-II, Indore Univ. July 1977)*

5. Two 15-kVA, 400-V, 3-ph alternators in parallel supply a total load of 25 kVA at 0.8 p.f. lagging. If one alternator shares half the power at unity p.f., determine the p.f. and kVA shared by the other alternator. **[0.5548; 18.03 kVA]** *(Electrical Technology-II, Madras Univ. Apr. 1977)*

6. Two 3-φ, 6,600-V, star-connected alternators working in parallel supply the following loads :
   (*i*) Lighting load of 400 kW (*ii*) 300 kW at p.f. 0.9 lagging
   (*iii*) 400 kW at p.f. 0.8 lagging (*iv*) 1000 kW at p.f. 0.71 lagging
   Find the output, armature current and the p.f. of the other machine if the armature current of one machine is 110 A at 0.9 p.f. lagging. **[970 kW, 116 A, 0.73 lagging]**

7. A 3-φ, star-connected, 11,000-V turbo-generator has an equivalent resistance and reactance of 0.5 Ω and 8 Ω respectively. It is delivering 200 A at u.p.f. when running on a constant-voltage and constant-frequency bus-bars. Assuming constant steam supply and unchanged efficiency, find the current and p.f. if the induced e.m.f. is raised by 25%. **[296 A, 0.67 lagging]**

8. Two similar 13,000-V, 3-ph alternators are operated in parallel on infinite bus-bars. Each machine has an effective resistance and reactance of 0.05 Ω and 0.5 Ω respectively. When equally excited, they share equally a total load of 18 MW at 0.8 p.f. lagging. If the excitation of one generator is adjusted until the armature current is 400 A and the steam supply to its turbine remains unaltered, find the armature current, the e.m.f. and the p.f. of the other generator.

[774.6 A; 0.5165 ; 13,470 V] *(Electric Machinery-II, Madras Univ. Nov. 1977)*

37.41. Time-period of Oscillation

Every synchronous machine has a natural time period of free oscillation. Many causes, including the variations in load, create phase-swinging of the machine. If the time period of these oscillations coincides with natural time period of the machine, then the amplitude of the oscillations may become so greatly developed as to swing the machine out of synchronism.

The expression for the natural time period of oscillations of a synchronous machine is derived below :

Let T = torque per mechanical radian (in N-m/mech. radian)

$J = \Sigma m r^2$ —moment of inertia in kg-m².

The period of undamped free oscillations is given by $t = 2\pi \sqrt{\dfrac{J}{T}}$.

We have seen in Art. 37.32 that when an alternator swings out of phase by an angle α (electrical radian), then synchronizing power developed is

$$P_{SY} = \alpha E^2/Z \qquad \text{—}\alpha \text{ in elect. radian}$$

$$= \frac{E^2}{Z} \text{ per electrical radian per phase.}$$

Now, 1 electrical radian $= \frac{P}{2} \times$ mechanical radian—where P is the number of poles.

$\therefore$ P_{SY} per mechanical radian displacement $= \frac{E^2 P}{2 Z}$.

The synchronizing or restoring torque is given by

$$T_{SY} = \frac{P_{SY}}{2\pi N_S} = \frac{E^2 P}{4\pi Z N_S} \qquad \text{—}N_S \text{ in r.p.s.}$$

Torque for three phases is $T = 3T_{SY} = \frac{3\, E^2 P}{4\,\pi\, Z N_S}$ where E is e.m.f. per phase ...(i)

Now E/Z = short-circuit current $= I_{SC}$

$f = PN_S/2$; hence $P/N_S = 2\,f/N_S^2$

Substituting these values in (i) above, we have

$$T_{SY} = \frac{3}{4\pi} \cdot \left(\frac{E}{Z}\right) \cdot E \cdot \frac{P}{N_S} = \frac{3}{4\,\pi} \cdot I_{SC} \cdot E \cdot \frac{2f}{N_S^2} = 0.477\, \frac{E I_{SC} f}{N_S^2}$$

Now,

$$t = 2\,\pi \sqrt{\frac{J}{0.477\; E\, I_{SC}\; f / N_S^2}} = 9.1\, N_S \sqrt{\frac{J}{E \cdot I_{SC} \cdot f}} \text{ second}$$

$$= 9.1\; N_S \sqrt{\frac{J}{\frac{1}{\sqrt{3}} \cdot E_L \cdot I \cdot (I_{SC} / I) \cdot f}}$$

$$= 9.1\; N_S \sqrt{\frac{J}{\frac{1}{2} \cdot \sqrt{3}\, E_L \cdot I \cdot (I_{SC} / I) \cdot f}}$$

$$= 9.1\; N_S \sqrt{\frac{J}{\frac{1000}{3} \cdot \frac{\sqrt{3} \cdot E_L \cdot I}{1000} \left(\frac{I_{SC}}{I}\right) \cdot f}}$$

$$= \frac{9.1 \times \sqrt{3}}{\sqrt{1000}} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC} / I)\; f}}$$

$$\therefore \quad t = 0.4984\; N_S \sqrt{\frac{J}{kVA \cdot (I_{SC} / I) \cdot f}}$$

where kVA = full-load kVA of the alternator; N_S = r.p.s. of the rotating system

If N_S represents the speed in r.p.m., then

$$t = \frac{0.4984}{60} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC} / I) \cdot f}} = 0.0083\;\; N_S \sqrt{\frac{J}{kVA\, (I_{SC} / I) \cdot f}} \text{ second}$$

Note. It may be proved that I_{SC}/I = 100/percentage reactance = 100/% X_S.

Proof. Reactance drop $= I X_S = \frac{V \times \%\, X_s}{100}$ $\quad \therefore \quad X_S = \frac{\text{reactance drop}}{\text{full-load current}} = \frac{V \times \%\, X_S}{100 \times I}$

Now $$I_{SC} = \frac{V}{X_S} = \frac{V \times 100 \times I}{V \times \% X_S} = \frac{100}{\% X_S} \times I \text{ ; or } \frac{I_{SC}}{I} = \frac{100}{\% X_S}$$

For example, if synchronous reactances is 25 per cent, then

I_{SC}/I =100/25 = 4 (please see Ex. 37.64)

Example 37.63. *A 5,000-kVA, 3-phase, 10,000-V, 50-Hz alternate runs at 1500 r.p.m. connected to constant-frequency, constant-voltage bus-bars. If the moment of inertia of entire rotating system is 1.5×10^4 kg.m^2 and the steady short-circuit current is 5 times the normal full-load current, find the natural time period of oscillation.* **(Elect. Engg. Grad. I.E.T.E. 1991)**

Solution. The time of oscillation is given by

$$t = 0.0083\, NS \sqrt{\frac{J}{kVA\,.\,(I_{SC}/I) f}} \text{ . second}$$

Here, $$N_S = 1500 \text{ r.p.m. ; } I_{SC}/I = 5^* \text{ ; } J = 1.5 \times 10^4 \text{ kg-m}^2 \text{ ; } \quad f = 50 \text{ Hz}$$

$$t = 0.0083 \times 1500 \sqrt{\frac{1.5 \times 10^4}{5000 \times 5 \times 50}} = \mathbf{1.364\ s.}$$

Example 37.64. *A 10,000-kVA, 4-pole, 6,600-V, 50-Hz, 3-phase star-connected alternator has a synchronous reactance of 25% and operates on constant-voltage, constant frequency bus-bars. If the natural period of oscillation while operating at full-load and unity power factor is to be limited to 1.5 second, calculate the moment of inertia of the rotating system.*

(Electric Machinery-II, Andhra Univ. 1990)

Solution. $$t = 0.0083\, N_S \sqrt{\frac{J}{kVA\,(I_{SC}/I) f}} \text{ second.}$$

Here $$I_{SC}/I = 100/25 = 4; N_S = 120 \times 50/4 = 1500 \text{ r.p.m.}$$

$$\therefore \quad 1.5 = 0.0083 \times 1500 \sqrt{\frac{J}{10,000 \times 4 \times 50}} = 12.45 \times \frac{\sqrt{J}}{10^3 \times \sqrt{2}}$$

$$\therefore \quad J = (1.5 \times 10^3 \times \sqrt{2}/12.45)^2 = \mathbf{2.9 \times 10^4\ kg\text{-}m^2}$$

Example 37.65. *A 10-MVA, 10-kV, 3-phase, 50-Hz, 1500 r.p.m. alternator is paralleled with others of much greater capacity. The moment of inertia of the rotor is 2×10^5 kg-m2 and the synchronous reactance of the machine is 40%. Calculate the frequency of oscillation of the rotor.*

(Elect. Machinery-III, Bangalore Univ. 1992)

Solution. Here, $$I_{SC}/I = 100/40 = 2.5$$

$$t = 0.0083 \times 1500 \sqrt{\frac{2 \times 10^5}{10^4 \times 2.5 \times 50}} = 5 \text{ second}$$

$$\text{Frequency} = 1/5 = \mathbf{0.2\ Hz.}$$

* It means that synchronous reactance of the alternator is 20 %.

Tutorial Problem No. 37.8

1. Show that an alternator running in parallel on constant-voltage and frequency bus-bars has a natural time period of oscillation. Deduce a formula for the time of one complete oscillation and calculate its value for a 5000-kVA, 3-phase, 10,000 V machine running at 1500 r.p.m. on constant 50-Hz bus-bars.

   The moment of inertia of the whole moving system is 14112 kg-m^2 and the steady short-circuit current is five times the normal full-load value. **[1.33 second]**

2. A 10,000-kVA, 5-kV, 3-phase, 4-pole, 50-Hz alternator is connected to infinite bus-bars. The short-circuit current is 3.5 times the normal full-load current and the moment of inertia of the rotating system is 21,000 kg-m^2. Calculate its normal period of oscillation. **[1.365 second]**

3. Calculate for full-load and unity p.f., the natural period of oscillation of a 50-Hz, 10,000-kVA, 11-kV alternator driven at 1500 r.p.m. and connected to an infinite bus-bar. The steady short-circuit current is four times the full-load current and the moment of the inertia of the rotating masses is 17,000 kg-m^2. **[1.148 s.]** *(Electrical Machinery-II, Madras Univ. Apr. 1976)*

4. Calculate the rotational inertia in kg-m^2 units of the moving system of 10,000 kVA, 6,600-V, 4-pole, turbo-alternator driven at 1500 r.p.m. for the set to have a natural period of 1 second when running in parallel with a number of other machines. The steady short-circuit current of the alternator is five times the full-load value. **[16,828 kg-m2]** *(City & Guilds, London)*

5. A 3-ϕ, 4-pole, 6,000 kVA, 5,000-V, 50-Hz star-connected alternator is running on constant-voltage and constant-frequency bus-bars. It has a short-circuit reactance of 25% and its rotor has a moment of inertia of 16,800 kg-m^2. Calculate its natural time period of oscillation. **[1.48 second]**

37.42. Maximum Power Output

For given values of terminal voltage, excitation and frequency, there is a maximum power that the alternator is capable of delivering. Fig. 37.103 (*a*) shows full-load conditions for a cylindrical rotor where IR_a drop has been neglected*.

The power output per phase is

$$P = VI\cos\phi = \frac{VIX_S\cos\phi}{X_S}$$

Now, from $\Delta\, OBC$, we get

$$\frac{IX_S}{\sin\alpha} = \frac{E}{\sin(90+\phi)} = \frac{E}{\cos\phi}$$

$$IX_S\cos\phi = E\sin\alpha$$

$$\therefore \quad P = \frac{EV\sin\alpha}{X_S}$$

Power becomes maximum when $\alpha = 90°$, if V, E and X_S are regarded as constant (of course, E is fixed by excitation).

$$\therefore \quad P_{max} = EV/X_S$$

It will be seen from Fig. 37.103 (*b*) that under maximum power output conditions, I leads V by ϕ and since IX_S leads I by 90°, angle ϕ and hence $\cos\phi$ is fixed $= E/IX_S$

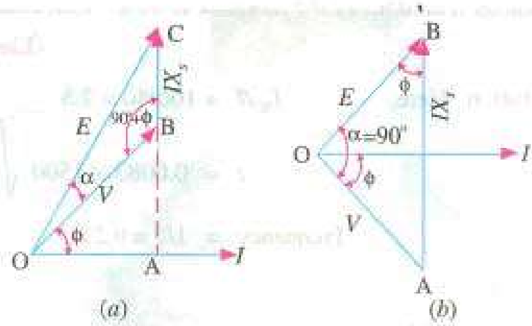


Fig. 37.103

* In fact, this drop can generally be neglected without sacrificing much accuracy of results.

Now, from right-angled ΔAOB, we have that $IX_S = \sqrt{E^2 + V^2}$. Hence, p.f. corresponding to maximum power output is

$$\cos\phi = \frac{E}{\sqrt{E^2 + V^2}}$$

The maximum power output per phase may also be written as

$$P_{max} = VI_{max}\cos\phi = VI_{max}\frac{E}{\sqrt{E^2 + V^2}}$$

where I_{max} represents the current/phase for maximum power output.

If I_f is the full-load current and $\% X_S$ is the percentage synchronous reactance, then

$$\% X_S = \frac{I_f X_S}{V}\times 100 \quad \therefore \quad \frac{V}{X_S} = \frac{I_f \times 100}{\% X_S}$$

Now, $$P_{max} = VI_{max}\frac{E}{\sqrt{E^2 + V^2}} = \frac{EV}{X_S} = \frac{EI_f \times 100}{\% X_S}$$

Two things are obvious from the above equations.

(i) $$I_{max} = \frac{100\, I_f}{\% X_S}\times\frac{\sqrt{E^2 + V^2}}{V}$$

Substituting the value of $\%X_S$ from above,

$$I_{max} = \frac{100\, I_f}{100\, I_f X_S}\times V\times\frac{\sqrt{E^2 + V^2}}{V} = \frac{\sqrt{E^2 + V^2}}{X_S}$$

(ii) $$P_{max} = \frac{100\, EI_f}{\% X_S} = \frac{E}{V}\cdot\frac{100}{\% X_S}\times VI_f \text{ per phase}$$

$$= \frac{E}{V}\cdot\frac{100}{\% X_S}\times \text{ F.L. power output at u.p.f.}$$

Total maximum power output of the alternator is

$$= \frac{E}{V}\cdot\frac{100}{\% X_S}\times \text{ F.L. power output at u.p.f.}$$

Example 37.66. *Derive the condition for the maximum output of a synchronous generator connected to infinite bus-bars and working at constant excitation.*

A 3-φ, 11-kV, 5-MVA, Y-connected alternator has a synchronous impedance of (1 + j 10) ohm per phase. Its excitation is such that the generated line e.m.f. is 14 kV. If the alternator is connected to infinite bus-bars, determine the maximum output at the given excitation

(Electrical Machines-III, Gujarat Univ. 1984)

Solution. For the first part, please refer to Art. 37.41

P_{max} per phase $= \dfrac{EV}{X_S}$ —if R_a is neglected $= \dfrac{V}{Z_s}(E - V\cos\theta)$ —if R_a is considered

Now, $$E = 14{,}000/\sqrt{3} = 8{,}083 \text{ V}\ ; \ V = 11{,}000/\sqrt{3} = 6352 \text{ V}$$

$$\cos\theta = R_a/Z_S = 1\sqrt{1^2 + 10^2} = 1/10.05$$

$\therefore$ $$P_{max} \text{ per phase} = \frac{8083\times 6352}{10\times 1000} = 5{,}135 \text{ kW}$$

Total $$P_{max} = 3\times 5{,}135 = \mathbf{15{,}405\ kW}$$

More accurately, $$P_{max}/\text{phase} = \frac{6352}{10.05}\left(8083 - \frac{6352}{10.05}\right) = \frac{6352}{10.05}\times\frac{7451}{1000} = 4{,}711 \text{ kW}$$

Total $$P_{max} = 4{,}711\times 3 = \mathbf{14{,}133\ kW.}$$

Example 37.67. *A 3-phase, 11-kVA, 10-MW, Y-connected synchronous generator has synchronous impedance of (0.8 + j 8.0) ohm per phase. If the excitation is such that the open circuit voltage is 14 kV, determine (i) the maximum output of the generator (ii) the current and p.f. at the maximum output.*

(Electrical Machines-III, Gujarat Univ. 1987)

Solution. (*i*) If we neglect R_a*, the P_{max} per phase = EV/X_S

where V is the terminal voltage (or bus-bar voltage in general) and E the e.m.f. of the machine.

$$\therefore \quad P_{max} = \frac{(11,000/\sqrt{3}) \times (14,000/\sqrt{3})}{8} = \frac{154,000}{24} \text{ kW/phase}$$

$$\text{Total} \quad P_{max} = 3 \times 154,000/24 = 19,250 \text{ kW} = \mathbf{19.25\ MW}$$

Incidentally, this output is nearly twice the normal output.

$$(ii) \quad I_{max} = \frac{\sqrt{E^2 + V^2}}{X_S} = \frac{\sqrt{[(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2]}}{8} = \mathbf{1287\ A}$$

$$(iii) \quad \text{p.f.} = \frac{E}{\sqrt{E^2 + V^2}} = \frac{14000/\sqrt{3}}{\sqrt{(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2}} = \mathbf{0.786\ (lead).}$$

QUESTIONS AND ANSWERS ON ALTERNATORS

Q. 1. What are the two types of turbo-alternators ?

Ans. Vertical and horizontal.

Q. 2. How do you compare the two ?

Ans. Vertical type requires less floor space and while step bearing is necessary to carry the weight of the moving element, there is very little friction in the main bearings. The horizontal type requires no step bearing, but occupies more space.

Q. 3. What is step bearing ?

Ans. It consists of two cylindrical cast iron plates which bear upon each other and have a central recess between them. Suitable oil is pumped into this recess under considerable pressure.

Q. 4. What is direct-connected alternator ?

Ans. One in which the alternator and engine are directly connected. In other words, there is no intermediate gearing such as belt, chain etc. between the driving engine and alternator.

Q. 5. What is the difference between direct-connected and direct-coupled units ?

Ans. In the former, alternator and driving engine are directly and permanently connected. In the latter case, engine and alternator are each complete in itself and are connected by some device such as friction clutch, jaw clutch or shaft coupling.

Q. 6. Can a d.c. generator be converted into an alternator ?

Ans. Yes.

Q. 7. How ?

Ans. By providing two collector rings on one end of the armature and connecting these two rings to two points in the armature winding 180° apart.

Q. 8. Would this arrangement result in a desirable alternator ?

Ans. No.

---

* If R_a is not neglected, then $P_{max} = \frac{V}{Z_S}(E - V\cos\theta)$ where $\cos\theta = R_a/Z_s$ (Ex. 37.66)

Q. 9. How is a direct-connected exciter arranged in an alternator ?

Ans. The armature of the exciter is mounted on the shaft of the alternator close to the spider hub. In some cases, it is mounted at a distance sufficient to permit a pedestal and bearing to be placed between the exciter and the hub.

Q. 10. Any advantage of a direct-connected exciter ?

Ans. Yes, economy of space.

Q. 11. Any disadvantage ?

Ans. The exciter has to run at the same speed as the alternator which is slower than desirable. Hence, it must be larger for a given output than the gear-driven type, because it can be run at high speed and so made proportionately smaller.

OBJECTIVE TESTS – 37

1. The frequency of voltage generated by an alternator having 4-poles and rotating at 1800 r.p.m. is ....hertz.
   (*a*) 60 (*b*) 7200
   (*c*) 120 (*d*) 450.
2. A 50-Hz alternator will run at the greatest possible speed if it is wound for .... poles.
   (*a*) 8 (*b*) 6
   (*c*) 4 (*d*) 2.
3. The main disadvantage of using short-pitch winding in alterators is that it
   (*a*) reduces harmonics in the generated voltage
   (*b*) reduces the total voltage around the armature coils
   (*c*) produces asymmetry in the three phase windings
   (*d*) increases Cu of end connections.
4. Three-phase alternators are invariably Y-connected because
   (*a*) magnetic losses are minimised
   (*b*) less turns of wire are required
   (*c*) smaller conductors can be used
   (*d*) higher terminal voltage is obtained.
5. The winding of a 4-pole alternator having 36 slots and a coil span of 1 to 8 is short-pitched by .... degrees.
   (*a*) 140 (*b*) 80
   (*c*) 20 (*d*) 40.
6. If an alternator winding has a fractional pitch of 5/6, the coil span is .... degrees.
   (*a*) 300 (*b*) 150
   (*c*) 30 (*d*) 60.
7. The harmonic which would be totally eliminated from the alternator e.m.f. using a fractional pitch of 4/5 is
   (*a*) 3rd (*b*) 7th
   (*c*) 5th (*d*) 9th.
8. For eliminating 7th harmonic from the e.m.f. wave of an alternator, the fractional-pitch must be
   (*a*) 2/3 (*b*) 5/6
   (*c*) 7/8 (*d*) 6/7.
9. If, in an alternator, chording angle for fundamental flux wave is α, its value for 5th harmonic is
   (*a*) 5α (*b*) $\alpha/5$
   (*c*) 25α (*d*) $\alpha/25$.
10. Regarding distribution factor of an armature winding of an alternator which statement is false?
   (*a*) it decreases as the distribution of coils (slots/pole) increases
   (*b*) higher its value, higher the induced e.m.f. per phase
   (*c*) it is not affected by the type of winding either lap, or wave
   (*d*) it is not affected by the number of turns per coil.
11. When speed of an alternator is changed from 3600 r.p.m. to 1800 r.p.m., the generated e.m.f./phases will become
   (*a*) one-half (*b*) twice
   (*c*) four times (*d*) one-fourth.
12. The magnitude of the three voltage drops in an alternator due to armature resistance, leakage reactance and armature reaction is solely determined by
   (*a*) load current, I_a

(b) p.f. of the load
(c) whether it is a lagging or leading p.f. load
(d) field construction of the alternator.

13. Armature reaction in an alternator primarily affects
(a) rotor speed
(b) terminal voltage per phase
(c) frequency of armature current
(d) generated voltage per phase.

14. Under no-load condition, power drawn by the prime mover of an alternator goes to
(a) produce induced e.m.f. in armature winding
(b) meet no-load losses
(c) produce power in the armature
(d) meet Cu losses both in armature and rotor windings.

15. As load p.f. of an alternator becomes more leading, the value of generated voltage required to give rated terminal voltage
(a) increases
(b) remains unchanged
(c) decreases
(d) varies with rotor speed.

16. With a load p.f. of unity, the effect of armature reaction on the main-field flux of an alternator is
(a) distortional (b) magnetising
(c) demagnetising (d) nominal.

17. At lagging loads, armature reaction in an alternator is
(a) cross-magnetising (b) demagnetising
(c) non-effective (d) magnetising.

18. At leading p.f., the armature flux in an alternator ...... the rotor flux.
(a) opposes (b) aids
(c) distorts (d) does not affect.

19. The voltage regulation of an alternator having 0.75 leading p.f. load, no-load induced e.m.f. of 2400V and rated terminal voltage of 3000V is .... percent.
(a) 20 (b) – 20
(c) 150 (d) – 26.7

20. If, in a 3-ϕ alternator, a field current of 50A produces a full-load armature current of 200 A on short-circuit and 1730 V on open circuit, then its synchronous impedance is .... ohm.
(a) 8.66 (b) 4
(c) 5 (d) 34.6

21. The power factor of an alternator is determined by its
(a) speed
(b) load
(c) excitation
(d) prime mover.

22. For proper parallel operation, a.c. polyphase alternators must have the same
(a) speed (b) voltage rating
(c) kVA rating (d) excitation.

23. Of the following conditions, the one which does not have to be met by alternators working in parallel is
(a) terminal voltage of each machine must be the same
(b) the machines must have the same phase rotation
(c) the machines must operate at the same frequency
(d) the machines must have equal ratings.

24. After wiring up two 3-ϕ alternators, you checked their frequency and voltage and found them to be equal. Before connecting them in parallel, you would
(a) check turbine speed
(b) check phase rotation
(c) lubricate everything
(d) check steam pressure.

25. Zero power factor method of an alternator is used to find its
(a) efficiency
(b) voltage regulation
(c) armature resistance
(d) synchronous impedance.

26. Some engineers prefer 'lamps bright' synchronization to 'lamps dark' synchronization because
(a) brightness of lamps can be judged easily
(b) it gives sharper and more accurate synchronization
(c) flicker is more pronounced
(d) it can be performed quickly.

27. It is never advisable to connect a stationary alternator to live bus-bars because it
(a) is likely to run as synchronous motor
(b) will get short-circuited
(c) will decrease bus-bar voltage though momentarily
(d) will disturb generated e.m.fs. of other alternators connected in parallel.

28. Two identical alternators are running in parallel and carry equal loads. If excitation of one alternator is increased without changing its steam supply, then
(*a*) it will keep supplying almost the same load
(*b*) kVAR supplied by it would decrease
(*c*) its p.f. will increase
(*d*) kVA supplied by it would decrease.

29. Keeping its excitation constant, if steam supply of an alternator running in parallel with another identical alternator is increased, then
(*a*) it would over-run the other alternator
(*b*) its rotor will fall back in phase with respect to the other machine
(*c*) it will supply greater portion of the load
(*d*) its power factor would be decreased.

30. The load sharing between two steam-driven alternators operating in parallel may be adjusted by varying the
(*a*) field strengths of the alternators
(*b*) power factors of the alternators
(*c*) steam supply to their prime movers
(*d*) speed of the alternators.

31. Squirrel-cage bars placed in the rotor pole faces of an alternator help reduce hunting
(*a*) above synchronous speed only
(*b*) below synchronous speed only
(*c*) above and below synchronous speeds both
(*d*) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

32. For a machine on infinite bus active power can be varied by
(*a*) changing field excitation
(*b*) changing of prime cover speed
(*c*) both (*a*) and (*b*) above
(*d*) none of the above .
(Elect. Machines, A.M.I.E. Sec. B, 1993)

ANSWERS

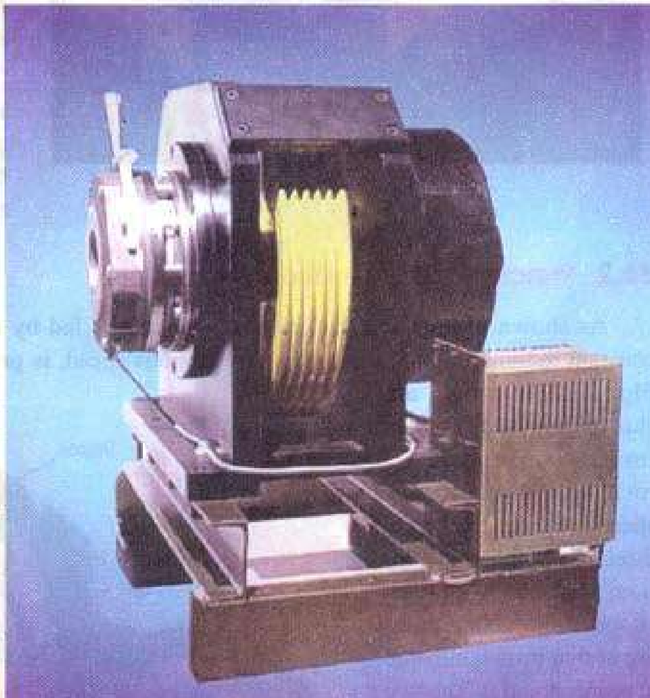
1. *a*	**2.** *d*	**3.** *b*	**4.** *d*	**5.** *d*	**6.** *b*	**7.** *c*	**8.** *d*	**9.** *a*	**10.** *b*	**11.** *a*
12. *a*	**13.** *d*	**14.** *b*	**15.** *c*	**16.** *a*	**17.** *d*	**18.** *b*	**19.** *b*	**20.** *c*	**21.** *b*	**22.** *b*
23. *d*	**24.** *b*	**25.** *b*	**26.** *b*	**27.** *b*	**28.** *a*	**29.** *c*	**30.** *c.*	**31.** *c*	**32.** *b*	

C H A P T E R 38

SYNCHRONOUS MOTOR

Learning Objectives

- Synchronous Motor-General
- Principle of Operation
- Method of Starting
- Motor on Load with Constant Excitation
- Power Flow within a Synchronous Motor
- Equivalent Circuit of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Synchronous Motor with Different Excitations
- Effect of Increased Load with Constant Excitation
- Effect of Changing Excitation of Constant Load
- Different Torques of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Alternative Expression for Power Developed
- Various Conditions of Maxima
- Salient Pole Synchronous Motor
- Power Developed by a Salient Pole Synchronous Motor
- Effects of Excitation on Armature Current and Power Factor
- Constant-Power Lines
- Construction of V-curves
- Hunting or Surging or Phase Swinging
- Methods of Starting
- Procedure for Starting a Synchronous Motor
- Comparison between Synchronous and Induction Motors
- Synchronous Motor Applications



Rotary synchronous motor for lift applications

38.1. Synchronous Motor—General

A synchronous motor (Fig. 38.1) is electrically identical with an alternator or a.c. generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of d.c. machines. Most synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m.

Synchronous motor

Some characteristic features of a synchronous motor are worth noting :

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency (because $Ns = 120 f / P$).
2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.
3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

38.2. Principle of Operation

As shown in Art. 34.7, when a 3-φ winding is fed by a 3-φ supply, then a magnetic flux of constant magnitude but ***rotating*** at synchronous speed, is produced. Consider a two-pole stator of Fig. 38.2, in which are shown two stator poles (marked N_S and S_S) rotating at synchronous speed, say, in clockwise direction. With the rotor position as shown, suppose the stator poles are at that instant situated at points *A* and *B*. The two similar poles, *N* (of rotor) and N_S (of stator) as well as *S* and S_S will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

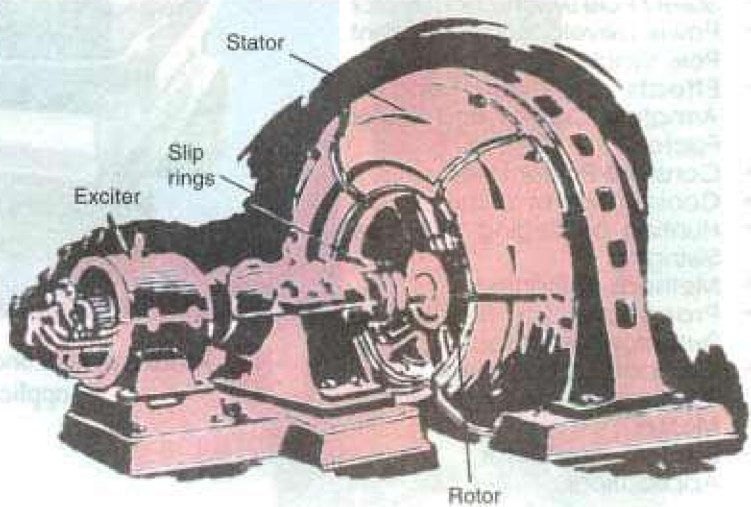


Fig. 38.1

But half a period later, stator poles, having rotated around, interchange their positions *i.e.* N_S is at point B and S_S at point A. Under these conditions, N_S attracts S and S_S attracts N. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.

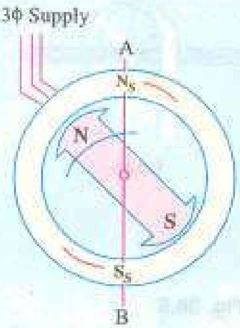


Fig. 38.2

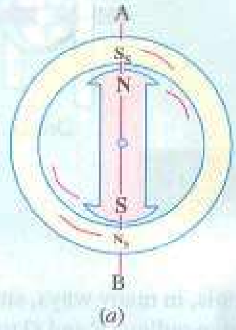


Fig. 38.3

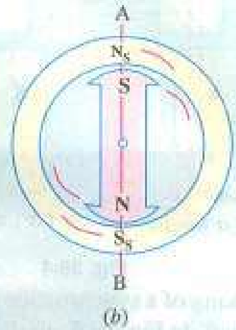


Fig. 38.3

Now, consider the condition shown in Fig. 38.3 (*a*). The stator and rotor poles are attracting each other. Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig. 38.3 (*b*). Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e.*, clockwise torque, as shown in Fig. 38.3.

The rotor and the stator parts of motor.

38.3. Method of Starting

The rotor (which is as yet unexcited) is speeded up to synchronous / near synchronous speed by some arrangement and then excited by the d.c. source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. The synchronous speed is given by the usual relation $N_S = 120 f / P$.

However, it is important to understand that the arrangement between the stator and rotor poles is ***not an absolutely rigid one***. As the load on the motor is increased, the rotor progressively tends to fall back ***in phase*** (but *not* in speed as in d.c. motors) by some angle (Fig. 38.4) ***but it still continues to run synchronously***. The value of this load angle or coupling angle (as it is called) depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, say, α.

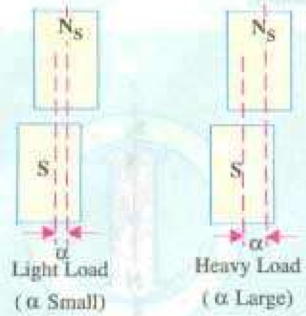


Fig. 38.4

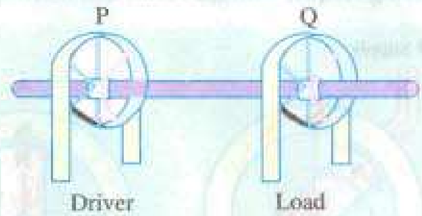


Fig. 38.5

The working of a synchronous motor is, in many ways, similar to the transmission of mechanical power by a shaft. In Fig. 38.5 are shown two pulleys P and Q transmitting power from the driver to the load. The two pulleys are assumed to be keyed together (just as stator and rotor poles are interlocked) hence they run at exactly the same (average) speed. When Q is loaded, it slightly falls behind owing to the twist in the shaft (twist angle corresponds to α in motor), the angle of twist, in fact, being a measure of the torque transmitted. It is clear that unless Q is so heavily loaded as to break the coupling, both ***pulleys must run at exactly the same (average) speed***.

38.4. Motor on Load with Constant Excitation

Before considering as to what goes on inside a synchronous motor, it is worthwhile to refer briefly to the d.c. motors. We have seen (Art. 29.3) that when a d.c. motor is running on a supply of, say, V volts then, on rotating, a back e.m.f. E_b is set up in its armature conductors. The resultant voltage across the armature is $(V - E_b)$ and it causes an armature current $I_a = (V - E_b)/ R_a$ to flow where R_a is armature circuit resistance. The value of E_b depends, among other factors, on the speed of the rotating armature. The mechanical power developed in armature depends on $E_b I_a$ (E_b and I_a being in opposition to each other).

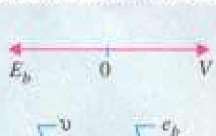


Fig. 38.6

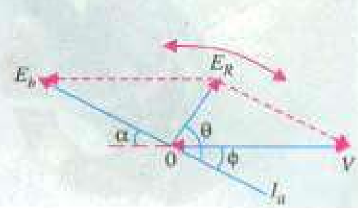


Fig. 38.7

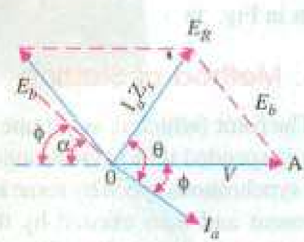
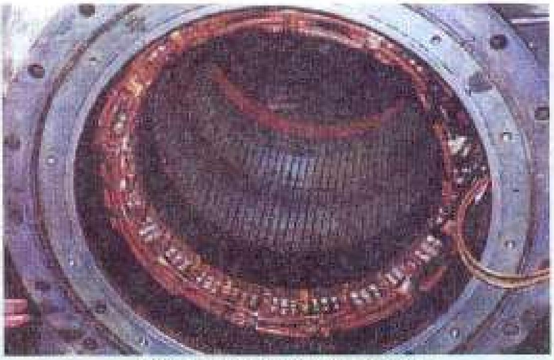


Fig. 38.8

Similarly, in a synchronous machine, a back e.m.f. E_b is set up in the armature (stator) by the rotor flux which opposes the applied voltage V. This back e.m.f. depends on rotor excitation only (and not on speed, as in d.c. motors). The net voltage in armature (stator) is the ***vector difference*** (not arithmetical, as in d.c. motors) of V and E_b. Armature current is obtained by dividing this *vector* difference of voltages by armature impedance (not resistance as in d.c. machines).

Fig. 38.6 shows the condition when the motor (properly synchronized to the supply) is running on ***no-load*** and has ***no losses***.* and is having field excitation which makes $E_b = V$. It is seen that vector difference of E_b and V is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be met by it. In other words, the motor just floats.


Stator of synchronous motor

If motor is on no-load, but it has losses, then the vector for E_b falls back (vectors are rotating anti-clockwise) by a certain small angle α (Fig. 38.7), so that a resultant voltage E_R and hence current I_a is brought into existence, which supplies losses.**

If, now, the motor is loaded, then its rotor will further fall back ***in phase*** by a greater value of angle α – called the load angle or coupling angle (corresponding to the twist in the shaft of the pulleys). The resultant voltage E_R is increased and motor draws an increased armature current (Fig. 38.8), though at a slightly decreased power factor.

38.5. Power Flow within a Synchronous Motor

Let R_a = armature resistance / phase ; X_S = synchronous reactance / phase

then $\mathbf{Z_S} = \mathbf{R}_a + j X_S$; $\mathbf{I}_a = \dfrac{\mathbf{E_R}}{\mathbf{Z_S}} = \dfrac{\mathbf{V} - \mathbf{E_b}}{\mathbf{Z_S}}$; Obviously, $\mathbf{V} = \mathbf{E}_b + \mathbf{I}_a \mathbf{Z_S}$

The angle θ (known as internal angle) by which I_a lags behind E_R is given by $\tan \theta = X_S / R_a$.

If R_a is negligible, then $\theta = 90°$. Motor input = $V I_a \cos \phi$ —per phase

Here, V is applied voltage / phase.

Total input for a star-connected, 3-phase machine is, $P = \sqrt{3}\, V_L \cdot I_L \cos \phi$.

The mechanical power developed in the rotor is

P_m = back e.m.f. × armature current × cosine of the angle between the two *i.e.*, angle between I_a and E_b reversed.

$= E_b I_a \cos(\alpha - \phi)$ per phase ...Fig. 38.8

Out of this power developed, some would go to meet iron and friction and excitation losses. Hence, the power available at the shaft would be less than the developed power by this amount.

Out of the input power / phase $V I_a \cos \phi$, and amount $I_a^2 R_a$ is wasted in,armature***, the rest ($V. I_a \cos \phi - I_a^2 R_a$) appears as mechanical power in rotor; out of it, iron, friction and excitation losses are met and the rest is available at the shaft. If power input / phase of the motor is P, then

$$P = P_m + I_a^2 R_a$$

or mechanical power in rotor $P_m = P - I_a^2 R_a$ —per phase

For three phases $P_m = \sqrt{3}\, V_L I_L \cos \phi - 3 I_a^2 R_a$

The per phase power development in a synchronous machine is as under :

* This figure is exactly like Fig. 37.74 for alternator except that it has been shown horizontally rather than vertically.

** It is worth noting that magnitude of E_b does not change, only its phase changes. Its magnitude will change only when rotor dc excitation is changed *i.e.*, when magnetic strength of rotor poles is changed.

*** The Cu loss in rotor is not met by motor ac input, but by the dc source used for rotor excitation.

Power input/phase in stator

$P = VI_a \cos\phi$

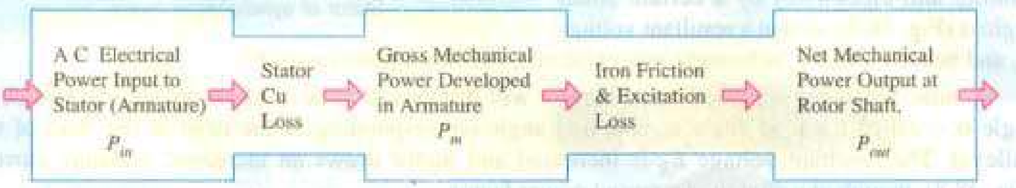
Armature (*i.e.*, stator) Cu loss $= I_a^2 R_a$

Mechanical power in armature $P_m = E_b I_a \cos(\alpha - \phi)$

Iron, excitation & friction losses

Output power P_{out}

Different power stages in a synchronous motor are as under :



38.6. Equivalent Circuit of a Synchronous Motor

Fig. 38.9 (*a*) shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. 38.9 (*b*) that the phase applied voltage V is the vector sum of reversed back e.m.f. *i.e.*, $-E_b$ and the impedance drop $I_a Z_S$. In other words, $V = (-E_b + I_a Z_S)$. The angle α* between the phasor for V and E_b is called the load angle or power angle of the synchronous motor.

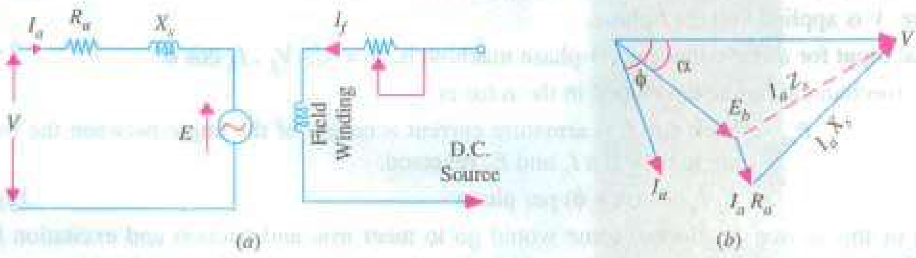


Fig. 38.9

38.7. Power Developed by a Synchronous Motor

Except for very small machines, the armature resistance of a synchronous motor is negligible as compared to its synchronous reactance. Hence, the equivalent circuit for the motor becomes as shown in Fig. 38.10 (*a*). From the phasor diagram of Fig. 38.10 (*b*), it is seen that

$$AB = E_b \sin\alpha = I_a X_S \cos\phi$$

$$\text{or} \quad VI_a \cos\phi = \frac{E_b V}{X_S} \sin\alpha$$

Now, $VI_a \cos\phi$ = motor power input/phase

* This angle was designated as δ when discussing synchronous generators.

$\therefore \quad P_{in} = \dfrac{E_b V}{X_S} \sin \alpha$ ...per phase*

$= 3\dfrac{E_b V}{X_S} \sin \alpha$ ... for three phases

Since stator Cu losses have been neglected, P_{in} also represents the gross mechanical power $\{P_m\}$ developed by the motor.

$$\therefore \quad P_m = \frac{3E_b V}{X_S} \sin \alpha$$

The gross torque developed by the motor is $T_g = 9.55\ P_m / N_s$ N–m ...Ns in rpm.

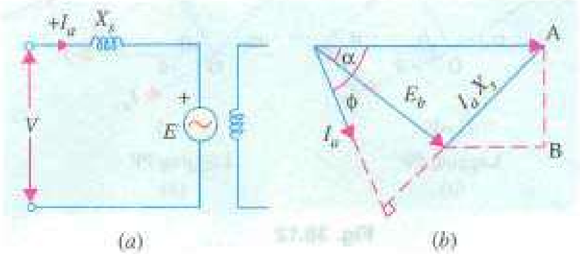


Fig. 38.10

Example 38.1. *A 75-kW, 3-φ, Y-connected, 50-Hz, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency excluding field and stator losses, is 95% and X_S = 2.5 Ω. Calculate (i) mechanical power developed (ii) armature current (iii) back e.m.f. (iv) power angle and (v) maximum or pull-out torque of the motor.*

Solution. $N_S = 120 \times 50/4 = 1500$ rpm = 25 rps

(i) $P_m = P_{in} = P_{out} / \eta = 75 \times 10^3/0.95 =$ **78,950 W**

(ii) Since power input is known

$\therefore \quad \sqrt{3} \times 440 \times I_a \times 0.8 = 78{,}950; \quad I_a =$ **129 A**

(iii) Applied voltage/phase = $440/\sqrt{3}$ = 254 V. Let V = 254 $\angle 0°$ as shown in Fig. 38.11.

Now, $V = E_b + j\, IX_S$ or $E_b = V - j\, I_a X_S = 254 \angle 0° - 129 \angle 36.9° \times 2.5 \angle 90° = 250 \angle 0° - 322 \angle 126.9° = 254 - 322$ (cos $126.9° + j$ sin $126.9°$) $= 254 - 322\,(-0.6 + j\,0.8) =$ **516 ∠–30°**

(iv) $\therefore \quad \alpha =$ **–30°**

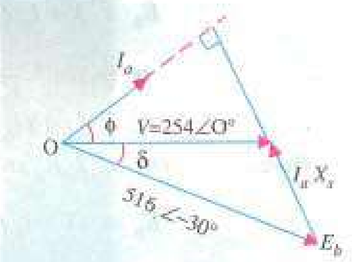


Fig. 38.11

(v) pull-out torque occurs when α = 90°

$$\text{maximum } P_m = 3\frac{E_b V}{X_S} \sin \delta = 3\ \frac{256 \times 516}{2.5} = \sin 90° = 157{,}275 \text{ W}$$

$\therefore$ pull-out torque = 9.55 × 157, 275/1500 = **1,000 N-m**

38.8. Synchronous Motor with Different Excitations

A synchronous motor is said to have normal excitation when its $E_b = V$. If field excitation is such that $E_b < V$, the motor is said to be ***under-excited***. In both these conditions, it has a lagging power factor as shown in Fig. 38.12.

On the other hand, if d.c. field excitation is such that $E_b > V$, then motor is said to be ***over-excited*** and draws a leading current, as shown in Fig. 38.13 (*a*). There will be some value of excitation for which armature current will be in phase with *V*, so that power factor will become unity, as shown in Fig. 38.13 (*b*).

* Strictly speaking, it should be $P_{in} = \dfrac{-E_b V}{X_S} \sin \alpha$

The value of α and back e.m.f. E_b can be found with the help of vector diagrams for various power factors, shown in Fig. 38.14.

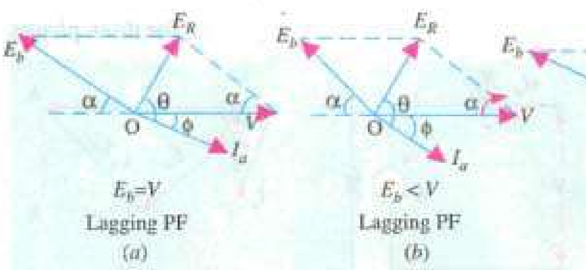


Fig. 38.12

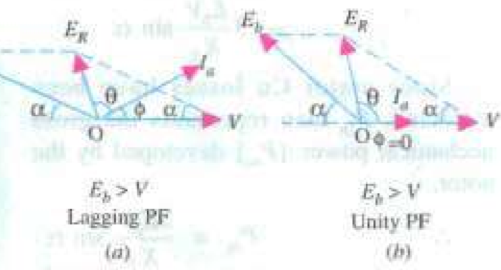


Fig. 38.13

(i) Lagging p.f. As seen from Fig. 38.14 (a)

$$AC^2 = AB^2 + BC^2 = [V - E_R \cos(\theta - \phi)]^2 + [E_R \sin(\theta - \phi)]^2$$

$$\therefore \quad E_b = \sqrt{[V - I_a Z_S \cos(\theta - \phi)]^2 + [I_a Z_S \sin(\theta - \phi)]^2}$$

Load angle $\quad \alpha = \tan^{-1}\left(\frac{BC}{AB}\right) = \tan^{-1}\left[\frac{I_a Z_S \sin(\theta - \phi)}{V - I_a Z_S \cos(\theta - \phi)}\right]$

(ii) Leading p.f. [38.14 (b)]

$$E_b = V + I_a Z_S \cos[180° - (\theta + \phi)] + j\, I_a Z_S \sin[180° - (\theta + \phi)]$$

$$\alpha = \tan^{-1} \frac{I_a Z_S \sin[180° - (\theta + \phi)]}{V + I_a Z_S \cos[180° - (\theta + \phi)]}$$

(iii) Unity p.f. [Fig. 38.14 (c)]

Here, $\quad OB = I_a R_a$ and $BC = I_a X_S$

$$\therefore \quad E_b = (V - I_a R_a) + j\, I_a X_S; \; \alpha = \tan^{-1}\left(\frac{I_a X_S}{V - I_a R_a}\right)$$

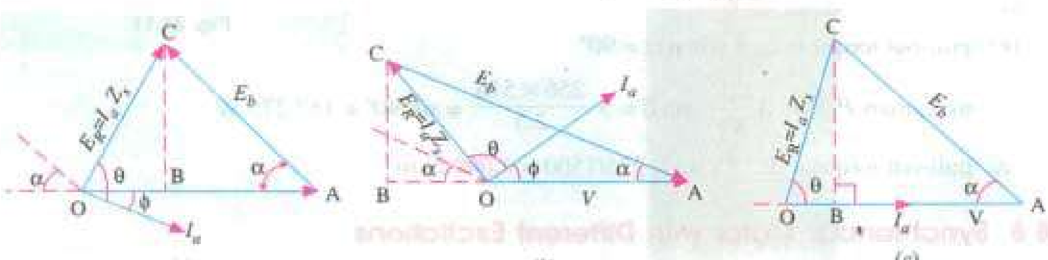


Fig. 38.14

38.9. Effect of Increased Load with Constant Excitation

We will study the effect of increased load on a synchronous motor under conditions of normal, under and over-excitation (ignoring the effects of armature reaction). With normal excitation, $E_b = V$, with under excitation, $E_b < V$ and with over-excitation, $E_b > V$. Whatever the value of excitation, it would be kept *constant* during our discussion. It would also be assumed that R_a is negligible as compared to X_S so that phase angle between E_R and I_a *i.e.*, $\theta = 90°$.

(i) Normal Excitation

Fig. 38.15. (a) shows the condition when motor is running with light load so that (i) torque angle

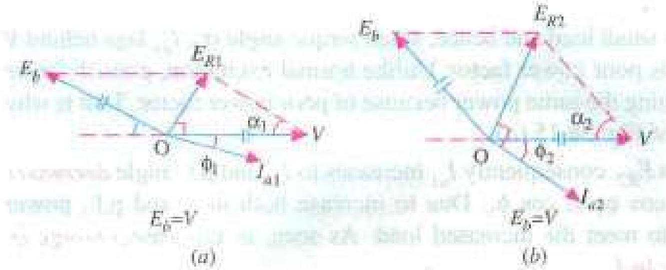


Fig. 38.15

α_1 is small ***(ii)*** so E_{R1} is small ***(iii)*** hence I_{a1} is small and *(iv)* ϕ_1 is small so that $\cos \phi_1$ is large.

Now, suppose that load on the motor is ***increased*** as shown in Fig. 38.15 (*b*). For meeting this extra load, motor must develop more torque by drawing more armature current. Unlike a d.c. motor, a synchronous motor cannot increase its I_a by decreasing its speed and hence E_b because both are constant in its case. What actually happens is as under :

1. rotor falls back ***in phase i.e.***, load angle increases to α_2 as shown in Fig. 38.15 (*b*),

2. the resultant voltage in armature is increased ***considerably*** to new value E_{R2},

3. as a result, I_{a1} increases to I_{a2}, thereby increasing the torque developed by the motor,

4. ϕ_1 increases to ϕ_2, so that power factor ***decreases*** from $\cos \phi_1$ to the new value $\cos \phi_2$.

Geared motor added to synchronous servo motor line offers a wide range of transmission ratios, and drive torques.

Since increase in I_a is much greater than the ***slight*** decrease in power factor, the torque developed by the motor is ***increased*** (on the whole) to a new value sufficient to meet the extra load put on the motor. It will be seen that essentially it is by increasing its I_a that the motor is able to carry the extra load put on it.

Fig. 38.16

A phase summary of the effect of increased load on a synchronous motor at ***normal excitation*** is shown in Fig. 38.16 (a) It is seen that there is a comparatively much greater ***increase*** in I_a than in ϕ.

(ii) Under-excitation

As shown in Fig. 38.16 (*b*), with a small load and hence, small torque angle α_1, I_{a1} lags behind V by a ***large*** phase angle ϕ_1 which means poor power factor. Unlike normal excitation, a much larger armature current must flow for developing the same power because of poor power factor. That is why I_{a1} of Fig. 38.16 (*b*) is larger than I_{a1} of Fig. 38.15 (*a*).

As load increases, E_{R1} increases to E_{R2}, consequently I_{a1} increases to I_{a2} and p.f. angle ***decreases*** from ϕ_1 to ϕ_2 or p.f. ***increases*** from cos ϕ_1 to cos ϕ_2. Due to increase both in I_a and p.f., power generated by the armature increases to meet the increased load. As seen, in this case, ***change in power factor is more than the change in I_a***.

(iii) Over-excitation

When running on light load, α_1 is small but I_{a1} is comparatively larger and ***leads V*** by a larger angle ϕ_1. Like the under-excited motor, as more load is applied, the power factor improves and ***approaches*** unity. The armature current also increases thereby producing the necessary increased armature power to meet the increased applied load (Fig. 38.17). However, it should be noted that in this case, power factor angle ϕ decreases (or p.f. increases) at a faster rate than the armature current thereby producing the necessary increased power to meet the increased load applied to the motor.

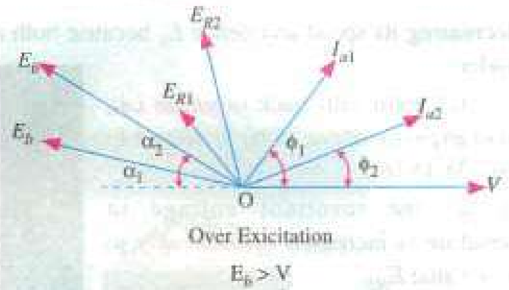


Fig. 38.17

Summary

The main points regarding the above three cases can be summarized as under :

1. As load on the motor increases, I_a ***increases regardless of excitation.***
2. For under-and over-excited motors, p.f. tends to approach unity with increase in load.
3. Both with under-and over-excitation, change in p.f. is greater than in I_a with increase in load.
4. With normal excitation, when load is increased change in I_a is greater than in p.f. which tends to become increasingly lagging.

Example 38.2. *A 20-pole, 693-V, 50-Hz, 3-ϕ, Δ-connected synchronous motor is operating at no-load with normal excitation. It has armature ressistance per phase of zero and synchronous reactance of 10 Ω. If rotor is retarded by 0.5° (mechanical) from its synchronous position, compute.*

(i) rotor displacement in electrical degrees
(ii) armature emf / phase *(iii) armature current / phase*
(iv) power drawn by the motor *(v) power developed by armature*

How will these quantities change when motor is loaded and the rotor displacement increases to 5° (mechanical) ?

(Elect. Machines, AMIE Sec. B, 1993)

Solution. (a) 0.5° (mech) Displacement [Fig 38.18 (*a*)]

(i) α (elect.) $= \dfrac{P}{2} \times \alpha$ (mech)

$\therefore$ α (elect)

$= \dfrac{20}{2} \times 0.5 =$ **5° (elect)**

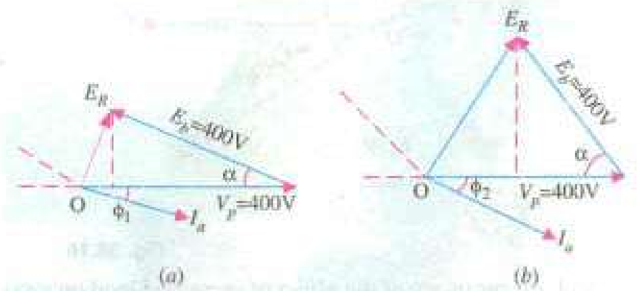


Fig. 38.18

(ii) $V_p = V_L / \sqrt{3} = 693 / \sqrt{3}$
$= 400$ V,

$E_b = V_p = 400$ V

$\therefore$ $E_R = (V_p - E_b \cos \alpha) + j E_b \sin \alpha = (400 - 400 \cos 5° + j\, 400 \sin 5°)$
$= 1.5 + j\, 35 =$ **$35 \angle 87.5$ V/phase**

(iii) $Z_S = 0 + j10 = 10 \angle 90°$; $I_a = E_R / Z_S = 35 \angle 87.5°/10 \angle 90° =$ **$3.5 \angle -2.5°$ A/phase**
Obviously, I_a lags behind V_p by 2.5°

(iv) Power input/phase $V_p I_a \cos \phi = 400 \times 3.5 \times \cos 2.5° = 1399$ W
Total input power $= 3 \times 1399 =$ **4197 W**

(v) Since R_a is negligible, armature Cu loss is also negligible. Hence 4197 W also represent power developed by armature.

(b) 5° (mech) Displacement – Fig. 38.18 (b)

(i) α (elect) $= \dfrac{20}{2} \times 5° =$ **50°**

(ii) $E_R = (400 - 400 \cos 50°) + j400 \sin 50° = 143 + j\, 306.4 =$ **$338.2 \angle 64.9°$**

(iii) $I_a = 338.2 \times 64.9°/10 \angle 90° =$ **$33.8 \angle -25.1°$ A/phase**

(iv) motor power/phase $= V_p I_a \cos \phi = 400 \times 33.8 \cos 25.1° = 12{,}244$ W
Total power $= 3 \times 12{,}244 = 36{,}732$ W $=$ **36.732 kW**

It is seen from above that as motor load is increased

1. rotor displacement increases from 5° (elect) to 50° (elect) *i.e.* E_b falls back in phase considerably.
2. E_R increases from 35 V to 338 V/phase
3. I_a increases from 3.5 A to 33.8 A
4. angle ϕ increases from 2.5° to 25.1° so that p.f. ***decreases*** from 0.999 (lag) to 0.906 (lag)
5. increase in power is almost ***directly proportional to increase in load angle***.

Obviously, increase in I_a is much more than decrease in power factor.

It is interesting to note that not only power but even I_a, E_R and ϕ also increase almost as many times as α.

Special Illustrative Example 38.3

Case of Cylindrical Rotor Machine :

A 3-Phase synchronous machine is worked as follows: Generator - mode : 400 V/Ph, 32 A/Ph, Unity p.f. X_S = 10 ohms. Motoring - mode : 400 V/Ph, 32 A/Ph, Unity p.f. , X_S = 10 ohms. Calculate E and δ in both the cases and comment.

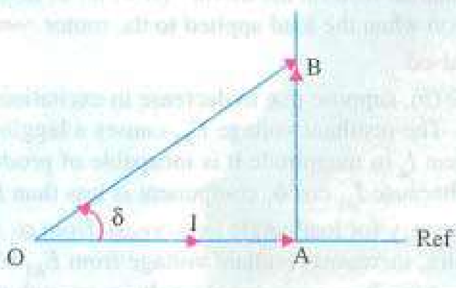


Fig. 38.19 (a) Generator-mode

Solution. In Fig. 38.19 (*a*), $V = OA = 400$, $IX_S = AB = 320$ V

$$E = OB = 512.25,\ \delta = \tan^{-1}\frac{320}{400} = 38.66^\circ$$

Total power in terms of parameters measurable at terminals (*i.e.*, V, I, and ϕ)

$$= 3\,V_{ph}\,I_{ph}\cos\phi = 3 \times 400 \times 32 = 38.4 \text{ kW}$$

Total power using other parameters $= 3 \times \left[\frac{VE}{X_S}\sin\delta\right] \times 10^{-3}$ kW

$$= 3 \times \frac{400 \times 512.25}{10} \times (\sin 38.66^\circ) \times 10^{-3} = 38.4 \text{ kW}$$

Since losses are neglected, this power is the electrical output of generator and also is the required mechanical input to the generator.

For motoring mode : $V = OA = 400$, $-IX_S = AB = 320$

$E = OB = 512.25$, as in Fig. 38.19 (*b*)

Hence, $|\delta| = 38.66^\circ$, as before.

Comments : The change in the sign of δ has to be noted in the two modes. It is +ve for generator and –ve for motor. E happens to be equal in both the cases due to unity p.f. At other p.f., this will be different.

As before, power can be calculated in two ways and it will be electrical power input to motor and also the mechanical output of the motor.

Naturally, Power = 38.4 kW

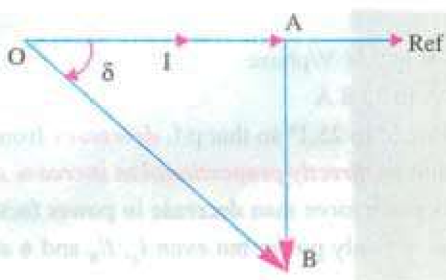


Fig. 38.19 (*b*) Motoring mode

38.10. Effect of Changing Excitation on Constant Load

As shown in Fig. 38.20 (*a*), suppose a synchronous motor is operating with normal excitation ($E_b = V$) at unity p.f. with a given load. If R_a is negligible as compared to X_S, then I_a lags E_R by 90° and is in phase with V because p.f. is unity. The armature is drawing a power of $V.I_a$ per phase which is enough to meet the mechanical load on the motor. Now, let us discuss the effect of decreasing or increasing the field excitation when the load applied to the motor ***remains constant***.

(*a*) Excitation Decreased

As shown in Fig. 38.20 (*b*), suppose due to decrease in excitation, back e.m.f. is reduced to Eb_1 at the ***same load angle*** α_1. The resultant voltage E_{R1} causes a lagging armature current I_{a1} to flow. Even though I_{a1} is larger than I_a in magnitude it is incapable of producing necessary power VI_a for carrying the ***constant*** load because $I_{a1}\cos\phi_1$ component is less than I_a so that $VI_{a1}\cos\phi_1 < VI_a$.

Hence, it becomes necessary for load angle to ***increase*** from α_1 to α_2. It increases back e.m.f. from E_{b1} to E_{b2} which, in turn, increases resultant voltage from E_{R1} to E_{R2}. Consequently, armature current increases to I_{a2} whose in-phase component produces enough power ($VI_{a2}\cos\phi_2$) to meet the constant load on the motor.

(b) Excitation Increased

The effect of increasing field excitation is shown in Fig. 38.20 (*c*) where increased E_{b1} is shown at the original load angle α_1. The resultant voltage E_{R1} causes a ***leading*** current I_{a1} whose in-phase component is larger than I_a. Hence, armature develops more power than the load on the motor. Accordingly, load angle ***decreases*** from α_1 to α_2 which decreases resultant voltage from E_{R1} to E_{R2}. Consequently, armature current decreases from I_{a1} to I_{a2} whose in-phase component $I_{a2} \cos \phi_2 = I_a$. In that case, armature develops power sufficient to carry the constant load on the motor.

Hence, we find that variations in the excitation of a synchronous motor running with a ***given*** load produce variations in its ***load angle only***.

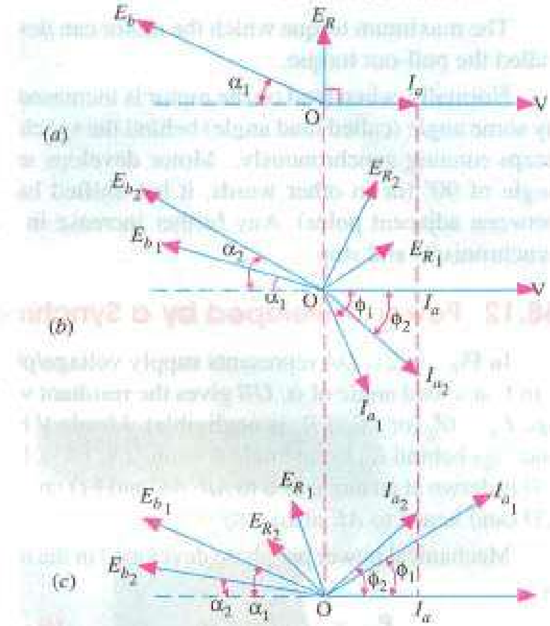


Fig. 38.20

38.11. Different Torques of a Synchronous Motor

Various torques associated with a synchronous motor are as follows:

1. starting torque
2. running torque
3. pull-in torque and
4. pull-out torque

(a) Starting Torque

It is the torque (or turning effort) developed by the motor when full voltage is applied to its stator (armature) winding. It is also sometimes called ***breakaway*** torque. Its value may be as low as 10% as in the case of centrifugal pumps and as high as 200 to 250% of full-load torque as in the case of loaded reciprocating two-cylinder compressors.


Torque motors are designed to privide maximum torque at locked rotor or near stalled conditions

(b) Running Torque

As its name indicates, it is the torque developed by the motor under running conditions. It is determined by the horse-power and speed of the ***driven*** machine. The peak horsepower determines the maximum torque that would be required by the driven machine. The motor must have a breakdown or a maximum running torque greater than this value in order to avoid stalling.

(c) Pull-in Torque

A synchronous motor is started as induction motor till it runs 2 to 5% below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously-rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

(d) Pull-out Torque

The maximum torque which the motor can develop without pulling out of step or synchronism is called the pull-out torque.

Normally, when load on the motor is increased, its rotor progressively tends to fall back ***in phase*** by some angle (called load angle) behind the synchronously-revolving stator magnetic field though it keeps running synchronously. Motor develops maximum torque when its rotor is retarded by an angle of 90° (or in other words, it has shifted backward by a distance equal to half the distance between adjacent poles). Any further increase in load will cause the motor to pull out of step (or synchronism) and stop.

38.12. Power Developed by a Synchronous Motor

In Fig. 38.21, OA represents supply voltage/phase and $I_a = I$ is the armature current, AB is back e.m.f. at a load angle of α. OB gives the resultant voltage $E_R = IZ_S$ (or IX_S if R_a is negligible). I leads V by ϕ and lags behind E_R by an angle $\theta = \tan^{-1}(X_S / R_a)$. Line CD is drawn at an angle of θ to AB. AC and ED are $\perp$ to CD (and hence to AE also).

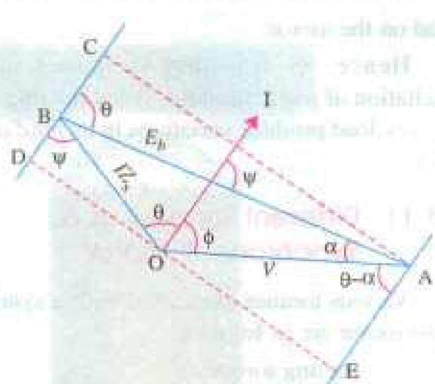


Fig. 38.21

Mechanical power per phase developed in the rotor is

$$P_m = E_b I \cos \psi \qquad ...(i)$$

In Δ OBD, $BD = I Z_S \cos \psi$

Now, $BD = CD - BC = AE - BC$

$$I Z_S \cos \psi = V \cos(\theta - \alpha) - E_b \cos \theta$$

$$\therefore \quad I \cos \psi = \frac{V}{Z_S} \cos(\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta$$

Substituting this value in (i), we get

$$P_m \text{ per phase} = E_b \left[\frac{V}{Z_S} \cos(\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta \right] = \frac{E_b V}{Z_S} \cos(\theta - \alpha) - \frac{E_b^2}{Z_S} \cos \theta^* \qquad ...(ii)$$

This is the expression for the mechanical power developed in terms of the load angle (α) and the internal angle (θ) of the motor for a constant voltage V and E_b (or excitation because E_b depends on excitation only).

If T_g is the gross armature torque developed by the motor, then

$$T_g \times 2\pi N_S = P_m \text{ or } T_g = P_m / \omega_s = P_m / 2\pi N_S \qquad -N_S \text{ in rps}$$

$$T_g = \frac{P_m}{2\pi N_S / 60} = \frac{60}{2\pi} \cdot \frac{P_m}{N_S} = 9.55 \frac{P_m}{N_S} \qquad -N_S \text{ in rpm}$$

Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\therefore \quad \frac{dP_m}{d\alpha} = -\frac{E_b V}{Z_S} \sin(\theta - \alpha) = 0 \quad \text{or} \quad \sin(\theta - \alpha) = 0 \quad \therefore \quad \theta = \alpha$$

---

* Since R_a is generally negligible, $Z_S = X_S$ so that $\theta \equiv 90°$. Hence

$$P_m = \frac{E_b V}{X_S} \cos(90° - \alpha) = \frac{E_b V}{X_S} \sin \alpha$$

This gives the value of mechanical power developed in terms of α – the basic variable of a synchronous machine.

$\therefore$ value of maximum power $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b^2}{Z_S}\cos\alpha$ or $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b^2}{Z_S}\cos\theta$. ...(*iii*)

This shows that the maximum power and hence torque ($\because$ speed is constant) depends on V and E_b *i.e.*, excitation. Maximum value of θ (and hence α) is 90°. For all values of V and E_b, this limiting value of α is the same but maximum torque will be proportional to the maximum power developed as given in equation (*iii*). Equation (*ii*) is plotted in Fig. 38.22.

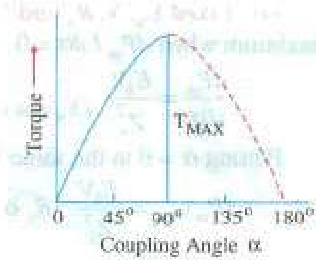


Fig. 38.22

If R_a is neglected, then $Z_S \cong X_S$ and $\theta = 90°$ $\therefore \cos\theta = 0$

$P_m = \dfrac{E_b V}{X_S}\sin\alpha$ ...(*iv*) $(P_m)_{max} = \dfrac{E_b V}{X_S}$ ... from equation (*iii*)* The same value can be otained by putting $\alpha = 90°$ in equation (*iv*). This corresponds to the 'pull-out' torque.

38.13. Alternative Expression for Power Developed

In Fig. 38.23, as usual, OA represents the supply voltage per phase *i.e.*, V and AB (= OC) is the induced or back e.m.f. per phase *i.e.*, E_b at an angle α with OA. The armature current I (or I_a) lags V by ϕ.

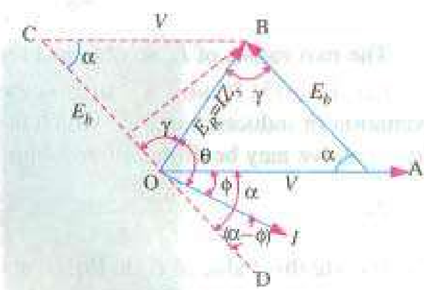


Fig. 38.23

Mechanical power developed is,

$$\begin{aligned} P_m &= E_b \, . \, I \times \text{cosine of the angle between } E_b \text{ and } I \\ &= E_b I \cos \angle DOI \\ &= E_b I \cos(\pi - \angle COI) \\ &= -E_b I \cos(\theta + \gamma) \\ &= -E_b\left(\frac{E_R}{Z_S}\right)(\cos\theta\cos\gamma - \sin\theta\sin\gamma) \qquad ...(i) \end{aligned}$$

Now, E_R and functions of angles θ and γ will be eliminated as follows :

From $\Delta\, OAB$; $V/\sin\gamma = E_R/\sin\alpha$ $\therefore$ $\sin\gamma = V\sin\alpha / E_R$

From $\Delta\, OBC$; $E_R\cos\gamma + V\cos\alpha = E_b$ $\therefore$ $\cos\gamma = (E_b - V\cos\alpha)/E_R$

Also $\cos\theta = R_a / Z_S$ and $\sin\theta = X_S / Z_S$

Substituting these values in Eq. (*i*) above, we get

$$\begin{aligned} P_m &= -\frac{E_b \, . \, E_R}{Z_S}\left(\frac{R_a}{Z_S} \, . \, \frac{E_b - V\cos\alpha}{E_R} - \frac{X_S}{Z_S} \, . \, \frac{V\sin\alpha}{E_R}\right) \\ &= \frac{E_b V}{Z_{S^2}}(R_a\cos\alpha + X_S\sin\alpha) - \frac{E_b^2 R_a}{Z_{S^2}} \qquad ...(ii) \end{aligned}$$

It is seen that P_m varies with E_b (which depends on excitation) and angle α (which depends on the motor load).

Note. If we substitute $R_a = Z_S\cos\theta$ and $X_S = Z_S\sin\theta$ in Eq. (*ii*), we get

$$P_m = \frac{E_b V}{Z_{S^2}}(Z_S\cos\theta\cos\alpha + Z_S\sin\theta\sin\alpha) - \frac{E_b^2 Z_S\cos\theta}{Z_{S^2}} = \frac{E_b V}{Z_S}\cos(\theta - \alpha) - \frac{E_{b^2}}{Z_S}\cos\theta$$

It is the same expression as found in Art. 38.10.

* It is the same expression as found for an alternator or synchronous generator in Art. 37.37.

38.14. Various Conditions of Maxima

The following two cases may be considered :

(i) Fixed E_b, V, R_a and X_S. Under these conditions, P_m will vary with load angle α and will be maximum when $dP_m / d\alpha = 0$. Differentiating Eq. (ii) in Art. 38.11, we have

$$\frac{dP_m}{d\alpha} = \frac{E_b V}{Z_s^2} (X_S \cos\alpha - R_a \sin\alpha) = 0 \quad \text{or} \quad \tan\alpha = X_S / R_a = \tan\theta \quad \text{or} \quad \alpha = \theta$$

Putting $\alpha = \theta$ in the same Eq. (ii), we get

$$(P_m)_{max} = \frac{E_b V}{Z_s^2}(R_a \cos\theta + X_S \sin\theta) - \frac{E_b^2 R_a}{Z_s^2} = \frac{E_b V}{Z_s^2}\left(R_a \cdot \frac{R_a}{Z_S} + X_S \cdot \frac{X_S}{Z_S}\right) - \frac{E_b^2 R_a}{Z_s^2}$$

$$= \frac{E_b V}{Z_s^2}\left(\frac{R_a^2 + X_s^2}{Z_S}\right) - \frac{E_b^2 R_a}{Z_s^2} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_s^2} \qquad \text{...(i)}$$

This gives the value of power at which the motor falls out of step.

Solving for E_b from Eq. (i) above, we get

$$E_b = \frac{Z_S}{2R_a}\left[V \pm \sqrt{V^2 - 4R_a \cdot (P_m)_{max}}\right]$$

The two values of E_b so obtained represent the excitation limits for any load.

(ii) Fixed V, R_a and X_S. In this case, P_m varies with excitation or E_b. Let us find the value of the excitation or induced e.m.f. E_b which is necessary for maximum power possible. For this purpose, Eq. (i) above may be differentiated with respect to E_b and equated to zero.

$$\therefore \qquad \frac{d(P_m)_{max}}{dE_b} = \frac{V}{Z_S} - \frac{2R_a E_b}{Z_S^2} = 0; \qquad E_b = \frac{V Z_S}{2R_a}^{*} \qquad \text{...(ii)}$$

Putting this value of E_b in Eq. (i) above, maximum power developed becomes

$$(P_m)_{max} = \frac{V^2}{2R_a} - \frac{V^2}{4R_a} = \frac{V^2}{4R_a}$$

38.15. Salient Pole Synchronous Motor

Cylindrical-rotor synchronous motors are much easier to analyse than those having salient-pole rotors. It is due to the fact that cylindrical-rotor motors have a uniform air-gap, whereas in salient-pole motors, air-gap is much greater between the poles than along the poles. Fortunately, cylindrical rotor theory is reasonably accurate in predicting the steady-state performance of salient-pole motors. Hence, salient-pole theory is required only when very high degree of accuracy is needed or when problems concerning transients or power system stability are to be handled.

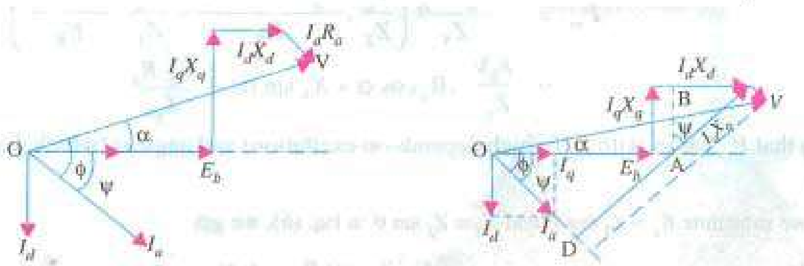


Fig. 38.24

* This is the value of induced e.m.f. to give maximum power, but it is not the maximum possible value of the generated voltage, at which the motor will operate.

The d-q currents and reactances for a salient-pole synchronous motor are exactly the same as discussed for salient-pole synchronous generator. The motor has d-axis reactance X_d and q-axis reactance X_q. Similarly, motor armature current I_a has two components : I_d and I_q. The complete phasor diagram of a salient-pole synchronous motor, for a lagging power factor is shown in Fig. 38.24 (a).

With the help of Fig. 38.24 (b), it can be proved that $\tan\psi = \dfrac{V\sin\phi - I_q X_q}{V\cos\phi - I_a R_a}$

If R_a is negligible, then $\tan\psi = (V\sin\phi + I_a X_q)/V\cos\phi$

For an overexcited motor *i.e.*, when motor has leading power factor,

$$\tan\psi = (V\sin\phi + I_a X_q)/V\cos\phi$$

The power angle α is given by $\alpha = \phi - \psi$

The magnitude of the excitation or the back e.m.f. E_b is given by

$$E_b = V\cos\alpha - I_q R_a - I_d X_d$$

Similarly, as proved earlier for a synchronous generator, it can also be proved from Fig. 38.24 (b) for a synchronous motor with $R_a = 0$ that

$$\tan\alpha = \frac{I_a X_q \cos\phi}{V - I_a X_q \sin\phi}$$

In case R_a is not negligible, it can be proved that

$$\tan\alpha = \frac{I_a X_q \cos\phi - I_a R_a \sin\phi}{V - I_a X_q \sin\phi - I_a R_a \cos\alpha}$$

38.16. Power Developed by a Salient Pole Synchronous Motor

The expression for the power developed by a salient-pole synchronous generator derived in Chapter 35 also applies to a salient-pole synchronous motor.

$$\therefore \quad P_m = \frac{E_b V}{X_d}\sin\alpha + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\alpha \quad \text{... per phase}$$

$$= 3\times\left[\frac{E_b V}{X_d}\sin\alpha + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\alpha\right] \quad \text{... per three phases}$$

$$T_g = 9.55\,P_m/N_S \quad \text{...}N_S \text{ in rps.}$$

As explained earlier, the power consists of two components, the first component is called excitation power or magnet power and the second is called reluctance power (because when excitation is removed, the motor runs as a reluctance motor).

Example 38.4. *A 3-ϕ, 150-kW, 2300-V, 50-Hz, 1000-rpm salient-pole synchronous motor has X_d = 32 Ω / phase and X_q = 20 Ω / phase. Neglecting losses, calculate the torque developed by the motor if field excitation is so adjusted as to make the back e.m.f. twice the applied voltage and $\alpha = 16°$.*

Solution. $V = 2300/\sqrt{3} = 1328$ V ; $E_b = 2\times 1328 = 2656$ V

$$\text{Excitation power / phase} = \frac{E_b V}{X_d}\sin\alpha = \frac{2656\times 1328}{32}\sin 16° = 30{,}382 \text{ W}$$

$$\text{Reluctance power / phase} = \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\alpha = \frac{1328^2(32-20)}{2\times 32\times 20}\sin 32° = 8760 \text{ W}$$

$$\text{Total power developed,} \quad P_m = 3(30382 + 8760) = 117{,}425 \text{ W}$$

$$T_g = 9.55\times 117{,}425/1000 = \mathbf{1120 \text{ N-m}}$$

Example 38.5. *A 3300-V, 1.5-MW, 3-ϕ, Y-connected synchronous motor has $X_d = 4\Omega$ / phase and $X_q = 3\ \Omega$/ phase. Neglecting all losses, calculate the excitation e.m.f. when motor supplies rated load at unity p.f. Calculate the maximum mechanical power which the motor would develop for this field excitation.* **(Similar Example, Swami Ramanand Teertha Marathwada Univ. Nanded 2001)**

Solution.

$$V = 3300/\sqrt{3} = 1905 \text{ V};\ \cos\phi = 1;\ \sin\phi = 0;\ \phi = 0°$$

$$I_a = 1.5 \times 10^6/\sqrt{3} \times 3300 \times 1 = 262 \text{ A}$$

$$\tan\psi = \frac{V\sin\phi - I_a X_q}{V\cos\phi} = \frac{1905\times 0 - 262\times 3}{1905} = -0.4125;\ \psi = -22.4°$$

$$\alpha = \phi - \psi = 0 - (-22.4°) = 22.4°$$

$$I_d = 262 \times \sin(-22.4°) = -100 \text{ A};\ I_q = 262\cos(-22.4°) = 242 \text{ A}$$

$$E_b = V\cos\alpha - I_d X_d = 1905\cos(-22.4°) - (-100\times 4) = 2160 \text{ V}$$

$$= 1029\sin\alpha + 151\sin 2\alpha$$

$$P_m = \frac{E_b V}{X_d}\sin\alpha + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\alpha \quad \text{... per phase}$$

$$= \frac{2160\times 1905}{4\times 1000} + \frac{1905^2(4-3)}{2\times 4\times 3\times 1000}\sin 2\alpha \quad \text{...kW/phase}$$

$$= 1029\sin\alpha + 151\sin 2\alpha \quad \text{...kW/phase}$$

If developed power has to achieve maximum value, then

$$\frac{dP_m}{d\alpha} = 1029\cos\alpha + 2\times 151\cos 2\alpha = 0$$

$$\therefore\quad 1029\cos\alpha + 302(2\cos^2\alpha - 1) = 0 \quad \text{or} \quad 604\cos^2\alpha + 1029\cos\alpha - 302 = 0$$

$$\therefore\quad \cos\alpha = \frac{-1029 \pm \sqrt{1029^2 + 4\times 604\times 302}}{2\times 604} = 0.285;\ \alpha = 73.4°$$

$$\therefore\quad \text{maximum } P_m = 1029\sin 73.4° + 151\sin 2\times 73.4° = 1070 \text{ kW/phase}$$

Hence, maximum power developed for three phases = 3 × 1070 = **3210 kW**

Example 38.6. *The input to an 11000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find (i) the power supplied to the motor (ii) mechanical power developed and (iii) induced emf for a power factor of 0.8 leading.* **(Elect. Engg. AMIETE (New Scheme) June 1990)**

Solution. (*i*) Motor power input = $\sqrt{3} \times 11000 \times 60 \times 0.8 = 915$ kW

(*ii*) stator Cu loss/phase = $60^2 \times 1 = 3600$ W; Cu loss for three phases = 3 × 3600 = 10.8 kW

$$P_m = P_2 - \text{rotor Cu loss} = 915 - 10.8 = 904.2 \text{ kW}$$

$$V_p = 11000/\sqrt{3} = 6350 \text{ V};\quad \phi = \cos^{-1} 0.8 = 36.9°;$$

$$\theta = \tan^{-1}(30/1) = 88.1°$$

$$Z_S \equiv 30\ \Omega;\ \text{stator impedance drop / phase} = I_a Z_S$$

$$= 60 \times 30 = 1800 \text{ V}$$

Fig. 38.25

As seen from Fig. 38.25,

$$E_b^2 = 6350^2 + 1800^2 - 2\times 6350\times 1800\times\cos(88.1° + 36.9°)$$

$$= 6350^2 + 1800^2 - 2\times 6350\times 1800\times -0.572$$

$$\therefore\quad E_b = 7528 \text{ V};\ \text{line value of } E_b = 7528\times\sqrt{3} = \mathbf{13042}$$

Special Example 38.7. Case of Salient - Pole Machines

A synchronous machine is operated as below :

As a Generator : 3 -Phase, $V_{ph} = 400, I_{ph} = 32$, *unity p.f.*

As a Motor : 3 - Phase, $V_{ph} = 400, I_{ph} = 32$, *unity p.f.*

Machine parameters : $X_d = 10\,\Omega, X_q = 6.5\,\Omega$

Calculate excitation emf and δ in the two modes and deal with the term power in these two cases.

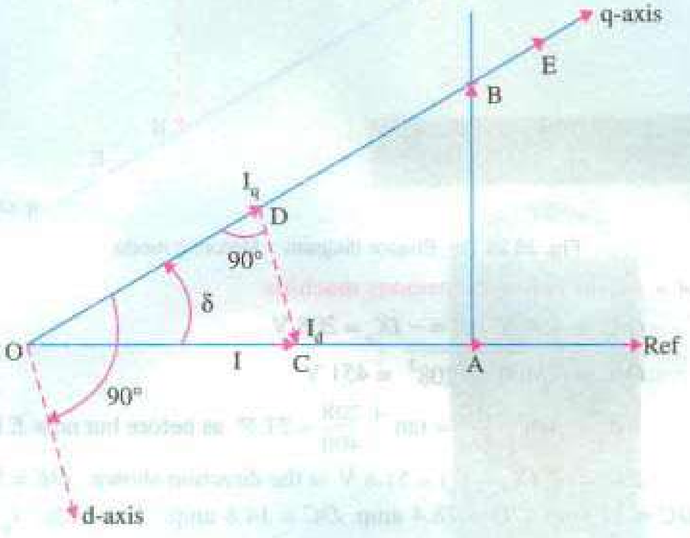


Fig. 38.26 (*a*) Generator-action

Solution.

Generating Mode :

Voltages :

$$OA = 400 \text{ V}, AB = IX_q = 32 \times 6.5 = 208 \text{ V}$$

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V},$$

$$\delta = \tan^{-1}\frac{AB}{OA} = \tan^{-1}\frac{208}{400} = 27.5°$$

$$BE = I_d(X_d - X_q) = 140 \times 3.5 = 51.8 \text{ V},$$

$$E = OE = OB + BE = 502.8 \text{ V}$$

Currents : $I = OC = 32$, $I_q = I\cos\delta = OD = 28.4$ amp., $I_d = DC = I\sin\delta = 14.8$ amp. E leads V in case of generator, as shown in Fig. 38.26 (*a*)

$$\text{Power (by one formula)} = 3 \times 400 \times 32 \times 10^{-3} = 38.4 \text{ kW}$$

or
$$\text{Power (by another formula)} = 3 \times \left[\frac{400 \times 502.8}{10}\sin 27.5° + \frac{400^2}{2} \times \left(\frac{3.5}{65}\right) \times \sin 55°\right]$$

$$= 38.44 \text{ kW}$$

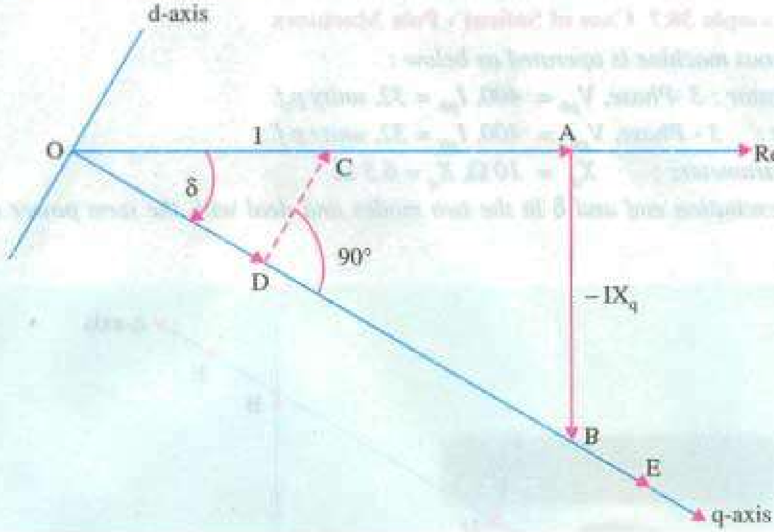


Fig. 38.26 (*b*) Phasor diagram : Motoring mode

Motoring mode of a salient pole synchronous machine

Voltages : $OA = 400$ V, $AB = -IX_q = 208$ V

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V}$$

$$\delta = \tan^{-1}\frac{AB}{OA} = \tan^{-1}\frac{208}{400} = 27.5^\circ \text{ as before but now } E \text{ lags behind } V.$$

$$BE = I_d(X_d - X_q) = 51.8 \text{ V in the direction shown. } OE = 502.8 \text{ V as before}$$

Currents : $OC = 32$ amp. $OD = 28.4$ amp. $DC = 14.8$ amp. Naturally, $I_q = 28.4$ amp. and $I_d = 14.8$ amp

Power (by one formula) = 38.4 kW

Power (by another formula) = 38.44 kW

Note. Numerical values of E and δ are same in cases of generator-mode and motor-mode, due to unity p.f. δ has different signs in the two cases.

Example 38.8. *A 500-V, 1-phase synchronous motor gives a net output mechanical power of 7.46 kW and operates at 0.9 p.f. lagging. Its effective resistance is 0.8 Ω. If the iron and friction losses are 500 W and excitation losses are 800 W, estimate the armature current. Calculate the commercial efficiency.* **(Electrical Machines-I, Gujarat Univ. 1988)**

Solution. Motor input $= VI_a \cos\phi$; Armature Cu loss $= I_a R_a^2$

Power developed in *armature* is $P_m = VI_a \cos\phi - I_a^2 R_a$

$$\therefore \quad I_a^2 R_a - VI_a\cos\phi + P_m = 0 \quad \text{or} \quad I_a = \frac{V\cos\phi \pm \sqrt{V^2\cos^2\phi - 4R_a P_m}}{2R_a}$$

Now,

$$P_{out} = 7.46 \text{ kW} = 7{,}460 \text{ W}$$

$$\text{P}_m = P_{out} + \text{iron and friction losses} + \text{excitation losses}$$

$$= 7460 + 500 + 800 = 8760 \text{ W} \qquad \text{... Art. 38.5}$$

$$I_a = \frac{500\times 0.9 \pm \sqrt{(500\times 0.9)^2 - 4\times 0.8\times 3760}}{2\times 0.8}$$

$$= \frac{450 \pm \sqrt{202{,}500 - 28{,}030}}{1.6} = \frac{450 \pm 417.7}{1.6} = \frac{32.3}{1.6} = \mathbf{20.2\ A}$$

$$\text{Motor input} = 500 \times 20.2 \times 0.9 = 9090 \text{ W}$$

$$\eta_c = \text{net output / input} = 7460 / 9090 = 0.8206 \quad \text{or} \quad \textbf{82.06\%}.$$

Example 38.9. *A 2,300-V, 3-phase, star-connected synchronous motor has a resistance of 0.2 ohm per phase and a synchronous reactance of 2.2 ohm per phase. The motor is operating at 0.5 power factor leading with a line current of 200 A. Determine the value of the generated e.m.f. per phase.* **(Elect. Engg.-I, Nagpur Univ. 1993)**

Solution. Here, $\phi = \cos^{-1}(0.5) = 60°$ (lead)

$$\theta = \tan^{-1}(2.2/0.2) = 84.8°$$

$$\therefore \quad (\theta + \phi) = 84.8° + 60° = 144.8°$$

$$\cos 144.8° = -\cos 35.2°$$

$$V = 2300/\sqrt{3} = 1328 \text{ volt}$$

$$Z_S = \sqrt{0.2^2 + 2.2^2} = 2.209\ \Omega$$

$$IZ_S = 200 \times 2.209 = 442 \text{ V}$$

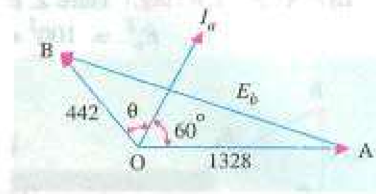


Fig. 38.27

The vector diagram is shown in Fig. 38.27.

$$E_b = \sqrt{V^2 + E_{R^2} - 2V \,.\, E_R \cos(\theta + \phi)}$$

$$= \sqrt{1328^2 + 442^2 + 2 \times 1328 \times 442 \times \cos 35.2°} = \textbf{1708 Volt / Phase}$$

Example 38.10. *A 3-phase, 6,600-volts, 50-Hz, star-connected synchronous motor takes 50 A current. The resistance and synchronous reactance per phase are 1 ohm and 20 ohm respectively. Find the power supplied to the motor and induced emf for a power factor of (i) 0.8 lagging and (ii) 0.8 leading.* **(Eect. Engg. II pune Univ. 1988)**

Solution. (*i*) **p.f. = 0.8 lag** (Fig. 38.28 (*a*)).

$$\text{Power input} = \sqrt{3} \times 6600 \times 50 \times 0.8 = 457{,}248 \text{ W}$$

$$\text{Supply voltage / phase} = 6600 / \sqrt{3} = 3810 \text{ V}$$

$$\phi = \cos^{-1}(0.8) = 36°52'; \theta = \tan^{-1}(X_S / R_a) = (20/1) = 87.8'$$

$$Z_S = \sqrt{20^2 + 1^2} = 20\ \Omega \text{ (approx.)}$$

$$\text{Impedance drop} = I_a Z_S = 50 \times 20 = 1000 \text{ V/phase}$$

$$\therefore \quad E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times \cos(87°8' - 36°52') \quad \therefore \quad E_b = 3263 \text{ V / phase}$$

$$\text{Line induced e.m.f.} = 3263 \times \sqrt{3} = \textbf{5651 V}$$

(*ii*) Power input would remain the same.

As shown in Fig. 38.28 (*b*), the current vector is drawn at a ***leading*** angle of

$$\phi = 36°52'$$

Now, $(\theta + \phi) = 87°8' + 36°52' = 124°$, $\cos 124° = -\cos 56°$

$\therefore\ E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times -\cos 56° \quad \therefore\ E_b = 4447$ V / phase

$$\text{Line induced e.m.f.} = \sqrt{3} \times 4447 = \textbf{7,700 V}$$

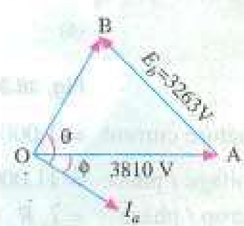


Fig. 38.28 (*a*)

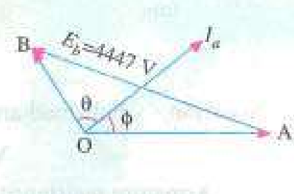


Fig. 38.28 (*b*)

Note. It may be noted that if $E_b > V$, then motor has a leading power factor and if $E_b < V$.

Example 38.11. *A synchronous motor having 40% reactance and a negligible resistance is to be operated at rated load at (i) u.p.f. (ii) 0.8 p.f. lag (iii) 0.8 p.f. lead. What are the values of induced e.m.f. ? Indicate assumptions made, if any.* **(Electrical Machines-II, Indore Univ. 1990)**

Solution. Let $V = 100$ V, then reactance drop $= I_a X_S = 40$ V

(i) **At unity p.f.**

Here, $\theta = 90°$, $E_b = \sqrt{100^2 + 40^2} = 108$ V ...Fig. 38.29 (*a*)

(ii) **At p.f. 0.8 (lag.)** Here $\angle BOA = \theta - \phi = 90° - 36°54' = 53°6'$

$$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 53°6';\ E_b = \mathbf{82.5\ V}, \text{ as in Fig. 38.29 } (b)$$

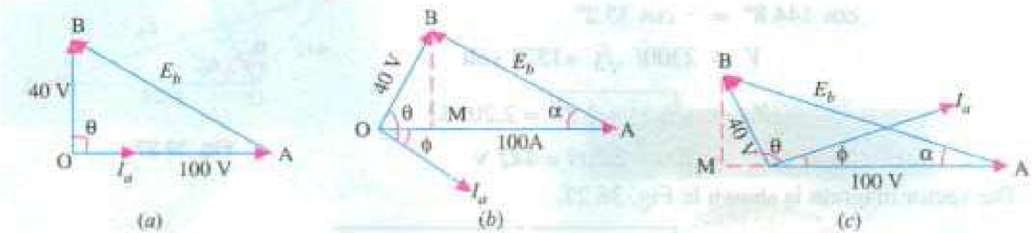


Fig. 38.29

Alternatively, $E_b = AB = \sqrt{AM^2 + MB^2} = \sqrt{76^2 + 32^2} = \mathbf{82.5\ V}$

(iii) **At p.f. 0.8 (lead.)** Here, $(\theta + \phi) = 90° + 36.9° = 126.9°$

$$E_b^2 = 100^2 + 40^2 - 2 \times 40 \times \cos 126.9° = \mathbf{128\ V}$$

Again from Fig. 38.29 (*c*), $E_b^2 = (OM + OA)^2 + MB^2 = 124^2 + 32^2$; $E_b = \mathbf{128\ V.}$

Example 38.12. *A 1,000-kVA, 11,000-V, 3-ϕ, star-connected synchronous motor has an armature resistance and reactance per phase of 3.5 Ω and 40 Ω respectively. Determine the induced e.m.f. and angular retardation of the rotor when fully loaded at (a) unity p.f. (b) 0.8 p.f. lagging (c) 0.8 p.f. leading.* **(Elect. Engineering-II, Bangalore Univ. 1992)**

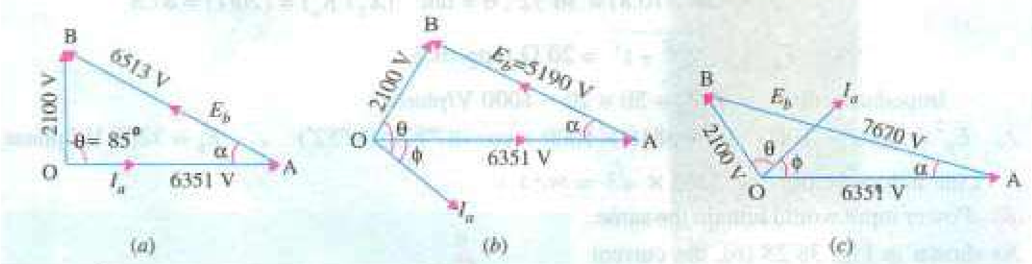


Fig. 38.30

Solution. Full-load armature current $= 1{,}000 \times 1{,}000 / \sqrt{3} \times 11{,}000 = 52.5$ A

Voltage / phase $= 11{,}000 / \sqrt{3} = 6{,}351$ V ; $\cos\phi = 0.8$ $\therefore \phi = 36°53'$

Armature resistance drop / phase $= I_a R_a = 3.5 \times 52.5 = 184$ V

reactance drop / phase $= I_a X_S = 40 \times 52.5 = 2{,}100$ V

$\therefore$ impedance drop / phase $= I_a Z_S = \sqrt{(184^2 + 2100^2)} = 2{,}100$ V (approx.)

$\tan\theta = X_S / R_a$ $\therefore$ $\theta = \tan^{-1}(40 / 3.5) = 85°$

(a) **At unity p.f.** Vector diagram is shown in Fig. 38.30 (*a*)

$E_b^2 = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \cos 85°$; $E_b = 6{,}513$ V per phase

Induced line voltage $= 6{,}513 \times \sqrt{3} =$ **11,280 V**

From ΔOAB, $\dfrac{2100}{\sin \alpha} = \dfrac{6153}{\sin 85°} = \dfrac{6153}{0.9961}$

$\sin \alpha = 2{,}100 \times 0.9961 / 6{,}513 = 0.3212$ $\therefore$ $\alpha =$ **18°44′**

(b) At p.f. 0.8 lagging – Fig. 38.30 (b)

$\angle BOA = \theta - \phi = 85° - 36°53' = 48°7'$

$E_b^2 = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \times \cos 48°7'$

$E_b = 5{,}190$ V per phase

Induced line voltage $= 5{,}190 \times \sqrt{3} =$ **8,989 V**

Again from the $\Delta\ OAB$ of Fig. 36.30 (b)

$\dfrac{2100}{\sin \alpha} = \dfrac{5190}{\sin 48°\ 7'} = \dfrac{5.190}{0.7443}$

$\therefore$ $\sin \alpha = 2100 \times 0.7443/5190 = 0.3012$ $\therefore$ $\alpha =$ **17°32′**

(c) At p.f. 0.8 leading [Fig. 38.30 (c)]

$\angle BOA = \theta + \phi = 85° + 36°53' = 121°53'$

$\therefore$ $E_b^2 = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \times \cos 121°53'$

$\therefore$ $E_b = 7{,}670$ volt per phase.

Induced line e.m.f $= 7{,}670 \times \sqrt{3} =$ **13,280 V**

Also, $\dfrac{2{,}100}{\sin \alpha} = \dfrac{7{,}670}{\sin 121°53'} = \dfrac{7{,}670}{0.8493}$

$\therefore$ $\sin \alpha = 2{,}100 \times 0.8493 / 7{,}670 = 0.2325$ $\therefore \alpha =$ **13°27′**

Special Example 38.13. *Both the modes of operation : Phase - angle = 20° Lag*

Part (a) : A three phase star-connected synchronous generator supplies a current of 10 A having a phase angle of 20° lagging at 400 volts/phase. Find the load angle and components of armature current (namely I_d and I_q) if X_d = 10 ohms, X_q = 6.5 ohms. Neglect r_a. Calculate voltage regulation.

Solution. The phasor diagram is drawn in Fig 38.31 (a)

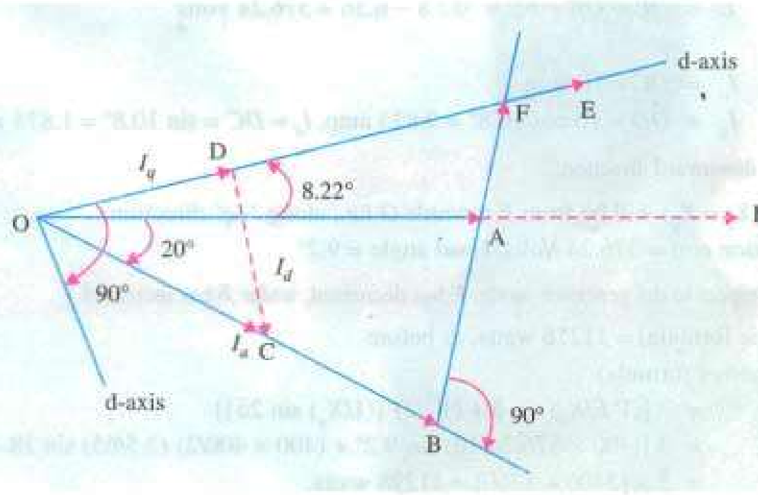


Fig. 38.31 (a) : Generator-mode

$$OA = 400 \text{ V}, OB = 400 \cos 20° = 376 \text{ V}, AB = 400 \sin 20° = 136.8 \text{ V}$$

$$AF = IX_q = 10 \times 6.5 = 65 \text{ V}, BF = BA + AF = 201.8 \text{ V}$$

$$OF = \sqrt{376^2 + 201.8^2} = 426.7 \text{ V}, \delta = 8.22°$$

$$DC = I_d = I_a \sin 28.22° = 4.73 \text{ amp}, DC \text{ perpendicular to } OD,$$

$$OD = I_q = I_a \cos 28.22° = 8.81 \text{ amp}$$

$$FE = I_d (X_d - X_q) = 4.73 \times 3.5 = 16.56 \text{ V. This is along the direction of '+q' –axis}$$

$$E = OE = OF + FE = 426.7 + 16.56 = 443.3 \text{ V}$$

$$\% \text{ Regulation} = \frac{443 - 400}{400} \times 100\% = \mathbf{10.75\ \%}$$

If the same machine is now worked as a synchronous motor with terminal voltage, supply-current and its power-factor kept unaltered, find the excitation emf and the load angle.

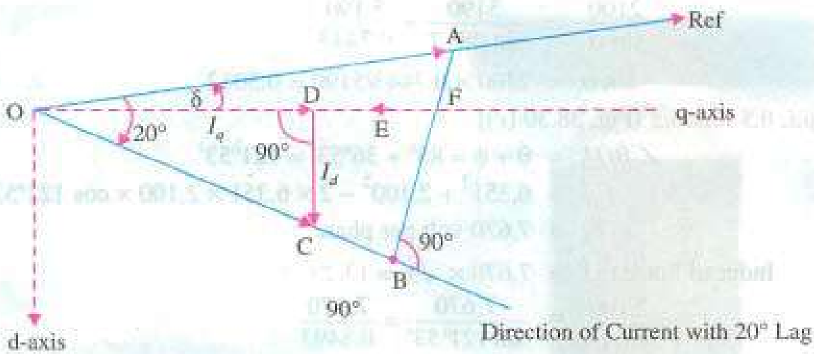


Fig. 38.31 (*b*) Motoring-mode

$$AF = -I_a X_q = -65 \text{ V}, AB = 136.8 \text{ V}, \text{FB} = 71.8 \text{ V}$$

$$OB = 400 \cos 20° = 376 \text{ V}$$

$$OF = \sqrt{376^2 + 71.8^2} = 382.8 \text{ V}$$

$$20° - \delta = \tan^{-1} BF/OB = \tan^{-1} 71.8/376 = 10.8°, \delta = 9.2°$$

$$FE = -I_d (X_d - X_q) = -1.874 \times (3.5) = -6.56 \text{ volts, as shown in Fig.38.31 } (b)$$

$$E = OE = OF + FE = 382.8 - 6.56 = 376.24 \text{ volts}$$

Currents :

$$I_a = OC = 10 \text{ amp}$$

$$I_q = OD = 10 \cos 10.8° = 9.823 \text{ amp}, I_d = DC = \sin 10.8° = 1.874 \text{ amp}$$

Note. I_d is in downward direction.

Hence, $-I_d (X_d - X_q)$ will be from *F* towards *O i.e.*, along '$-q$' direction.

Thus, Excitation emf = 376.24 Volts, Load angle = 9.2°

(**Note.** With respect to the generator mode, *E* has decreased, while δ has increased.)

Power (by one formula) = 11276 watts, as before

Power (by another formula)

$$= 3 [(V E/X_d) \sin \delta + (V^2/2) \{(1/X_q) \sin 2\delta\}]$$

$$= 3 [(400 \times 376.24/10) \sin 9.2° + (400 \times 400/2) (3.5/65) \sin 18.4°]$$

$$= 3 \times [2406 + 1360] = 11298 \text{ watts.}$$

[This matches quite closely to the previous value calculated by other formula.]

Example 38.14. *A 1-φ alternator has armature impedance of (0.5 + j0.866). When running as a synchronous motor on 200-V supply, it provides a net output of 6 kW. The iron and friction losses amount to 500 W. If current drawn by the motor is 50 A, find the two possible phase angles of current and two possible induced e.m.fs.* **(Elec. Machines-I, Nagpur Univ. 1990)**

Solution. Arm. Cu loss/phase $= I_a^2 R_a = 50^2 \times 0.5 = 1250$ W

Motor intake $= 6000 + 500 + 1250 = 7750$ W

p.f. $= \cos\phi = $ Watts / VA $= 7750 / 200 \times 50 = 0.775 \therefore \phi = 39°$ **lag or lead.**

$\theta = \tan^{-1}(X_S / R_a) = \tan^{-1}(0.866/0.5) = 60°$;

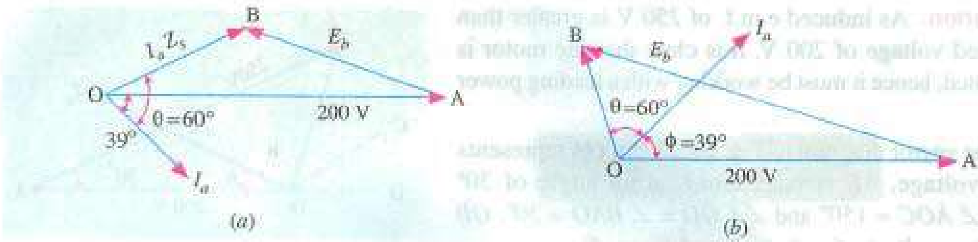


Fig. 38.32

$\angle BOA = 60° - 39° = 21°$ – Fig. 38.32 (*a*)

$Z_S = \sqrt{0.5^2 + 0.866^2} = 1\ \Omega$; $I_a Z_S = 50 \times 1 = 50$ V

$AB = E_b = \sqrt{200^2 + 50^2 - 2 \times 200 \times 50 \cos 21°}$; $E_b =$ **154 V.**

In Fig. 38.32 (*b*), $\angle BOA = 60° + 39° = 99°$

$\therefore \quad AB = E_b = \sqrt{(200^2 + 50^2) - 2 \times 200 \times 50 \cos 99°}$; $E_b =$ **214 V.**

Example 38.15. *A 2200-V, 3-φ, Y-connected, 50-Hz, 8-pole synchronous motor has $Z_S = (0.4 + j\,6)$ ohm/phase. When the motor runs at no-load, the field excitation is adjusted so that E is made equal to V. When the motor is loaded, the rotor is retarded by 3° mechanical.*

Draw the phasor diagram and calculate the armature current, power factor and power of the motor. What is the maximum power the motor can supply without falling out of step?

(Power Apparatus-II, Delhi Univ. 1988)

Solution. Per phase $E_b = V = 2200/\sqrt{3} = 1270$ V

$\alpha = 3°$ (mech) $= 3° \times (8/2) = 12°$ (elect).

As seen from Fig 38.33 (*a*),

$E_R = (1270^2 + 1270^2 - 2 \times 1270 \times 1270 \times \cos 12°)^{1/2}$

$= 266$ V; $Z_S = \sqrt{0.4^2 + 6^2} = 6.013\ \Omega$

$I_a = E_R / Z_S = 266 / 6.013 =$ **44.2 A.** From Δ *OAB*,

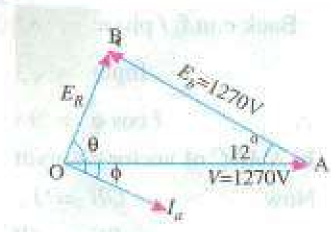


Fig. 38.33 (*a*)

we get, $\dfrac{1270}{\sin(\theta - \phi)} = \dfrac{266}{\sin 12°}$

$\therefore \quad \sin(\theta - \phi) = 1270 \times 0.2079/266 = 0.9926 \qquad \therefore (\theta - \phi) = 83°$

Now, $\theta = \tan^{-1}(X_S / R_a) = \tan^{-1}(6/0.4) = 86.18°$

$\phi = 86.18° - 83° = 3.18° \qquad \therefore$ p.f. $= \cos 3.18° =$ **0.998(lag)**

Total motor power input $= 3\,VI_a \cos\phi = 3 \times 1270 \times 44.2 \times 0.998 = 168$ kW

$$\text{Total Cu loss} = 3 I_a^2 R_a = 3 \times 44.2^2 \times 0.4 = 2.34 \text{ kW}$$

$$\text{Power developed by motor} = 168 - 2.34 = \mathbf{165.66\ kW}$$

$$P_{m(max)} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_s^2} = \frac{1270 \times 1270}{6.013} - \frac{1270^2 \times 0.4}{6.013^2} = \mathbf{250\ kW}$$

Example 38.16. *A 1–φ, synchronous motor has a back e.m.f. of 250 V, leading by 150 electrical degrees over the applied voltage of 200 volts. The synchronous reactance of the armature is 2.5 times its resistance. Find the power factor at which the motor is operating and state whether the current drawn by the motor is leading or lagging.*

Solution. As induced e.m.f. of 250 V is greater than the applied voltage of 200 V, it is clear that the motor is over-excited, hence it must be working with a leading power factor.

In the vector diagram of Fig. 38.33 (*b*), *OA* represents applied voltage, *AB* is back e.m.f. at an angle of 30° because ∠ *AOC* = 150° and ∠ *COD* = ∠ *BAO* = 30°. *OB* represents resultant of voltage *V* and E_b *i.e.* E_R

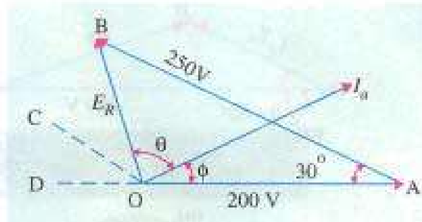


Fig. 38.33 (*b*)

In Δ *OBA*,

$$E_R = \sqrt{(V^2 + E_b^2 - 2V\,E_b \cos 30°)}$$

$$= \sqrt{(220^2 + 250^2 - 2 \times 200 \times 250 \times 0.866)} = 126 \text{ V}$$

Now, $\dfrac{E_R}{\sin 30°} = \dfrac{E_b}{\sin(\theta + \phi)}$ or $\dfrac{126}{0.5} = \dfrac{250}{\sin(\theta + \phi)}$

∴ $\sin(\theta + \phi) = 125/126$ (approx.) ∴ $(\theta + \phi) = 90°$

Now $\tan\theta = 2.5$ ∴ $\theta = 68°12'$ ∴ $\phi = 90° - 68°12' = 21°48'$

∴ p.f. of motor = cos 21°48′ = **0.9285 (leading)**

Example 38.17. *The synchronous reactance per phase of a 3-phase star-connected 6,600 V synchronous motor is 10 Ω. For a certain load, the input is 900 kW and the induced line e.m.f. is 8,900 V. (line value). Evaluate the line current. Neglect resistance.*

(Basic Elect. Machines, Nagpur Univ. (1993)

Solution. Applied voltage / phase = $6,600/\sqrt{3}$ = 3,810 V

Back e.m.f. / phase = $8,900/\sqrt{3}$ = 5,140 V

Input = $\sqrt{3}\, V_L \cdot I \cos\phi = 900,000$

∴ $I \cos\phi = 9 \times 10^5 / \sqrt{3} \times 6,600 = 78.74$ A

In Δ *ABC* of vector diagram in Fig. 38.34, we have $AB^2 = AC^2 + BC^2$

Now $OB = I \cdot X_S = 10\,I$

$BC = OB \cos\phi = 10\,I \cos\phi$

$= 10 \times 78.74 = 787.4$ V

∴ $5,140^2 = 787.4^2 + AC^2$ ∴ $AC = 5,079$ V

∴ $OC = 5,079 - 3,810 = 1,269$ V

$\tan\phi = 1269/787.4 = 1.612$; $\phi = 58.2°$, $\cos\phi = 0.527$

Fig. 38.34

Now $I \cos\phi = 78.74$; I = 78.74/0.527 = **149.4 A**

Example 38.18. *A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohms per phase and armature resistance negligible when the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.*

(Elect. Machines, AMIE Sec. B 1991)

Solution. When Power Input is 1000 kW (Fig. 38.35 (*a*))

$$\sqrt{3} \times 6600 \times I_{a1} \times 0.8 = 1000{,}000;\ I_{a1} = 109.3 \text{ A}$$

$$Z_S = X_S = 20\ \Omega\ ;\ I_{a1} Z_S = 109.3 \times 20 = 2186 \text{ V}\ ;\ \phi_1 = \cos^{-1} 0.8 = 36.9^\circ;\ \theta = 90^\circ$$

$$E_b^2 = 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times \cos(90^\circ + 36.9^\circ)$$

$$= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times -\cos 53.1^\circ;\ \therefore\ E_b = 5410 \text{ V}$$

Since excitation remains constant, E_b in the second case would remain the same *i.e.*, 5410 V.

When Power Input is 1500 kW :

$\sqrt{3} \times 6600 \times I_{a2} \cos \phi_2 = 1500{,}000$; $I_{a2} \cos \phi_2 = 131.2$ A

As seen from Fig. 38.35 (*b*), $OB = I_{a2} Z_S = 20\ I_a$

$BC = OB \cos \phi_2 = 20\ I_{a2}$

$\cos \phi_2 = 20 \times 131.2 = 2624$ V

In ΔABC, we have, $AB^2 = AC^2 + BC^2$ or $5410^2 = AC^2 + 2624^2$

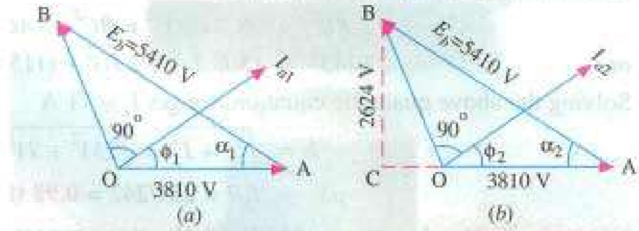


Fig. 38.35

$$\therefore \quad AC = 4730 \text{ V};\ OC = 4730 - 3810 = 920 \text{ V}$$

$$\tan \phi_2 = 920 / 2624\ ;\ \phi_2 = 19.4^\circ;\ \text{p.f.} = \cos \phi_2 = \cos 19.4^\circ = 0.9432 \text{ (lead)}$$

$$\tan \alpha_2 = BC / AC = 2624/4730;\ \alpha_2 = \mathbf{29^\circ}$$

Example 38.19. *A 3-phase, star-connected 400-V synchronous motor takes a power input of 5472 watts at rated voltage. Its synchronous reactance is 10 Ω per phase and resistance is negligible. If its excitation voltage is adjusted equal to the rated voltage of 400 V, calculate the load angle, power factor and the armature current.* **(Elect. Machines AMIE Sec. B, 1990)**

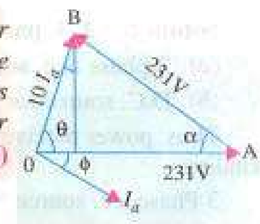


Fig. 38.36

Solution. $\sqrt{3} \times 400 \times I_a \cos \phi = 5472$; $I_a \cos \phi = 7.9$ A

$$Z_S = 10\ \Omega;\ E_R = I_a Z_S = 10\ I_a$$

As seen from Fig. 38.36, $BC = OB \cos \phi = 10,\ I_a \cos \phi = 79$ V

$$AC = \sqrt{231^2 - 79^2} = 217 \text{ V};\ OC = 231 - 217 = 14 \text{ V}$$

$$\tan \phi = 14 / 79;\ \phi = 10^\circ;\ \cos \phi = 0.985 \text{ (lag)}$$

$$I_a \cos \phi = 7.9;\ I_a = 7.9 / 0.985 = 8 \text{ A};\ \tan \alpha = BC / AC = 79 / 217;\ \alpha = \mathbf{20^\circ}$$

Example 38.20. *A 2,000-V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0.2 Ω and 2.2 Ω respectively. The input is 800 kW at normal voltage and the induced e.m.f. is 2,500 V. Calculate the line current and power factor.*

(Elect. Engg. A.M.I.E.T.E., June 1992)

Solution. Since the induced e.m.f. is greater than the applied voltage, the motor must be running with a leading p.f. If the motor current is *I*, then its in-phase or power component is $I \cos \phi$ and reactive component is $I \sin \phi$.

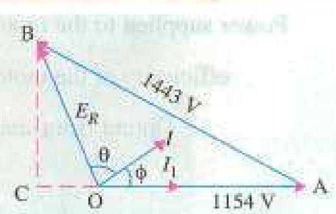


Fig. 38.37

Let $I \cos \phi = I_1$ and $I \sin \phi = I_2$ so that $I = (I_1 + jI_2)$

$$I \cos \phi = I_1 = 800{,}00 / \sqrt{3} = 231 \text{ A}$$

$$\text{Applied voltage / phase} = 2{,}000 / \sqrt{3} = 1{,}154 \text{ V}$$

$$\text{Induced e.m.f. / phase} = 2500 / \sqrt{3} = 1{,}443 \text{ V}$$

In Fig. 38.37 OA = 1154 V and

AB = 1443 V, OI leads OA by ϕ

$E_R = I Z_S$ and $\theta = \tan^{-1}(2.2 / 0.2) = 84.8°$

BC is $\perp AO$ produced.

Now,
$$\mathbf{E_R} = \mathbf{I Z_S} = (I_1 + jI_2)(0.2 + j2.2)$$
$$= (231 + jI_2)(0.2 + j2.2) = (46.2 - 2.2\, I_2) + j(508.2 + 0.2\, I_2)$$

Obviously, $OC = (46.2 - 2.2\, I_2)$; $BC = j(508.2 + 0.2\, I_2)$

From the right-angled ΔABC, we have

$$AB^2 = BC^2 + AC^2 = BC^2 + (AO + OC)^2$$

or
$$1443^2 = (508.2 + 0.2\, I_2)^2 + (1154 + 46.2 - 2.2\, I_2)^2$$

Solving the above quadratic equation, we get $I_2 = 71$ A

$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{231^2 + 71^2} = \mathbf{242 \text{ A}}$$

$$\text{p.f.} = I_1/I = 231/242 = \mathbf{0.95 \text{ (lead)}}$$

Example 38.21. *A 3 phase, 440-V, 50 Hz, star-connected synchronous motor takes 7.46 kW from the three phase mains. The resistance per phase of the armature winding is 0.5 ohm. The motor operates at a p. f. of 0.75 lag. Iron and mechanical losses amount to 500 watts. The excitation loss is 650 watts. Assume the source for excitation to be a separate one.*

Calculate. (i) armature current, (ii) power supplied to the motor, (iii) efficiency of the motor

(Amravati University 1999)

Solution. A 3 -phase synchronous motor receives power from two sources :

(*a*) 3-phase a. c. source feeding power to the armature.

(*b*) D.C. source for the excitation, feeding electrical power only to the field winding.

Thus, power received from the d. c. source is utilized only to meet the copper-losses of the field winding.

3 Phase a.c. source feeds electrical power to the armature for following components of power:

(*i*) Net mechanical power output from the shaft

(*ii*) Copper-losses in armature winding

(*iii*) Friction, and armature-core-losses.

In case of the given problem

$$\sqrt{3} \times I_a \times 440 \times 0.75 = 7460$$

$$I_a = 13.052 \text{ amp}$$

Total copper-loss in armature winding $= 3 \times 13.052^2 \times 0.50 = 255$ watts

Power supplied to the motor = 7460 + 650 = **8110 watts**

$$\text{efficiency of the motor} = \frac{\text{Output}}{\text{Input}}$$

Output from shaft = (Armature Input) − (Copper losses in armature winding) − (friction and iron losses)

= 7460 − 255 − 500 = 6705 watts

$$\text{Efficiency of the motor} = \frac{6705}{8110} \times 100\% = \mathbf{82.7\%}$$

Example 38.22. *Consider a 3300 V delta connected synchronous motor having a synchronous reactance per phase of 18 ohm. It operates at a leading pf of 0.707 when drawing 800 kW from mains. Calculate its excitation emf and the rotor angle (= delta), explaining the latter term.* **(Elect. Machines Nagpur Univ. 1993)**

Fig. 38.38

Solution. $\sqrt{3} \times 3300 \times I_a \times 0.707 = 800{,}000$

$\therefore$ Line current = 198 A, phase current, $I_a = 198 / \sqrt{3} = 114.3$ A;

$$Z_S = 18\,\Omega \,;\, I_a Z_S = 114.3 \times 18 = 2058 \text{ V}$$
$$\phi = \cos^{-1} 0.707;\ \phi = 45°;\ \theta = 90°;$$
$$\cos(\theta + \phi) = \cos 135° = -\cos 45° = -0.707$$

From Fig. 38.38, we find

$$E_b^2 = 3300^2 + 2058^2 - 2 \times 3300 \times 2058 \times -0.707$$
$$\therefore \quad E_b = \mathbf{4973\ V}$$

From $\Delta\, OAB$, we get 2058/sin α = 4973/sin 135°. Hence, α = **17°**

Example 38.23. *A 75-kW, 400-V, 4-pole, 3-phase star connected synchronous motor has a resistance and synchronous reactance per phase of 0.04 ohm and 0.4 ohm respectively. Compute for full-load 0.8 p.f. lead the open circuit e.m.f. per phase and mechanical power developed. Assume an efficiency of 92.5%.* **(Elect. Machines AMIE Sec. B 1991)**

Solution. Motor input = 75,000 / 0.925 = 81,080 Ω

Fig. 38.39

$I_a = 81{,}080/\sqrt{3} \times 400 \times 0.8 = 146.3 \text{ A}\,;\, Z_S = \sqrt{0.04^2 + 0.4^2} = 0.402\,\Omega$

$I_a Z_S = 146.3 \times 0.402 = 58.8$ V; $\tan\phi = 0.4 / 0.04 = 10$;
$\theta = 84.3°$; $\phi = \cos^{-1} 0.8$; $\phi = 36.9°$; $(\theta + \phi) = 121.2°$;

$V_{ph} = 400 / \sqrt{3} = 231$ V

As seen from Fig. 36.39,

$E_b^2 = 231^2 + 58.8^2 - 2 \times 231 \times 58.8 \times \cos 121.2$; E_b / phase = 266 V

Stator Cu loss for 3 phases = $3 \times 146.3^2 \times 0.04 = 2570$ W;

$N_s = 120 \times 50/40 = 1500$ r.p.m.

$P_m = 81080 - 2570 =$ **78510 W** ; $T_g = 9.55 \times 78510/1500 =$ **500 N-m.**

Example 38.24. *A 400-V, 3-phase, 50-Hz, Y-connected synchronous motor has a resistance and synchronous impedance of 0.5 Ω and 4 Ω per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is increased to 60 A, the field current remaining unchanged, calculate the gross torque developed and the new power factor.* **(Elect. Machines, AMIE Sec. B 1992)**

Solution. The conditions corresponding to the first case are shown in Fig. 38.40.

$$\text{Voltage/phase} = 400/\sqrt{3} = 231 \text{ V}\,;\, I_a Z_S = OB = 15 \times 4 = 60 \text{ V}$$
$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{4^2 - 0.5^2} = 3.968\,\Omega$$
$$\theta = \tan^{-1}(3.968/0.5) = \tan^{-1}(7.936) = 81.8°$$
$$E_b^2 = 231^2 + 60^2 - 2 \times 231 \times 60 \times \cos 81°\,48';\ E_b = 231\ V$$

It is obvious that motor is running with normal excitation because $E_b = V$

When the motor load is increased, the phase angle between the applied voltage and the induced (or back) e.m.f. is increased. Art (38.7). The vector diagram is as shown in Fig. 38.41.

Let ϕ be the new phase angle.

$$I_a Z_S = 60 \times 4 = 240 \text{ V}$$

$$\angle BOA = (81°48' - \phi).$$

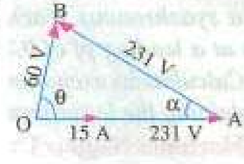


Fig. 38.40

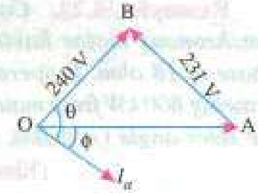


Fig. 38.41

Since the field current remains constant, the value of E_b remains the same.

$\therefore \quad 231^2 = 231^2 + 240^2 - 2 \times 231 \times 240 \cos (81°48' - \phi)$

$\therefore \quad \cos (81°48' - \phi) = 0.4325$ or $81°48' \quad \phi = 64°24'$

$\therefore \quad \phi = 81°48' - 64°24' = 17°24'$. New p.f. $= \cos 17°24' =$ **0.954 (lag)**

$$\text{Motor input} = \sqrt{3} \times 400 \times 60 \times 0.954 = 39{,}660 \text{ W}$$

$$\text{Total armature Cu loss} = 3 \times 60^2 \times 0.5 = 5{,}400 \text{ W}$$

Electrical power converted into mechanical power = 39,660 − 5,400 − 34,260 W

$N_S = 120 \times 50/6 = 1000$ r.p.m. $\quad T_g = 9.55 \times 34{,}260/1000 =$ **327 N-m**

Example 38.25. *A 400-V, 10 h.p. (7.46 kW), 3-phase synchronous motor has negligible armature resistance and a synchronous reactance of 10 W / phase. Determine the minimum current and the corresponding induced e.m.f. for full-load conditions. Assume an efficiency of 85%.*

(A.C. Machines-I, Jadavpur Univ. 1987)

Solution. The current is minimum when the power factor is unity *i.e.*, when $\cos \phi = 1$. The vector diagram is as shown in Fig. 38.42.

$$\text{Motor input} = 7460 / 0.85 = 8{,}775 \text{ W}$$

$$\text{Motor line current} = 8{,}775 / \sqrt{3} \times 400 \times 1 = \mathbf{12.67\ A}$$

$$\text{Impedance drop} = I_a X_S = 10 \times 12.67 = 126.7 \text{ V}$$

$$\text{Voltage / phase} = 400 / \sqrt{3} = 231 \text{ V}$$

$$E_b = \sqrt{231^2 + 126.7^2} = \mathbf{263.4\ V}$$

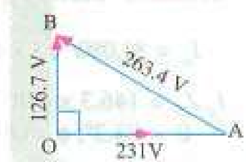


Fig. 38.42

Example 38.26. *A 400-V, 50-Hz, 3-phase, 37.5 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is (0.2 + j 1.6) ohm per phase. If the excitation of the motor is adjusted to give a leading power factor of 0.9, calculate the following for full load :*

(*i*) *the excitation e.m.f.*

(*ii*) *the total mechanical power developed* **(Elect.Machines, A.M.I.E. Sec. B, 1989)**

Solution. Motor input = 37.5/0.88 = 42.61 kW; $I_a = 42{,}610 / \sqrt{3} \times 400 \times 0.9 = 68.3$ A

$$V = 400 / \sqrt{3} = 231 \text{ V}; \; Z_S = 0.2 + j\,1.6 = 1.612 \angle 82.87°$$

$$E_R = I_a Z_S = 68.3 \times 1.612 = 110 \text{ V};$$

$$\phi = \cos^{-1}(0.9) = 25.84°$$

Now, $(\phi + \theta) = 25.84° + 82.87° = 108.71°$

$\cos (\phi + \theta) = \cos 108.71° = -0.32$

(*a*) $\therefore \; E_b^{\,2} = V^2 + E_R^{\,2} - 2\,VE_R \cos 108.71°$ or $E_b = 286$ V

Line value of excitation voltage $= \sqrt{3} \times 285 =$ **495 V**

Fig. 38.43

(*b*) From $\Delta\, OAB$, (Fig. 38.43) $E_R / \sin \alpha = E_b / \sin (\phi + \theta)$, $\alpha = 21.4°$

$$P_m = 3 \frac{E_b V}{Z_S} \sin \alpha = 3 \frac{286 \times 231}{1.612} \sin 21.4° = \mathbf{14{,}954\ W}$$

Example 38.27. *A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohm per phase and armature resistance negligible. When the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.*

(Elect. Machines, A.M.I.E., Sec. B, 1991)

Solution. $V = 6600 / \sqrt{3} = 3810$ V, $I_a = 1000 \times 10^3 / \sqrt{3} \times 6600 \times 0.8 = 109.3$ A

The phasor diagram is shown in Fig. 38.44. Since R_a is negligible, $\theta = 90^\circ$

$$E_R = I_a X_S = 109.3 \times 20 = 2186 \text{ V}$$

$$\cos = 0.8,\ \phi = 36.87^\circ$$

$$E_b^2 = V^2 + E_R^2 - 2E_b V \cos(90^\circ + 36.87^\circ);$$

$$E_b = 5410 \text{ V}$$

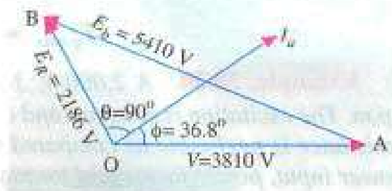


Fig. 38.44

Now, excitation has been kept constant but power has been increased to 1500 kW

$$\therefore \quad 3\,\frac{E_b V}{Z_S}\sin\alpha = P;\ 3 \times \frac{5410 \times 3810}{20}\sin\alpha = 1500 \times 10^3;\ \alpha = 29^\circ$$

Also,
$$\frac{E_b}{(\sin 90^\circ + \phi)} = \frac{V}{\sin[180^\circ - (\alpha + 90 + \phi)]} = \frac{V}{\cos(\alpha + \phi)}$$

or
$$\frac{E_b}{\cos\phi} = \frac{V}{\cos(\alpha + \phi)} \quad \text{or} \quad \frac{V}{E_b} = \frac{\cos(29^\circ + \phi)}{\cos\phi} = 0.3521$$

$$\therefore \quad \phi = 19.39^\circ,\ \cos\phi = \cos 19.39^\circ = \mathbf{0.94\ (lead)}$$

Example 38.28. *A 400-V, 50-Hz, 6-pole, 3-phase, Y-connected synchronous motor has a synchronous reactance of 4 ohm/phase and a resistance of 0.5 ohm/phase. On full-load, the excitation is adjusted so that machine takes an armature current of 60 ampere at 0.866 p.f. leading.*

Keeping the excitation unchanged, find the maximum power output. Excitation, friction, windage and iron losses total 2 kW. **(Electrical Machinery-III, Bangalore Univ. 1990)**

Solution. $V = 400/\sqrt{3} = 231$ V/phase; $Z_S = 0.5 + j4 = 4.03 \angle 82.9^\circ$; $\theta = 82.9^\circ$

$$I_a Z_S = 60 \times 4.03 = 242 \text{ V};\ \cos\phi = 0.866$$

$$\phi = 30^\circ \text{ (lead)}$$

As seen from Fig. 36.45,

$$E_b^2 = 231^2 + 242^2 - 2 \times 231 \times 242 \cos 112.9^\circ$$

$$E_b = \mathbf{394\ V}$$

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_{S^2}} = \frac{394 \times 231}{4.03} - \frac{394^2 \times 0.5}{4.03^2}$$

$$= 17{,}804 \text{ W/phase.} \qquad \text{–Art. 38.12}$$

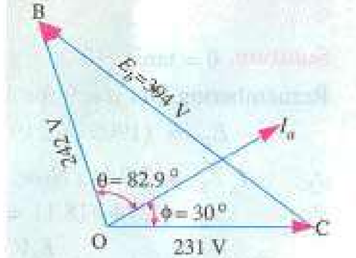


Fig. 38.45

Maximum power developed in armature for 3 phases

$$= 3 \times 17{,}804 = 52{,}412 \text{ W}$$

Net output $= 52{,}412 - 2{,}000 = 50{,}412$ W $=$ **50.4 kW**

Example 38.29. *A 6-pole synchronous motor has an armature impedance of 10 Ω and a resistance of 0.5 Ω. When running on 2,000 volts, 25-Hz supply mains, its field excitation is such that the e.m.f. induced in the machine is 1600 V. Calculate the maximum total torque in N-m developed before the machine drops out of synchronism.*

Solution. Assuming a three-phase motor,

$V = 2000$ V, $E_b = 1600$ V ; $R_a = 0.5\ \Omega$; $Z_S = 10\ \Omega$; $\cos\theta = 0.5/10 = 1/20$

Using equation (*iii*) of Art. 37-10, the total max. power for 3 phases is

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2}{Z_S}\cos\theta = \frac{2000 \times 1600}{10} - \frac{1600^2 \times 1}{10 \times 20} = 307{,}200 \text{ watt}$$

Now, $N_S = 120\, f/P = 120 \times 25/6 = 500$ r.p.m.

Let $T_{g\text{-}max}$ be the maximum gross torque, then

$$T_{g\,max} = 9.55 \times \frac{307200}{500} = \mathbf{5{,}868\ N\text{-}m}$$

Example. 38.30. *A 2,000-V, 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m. The excitation is constant and corresponds to an open-circuit terminal voltage of 2,000 V. The resistance is negligible as compared with synchronous reactance of 3 Ω per phase. Determine the power input, power factor and torque developed for an armature current of 200 A.*

(Elet. Engg.-I, Nagpur Univ. 1993)

Solution. Voltage/phase $= 2000/\sqrt{3} = 1150$ V

Induced e.m.f. $= 1150$ V — given

Impedance drop $= 200 \times 3 = 600$ V

As shown in Fig. 38.46, the armature current is assumed to lag behind V by an angle ϕ. Since R_a is negligible, $\theta = 90°$.

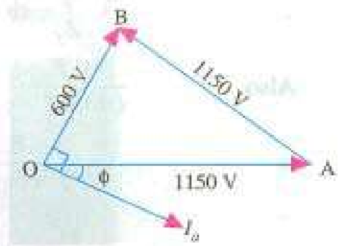


Fig. 38.46

$\angle BOA = (90° - \phi)$

Considering ΔBOA, we have

$$1150^2 = 1150^2 + 600^2 - 2 \times 600 \times 1150 \cos(90 - \phi°)$$

$\sin\phi = 0.2605$; $\phi = 16.2°$; p.f. $= \cos 16.2° = \mathbf{0.965\ (lag)}$

Power input $= \sqrt{3} \times 2{,}000 \times 200 \times 0.965 = \mathbf{668.5\ kW}$

$N_S = 1500$ r.p.m. $\therefore$ $T_g = 9.55 \times 66{,}850/1500 = \mathbf{4{,}255\ N\text{-}m.}$

Example 38.31. *A 3-φ, 3300-V, Y-connected synchronous motor has an effective resistance and synchronous reactance of 2.0 Ω and 18.0 Ω per phase respectively. If the open-circuit generated e.m.f. is 3800 V between lines, calculate (i) the maximum total mechanical power that the motor can develop and (ii) the current and p.f. at the maximum mechanical power.*

(Electrical Machines-III, Gujarat Univ. 1988)

Solution. $\theta = \tan^{-1}(18/2) = 83.7°$; $V_{ph} = 3300/\sqrt{3} = 1905$ V; $E_b = 3800/\sqrt{3} = 2195$ V

Remembering that $\alpha = \theta$ for maximum power development (Ar. 38-10)

$$E_R = (1905^2 + 2195^2 - 2 \times 1905 \times 2195 \times \cos 83.7°)^{1/2} = 2744 \text{ volt per phase}$$

$\therefore$ $I_a Z_S = 2{,}744$; Now, $Z_S = \sqrt{2^2 + 18^2} = 18.11\ \Omega$

$\therefore$ $I_a = 2744/18.11 = 152$ A/phase ; line current $= \mathbf{152\ A}$

$$(P_m)_{max} \text{ per phase} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{2195 \times 1905}{18.11} - \frac{2195^2 \times 2}{18.11^2}$$

$$= 230{,}900 - 29{,}380 = 201520 \text{ W per phase}$$

Maximum power for three phases that the motor can develop *in its armature*

$$= 201{,}520 \times 3 = \mathbf{604{,}560\ W}$$

Total Cu losses $= 3 \times 152^2 \times 2 = 138{,}700$ W

Motor input $= 604{,}560 + 138{,}700 = 743{,}260$ W

$\therefore$ $\sqrt{3} \times 3300 \times 152 \times \cos\phi = 743{,}260$ $\therefore$ $\cos\phi = \mathbf{0.855\ (lead).}$

Example 38.32. *The excitation of a 415-V, 3-phase, mesh-connected synchronous motor is such that the induced e.m.f. is 520 V. The impedance per phase is (0.5 + j4.0) ohm. If the friction and iron losses are constant at 1000 W, calculate the power output, line current, power factor and efficiency for maximum power output.* **(Elect. Machines-I, Madras Univ. 1987)**

Solution. As seen from Art. 38-12, for fixed E_b, V, R_a and X_S, maximum power is developed when $\alpha = \theta$.

Now, $\theta = \tan^{-1}(4/0.5) = \tan^{-1}(8) = 82.90° = \alpha$

$$E_R = \sqrt{415^2 + 520^2 - 2 \times 415 \times 520 \times \cos 82.9°} = 625 \text{ V per phase}$$

Now, $IZ_S = 625$; $Z_S = \sqrt{4^2 + 0.5^2} = 4.03\ \Omega$ $\therefore$ $I = 625/4.03 = 155$ A

Line current $= \sqrt{3} \times 155 = 268.5$ A

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{520 \times 415}{4.03} - \frac{520^2 \times 0.5}{16.25} = 45{,}230 \text{ W}$$

Max. power for 3 phases $= 3 \times 45{,}230 = 135{,}690$ W

Power output = power developed − iron and friction losses

$= 135{,}690 - 1000 = 134{,}690$ W = **134.69 kW**

Total Cu loss $= 3 \times 155^2 \times 0.5 = 36{,}080$ W

Total motor input $= 135{,}690 + 36{,}080 = 171{,}770$ W

$\therefore$ $\sqrt{3} \times 415 \times 268..5 \times \cos\phi = 171{,}770$; $\cos\phi =$ **0.89 (lead)**

Efficiency $= 134{,}690/171{,}770 = 0.7845$ or **78.45%**

Tutorial Problems 38.1

1. A 3-phase, 400-V, synchronous motor takes 52.5 A at a power factor of 0.8 leading. Calculate the power supplied and the induced e.m.f. The motor impedance per phase is (0.25 + j3.2) ohm.
[29.1 kW; 670V]

2. The input to a 11-kV, 3φ, Y-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are *1* Ω and 30 Ω respectively. Find (*a*) power supplied to the motor and (*b*) the induced e.m.f. for a p.f. of 0.8 leading.
[(*a*) 915 kW (*b*) 13kV] *(Grad. I.E.T.E. Dec. 1978)*

3. A 2,200-V, 3-phase, star-connected synchronous motor has a resistance of 0.6 Ω and a synchronous reactance of 6 Ω. Find the generated e.m.f. and the angular retardation of the motor when the input is 200 kW at (*a*) power factor unity and (*b*) power factor 0.8 leading.
[(*a*) 2.21 kV; 14.3° (*b*) 2.62 kV; 12.8°]

4. A 3-phase, 220-V, 50-Hz, 1500 r.p.m., mesh-connected synchronous motor has a synchronous impedance of 4 ohm per phase. It receives an input line current of 30 A at a leading power factor of 0.8. Find the line value of the induced e.m.f. and the load angle expressed in mechanical degrees.
If the mechanical load is thrown off without change of excitation, determine the magnitude of the current under the new conditions. Neglect losses. **[268 V; 6°, 20.6 A]**

5. A 400-V, 3-phase, Y-connected synchronous motor takes 3.73 kW at normal voltage and has an impedance of (1 + j8) ohm per phase. Calculate the current and p.f. if the induced e.m.f. is 460 V.
[6.28 A; 0.86 lead] *(Electrical Engineering, Madras Univ. April 1979)*

6. The input to 6600-V, 3-phase, star-connected synchronous motor is 900 kW. The synchronous reactance per phase is 20 Ω and the effective resistance is negligible. If the generated voltage is 8,900 V

(line), calculate the motor current and its power factor.

[Hint. See solved Ex. 38.17] (*Electrotechnics, M.S. Univ. April 1979*)

7. A 3-phase synchronous motor connected to 6,600-V mains has a star-connected armature with an impedance of (2.5 + *j*15) ohm per phase. The excitation of machine gives 7000 V. The iron, friction and excitation losses are 12 kW. Find the maximum output of the motor. **[153.68 kW]**

8. A 3300-V, 3-phase, 50-Hz, star-connected synchronous motor has a synchronous impedance of (2 + *j*15) ohm. Operating with an excitation corresponding to an e.m.f. of 2,500 V between lines, it just falls out of step at full-load. To what open-circuit e.m.f. will it have to be excited to stand a 50% excess torque. **[4 kV]**

9. A 6.6 kV, star-connected, 3-phase, synchronous motor works at constant voltage and constant excitation. Its synchronous impedance is (2.0 + *j* 20) per phase. When the input is 1000 kW, its power factor is 0.8 leading. Find the power factor when the input is increased to 1500 kW (solve graphically or otherwise).

[0.925 lead] (*AMIE Sec. B Advanced Elect. Machines (E-9) Summer 1991*)

10. A 2200-V, 373 kW, 3-phase, star-connected synchronous motor has a resistance of 0.3 Ω and a synchronous reactance of 3.0 Ω per phase respectively. Determine the induced e.m.f. per phase if the motor works on full-load with an efficiency of 94 per cent and a p.f. of 0.8 leading.

[1510 V] (*Electrical Machinery, Mysore Univ. 1992*)

11. The synchronous reactance per phase of a 3-phase star-connected 6600 V synchronous motor is 20 Ω. For a certain load, the input is 915 kW at normal voltage and the induced line e.m.f. is 8,942 V. Evaluate the line current and the p.f. Neglect resistance. **[97 A; 0.8258 (lead)]**

12. A synchronous motor has an equivalent armature reactance of 3.3 Ω. The exciting current is adjusted to such a value that the generated e.m.f. is 950 V. Find the power factor at which the motor would operate when taking 80 kW from a 800-V supply mains. **[0.965 leading]** (*City & Guilds, London*)

13. The input to an 11000 V, 3-phase star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohms. Find the power supplied to the motor and the induced electromotive force for a power factor of 0.8 leading.

[914.5 kW, 13 kV] (*Elect. Machines, A.M.I.E. Sec. B, 1990*)

14. A 400-V, 6-pole, 3-phase, 50-Hz, star-connected synchronous motor has a resistance and synchronous reactance of 0.5 ohm per phase and 4-ohm per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 60 A, the field current remaining unchanged, find the gross torque developed, and the new power factor. **[354 Nm; 0.93]** (*Elect. Engg. AMIETE Dec. 1990*)

15. The input to a 11,000-V, 3-phase, star-connected synchronous motor is 60 amperes. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find the power supplied to the motor and the induced e.m.f. for power factor of 0.8 (*a*) leading and (*b*) lagging.

[915 kW (a) 13 kV (*b*) 9.36 kV] (*Elect. Machines-II, South Gujarat Univ. 1981*)

16. Describe with the aid of a phasor diagram the behaviour of a synchronous motor starting from no-load to the pull-out point.

What is the output corresponding to a maximum input to a 3-ϕ delta-connected 250-V, 14.92 kW synchronous motor when the generated e.m.f. is 320 V ? The effective resistance and synchronous reactance per phase are 0.3 Ω and 4.5 Ω respectively. The friction, windage, iron and excitation losses total 800 watts and are assumed to remain constant. Give values for (*i*) output (*ii*) line current (*iii*) p.f.

[(*i*) 47.52 kW (*ii*) 161 A (*iii*) 0.804] (*Elect. Machines, Indore Univ. Feb. 1982*)

17. A synchronous motor takes 25 kW from 400 V supply mains. The synchronous reactance of the motor is 4 ohms. Calculate the power factor at which the motor would operate when the field excitation is so adjusted that the generated EMF is 500 volts.

[0.666 Leading] (*Rajiv Gandhi Technical University, Bhopal, 2000*)

38.17. Effect of Excitation on Armature Current and Power Factor

The value of excitation for which back e.m.f. E_b is equal (in magnitude) to applied voltage V is known as 100% excitation. We will now discuss what happens when motor is either over-excited or under-exicted although we have already touched this point in Art. 38-8.

Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected).

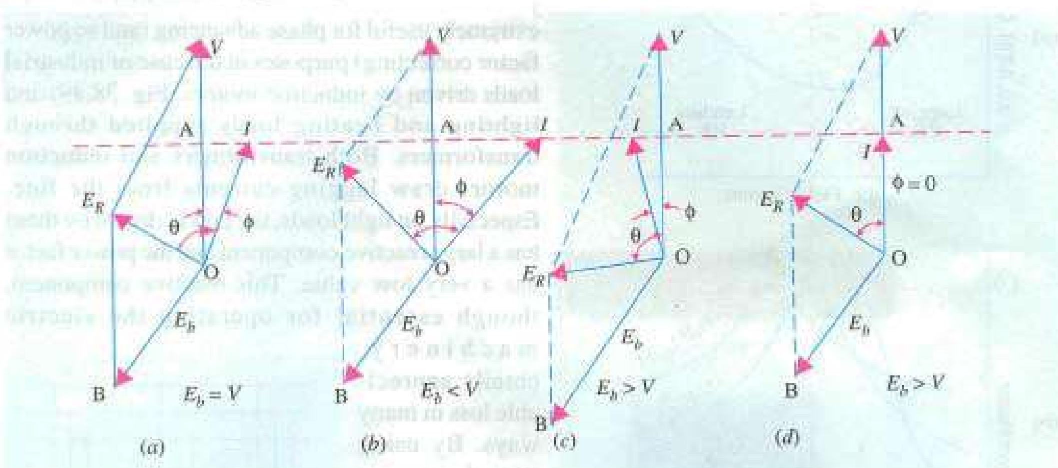


Fig. 38.47

Fig. 38.47 (*a*) shows the case for 100% excitation *i.e.*, when $E_b = V$. The armature current I lags behind V by a small angle ϕ. Its angle θ with E_R is fixed by stator constants *i.e.* $\tan \theta = X_S / R_a$.

In Fig. 38.47 (*b*)* excitation is less than 100% *i.e.*, $E_b < V$. Here, E_R is advanced clockwise and so is armature current (because it lags behind E_R by fixed angle θ). We note that the magnitude of I is increased but its power factor is decreased (ϕ has increased). Because input as well as V are constant, hence the power component of I *i.e.*, $I \cos \phi$ remains the same as before, but wattless component $I \sin \phi$ is increased. Hence, as excitation is decreased, I will increase but p.f. will decrease so that power component of I *i.e.*, $I \cos \phi = OA$ will remain constant. In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.

Incidentally, it may be noted that when field current is reduced, the motor pull-out torque is also reduced in proportion.

Fig. 38.47 (*c*) represents the condition for overexcited motor *i.e.* when $E_b > V$. Here, the resultant voltage vector E_R is pulled anticlockwise and so is I. It is seen that now motor is drawing a leading current. It may also happen for some value of excitation, that I may be in phase with V *i.e.*, p.f. is unity [Fig. 38.47 (*d*)]. At that time, the current drawn by the motor would be ***minimum***.

Two important points stand out clearly from the above discussion :

(*i*) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of I with excitation are shown in Fig. 38.48 (*a*) which are known as 'V' curves because of their shape.

(*ii*) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly. When over-excited, motor runs with leading p.f. and with lagging p.f. when under-excited. In between, the p.f. is unity. The variations of *p.f.* with excitation

* These are the same diagrams as given in Fig. 38.7 and 8 expect that vector for V has been shown vertical.

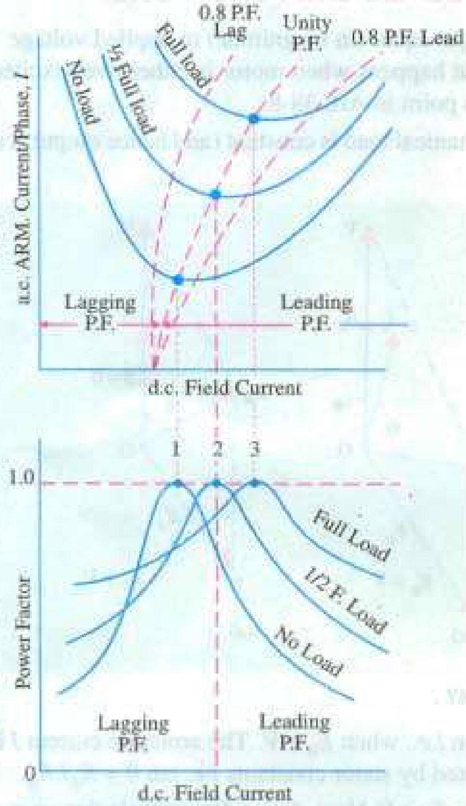


Fig. 38.48

are shown in Fig. 38.48 (*b*). The curve for p.f. looks like inverted '*V*' curve. It would be noted that ***minimum armature current corresponds to unity power factor***

It is seen (and it was pointed out in Art. 38.1) that an over-excited motor can be run with leading power factor. This property of the motor renders it extremely useful for phase advancing (and so power factor correcting) purposes in the case of industrial loads driven by induction motors (Fig. 38.49) and lighting and heating loads supplied through transformers. Both transformers and induction motors draw lagging currents from the line. Especially on light loads, the power drawn by them has a large reactive component and the power factor has a very low value. This reactive component, though essential for operating the electric machinery, entails appreciable loss in many ways. By using synchronous motors in conjunction with induction motors and transformers, the lagging reactive power required by the latter is supplied locally by the leading reactive component taken by the former, thereby relieving the line and generators of much of the reactive component. Hence, they now supply only the active component of the load current. When used in this way, a synchronous motor is called a ***synchronous capacitor***, because it draws, like a capacitor, leading current from the line. Most synchronous capacitors are rated between 20 MVAR and 200 MVAR and many are hydrogen-cooled.

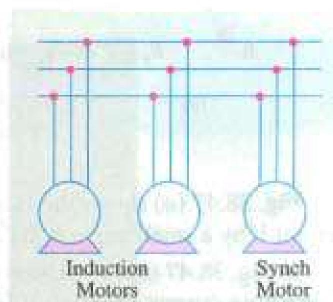


Fig. 38.49

Inductor motor

Example 38.33. *Describe briefly the effect of varying excitation upon the armature current and p.f. of a synchronous motor when input power to the motor is maintained constant.*

A 400-V, 50-Hz, 3-ϕ, 37.3 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is (0.2 + j 1.6) Ω per phase. If the excitation of the motor is adjusted to give a leading p.f. of 0.9, calculate for full-load (a) the induced e.m.f. (b) the total mechanical power developed.

Solution. Voltage / phase = $400/\sqrt{3} = 231$ V;

$$Z_S = \sqrt{(1.6^2 + 0.2^2)} = 1.61\ \Omega;$$

$$\text{Full-load current} = \frac{37,300}{\sqrt{3} \times 400 \times 0.88 \times 0.9}$$

$$= 68\text{ A}$$

$\therefore \quad I Z_S = 1.61 \times 68 = 109.5$ V

Fig. 38.50

With reference to Fig. 38.50

$\tan\theta = 1.6 / 0.2 = 8,\ \theta = 82°54'$

$\cos\phi = 0.9,\ \phi = 25°50'.$

$\therefore \quad (\theta + \phi) = 82°54' + 25°50' = 108°44'$

Now $\cos 108°44' = -0.3212$

(*a*) In $\Delta\, OAB$, $E_b^2 = 231^2 + 109.5^2 - 2 \times 231 \times 109.5 \times (-0.3212) = 285.6^2$; $E_b = 235.6$ V

Line value of $E_b = \sqrt{3} \times 285.6 =$ **495 V**

(*b*) Total motor input = 37,300 / 0.88 = 42,380 W

Total Cu losses = $3 \times I^2 R_a = 3 \times 682 \times 0.2 = 2,774$ W

$\therefore$ Electric power converted into mechanical power = 42,380 − 2,774 = **39.3 kW.**

Example 38.34. *A 3-φ, star-connected synchronous motor takes 48 kW at 693 V, the power factor being 0.8 lagging. The induced e.m.f. is increased by 30%, the power taken remaining the same. Find the current and the p.f. The machine has a synchronous reactance of 2 W per phase and negligible resistance.*

Solution. Full-load current

$= 48.000/\sqrt{3} \times 693 \times 0.8 = 50$ A

Voltage/phase = $693/\sqrt{3} = 400$ V

$Z_S = X_S = 2\ \Omega \quad \therefore \quad IZ_S = 50 \times 2 = 100$ V

$\tan\theta = 2/0 = \infty \quad \therefore\ \theta = 90°;\ \cos\phi = 0.8,\ \sin\phi = 0.6$

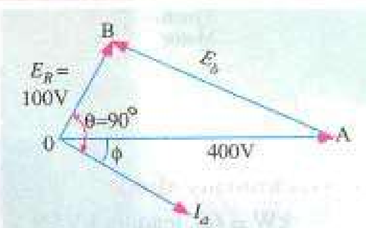


Fig. 38.51

The vector diagram is shown in Fig. 38.51. In $\Delta\, OAB$,

$E_b^2 = 400^2 + 100^2 - 2 \times 400 \times 100 \times \cos(90° - \phi)$

$= 400^2 + 100^2 - 2 \times 400 \times 100 \times 0.6 = 349^2 \quad \therefore \quad E_b = 349$ V.

The vector diagram for increased e.m.f. is shown in Fig. 38.52. Now, $E_b = 1.3 \times 349 = 454$ V. It can be safely assumed that in the second case, current is leading V by some angle ϕ'.

Let the new current and the leading angle of current by I' and ϕ' respectively. As power input remains the same and V is also constant, $I \cos\phi$ should be the same far the same input.

$\therefore \quad I\cos\phi = 50 \times 0.8 = 40 = I'\cos\phi'$

In $\quad \Delta\, ABC,\ AB^2 = AC^2 + BC^2$

Now $\quad BC = I' X_S \cos\phi' \quad (\because OB = I' X_S)$

$= 40 \times 2 = 80$ V

$\therefore \quad 454^2 = AC^2 + 80^2$ or $AC = 447$ V

$\therefore \quad OC = 447 - 400 = 47$ V

$\therefore \quad \tan\phi' = 47/80,\ \phi' = 30°26'$

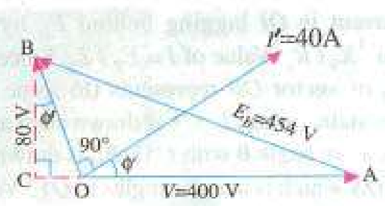


Fig. 38.52

$\therefore$ New $\quad$ p.f. $= \cos 30°26'$

$=$ **0.8623 (leading)**

Also, $\quad I'\cos\phi' = 40 \quad \therefore \quad I' = 40/0.8623 =$ **46.4 A.**

Example 38.35. *A synchronous motor absorbing 60 kW is connected in parallel with a factory load of 240 kW having a lagging p.f. of 0.8. If the combined load has a p.f. of 0.9, what is the value of the leading kVAR supplied by the motor and at what p.f. is it working ?*

(Electrical Engineering-II, Banglore Univ. 1990)

Solution. Load connections and phase relationships are shown in Fig. 38.53.

Total load = 240 + 60 = 300 kW; combined p.f. = 0.9 (lag)

ϕ = 25.8°, tan ϕ = 0.4834, combined kVAR = 300 × 0.4834 = 145(lag)

Factory Load

$\cos \phi_L$ = 0.8, ϕ_L = 36.9°, tan ϕ_L = 0.75, load kVAR = 240 × 0.75 = 180 (lag)

or load kVA = 240/0.8 = 300, kVAR = 300 × sin ϕ_L = 300 × 0.6 = 180

∴ *leading* kVAR supplied by synchronous motor = 180 − 145 = **35.**

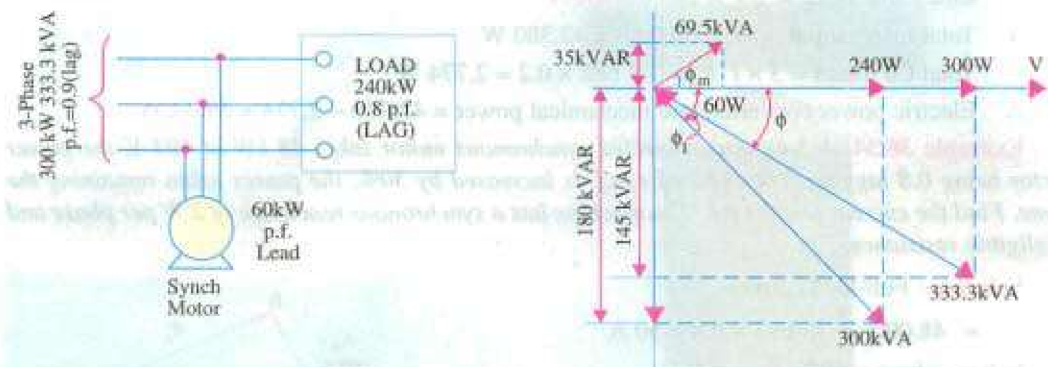


Fig. 38.53

For Synchronous Motor

kW = 60, leading kVAR = 35, tan ϕ_m = 35/60 ; ϕ_m = 30.3° ; cos 30.3° = 0.863

∴ motor p.f. = **0.863 (lead).** Incidentally, motor kVA = $\sqrt{60^2 + 35^2}$ = **69.5.**

38.18. Constant-power Lines

In Fig. 38.54, *OA* represents applied voltage / phase of the motor and *AB* is the back e.m.f. / phase, E_b. *OB* is their resultant voltage E_R. The armature current is *OI* lagging behind E_R by an angle $\theta = \tan^{-1} X_S / R_a$. Value of $I = E_R / Z_S$. Since Z_S is constant, E_R or vector *OB* represents (to some suitable scale) the main current *I*. *OY* is drawn at an angle ϕ with *OB* (or at an angle θ with *CA*). *BL* is drawn perpendicular to *OX* which is at right angles to *OY*. Vector *OB*, when referred to *OY*, also represents, on a different scale, the current both in magnitude and phase.

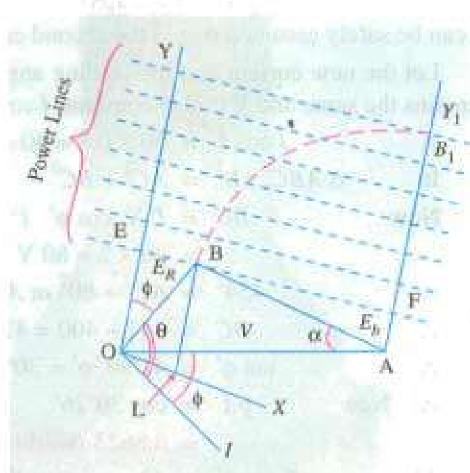


Fig. 38.54

Hence, $OB \cos \phi = I \cos \phi = BL$

The power input / phase of the motor

$= VI \cos \phi = V \times BL.$

As V is constant, power input is dependent on BL. If motor is working with a constant intake, then locus of B is a straight line ∥ to OX and ⊥ to OY *i.e.* line EF for which BL is constant. Hence, EF, represents a constant-power input line for a given voltage but varying excitation. Similarly, a series of such parallel lines can be drawn each representing a definite power intake of the motor. As regards these constant-power lines, it is to be noted that

1. for equal increase in intake, the power lines are parallel and equally-spaced
2. zero power line runs along OX
3. the perpendicular distance from B to OX (or zero power line) represents the motor intake
4. If excitation is fixed *i.e.* AB is constant in length, then as the load on motor is increased, increases. In other words, locus of B is a circle with radius = AB and centre at A. With increasing load, B goes on to lines of higher power till point B_1 is reached. Any further increase in load on the motor will bring point B down to a lower line. It means that as load increases beyond the value corresponding to point B_1, the motor intake decreases which is impossible. The area to the right of AY_1 represents unstable conditions. For a given voltage and excitation, the maximum power the motor can develop, is determined by the location of point B_1 beyond which the motor pulls out of synchronism.

38.19. Construction of V-curves

The V-curves of a synchornous motor show how armature current varies with its field current when motor ***input is kept constant***. These are obtained by plotting a.c. armature current against d.c. field current while motor input is kept constant and are so called because of their shape (Fig. 38.55). There is a family of such curves, each corresponding to a definite power intake.

In order to draw these curves experimentally, the motor is run from constant voltage and constant-frequency bus-bars. Power input to motor is kept constant at a definite value. Next, field current is increased in small steps and corresponding armature currents are noted. When plotted, we get a V-curve for a particular constant motor input. Similar curves can be drawn by keeping motor input constant at different values. A family of such curves is shown in Fig. 38.55.

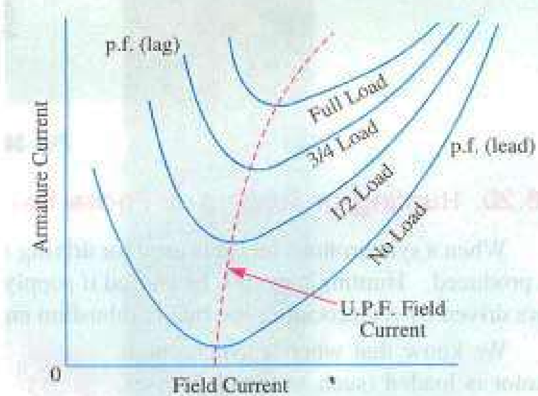


Fig. 38.55

Detailed procedure for graphic construction of V-curves is given below :

1. First, constant-power lines are drawn as discussed in Art. 38.14.
2. Then, with A as the centre, concentric circles of different radii AB, AB_1, AB_2, etc. are drawn where AB, AB_1, AB_2, etc., are the back e.m.fs corresponding to different excitations. The intersections of these circles with lines of constant power give positions of the working points for specific loads and excitations (hence back e.m.fs). The vectors OB, OB_1, OB_2 etc., represent different values of E_R (and hence currents) for different excitations. Back e.m.f. vectors AB, AB_1 etc., have not been drawn purposely in order to avoid confusion (Fig. 38.56).

3. The different values of back e.m.fs like AB, AB_1, AB_2, etc., are projected on the magnetisation and corresponding values of the field (or exciting) amperes are read from it.
4. The field amperes are plotted against the corresponding armature currents, giving us 'V' curves.

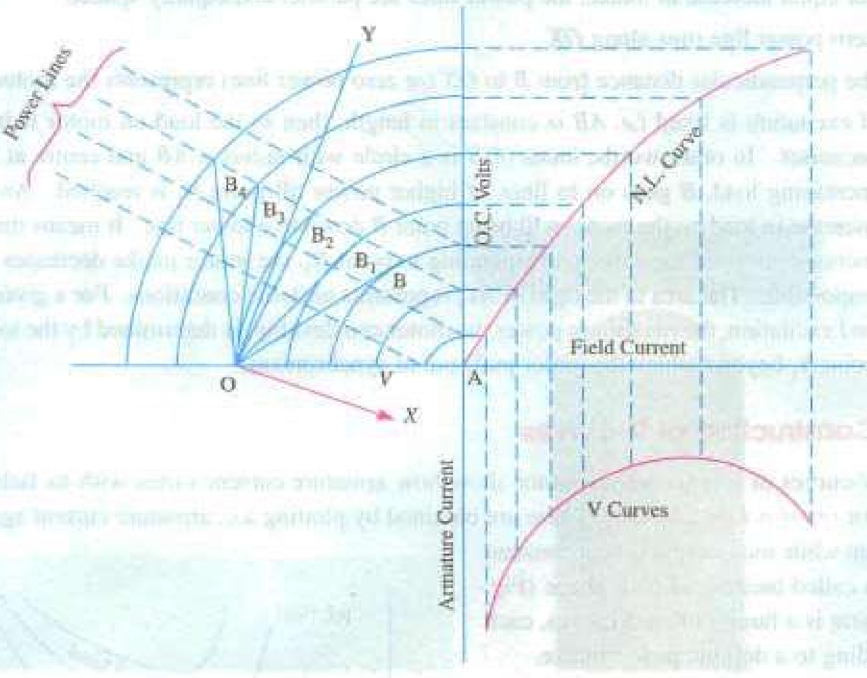


Fig. 38.56

38.20. Hunting or Surging or Phase Swinging

When a synchronous motor is used for driving a varying load, then a condition known as hunting is produced. Hunting may also be caused if supply frequency is pulsating (as in the case of generators driven by reciprocating internal combustion engines).

We know that when a synchronous motor is loaded (such as punch presses, shears, compressors and pumps etc.), its rotor falls back in phase by the coupling angle α. As load is progressively increased, this angle also increases so as to produce more torque for coping with the increased load. If now, there is sudden decrease in the motor load, the motor is immediately pulled up or advanced to a new value of α corresponding to the new load. But in this process, the rotor overshoots and hence is again pulled back. In this way, the rotor starts oscillating (like a pendulum) about its new position of

Fig. 38.57

Salient - poled squirrel cage motor

equilibrium corresponding to the new load. If the time period of these oscillations happens to be equal to the natural time period of the machine (refer Art. 37.36) then mechanical resonance is set up. The amplitude of these oscillations is built up to a large value and may eventually become so great as to throw the machine out of synchronism. To stop the build-up of these oscillations, dampers or damping grids (also known as squirrel-cage winding) are employed. These dampers consist of short-circuited Cu bars embedded in the faces of the field poles of the motor (Fig. 38.57). The oscillatory motion of the rotor sets up eddy currents in the dampers which flow in such a way as to suppress these oscillations.

But it should be clearly understood that dampers do not completely prevent hunting because their operation depends upon the presence of some oscillatory motion. However, they serve the additional purpose of making the synchronous motor self-starting.

38.21. Methods of Starting

As said above, almost all synchronous motors are equipped with dampers or squirrel cage windings consisting of Cu bars embedded in the pole-shoes and short-circuited at both ends. Such a motor starts readily, acting as an induction motor during the starting period. The procedure is as follows :

The line voltage is applied to the armature (stator) terminals and the field circuit is left *unexcited*. Motor starts as an induction motor and while it reaches nearly 95% of its synchronous speed, the d.c. field is excited. At that moment the stator and rotor poles get engaged or interlocked with each other and hence pull the motor into synchronism.

However, two points should be noted :*

1. At the beginning, when voltage is applied, the rotor is stationary. The rotating field of the stator winding induces a very large e.m.f. in the rotor during the starting period, though the value of this e.m.f. goes on decreasing as the rotor gathers speed.

   Normally, the field windings are meant for 110-V (or 250 V for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.

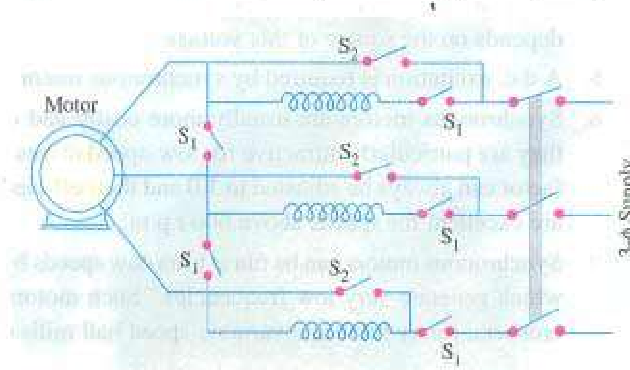


Fig. 38.58

2. When full line voltage is switched on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor. In some cases, this may not be objectionable but where it is, the applied voltage at starting, is reduced by using auto-transformers (Fig. 38.58). However, the voltage should not be reduced to a very low value because the starting torque of an induction motor varies approximately as the square of the applied voltage. Usually, a value of 50% to 80% of the full-line voltage is satisfactory.

   Auto-transformer connections are shown in Fig. 38.58. For reducing the supply voltage, the switches S_1 are closed and S_2 are kept open. When the motor has been speeded-up, S_2 are closed and S_1 opened to cut out the transformers.

38.22. Procedure for Starting a Synchronous Motor

While starting a modern synchronous motor provided with damper windings, following procedure is adopted.

1. First, main field winding is short-circuited.
2. Reduced voltage with the help of auto-transformers is applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak d.c. excitation is applied by removing the short-circuit on the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the auto-transformers.
5. The motor may be operated at any desired power factor by changing the d.c. excitation.

38.23. Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.
4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

38.24. Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation

(a) Power factor correction

Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors (Fig. 38.49) and other devices having lagging p.f. such as welders and flourescent lights etc.

(b) Constant-speed applications

Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc.

Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

(c) Voltage regulation

The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor with a field regulator (for varying its excitation), this voltage rise can be controlled.

When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.

QUESTIONS AND ANSWERS ON SYNCHRONOUS MOTORS

Q. 1. Does change in excitation affect the synchronous motor speed ?

Ans. No.

Q. 2. The power factor ?

Ans. Yes.

Q. 3. How ?

Ans. When over-excited, synchronous motor has leading power factor. However, when underexcited, it has lagging power factor.

Q. 4. For what service are synchronous motors especially suited ?

Ans. For high voltage service.

Q. 5. Which has more efficiency; synchronous or induction motor ?

Ans. Synchronous motor.

Q. 6. Mention some specific applications of synchronous motor ?

Ans. 1. constant speed load service 2. reciprocating compressor drives 3. power factor correction 4. voltage regulation of transmission lines.

Q. 7. What is a synchronous capacitor ?

Ans. An overexcited synchronous motor is called synchronous capacitor, because, like a capacitor, it takes a leading current.

Q. 8. What are the causes of faulty starting of a synchronous motor ?

Ans. It could be due to the following causes :

1. voltage may be too low – at least half voltage is required for starting
2. there may be open-circuit in one phase – due to which motor may heat up
3. static friction may be large – either due to high belt tension or too tight bearings
4. stator windings may be incorrectly connected
5. field excitation may be too strong.

Q. 9. What could be the reasons if a synchronous motor fails to start ?

Ans. It is usually due to the following reasons :

1. voltage may be too low
2. some faulty connection in auxiliary apparatus
3. too much starting load
4. open-circuit in one phase or short-circuit
5. field excitation may be excessive.

Q. 10. A synchronous motor starts as usual but fails to develop its full torque. What could it be due to ?

Ans. **1.** exciter voltage may be too low **2.** field spool may be reversed **3.** there may be either open-circuit or short-circuit in the field.

Q. 11. Will the motor start with the field excited ?

Ans. No.

Q. 12. Under which conditions a synchronous motor will fail to pull into step ?

Ans. **1.** no field excitation **2.** excessive load **3.** excessive load inertia

OBJECTIVE TESTS – 38

1. In a synchronous motor, damper winding is provided in order to
   (*a*) stabilize rotor motion
   (*b*) suppress rotor oscillations
   (*c*) develop necessary starting torque
   (*d*) both (*b*) and (*c*)
2. In a synchronous motor, the magnitude of stator back e.m.f. E_b depends on
   (*a*) speed of the motor
   (*b*) load on the motor
   (*c*) both the speed and rotor flux
   (*d*) d.c. excitation only
3. An electric motor in which both the rotor and stator fields rotates with the same speed is called a/an ....motor.
   (*a*) d.c.
   (*b*) chrage
   (*c*) synchronous
   (*d*) universal
4. While running, a synchronous motor is compelled to run at synchronous speed because of
   (*a*) damper winding in its pole faces
   (*b*) magnetic locking between stator and rotor poles
   (*c*) induced e.m.f. in rotor field winding by stator flux
   (*d*) compulsion due to Lenz's law
5. The direction of rotation of a synchronous motor can be reversed by reversing
   (*a*) current to the field winding
   (*b*) supply phase sequence
   (*c*) polarity of rotor poles
   (*d*) none of the above

6. When running under no-load condition and with normal excitation, armature current I_a drawn by a synchronous motor
   (*a*) leads the back e.m.f. E_b by a small angle
   (*b*) is large
   (*c*) lags the applied voltage V by a small angle
   (*d*) lags the resultant voltage E_R by 90°.
7. The angle between the synchronously-rotating stator flux and rotor poles of a synchronous motor is called........ angle.
   (*a*) synchronizing
   (*b*) torque
   (*c*) power factor
   (*d*) slip
8. If load angle of a 4-pole synchronous motor is 8° (elect), its value in mechanical degrees is ....
   (*a*) 4
   (*b*) 2
   (*c*) 0.5
   (*d*) 0.25
9. The maximum value of torque angle a in a synchronous motor is ....degrees electrical.
   (*a*) 45
   (*b*) 90
   (*c*) between 45 and 90
   (*d*) below 60
10. A synchronous motor running with normal excitation adjusts to load increases *essentially* by increase in its
   (a) power factor
   (b) torque angle
   (c) back e.m.f.
   (d) armature current.
11. When load on a synchronous motor running with normal excitation is increased, armature current drawn by it increases because
   (a) back e.m.f. E_b becomes less than applied voltage V
   (*b*) power factor is decreased
   (*c*) net resultant voltage E_R in armature is increased
   (*d*) motor speed is reduced
12. When load on a normally-excited synchronous motor is increased, its power factor tends to
   (*a*) approach unity
   (*b*) become increasingly lagging
   (*c*) become increasingly leading
   (*d*) remain unchanged.
13. The effect of increasing load on a synchronous motor running with normal excitation is to
   (*a*) increase both its I_a and p.f.
   (*b*) decrease I_a but increase p.f.
   (*c*) increase I_a but decrease p.f.
   (*d*) decrease both I_a and p.f.
14. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is increased, its torque angle must necessarily
   (*a*) decrease
   (*b*) increase
   (*c*) remain constant
   (*d*) become twice the no-load value.
15. If the field of a synchronous motor is under-excited, the power factor will be
   (*a*) lagging (*b*) leading
   (*c*) unity (*d*) more than unity
16. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is decreased from its normal value, it leads to
   (*a*) increase in but decrease in E_b
   (*b*) increase in E_b but decrease in I_a
   (*c*) increase in both I_a and p.f. which is lagging
   (*d*) increase in both I_a and ϕ
17. A synchronous motor connected to infinite bus-bars has at *constant* full-load, 100% excitation and unity p.f. On changing the excitation only, the armature current will have
   (*a*) leading p.f. with under-excitation
   (*b*) leading p.f. with over-excitation
   (*c*) lagging p.f. with over-excitation
   (*d*) no change of p.f.

   (Power App.-II, Delhi Univ. Jan 1987)
18. The *V*-curves of a synchronous motor show relationship between
   (*a*) excitation current and back e.m.f.
   (*b*) field current and p.f.
   (*c*) d.c. field current and a.c. armature current
   (*d*) armature current and supply voltage.

19. When load on a synchronous motor is increased, its armature currents is increas- ed provided it is
    (*a*) normally-excited
    (*b*) over-excited
    (*c*) under-excited
    (*d*) all of the above
20. If main field current of a salient-pole synchronous motor fed from an infinite bus and running at no-load is reduced to zero, it would
    (*a*) come to a stop
    (*b*) continue running at synchronous speed
    (*c*) run at sub-synchronous speed
    (*d*) run at super-synchronous speed
21. In a synchronous machine when the rotor speed becomes more than the synchronous speed during hunting, the damping bars develop
    (*a*) synchronous motor torque
    (*b*) d.c. motor torque
    (*c*) induction motor torque
    (*d*) induction generator torque

    (Power App.-II, Delhi Univ. Jan. 1987)
22. In a synchronous motor, the rotor Cu losses are met by
    (*a*) motor input
    (*b*) armature input
    (*c*) supply lines
    (*d*) d.c. source
23. A synchronous machine is called a doubly-excited machine because
    (*a*) it can be overexcited
    (*b*) it has two sets of rotor poles
    (*c*) both its rotor and stator are excited
    (*d*) it needs twice the normal exciting current.
24. Synchronous capacitor is
    (*a*) an ordinary static capacitor bank
    (*b*) an over-excited synchronous motor driving mechanical load
    (*c*) an over-excited synchronous motor running without mechanical load
    (*d*) none of the above 623

    (Elect. Machines, A.M.I.E. Sec. B, 1993)

ANSWERS

1. *d*	2. *d*	3. *c*	4. *b*	5. *b*	6. *c*	7. *b*	8. *a*	9. *b*	10. *d*	11. *c*
12. *b*	13. *c*	14. *a*	15. *a*	16. *d*	17. *b*	18. *c*	19. *d*	20. *b*	21. *d*	22. *d*
23. *c*	24. *c*									

C H A P T E R 39

SPECIAL MACHINES

Learning Objectives

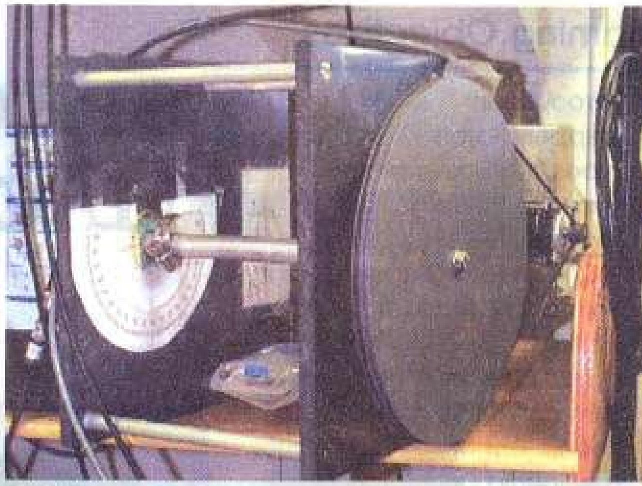
- Introduction
- Stepper Motors
- Types of Stepper Motors
- Variable Reluctance Stepper Motors
- Multi-stack VR Stepper Motor
- Permanent-Magnet Stepping Motor
- Hybrid Stepper Motor
- Summary of Stepper Motors
- Permanent-Magnet DC Motor
- Low-inertia DC Motors
- Shell-type Low-intertia DC Motor
- Printed-circuit (Disc) DC Motor
- Permanent-Magnet Synchronous Motors
- Synchros
- Types of Synchros
- Applications of Synchros
- Control Differential Transmitter
- Control Differential Receiver
- Switched Reluctance Motor
- Comparison between VR Stepper Motor and SR Motor
- The Resolver
- Servomotors
- DC Servomotors
- AC Servomotors

Stepper motor

39.1. Introduction

This chapter provides a brief introduction to electrical machines which have special applications. It includes machines whose stator coils are energized by electronically switched currents. The examples are: various types of stepper motors, brushless d.c. motor and switched reluctance motor etc. There is also a brief description of d.c./a.c. servomotors, synchro motors and resolvers. These motors are designed and built primarily for use in feedback control systems.

39.2. Stepper Motors



Stepper Motor

These motors are also called stepping motors or step motors. The name stepper is used because this motor rotates through a fixed angular step in response to each input current pulse received by its controller. In recent years, there has been wide-spread demand of stepping motors because of the explosive growth of the computer industry. Their popularity is due to the fact that they can be controlled directly by computers, microprocessors and programmable controllers.

As we know, industrial motors are used to convert electric energy into mechanical energy but they cannot be used for precision positioning of an object or precision control of speed without using closed-loop feedback. Stepping motors are ideally suited for situations where either precise positioning or precise speed control or both are required in automation systems.

Apart from stepping motors, other devices used for the above purposes are synchros and resolvers as well as dc/ac servomotors (discussed later).

The unique feature of a stepper motor is that its output shaft rotates in a series of discrete angular intervals or steps, one step being taken each time a command pulse is received. When a definite number of pulses are supplied, the shaft turns through a definite known angle. This fact makes the motor well-suited for open-loop position control because no feedback need be taken from the output shaft.

Such motors develop torques ranging from 1 μN-m (in a tiny wrist watch motor of 3 mm diameter) upto 40 N-m in a motor of 15 cm diameter suitable for machine tool applications. Their power output ranges from about 1 W to a maximum of 2500 W. The only moving part in a stepping motor is its rotor which has no windings, commutator or brushes. This feature makes the motor quite robust and reliable.

Step Angle

The angle through which the motor shaft rotates for each command pulse is called the step angle β. Smaller the step angle, greater the number of steps per revolution and higher the resolution or accuracy of positioning obtained. The step angles can be as small as 0.72° or as large as 90°. But the most common step sizes are 1.8°, 2.5°, 7.5° and 15°.

The value of step angle can be expressed either in terms of the rotor and stator poles (teeth) N_r and N_s respectively or in terms of the number of stator phases (m) and the number of rotor teeth.

$$\beta = \frac{(N_s - N_r)}{N_s \cdot N_r} \times 360^\circ$$

or

$$\beta = \frac{360^\circ}{m N_r} = \frac{360^\circ}{\text{No. of stator phases} \times \text{No. of rotor teeth}}$$

For example, if $N_s = 8$ and $N_r = 6$, $\beta = (8 - 6) \times 360 / 8 \times 6 = 15^\circ$

Resolution is given by the number of steps needed to complete one revolution of the rotor shaft. Higher the resolution, greater the accuracy of positioning of objects by the motor

$\therefore$ Resolution = No. of steps / revolution = $360^\circ / \beta$

A stepping motor has the extraordinary ability to operate at very high stepping rates (upto 20,000 steps per second in some motors) and yet to remain fully in synchronism with the command pulses. When the pulse rate is high, the shaft rotation seems continuous. Operation at high speeds is called 'slewing'. When in the slewing range, the motor generally emits an audible whine having a fundamental frequency equal to the stepping rate. If f is the stepping frequency (or pulse rate) in pulses per second (pps) and β is the step angle, then motor shaft speed is given by

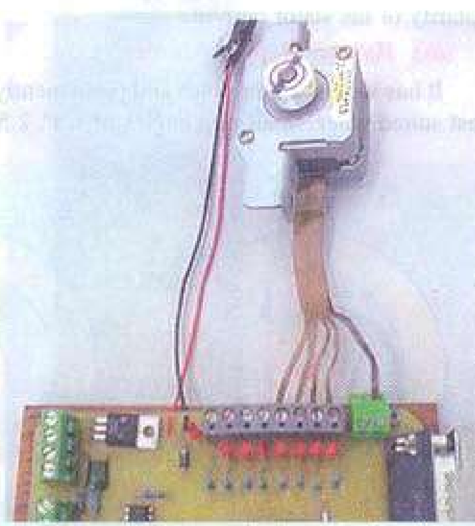
$$n = \beta \times f / 360 \text{ rps} = \text{pulse frequency resolution}$$

If the stepping rate is increased too quickly, the motor loses synchronism and stops. Same thing happens if when the motor is slewing, command pulses are suddenly stopped instead of being progressively slowed.

Stepping motors are designed to operate for long periods with the rotor held in a fixed position and with rated current flowing in the stator windings. It means that stalling is no problem for such motors whereas for most of the other motors, stalling results in the collapse of back emf (E_b) and a very high current which can lead to a quick burn-out.

Applications :

Such motors are used for operation control in computer peripherals, textile industry, IC fabrications and robotics etc. Applications requiring incremental motion are typewriters, line printers, tape drives, floppy disk drives, numerically-controlled machine tools, process control systems and X-Y plotters. Usually, position information can be obtained simply by keeping count of the pulses sent to the motor thereby eliminating the need for expensive position sensors and feedback controls. Stepper motors also perform countless tasks outside the computer industry. It includes commercial, military and medical applications where these motors perform such functions as mixing, cutting, striking, metering, blending and purging. They also take part in the manufacture of packed food stuffs, commercial end-products and even the production of science fiction movies.



Connecting a stepper motor to the interface

Example 39.1. *A hybrid VR stepping motor has 8 main poles which have been castleated to have 5 teeth each. If rotor has 50 teeth, calculate the stepping angle.*

Solution. $N_s = 8 \times 5 = 40; \quad N_r = 50$

$\therefore \quad \beta = (50 - 40) \times 360 / 50 \times 40 = 1.8^\circ$.

Example 39.2. *A stepper motor has a step angle of 2.5°. Determine (a) resolution (b) number of steps required for the shaft to make 25 revolutions and (c) shaft speed, if the stepping frequency is 3600 pps.*

Solution. (*a*) Resolution = $360° / \beta = 360° / 2.5° = 144$ steps / revolution.

(*b*) Now, steps / revolution = 144. Hence, steps required for making 25 revolutions = $144 \times 25 = 3600$.

(*c*) $n = \beta \times f / 360° = 2.5 \times 3600 / 360° = 25$ rps

39.3. Types of Stepper Motors

There is a large variety of stepper motors which can be divided into the following three basic categories :

(*i*) Variable Reluctance Stepper Motor

It has wound stator poles but the rotor poles are made of a ferromagnetic material as shown in Fig.39.1 (*a*). It can be of the single stack type (Fig.39.2) or multi-stack type (Fig.39.5) which gives smaller step angles. Direction of motor rotation is independent of the polarity of the stator current. It is called variable reluctance motor because the reluctance of the magnetic circuit formed by the rotor and stator teeth varies with the angular position of the rotor.

Permanent magnet stepper motor

(*ii*) Permanent Magnet Stepper Motor

It also has wound stator poles but its rotor poles are permanently magnetized. It has a cylindrical rotor as shown in Fig. 39.1 (*b*). Its direction of rotation depends on the polarity of the stator current.

(*iii*) Hybrid Stepper Motor

It has wound stator poles and permanently-magnetized rotor poles as shown in Fig.39.1(*c*). It is best suited when small step angles of 1.8°, 2.5° etc. are required.

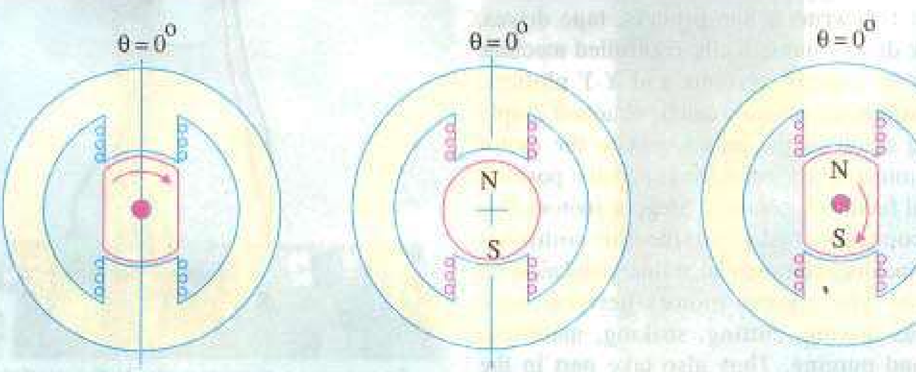
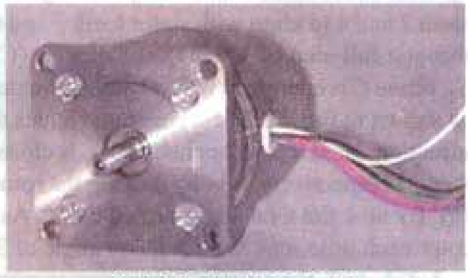


Fig. 39.1

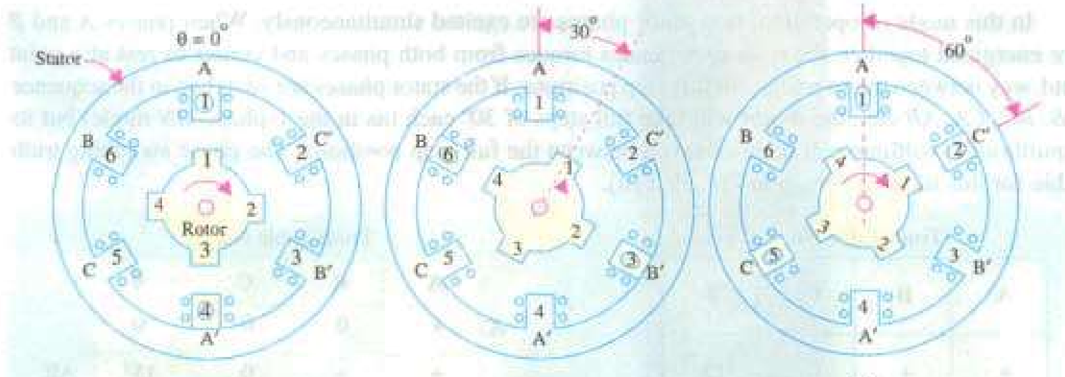
As a variable speed machine, *VR* motor is sometime designed as a switched-reluctance motor. Similarly, *PM* stepper motor is also called variable speed brushless dc motor. The hybrid motor combines the features of *VR* stepper motor and *PM* stepper motor. Its stator construction is similar to the single-stack *VR* motor but the rotor is cylindrical and is composed of radially magnetized permanent magnets. A recent type uses a disc rotor which is magnetized axially to give a small stepping angle and low inertia.

39.4. Variable Reluctance Stepper Motors

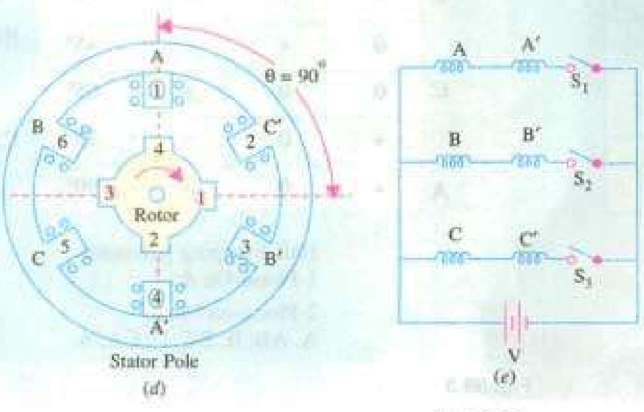
Construction : A variable-reluctance motor is constructed from ferromagnetic material with salient poles as shown in Fig. 39.2. The stator is made from a stack of steel laminations and has six equally-spaced projecting poles (or teeth) each wound with an exciting coil. The rotor which may be solid or laminated has four projecting teeth of the same width as the stator teeth. As seen, there are three independent stator circuits or phases *A*, *B* and *C* and each one can be energised by a direct current pulse from the drive circuit (not shown in the figure).


Variable reluctance motor

A simple circuit arrangement for supplying current to the stator coils in proper sequence is shown in Fig. 39.2 (*e*). The six stator coils are connected in 2-coil groups to form three separate circuits called phases. Each phase has its own independent switch. Diametrically opposite pairs of stator coils are connected in series such that when one tooth becomes a *N*-pole, the other one becomes a *S*-pole. Although shown as mechanical switches in Fig. 39.2 (*e*), in actual practice, switching of phase currents is done with the help of solid-state control. When there is no current in the stator coils, the rotor is completely free to rotate. Energising one or more stator coils causes the rotor to step forward (or backward) to a position that forms a path of least reluctance with the magnetized stator teeth. The step angle of this three-phase, four rotor teeth motor is $\beta = 360/4 \times 3 = 30°$.



(*a*) (*b*) (*c*)



(*d*) (*e*)

Truth Table No 1

A	B	C	θ
+	0	0	0°
0	+	0	30°
0	0	+	60°
+	0	0	90°

1-Phase-ON Mode, ABCA

(*f*)

Fig. 39.2

Working. The motor has following modes of operation :

(*a*) 1-phase-ON or Full-step Operation

Fig. 39.2 (*a*) shows the position of the rotor when switch S_1 has been closed for energising phase *A*. A magnetic field with its axis along the stator poles of phase *A* is created. The rotor is therefore, attracted into a position of minimum reluctance with diametrically opposite rotor teeth 1 and 3 lining up with stator teeth 1 and 4 respectively. Closing S_2 and opening S_1 energizes phase *B* causing rotor teeth 2 and 4 to align with stator teeth 3 and 6 respectively as shown in Fig. 39.2 (*b*). The rotor rotates through full-step of 30° in the clockwise (CW) direction. Similarly, when S_3 is closed after opening S_2, phase *C* is energized which causes rotor teeth 1 and 3 to line up with stator teeth 2 and 5 respectively as shown in Fig. 39.2 (*c*). The rotor rotates through an additional angle of 30° in the clockwise (CW) direction. Next if S_3 is opened and S_1 is closed again, the rotor teeth 2 and 4 will align with stator teeth 4 and 1 respectively thereby making the rotor turn through a further angle of 30° as shown in Fig. 39.2 (*d*). By now the total angle turned is 90°. As each switch is closed and the preceding one opened, the rotor each time rotates through an angle of 30°. By repetitively closing the switches in the sequence 1-2-3-1 and thus energizing stator phases in sequence *ABCA* etc., the rotor will rotate clockwise in 30° steps. If the switch sequence is made 3-2-1-3 which makes phase sequence *CBAC* (or *ACB*), the rotor will rotate anticlockwise. This mode of operation is known as 1-phase-ON mode or full-step operation and is the simplest and widely-used way of making the motor step. The stator phase switching truth table is shown in Fig. 39.2 (*f*). It may be noted that the direction of the stator magnetizing current is not significant because a stator pole of either magnetic polarity will always attract the rotor pole by inducing opposite polarity.

(*b*) 2-phase-ON Mode

In this mode of operation, two stator phases are excited simultaneously. When phases *A* and *B* are energized together, the rotor experiences torques from both phases and comes to rest at a point mid-way between the two adjacent full-step positions. If the stator phases are switched in the sequence *AB*, *BC*, *CA*, *AB* etc., the motor will take full steps of 30° each (as in the 1-phase-*ON* mode) but its equilibrium positions will be interleaved between the full-step positions. The phase switching truth table for this mode is shown in Fig. 39.3 (*a*).

Truth Table No. 2

A	B	C	θ
+	+	0	15°
0	+	+	45°
+	0	+	75°
+	+	0	105°

2 Phase-ON Mode
AB, BC, CA, AB

Truth Table No. 3

	A	B	C	θ	
A	+	0	0	0°	
	+	+	0	15°	AB
B	0	+	0	30°	
	0	+	+	45°	BC
C	0	0	+	65°	
	+	0	+	75°	CA
A	+	0	0	90°	

Half-Stepping Alternate
1-Phase-On &
2-Phase-on Mode
A, AB, B, BC, C, CA, A

Fig. 39.3

The 2-phase-ON mode provides greater holding torque and a much better damped single-stack response than the 1-phase-ON mode of operation.

(c) Half-step Operation

Half-step operation or 'half-stepping' can be obtained by exciting the three phases in the sequence A, AB, B, BC, C etc. *i.e.* alternately in the 1-phase-ON and 2-phase-ON modes. It is sometime known as 'wave' excitation and it causes the rotor to advance in steps of 15° *i.e.* half the full-step angle. The truth table for the phase pulsing sequence in half-stepping is shown in Fig. 39.3 (*b*).

Half-stepping can be illustrated with the help of Fig. 39.4 where only three successive pulses have been considered. Energizing only phase A causes the rotor position shown in Fig. 39.4 (*a*). Energising phases A and B simultaneously moves the rotor to the position shown in Fig. 39.4 (*b*) where rotor has moved through half a step only. Energising only phase B moves the rotor through another half-step as shown in Fig. 39.4 (*c*). With each pulse, the rotor moves $30 / 2 = 15°$ in the CCW direction.

It will be seen that in half-stepping mode, the step angle is halved thereby doubling the resolution. Moreover, continuous half-stepping produces a smoother shaft rotation.

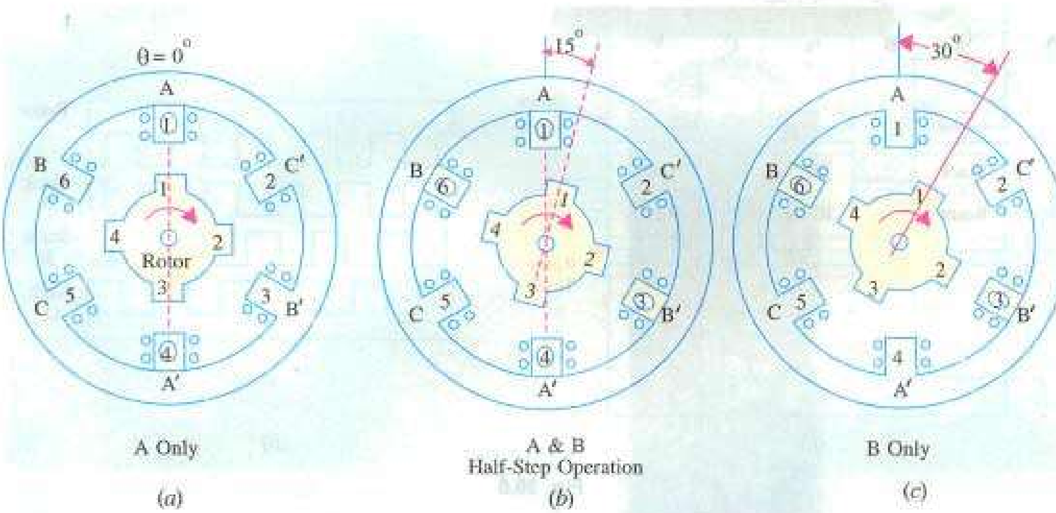


Fig. 39.4

(d) Microstepping

It is also known as mini-stepping. It utilizes two phases simultaneously as in 2-phase-*ON* mode but with the two currents deliberately made unequal (unlike in half-stepping where the two phase currents have to be kept equal). The current in phase A is held constant while that in phase B is increased in very small increments until maximum current is reached. The current in phase A is then reduced to zero using the same very small increments. In this way, the resultant step becomes very small and is called a microstep. For example, a *VR* stepper motor with a resolution of 200 steps / rev ($\beta = 1.8°$) can with microstepping have a resolution of 20,000 steps / rev ($\beta = 0.018°$). Stepper motors employing microstepping technique are used in printing and phototypesetting where very fine resolution is called for. As seen, microstepping provides smooth low-speed operation and high resolution.

Torque. If I_a is the d.c. current pulse passing through phase A, the torque produced by it is given by $T = (1 / 2)\, I_a^2\, dL / d\theta$. *VR* stepper motors have a high (torque / inertia) ratio giving high rates of acceleration and fast response. A possible disadvantage is the absence of detent torque which is necessary to retain the rotor at the step position in the event of a power failure.

39.5. Multi-stack VR Stepper Motor

So, far, we have discussed single-stack *VR* motors though multi-stack motors are also available which provide smaller step angles. The multi-stack motor is divided along its axial length into a number of magnetically-isolated sections or stacks which can be excited by a separate winding or phase. Both stator and rotor have the same number of poles. The stators have a common frame while rotors have a common shaft as shown in Fig. 39.5 (*a*) which represents a three-stack *VR* motor. The teeth of all the rotors are perfectly aligned with respect to themselves but the stator teeth of various stacks have a progressive angular displacement as shown in the developed diagram of Fig. 39.5 (*b*) for phase excitation.

Three-stack motors are most common although motors with upto seven stacks and phases are available. They have step angles in the range of 2° to 15°. For example, in a six-stack *VR* motor having 20 rotor teeth, the step angle $\beta = 360° / 6 \times 20 = 3°$.

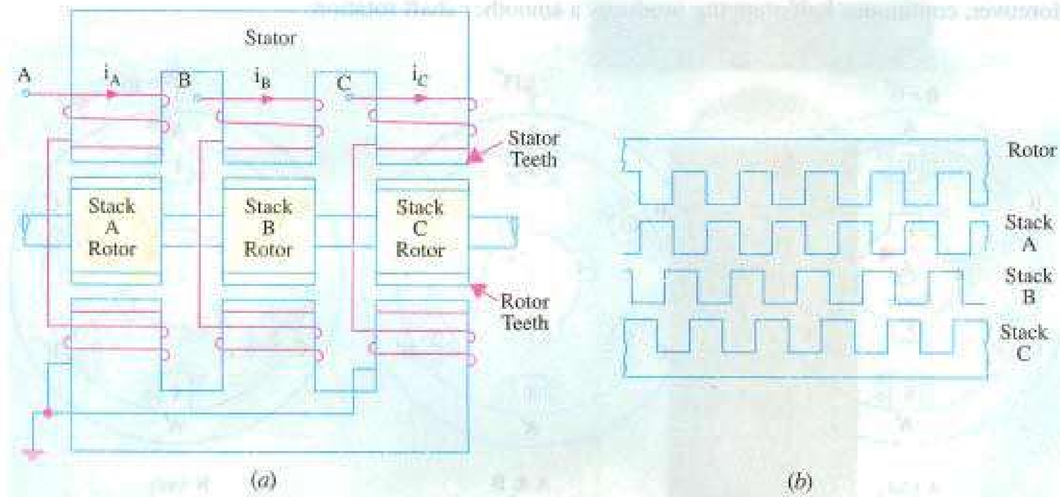
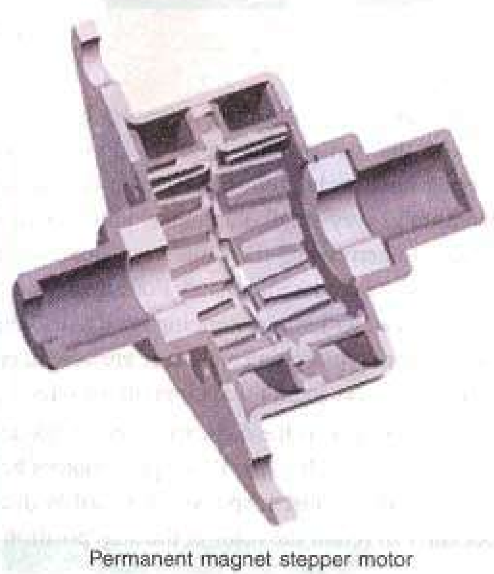


Fig. 39.5

39.6. Permanent-Magnet Stepping Motor

(*a*) **Construction.** Its stator construction is similar to that of the single-stack *VR* motor discussed above but the rotor is made of a permanent-magnet material like magnetically 'hard' ferrite. As shown in the Fig. 39.6 (*a*), the stator has projecting poles but the rotor is cylindrical and hasradially magnetized permanent magnets. The operating principle of such a motor can be understood with the help of Fig. 39.6 (*a*) where the rotor has two poles and the stator has four poles. Since two stator poles are energized by one winding, the motor has two windings or phases marked *A* and *B*. The step angle of this motor $\beta = 360° / mN_r = 360° / 2 \times 2 = 90°$ or $\beta = (4 - 2) \times 360° / 2 \times 4 = 90°$.


Permanent magnet stepper motor

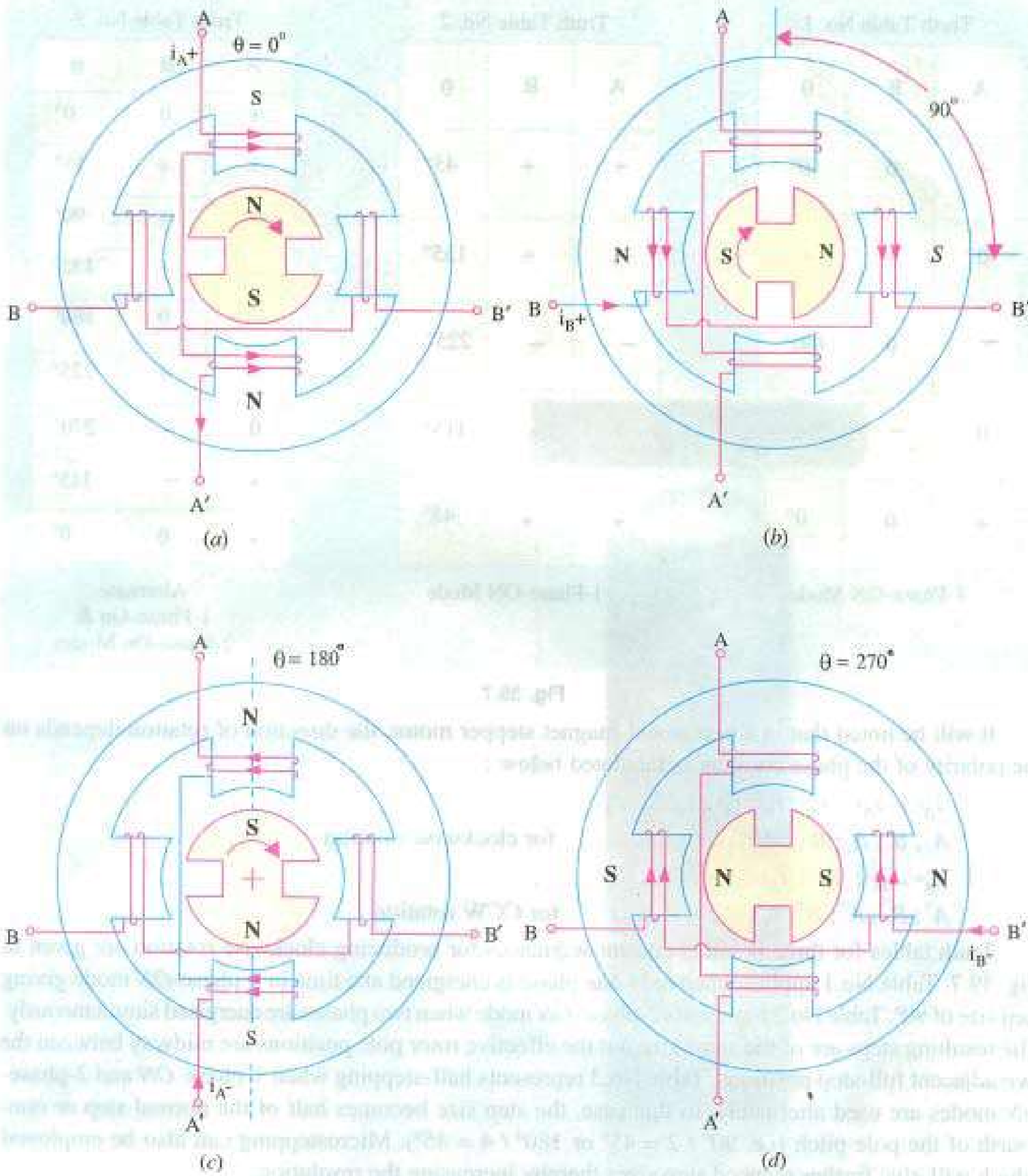


Fig. 39.6

(b)* Working.** When a particular stator phase is energized, the rotor magnetic poles move into alignment with the excited stator poles. The stator windings A and B can be excited with either polarity current (A^+ refers to positive current i_{A^+} in the phase A and A^- to ***negative current i_A-). Fig.39.6 (a) shows the condition when phase A is excited with positive current i_A+. Here, $\theta = 0°$. If excitation is now switched to phase B as in Fig. 39.6 (b), the rotor rotates by a full step of 90° in the clockwise direction. Next, when phase A is excited with negative current i_A-, the rotor turns through another 90° in CW direction as shown in Fig. 39.6 (c). Similarly, excitation of phase B with i_B- further turns the rotor through another 90° in the same direction as shown in Fig. 39.6 (d). After this, excitation of phase A with i_A+ makes the rotor turn through one complete revolution of 360°.

Truth Table No. 1

A	B	θ
+	0	0°
0	+	90°
–	0	180°
0	–	270°
+	0	0°

1-Phase-ON Mode

Truth Table No. 2

A	B	θ
+	+	45°
–	+	135°
–	–	225°
+	–	315°
+	+	45°

1-Phase-ON Mode

Truth Table No. 3

A	B	θ
+	0	0°
+	+	45°
0	+	90°
–	+	135°
–	0	180°
–	–	225°
0	–	270°
+	–	315°
+	0	0°

Alternate
1-Phase-On &
2-Phase-On Modes

Fig. 39.7

It will be noted that in a permanent-magnet stepper motor, the direction of rotation depends on the polarity of the phase currents as tabulated below :

i_{A^+}; i_{B^+}; i_{A^-}; i_{B^-}; i_{A^+}, ...

A^+; B^+; A^-; B^-; A^+; .... for clockwise rotation

i_{A^+}; i_{B^-}; i_{A^-}; i_{B^+}; i_{A^+}; ....

A^+; B^-; A^-; B^+; A^+; .... for CCW rotation

Truth tables for three possible current sequences for producing clockwise rotation are given in Fig. 39.7. Table No.1 applies when only one phase is energized at a time in 1-phase-*ON* mode giving step size of 90°. Table No.2 represents 2-phase-*ON* mode when two phases are energised simultaneously. The resulting steps are of the same size but the effective rotor pole positions are midway between the two adjacent full-step positions. Table No.3 represents half-stepping when 1-phase-*ON* and 2-phase-*ON* modes are used alternately. In this case, the step size becomes half of the normal step or one-fourth of the pole-pitch (*i.e.* 90° / 2 = 45° or 180° / 4 = 45°). Microstepping can also be employed which will give further reduced step sizes thereby increasing the resolution.

(c) Advantages and Disadvantages. Since the permanent magnets of the motor do not require external exciting current, it has a low power requirement but possesses a high detent torque as compared to a *VR* stepper motor. This motor has higher inertia and hence slower acceleration. However, it produces more torque per ampere stator current than a *VR* motor. Since it is difficult to manufacture a small permanent-magnet rotor with large number of poles, the step size in such motors is relatively large ranging from 30° to 90°. However, recently disc rotors have been manufactured which are magnetized axially to give a small step size and low inertia.

Example 39.3. *A single-stack, 3-phase VR motor has a step angle of 15°. Find the number of its rotor and stator poles.*

Solution. Now, $\beta = 360^\circ / mN_r$ or $15^\circ = 360^\circ / 3 \times N_r$; $\therefore N_r = 8$.

For finding the value of N_s, we will use the relation $\beta = (N_s - N_r) \times 360^\circ / N_s \cdot N_r$

(*i*) **When $N_s > N_r$.** Here, $\beta = (N_s - N_r) \times 360^\circ / N_s \cdot N_r$

or $\quad 15^\circ = (N_s - 8) \times 360^\circ / 8\,N_s$; $\quad \therefore \quad N_s = \mathbf{12}$

(*ii*) **When $N_s < N_r$.** Here, $15^\circ = (8 - N_s) \times 360^\circ / 8\,N_s$; $\quad \therefore \quad N_s = \mathbf{6}$.

Example 39.4. *A four-stack VR stepper motor has a step angle of 1.8°. Find the number of its rotor and stator teeth.*

Solution. A four-stack motor has four phases. Hence, $m = 4$.

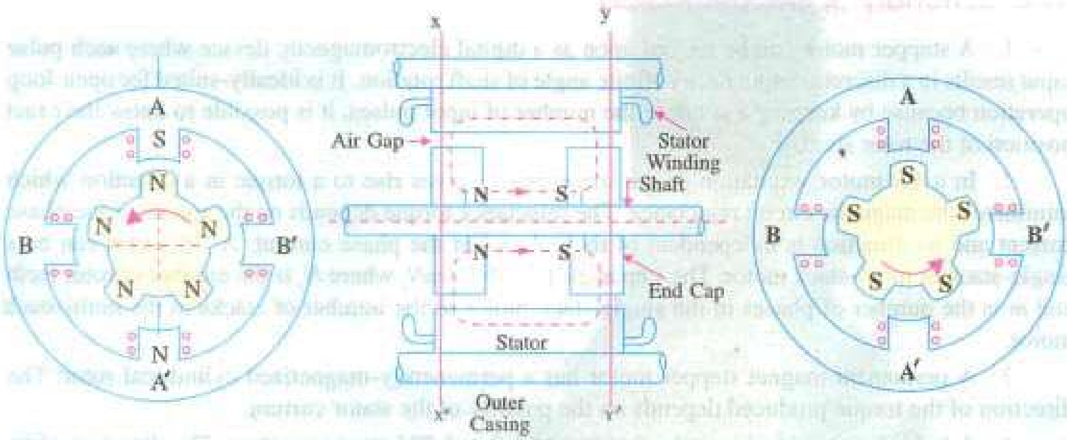
$\therefore \quad 1.8^\circ = 360^\circ / 4 \times N_r$; $\quad \therefore \quad N_r = \mathbf{50}$.

Since in multi-stack motors, rotor teeth equal the stator teeth, hence $N_s = \mathbf{50}$.

39.7. Hybrid Stepper Motor

Hybrid stepper motor

(*a*) **Construction.** It combines the features of the variable reluctance and permanent-magnet stepper motors. The rotor consists of a permanent-magnet that is magnetized axially to create a pair of poles marked *N* and *S* in Fig. 39.8 (*b*). Two end-caps are fitted at both ends of this axial magnet. These end-caps consist of equal number of teeth which are magnetized by the respective polarities of the axial magnet. The rotor teeth of one end-cap are offset by a half tooth pitch so that a tooth at one end-cap coincides with a slot at the other. The cross-sectional views perpendicular to the shaft along *X-X′* and *Y-Y′* axes are shown in Fig. 39.8 (*a*) and (*c*) respectively. As seen, the stator consists of four stator poles which are excited by two stator windings in pairs. The rotor has five *N*-poles at one end and five *S*-poles at the other end of the axial magnet. The step angle of such a motor is $= (5 - 4) \times 360^\circ / 5 \times 4 = 18^\circ$.



(*a*) (*b*) (*c*)

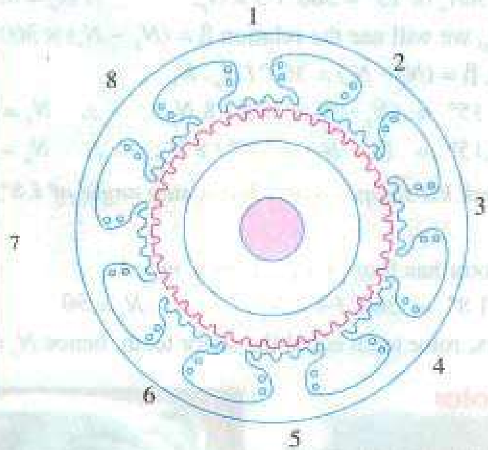
Fig. 39.8

(*b*) **Working.** In Fig.39.8 (*a*), phase *A* is shown excited such that the top stator pole is a *S*-pole so that it attracts the top *N*-pole of the rotor and brings it in line with the *A-A′* axis. To turn the rotor, Truth Table

A	B	
+	0	0°
0	+	18°
–	0	36°
0	–	54°
+	0	72°

1-Phase ON
Full-Step Mode

(*a*)



(*b*)

Fig. 39.9

phase *A* is denergized and phase *B* is excited positively. The rotor will turn in the CCW direction by a full step of 18°.

Next, phase *A* and *B* are energized negatively one after the other to produce further rotations of 18° each in the same direction. The truth table is shown in Fig. 39.9 (*a*). For producing clockwise rotation, the phase sequence should be $A^+; B^-; A^-; B^+; A^+$ etc.

Practical hybrid stepping motors are built with more rotor poles than shown in Fig. 39.9 in order to give higher angular resolution. Hence, the stator poles are often slotted or castleated to increase the number of stator teeth. As shown in Fig. 39.9 (*b*), each of the eight stator poles has been alloted or castleated into five smaller poles making $N_s = 8 \times 5 = 40°$. If rotor has 50 teeth, then step angle = $(50 - 40) \times 360° / 50 \times 40 = 1.8°$. Step angle can also be decreased (and hence resolution increased) by having more than two stacks on the rotor.

This motor achieves small step sizes easily and with a simpler magnet structure whereas a purely *PM* motor requires a multiple permanent-magnet. As compared to *VR* motor, hybrid motor requires less excitation to achieve a given torque. However, like a *PM* motor, this motor also develops good detent torque provided by the permanent-magnet flux. This torque holds the rotor stationary while the power is switched off. This fact is quite helpful because the motor can be left overnight without fear of its being accidentally moved to a new position.

39.8. Summary of Stepper Motors

1. A stepper motor can be looked upon as a digital electromagnetic device where each pulse input results in a discrete output *i.e.* a definite angle of shaft rotation. It is ideally-suited for open-loop operation because by keeping a count of the number of input pulses, it is possible to know the exact position of the rotor shaft.

2. In a *VR* motor, excitation of the stator phases gives rise to a torque in a direction which minimizes the magnetic circuit reluctance. The reluctance torque depends on the square of the phase current and its direction is independent of the polarity of the phase current. A *VR* motor can be a single-stack or multi-stack motor. The step angle $\beta = 360° / mN_r$ where N_r is the number of rotor teeth and *m* is the number of phases in the single-stack motor or the number of stacks in the multi-stack motor.

3. A permanent-magnet stepper motor has a permanently-magnetized cylindrical rotor. The direction of the torque produced depends on the polarity of the stator current.

4. A hybrid motor combines the features of *VR* and *PM* stepper motors. The direction of its torque also depends on the polarity of the stator current. Its step angle $\beta = 360° / mN_r$.

5. In the 1-phase *ON* mode of excitation, the rotor moves by one full-step for each change of excitation. In the 2-phase-*ON* mode, the rotor moves in full steps although it comes to rest at a point midway between the two adjacent full-step positions.

6. Half-stepping can be achieved by alternating between the 1-phase-*ON* and 2-phase-*ON* modes. Step angle is reduced by half.

7. Microstepping is obtained by deliberately making two phase currents unequal in the 2-phase-*ON* mode.

Tutorial Problems 39.1

1. A stepper motor has a step angle of 1.8°. What number should be loaded into the encoder of its drive system if it is desired to turn the shaft ten complete revolutions ? **[2000]**
2. Calculate the step angle of a single-stack, 4-phase, 8/6-pole *VR* stepper motor. What is its resolution ? **[15°; 24 steps/rev]**
3. A stepper motor has a step angle of 1.8° and is driven at 4000 pps. Determine (*a*) resolution (*b*) motor speed (*c*) number of pulses required to rotate the shaft through 54°. **[(*a*) 200 steps/rev (*b*) 1200 rpm (*c*) 30]**
4. Calculate the pulse rate required to obtain a rotor speed of 2400 rpm for a stepper motor having a resolution of 200 steps/rev. **[4000 pps]**
5. A stepper motor has a resolution of 500 steps/rev in the 1-phase-*ON* mode of operation. If it is operated in half-step mode, determine (*a*) resolution (*b*) number of steps required to turn the rotor through 72°. **[(*a*) 1000 steps/rev (*b*) 200]**
6. What is the required resolution for a stepper motor that is to operate at a pulse frequency of 6000 pps and a travel 180° in 0.025 s ? **[300 steps/rev]**

39.9. Permanent-Magnet DC Motor

A permanent-magnet d.c. (PMDC) motor is similar to an ordinary d.c. shunt motor except that its field is provided by permanent magnets instead of salient-pole wound-field structure. Fig. 39.10 (*a*) shows 2-pole PMDC motor whereas Fig. 39.10 (*b*) shows a 4-pole wound-field d.c. motor for comparison purposes.

(*a*) Construction

As shown in Fig. 39.10 (*a*), the permanent magnets of the PMDC motor are supported by a cylindrical steel stator which also serves as a return path for the magnetic flux. The rotor (*i.e.* armature) has winding slots, commutator segments and brushes as in conventional d.c. machines.

Permanent magnet DC - motor

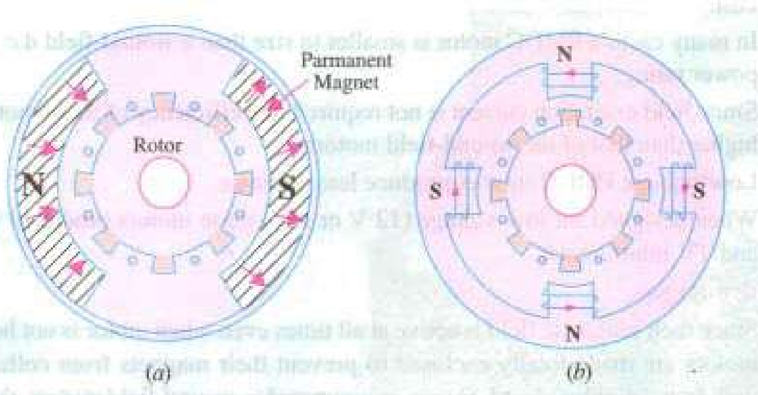


Fig. 39.10

There are three types of permanent magnets used for such motors. The materials used have residual flux density and high coercivity.

(*i*) Alnico magnets – They are used in motors having ratings in the range of 1 kW to 150 kW.

(*ii*) Ceramic (ferrite) magnets – They are much economical in fractional kilowatt motors.

(*iii*) Rare-earth magnets – Made of samarium cobalt and neodymium iron cobalt which have the highest energy product. Such magnetic materials are costly but are best economic choice for small as well as large motors.

Another form of the stator construction is the one in which permanent-magnet material is cast in the form of a continuous ring instead of in two pieces as shown in Fig. 39.10 (*a*).

(*b*) Working

Most of these motors usually run on 6 V, 12 V or 24 V dc supply obtained either from batteries or rectified alternating current. In such motors, torque is produced by interaction between the axial current-carrying rotor conductors and the magnetic flux produced by the permanent magnets.

(*c*) Performance

Fig. 39.11 shows some typical performance curves for such a motor. Its speed-torque curve is a straight line which makes this motor ideal for a servomotor. Moreover, input current increases linearly with load torque. The efficiency of such motors is higher as compared to wound-field dc motors because, in their case, there is no field Cu loss.

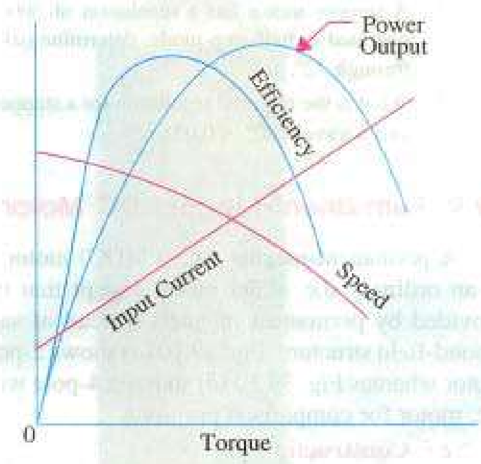


Fig. 39.11

(*d*) Speed Control

Since flux remains constant, speed of a PMDC motor cannot be controlled by using Flux Control Method (Art 33.2). The only way to control its speed is to vary the armature voltage with the help of an armature rheostat (Art 33.2) or electronically by using *x*-choppers. Consequently, such motors are found in systems where speed control below base speed only is required.

(*e*) Advantages

(*i*) In very small ratings, use of permanent-magnet excitation results in lower manufacturing cost.

(*ii*) In many cases a PMDC motor is smaller in size than a wound-field d.c. motor of equal power rating.

(*iii*) Since field excitation current is not required, the efficiency of these motors is generally higher than that of the wound-field motors.

(*iv*) Low-voltage PMDC motors produce less air noise.

(*v*) When designed for low-voltage (12 V or less) these motors produced very little radio and TV interference.

(*f*) Disadvantages

(*i*) Since their magnetic field is active at all times even when motor is not being used, these motors are made totally enclosed to prevent their magnets from collecting magnetic junk from neighbourhood. Hence, as compared to wound-field motors, their temperature tends to be higher. However, it may not be much of a disadvantage in situations where motor is used for short intervals.

(*ii*) A more serious disadvantage is that the permanent magnets can be demagnetized by armature reaction mmf causing the motor to become inoperative. Demagnetization can result from (*a*) improper design (*b*) excessive armature current caused by a fault or transient or improper connection in the armature circuit (*c*) improper brush shift and (*d*) temperature effects.

(*g*) Applications

(*i*) Small, 12-V PMDC motors are used for driving automobile heater and air conditioner blowers, windshield wipers, windows, fans and radio antennas etc. They are also used for electric fuel pumps, marine engine starters, wheelchairs and cordless power tools.

(*ii*) Toy industry uses millions of such motors which are also used in other appliances such as the toothbrush, food mixer, ice crusher, portable vacuum cleaner and shoe polisher and also in portable electric tools such as drills, saber saws and hedge trimmers etc.

39.10. Low-inertia DC Motors

These motors are so designed as to make their armature mass very low. This permits them to start, stop and change direction and speed very quickly making them suitable for instrumentation applications. The two common types of low-inertia motors are (*i*) shell-type motor and (*ii*) printed-circuit (PC) motor.

39.11. Shell-type Low-inertia DC Motor

Its armature is made up of flat aluminium or copper coils bonded together to form a hollow cylinder as shown in Fig. 39.12. This hollow cylinder is not attached physically to its iron core which is stationary and is located inside the shell-type rotor. Since iron does not form part of the rotor, the rotor inertia is very small.

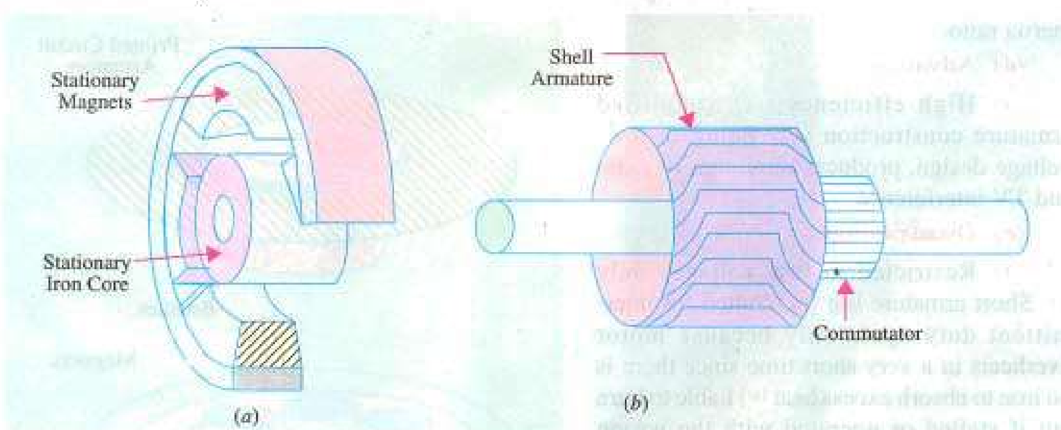


Fig. 39.12

39.12. Printed-circuit (Disc) DC Motor

(*a*) Constructional Details

It is a low-voltage dc motor which has its armature (rotor) winding and commutator printed on a thin disk of non-magnetic insulating material. This disk-shaped armature contains no iron and etched-copper conductors are printed on its both sides. It uses permanent magnets to produce the necessary magnetic field. The magnetic circuit is completed through the flux-return plate which also supports the brushes. Fig. 39.13 shows an 8-pole motor having wave-wound armature. Brushes mounted in an axial direction bear directly on the inner parts of the armature conductors which thus serve as a commutator. Since the number of armature conductors is very large, the torque produced is uniform even at low speeds. Typical sizes of these motors are in the fractional and subfractional horsepower ranges. In many applications, acceleration from zero to a few thousand rpm can be obtained within 10 ms.

Low voltage DC motor

(b) Speed Control

The speed can be controlled by varying either the applied armature voltage or current. Because of their high efficiency, fan cooling is not required in many applications. The motor brushes require periodic inspection and replacement. The rotor disk which carries the conductors and commutator, being very thin, has a limited life. Hence, it requires replacing after some time.

(c) Main Features

The main features of this motor are (*i*) very low-inertia (*ii*) high overload current capability (*iii*) linear speed-torque characteristic (*iv*) smooth torque down to near-zero speed (*v*) very suitable for direct-drive control applications (*vi*) high torque/inertia ratio.

(d) Advantages

(*i*) High efficiency (*ii*) Simplified armature construction (*iii*) Being of low-voltage design, produces minimum of radio and TV interference.

(e) Disadvantages

(*i*) Restricted to low voltages only (*ii*) Short armature life (*iii*) Suited for intermittent duty cycle only because motor overheats in a very short time since there is no iron to absorb excess heat (*v*) liable to burn out if stalled or operated with the wrong supply voltage.

(f) Applications

These low-inertia motors have been developed specifically to provide high performance characteristics when used in direct-drive control applications. Examples are :

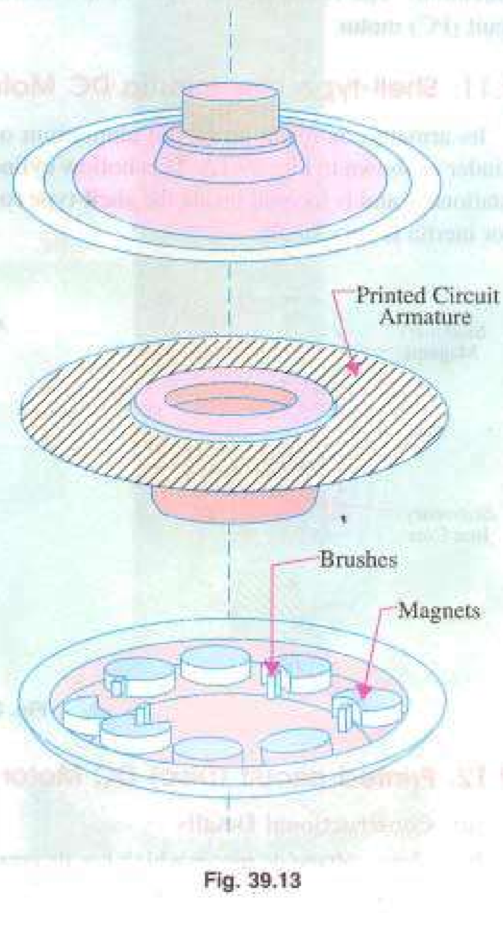


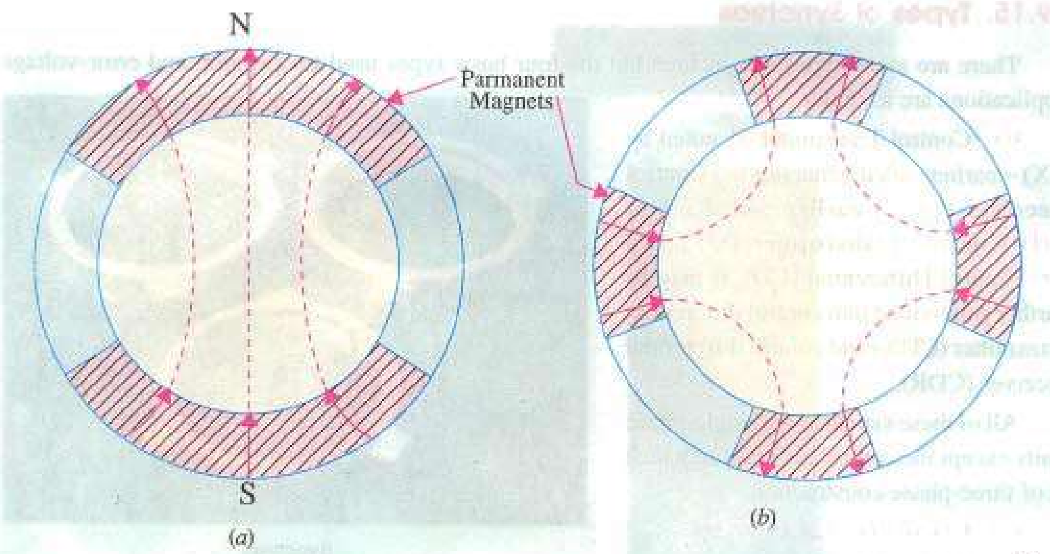
Fig. 39.13

(*i*) high speed paper tape readers (*ii*) oscillographs (*iii*) *X-Y* recorders (*iv*) layer winders (*v*) point-to-point tool positioners *i.e.* as positioning servomotors (*vi*) with in-built optical position encoder, it competes with stepping motor (*vii*) in high rating is being manufactured for heavy-duty drives such as lawn mowers and battery-driven vehicles etc.

39.13. Permanent-Magnet Synchronous Motors

(*a*) Construction and Performance

Such motors have a cage rotor having rare-earth permanent magnets instead of a wound field. Such a motor starts like an induction motor when fed from a fixed-frequency supply. A typical 2-pole and 4-pole surface-mounted versions of the rotor are shown in Fig. 39.14. Since no d.c. supply is needed for exciting the rotor, it can be made more robust and reliable. These motors have outputs ranging from about 100 W upto 100 kW. The maximum synchronous torque is designed to be around



(*a*) (*b*)

Fig. 39.14

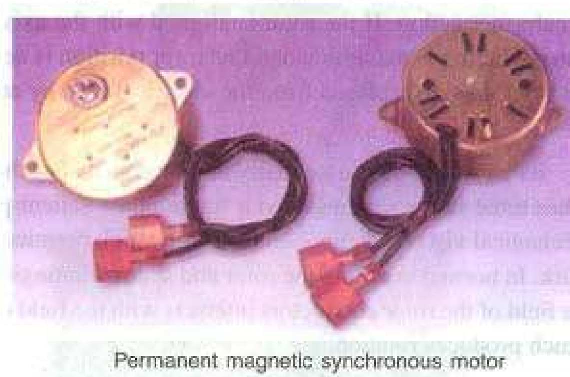
54303

150 per cent of the rated torque. If loaded beyond this point, the motor loses synchronism and will run either as an induction motor or stall.

These motors are usually designed for direct-on-line (DOL) starting. The efficiency and power factor of the permanent-magnet excited synchronous motors are each 5 to 10 points better than their reluctance motor counterparts.

(*b*) Advantages

Since there are no brushes or slip-rings, there is no sparking. Also, brush maintenance is eliminated. Such motors can pull into synchronism with inertia loads of many times their rotor inertia.



Permanent magnetic synchronous motor

(*c*) Applications

These motors are used where precise speed must be maintained to ensure a consistent product. With a constant load, the motor maintains a constant speed.

Hence, these motors are used for synthetic-fibre drawing where constant speeds are absolutely essential.

39.14. Synchros

It is a general name for self-synchronizing machines which, when electrically energized and electrically interconnected, exert torques which cause two mechanically independent shafts either to run in synchronism or to make the rotor of one unit follow the rotor position of the other. They are also known by the trade names of selsyns and autosyns. Synchros, in fact, are small cylindrical motors varying in diameter from 1.5 cm to 10 cm depending on their power output. They are low-torque devices and are widely used in control systems for transmitting shaft position information or for making two or more shafts to run in synchronism. If a large device like a robot arm is to be positioned, synchros will not work. Usually, a servomotor is needed for a higher torque.

39.15. Types of Synchros

There are many types of synchros but the four basic types used for position and error-voltage applications are as under :

(*i*) Control Transmitter (denoted by CX) – earlier called generator (*ii*) Control Receiver (CR) – earlier called motor (*iii*) Control-Transformer (CT) and (*iv*) Control Differential (CD). It may be further subdivided into control differential transmitter (CDX) and control differential receiver (CDR).

All of these synchros are single-phase units except the control differential which is of three-phase construction.

Synchros

(*a*) Constructional Features

1. Control Transmitter

Its constructional details are shown in Fig. 39.15 (*a*). It has a three-phase stator winding similar to that of a three-phase synchronous generator. The rotor is of the projecting-pole type using dumbell construction and has a single-phase winding. When a single-phase ac voltage is applied to the rotor through a pair of slip rings, it produces an alternating flux field along the axes of the rotor. This alternating flux induces three unbalanced single phase/voltage in the three stator windings by transformer action. If the rotor is aligned with the axis of the stator winding 2, flux linkage of this stator winding is maximum and this rotor position is defined as the electrical zero. In Fig. 39.15 (*b*), the rotor axis is displaced from the electrical zero by an angle displaced 120° apart.

(*b*) Control Receiver (CR)

Its construction is essentially the same as that of the control transmitter shown in Fig. 39.15 (*a*). It has three stator windings and a single-phase salient-pole rotor. However, unlike a *CX*, a *CR* has a mechanical viscous damper on the shaft which permits *CR* rotor to respond without overshooting its mark. In normal use, both the rotor and stator windings are excited with single-phase currents. When the field of the rotor conductors interacts with the field of the stator conductors, a torque is developed which produces rotation.

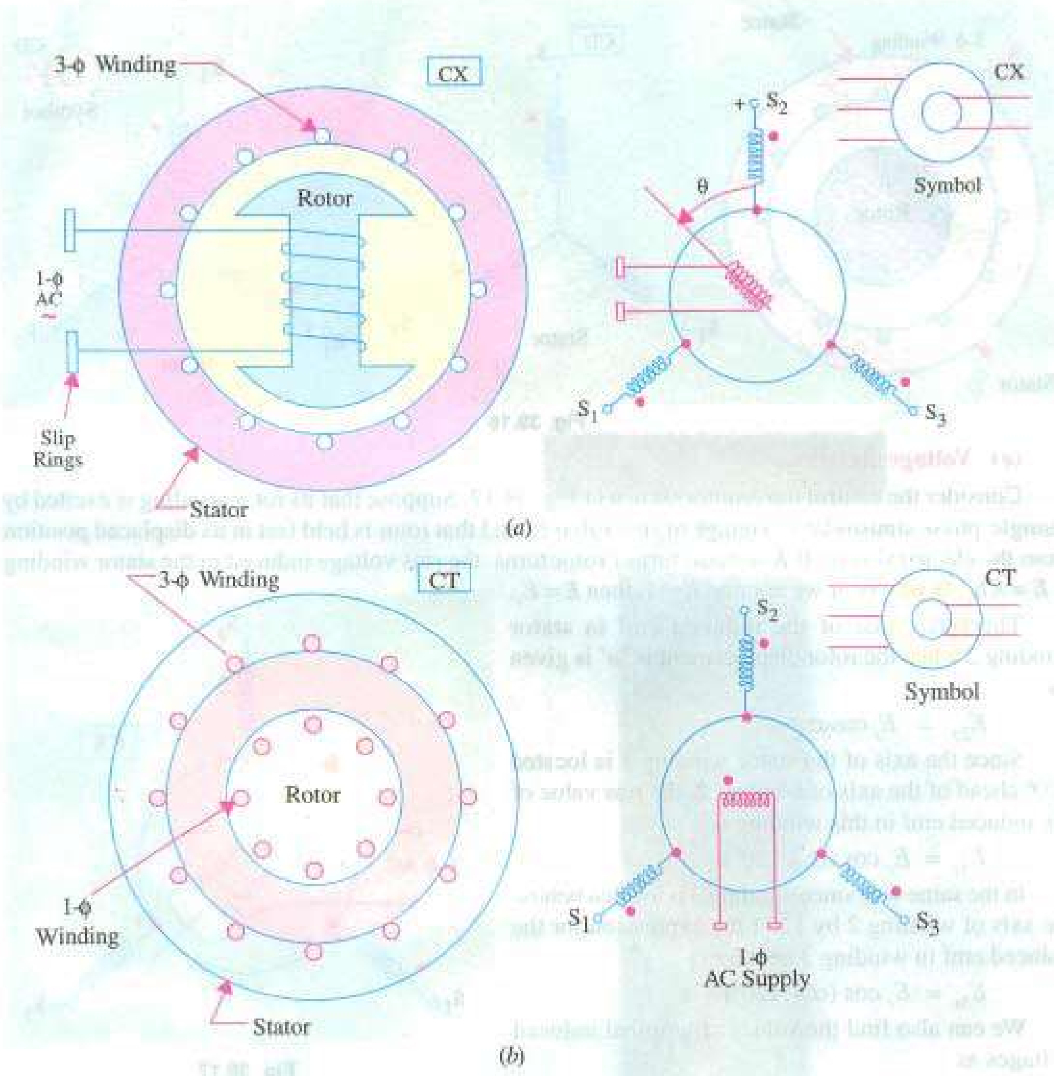


Fig. 39.15

(c) Control Transformer (CT)

As shown in Fig. 39.15 (*b*) its stator has a three-phase winding whereas the cylindrical rotor has a single-phase winding. In this case, the electrical zero is defined as that position of the rotor that makes the flux linkage with winding 2 of the stator zero. This rotor position has been shown in Fig. 39.15 (*b*) and is different from that of a control transmitter.

(d) Control Differential (CD)

The differential synchro has a balanced three-phase distributed winding in both the stator and the rotor. Moreover, it has a cylindrical rotor as shown in Fig. 39.16 (*a*). Although three-phase windings are involved, it must be kept in mind that these units deal solely with single-phase voltages. The three winding voltages are not polyphase voltages. Normally, the three-phase voltages are identical in magnitude but are separated in phase by 120°. In synchros, these voltages are in phase but differ in magnitude because of their physical orientation.

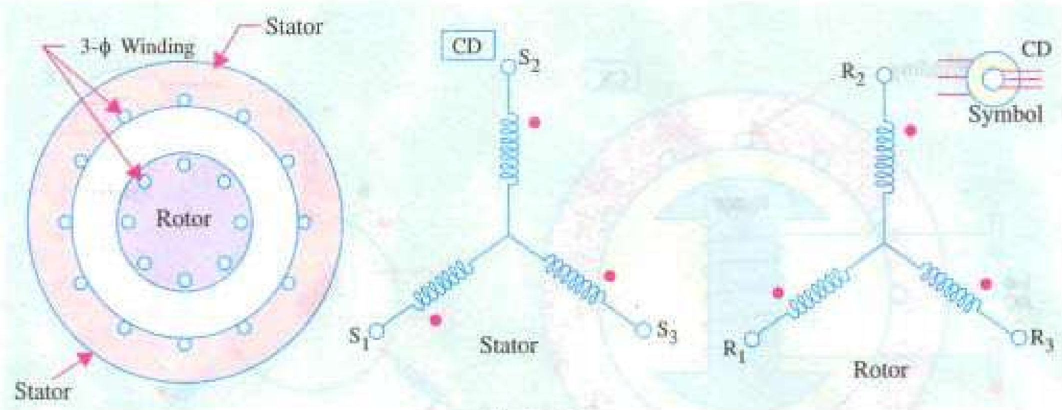


Fig. 39.16

(e) Voltage Relations

Consider the control transmitter shown in Fig. 39.17. Suppose that its rotor winding is excited by a single-phase sinusoidal ac voltage of rms value E_r and that rotor is held fast in its displaced position from the electrical zero. If K = stator turns / rotor turns, the rms voltage induced in the stator winding is $E = KE_r$. However, if we assume $K = 1$, then $E = E_r$.

The rms value of the induced emf in stator winding 2 when the rotor displacement is 'a' is given by

$$E_{2s} = E_r \cos \alpha.$$

Since the axis of the stator winding 1 is located 120° ahead of the axis of winding 2, the rms value of the induced emf in this winding is

$$E_{1s} = E_r \cos (\alpha - 120°).$$

In the same way since winding 3 is located behind the axis of winding 2 by 120°, the expression for the induced emf in winding 3 becomes

$$E_{3s} = E_r \cos (\alpha + 120°).$$

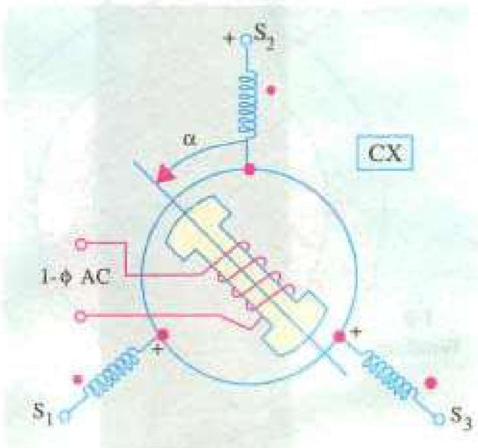


Fig. 39.17

We can also find the values of terminal induced voltages as

$$\begin{aligned} E_{12} &= E_{1s} + E_{s2} = E_{1s} - E_{2s} \\ &= E_r \cos \alpha \cos 120° + E_r \sin \alpha \sin 120° - E_r \cos \alpha \\ &= E_r \left(-\frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) \\ &= \sqrt{3}\, E_r \left(-\frac{1}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) \\ &= \sqrt{3}\, E_r \cos (\alpha - 150°) \end{aligned}$$

$$\begin{aligned} E_{23} &= E_{2s} + E_{s3} = E_{2s} - E_{3s} \\ &= E_r \left(\frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) = \sqrt{3}\, E_r \left(\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) = \sqrt{3}\, E_r \cos (\alpha - 30°) \end{aligned}$$

$$\begin{aligned} E_{31} &= E_{3s} + E_{s1} = E_{3s} - E_{1s} \\ &= E_r \cos (\alpha + 120°) - E_r \cos (\alpha - 120°) \\ &= -\sqrt{3}\, E_r \sin \alpha = \sqrt{3}\, E_r \cos (\alpha + 90°) \end{aligned}$$

Example 39.5. *The rotor of a control transmitter (CX) is excited by a single-phase ac voltage of rms value 20 V. Find the value of E_{1s}, E_{2s} and E_{3s} for rotor angle $\alpha = +40^o$ and -40^o. Assume the stator/rotor turn ratio as unity. Also, find the values of terminal voltages when $\alpha = +30^o$.*

Solution. Since $K = 1$, the voltage relations derived in will be used.

(*a*) $\alpha = +40^o$

$$E_{2s} = E_r \cos\alpha = 20\cos 40^o = 15.3 \text{ V}$$
$$E_{1s} = E_r \cos(\alpha - 120^o) = 20\cos(40^o - 120^o) = \mathbf{3.5\ V}$$
$$E_{3s} = E_r \cos(\alpha + 120^o) = 20\cos 160^o = \mathbf{-18.8\ V}$$

(*b*) $\alpha = -40^o$

$$E_{2s} = 20\cos(-40^o) = 15.3 \text{ V}$$
$$E_{1s} = 20\cos(-40^o - 120^o) = 20\cos(-160^o) = -18.8 \text{ V}$$
$$E_{3s} = 20\cos(-40^o + 120^o) = 20\cos 80^o = \mathbf{3.5\ V}$$

(*c*)
$$E_{12} = \sqrt{3} \times 20 \times \cos(30^o - 150^o) = -17.3 \text{ V}$$
$$E_{23} = \sqrt{3}\ E_r \cos(\alpha - 30^o) = \sqrt{3}\ E_r \cos(30^o - 30^o) = 34.6 \text{ V}$$
$$E_{31} = \sqrt{3}\ E_r \cos(\alpha + 90^o) = \sqrt{3} \times 20 \times \cos(30^o + 90^o) = \mathbf{-17.3\ V}$$

39.16. Applications of Synchros

The synchros are extensively used in servomechanism for torque transmission, error detection and for adding and subtracting rotary angles. We will consider these applications one by one.

(*a*) Torque Transmission

Synchros are used to transmit torque over a long distance without the use of a rigid mechanical connection. Fig. 39.18 represents an arrangement for maintaining alignment of two distantly-located shafts. The arrangement requires a control transmitter (CX) and a control receiver (CR) which acts as a torque receiver. As *CX* is rotated by an angle α, *CR* also rotates through the same angle α. As shown, the stator windings of the two synchros are connected together and their rotors are connected to the same single-phase ac supply.

Working. Let us suppose that *CX* rotor is displaced by an angle α and switch SW_1 is closed to energize the rotor winding. The rotor winding flux will induce an unbalanced set of three single-phase voltages (in time phase with the rotor voltage) in the *CX* stator phase windings which will circulate currents in the *CR* stator windings. These currents produce the *CR* stator flux field whose axis is fixed by the angle α. If the *CR* rotor winding is now energized by closing switch SW_2, its flux field will interact with the flux field of the stator winding and thereby produce a torque. This torque will rotate the freely-moving *CR* rotor to a position which exactly corresponds with the *CT* rotor *i.e.* it will be displaced by the same angle α as shown in Fig. 39.18. It should be noted that if the two rotors are in the same relative positions, the stator voltages in the two synchros will be exactly equal and opposite. Hence, there will be no current flow in the two stator windings and so no torque will be produced and the system will achieve equilibrium. If now, the transmitter rotor angle changes to a new value, then new set of voltages would be induced in the transmitter stator windings which will again drive currents through the receiver stator windings. Hence, necessary torque will be produced which will turn the *CR* rotor through an angle corresponding to that of the *CT* rotor. That is why the transmitter rotor is called the master and the receiver rotor as the slave, because it follows its master. It is worth noting that this master-slave relationship is reversible because when the receiver rotor is displaced through a certain angle, it causes the transmitter rotor to turn through the same angle.

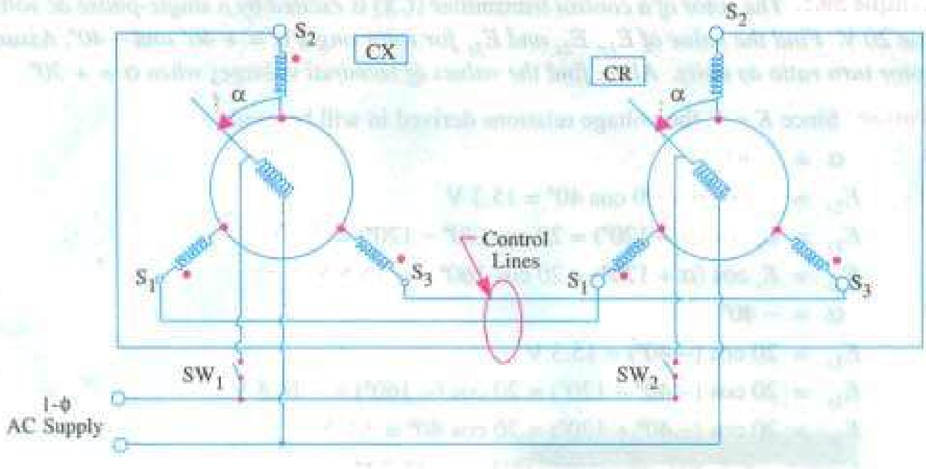


Fig. 39.18

(*b*) Error Detection

Synchros are also used for error detection in a servo control system. In this case, a command in the form of a mechanical displacement of the *CX* rotor is converted to an electrical voltage which appears at the *CT* rotor winding terminals which can be further amplified by an amplifier.

For this purpose, we require a *CX* synchro and a *CT* synchro as shown in Fig. 39.19. Only the *CX* rotor is energized from the single-phase ac voltage supply which produces an alternating air-gap flux field. This time-varying flux field induces voltages in the stator windings whose values for $\alpha = 30°$ are as indicated in the Fig. 39.19. The *CX* stator voltages supply magnetizing currents in the *CT* stator

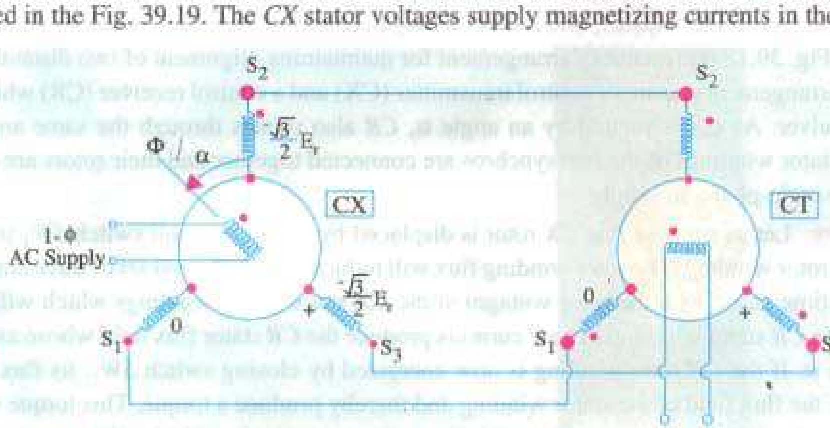


Fig. 39.19

windings which, in turn, create an alternating flux field in their own air-gap. The values of the *CT* stator phase currents are such that the air-gap flux produced by them induces voltages that are equal and opposite to those existing in the *CX* stator. Hence, the direction of the resultant flux produced by the *CX* stator phase currents is forced to take a position which is exactly identical to that of the rotor axis of the *CT*.

If the *CT* rotor is assumed to be held fast in its electrical zero position as shown in Fig.39.19, then the rms voltage induced in the rotor is given by $E = E_{max} \sin \alpha$, where E_{max} is the maximum voltage induced by the *CT* air-gap flux when coupling with the rotor windings is maximum and α is the displacement angle of the *CT* rotor.

In general, the value of the rms voltage induced in the *CT* rotor winding when the displacement of the *CX* rotor is α_x and that of the *CT* rotor is α_T is given by

$$E = E_{max} \sin (\alpha_x - \alpha_T)$$

39.17. Control Differential Transmitter

It can be used to produce a rotation equal to the sum of difference of the rotations of two shafts. The arrangement for this purpose is shown in Fig. 39.20 (*a*). Here, a CDX is coupled to a control transmitter on one side and a control receiver on the other. The *CX* and *CR* rotor windings are energized from the same single-phase voltage supply.

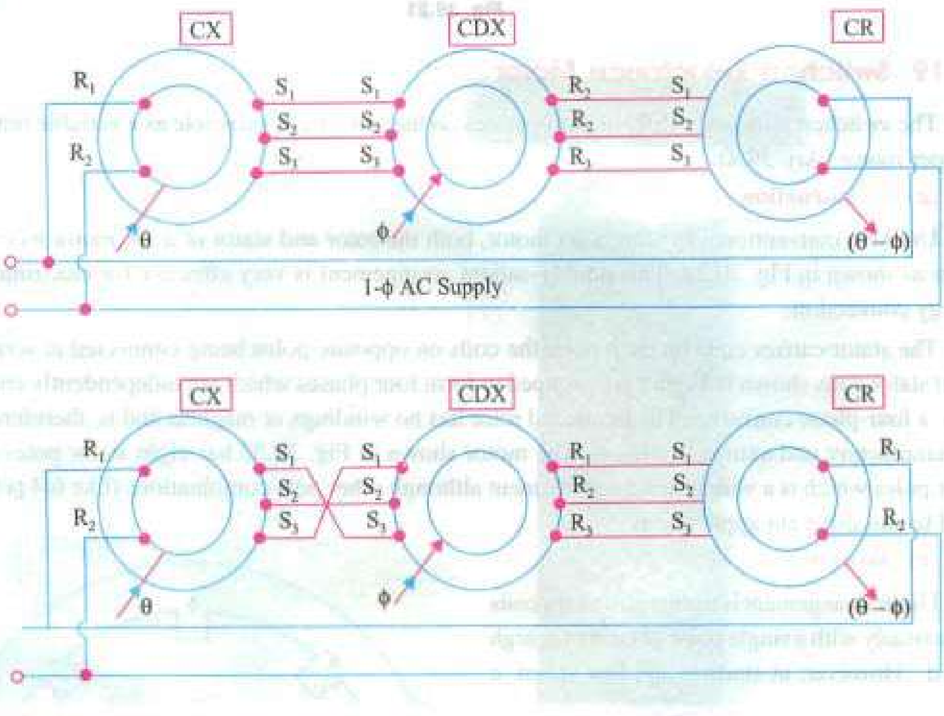


Fig. 39.20

It has two inputs : Mechanical θ and Electrical ϕ and the output is Machnical (θ – ϕ). The mechanical input (θ) to *CX* is converted and applied to the *CDX* stator. With a rotor input (ϕ), the electrical output of the *CDX* is applied to the *CR* stator which provides the mechanical output (θ – ϕ).

As shown in Fig. 39.20 (*b*), if any two stator connections between *CX* and *CDX* are transposed, the electrical input from *CX* to *CDX* becomes –θ, hence the output becomes (–θ – ϕ) = – (θ + ϕ).

39.18. Control Differential Receiver

In construction, it is similar to a *CDX* but it accepts two electrical input angles and provide the difference angle as a mechanical output (Fig. 39.21).

The arrangement consists of two control transmitters coupled to a *CDR*. The two control transmitters provide inputs to the CDX, one (θ) to the stator and the other (ϕ) to the rotor. The *CDX* output is the difference of the two inputs *i.e.* (θ – ϕ).

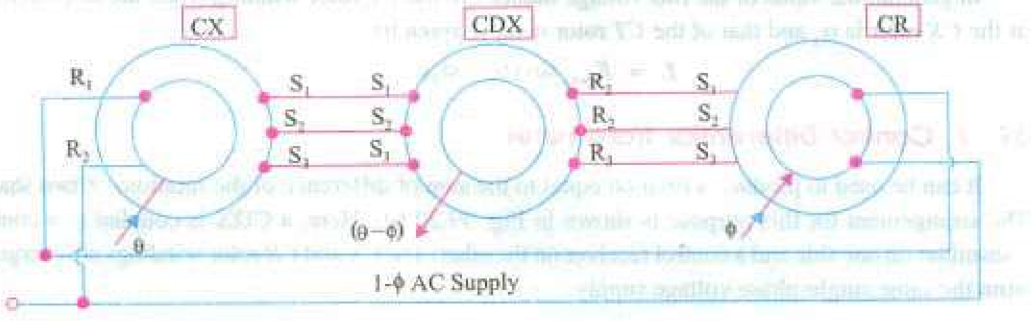


Fig. 39.21

39.19. Switched Reluctance Motor

The switched reluctance (SR) motor operates on the same basic principle as a variable reluctance stepper motor (Art. 39.4).

(a) Construction

Unlike a conventional synchronous motor, both the rotor and stator of a SR motor have salient poles as shown in Fig. 39.22. This doubly-salient arrangement is very effective for electromagnetic energy conversion.

The stator carries coils on each pole, the coils on opposite poles being connected in series. The eight stator coils shown in Figure are grouped to form four phases which are independently energized from a four-phase converter. The laminated rotor has no windings or magnets and is, therefore cheap to manufacture and extremely robust. The motor shown in Fig. 39.22 has eight stator poles and six rotor poles which is a widely-used arrangement although other pole combinations (like 6/4 poles) are used to suit different applications.

(b) Working

Usual arrangement is to energize stator coils sequentially with a single pulse of current at high speed. However, at starting and low speed, a current-chopper type control is used to limit the coil current.

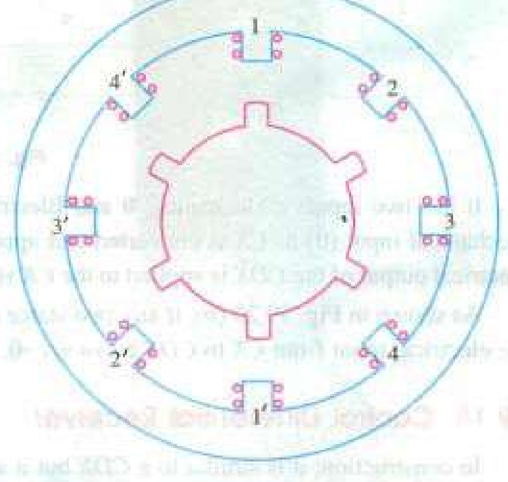
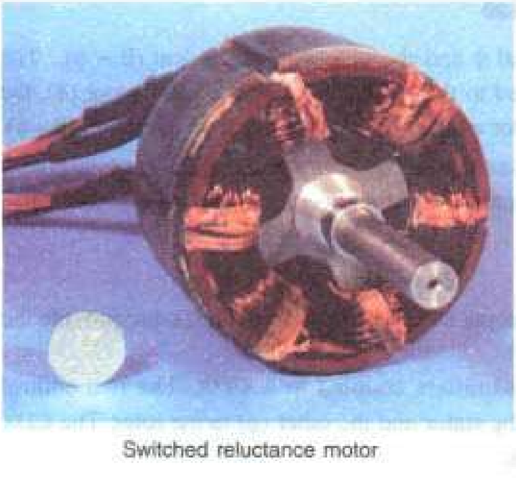


Fig. 39.22



Switched reluctance motor

The motor rotates in the anticlockwise direction when the stator phases are energized in the sequence 1, 2, 3, 4 and in clockwise direction when energized in the sequence 1, 4, 3, 2. When the stator coils are energized, the nearest pair of rotor poles is pulled into alignment with the appropriate stator poles by reluctance torque.

Closed-loop control is essential to optimize the switching angles of the applied coil voltages. The stator phases are switched by signals derived from a shaft-mounted rotor position detectors such as Hall-effect devices or optical sensors Fig. (39.23). This causes the behaviour of the SR motor to resemble that of a dc motor.

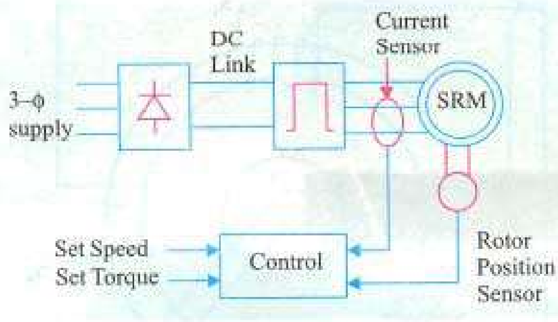


Fig. 39.23

(c) Advantages and Disadvantages

Although the newest arrival on the drives scene, the SR motor offers the following advantages: (*i*) higher efficiency (*ii*) more power per unit weight and volume (*iii*) very robust because rotor has no windings or slip rings (*iv*) can run at very high speed (upto 30,000 rpm) in hazardous atmospheres (*v*) has versatile and flexible drive features and (*vi*) four-quadrant operation is possible with appropriate drive circuitry.

However, the drawbacks are that it is (*i*) relatively unproven (*ii*) noisy and (*iii*) not well-suited for smooth torque production.

(d) Applications

Even though the SR technology is still in its infancy, it has been successfully applied to a wide range of applications such as (*i*) general purpose industrial drives (*ii*) traction (*iii*) domestic appliances like food processors, vacuum cleaners and washing machines etc., and (*iv*) office and business equipment.

39.20. Comparison between *VR* Stepper Motor and SR Motor

VR Stepper Motor	*SR Motor*
1. It rotates in steps.	It is meant for continuous rotation.
2. It is designed first and foremost for open-loop operation.	Closed-loop control is essential for its optimal working.
3. Its rotor poles are made of ferromagnetic material.	Its rotor poles are also made of ferromagnetic material.
4. It is capable of half-step operation and microstepping.	It is not designed for this purpose.
5. Has low power rating.	Has power ratings upto 75 kW (100 hp).
6. Has lower efficiency.	Has higher overall efficiency.

39.21. The Resolver

In many ways, it is similar to a synchro but differs from it in the following respects : *(i)* Electrical displacement between stator windings is 90° and not 120° *(ii)* It has two stator windings and two rotor windings (Fig. 39.24) *(iii)* Its input can be either to the stator or to the rotor *(iv)* They are usually not used as followers because their output voltage is put to further use.

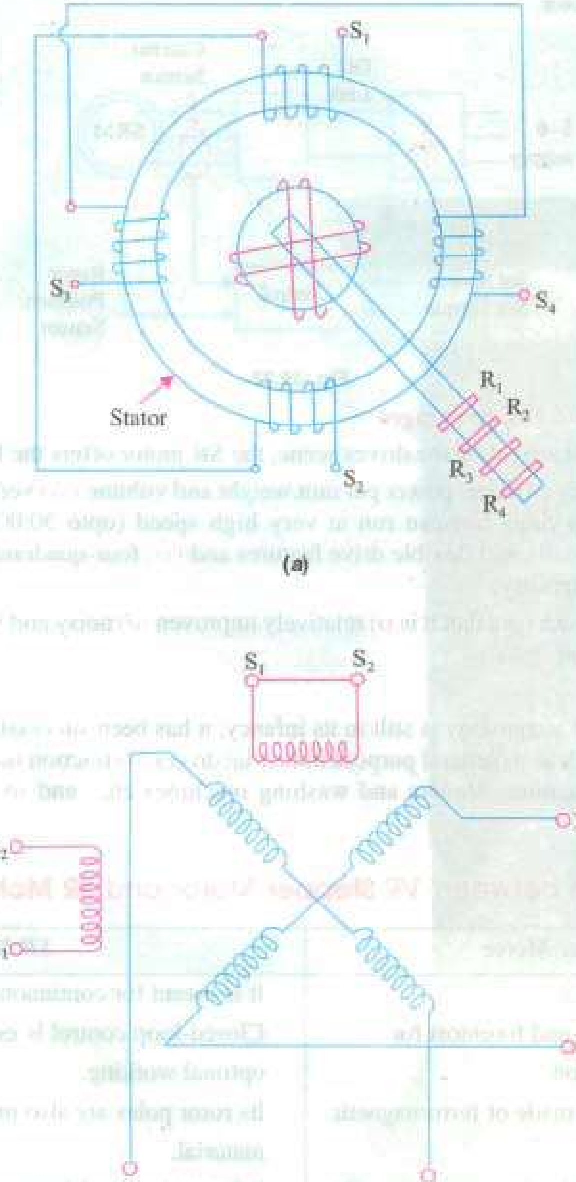


Fig. 39.24

(a) Construction

The main constructional features and the symbol for a resolver are shown in Fig. 39.24. There are two stator windings which are wound 90º apart. In most applications, only one stator winding is used, the other being short-circuited. The two rotor winding connections are brought out through slip rings and brushes.

(b) Applications

Resolvers find many applications in navigation and height determination as shown in Fig. 39.25 (a) and (c) where Fig. 39.25 (b) provides the key.

(i) Navigation Application

As shown in Fig. 39.25 (a), the purpose is to determine the distance D to the destination. Suppose the range R to a base station as found by a radar ranging device is 369 km. The angle θ is also determined directly. If the amplifier scale is 4.5 V per 100 km, the range would be represented by $369 \times (4.5 / 100) = 16.6$ V. Further suppose that angle θ is found to be 52.5º. Now, set the resolver at 52.5º and apply 16.6 V to rotor terminals $R_3 R_4$. The voltage which appears at terminals $S_1 S_2$ represents D. If we assume K = stator turns / rotor turns = 1, the voltage available at $S_1 S_2$ will be $= 16.6 / \cos 52.5° = 16.6 / 0.6088 = 27.3$ V. Since 4.5 V represents 100 km, 27.3 V represents $27.3 \times 100 / 4.5 =$ 607 km.

(ii) Height Determination

Suppose the height H of a building is to be found. First of all, the oblique distance D to the top of the building is found by a range finder. Let $D = 210$ m and the scale of the amplifier to the resolver stator be 9 V per 100 m. The equivalent voltage is $9 \times 210 / 100 = 18.9$ V. This voltage is applied to stator terminals is $S_1 S_2$ of the resolver. Suppose the angle θ read from the resolver scale is 61.3º. The height of the building is given in the form of voltage which appears across the rotor terminals $R_1 R_2$. Assuming stator / rotor turn ratio as unity and the same amplifier ratio for the rotor output, the voltage across $R_1 R_2 = 18.9 \times \sin 61.3° = 16.6$ V. Hence, $H = 16.6 \times 100 / 9 = 184$ m. It would be seen that in using the resolver, there is no need to go through trigonometric calculations because the answers come out directly.

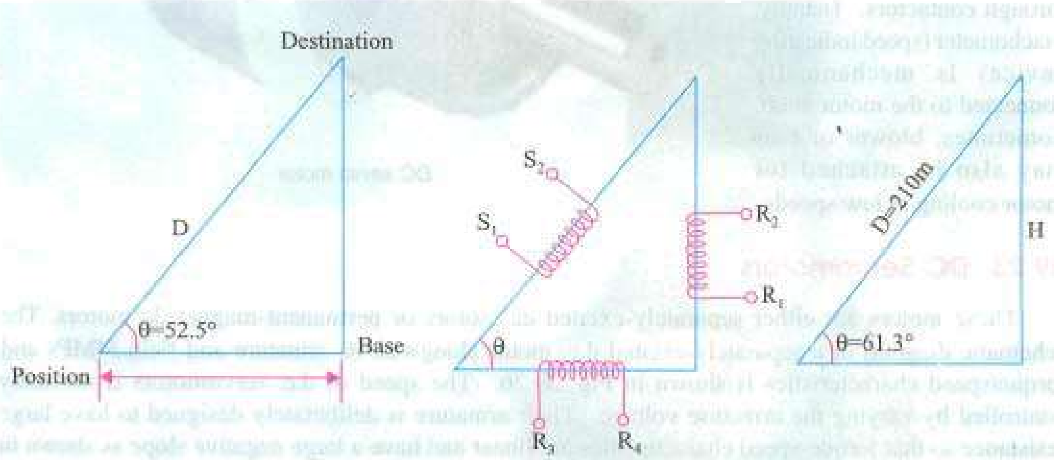


Fig. 39.25

39.22. Servomotors

They are also called control motors and have high-torque capabilities. Unlike large industrial motors, they are not used for continuous energy conversion but only for precise speed and precise position control at high torques. Of course, their basic principle of operation is the same as that of other electromagnetic motors. However, their construction, design and mode of operation are different. Their power ratings vary from a fraction of a watt upto a few 100 W. Due to their low-inertia, they have high speed of response. That is why they are smaller in diameter but longer in length. They generally operate at vary low speeds or sometimes zero speed. They find wide applications in radar, tracking and guidance systems, process controllers, computers and machine tools. Both dc and a.c. (2-phase and 3-phase) servomotors are used at present.

Servomotors differ in application capabilities from large industrial motors in the following respects :

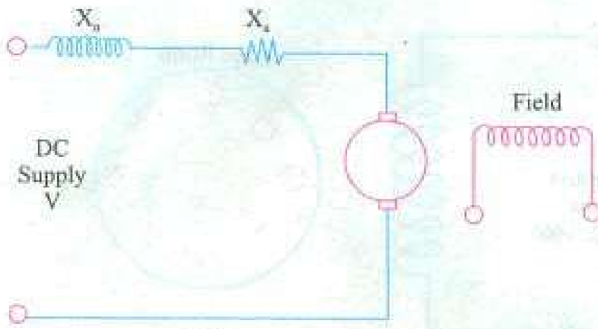
1. They produce high torque at all speeds including zero speed.
2. They are capable of holding a static (*i.e.* no motion) position.
3. They do not overheat at standstill or lower speeds.
4. Due to low-inertia, they are able to reverse directions quickly.
5. They are able to accelerate and deaccelerate quickly.
6. They are able to return to a given position time after time without any drift.

These motors look like the usual electric motors. Their main difference from industrial motors is that more electric wires come out of them for power as well as for control. The servomotor wires go to a controller and not to the electrical line through contactors. Usually, a tachometer (speed indicating device) is mechanically connected to the motor shaft. Sometimes, blower or fans may also be attached for motor cooling at low speeds.

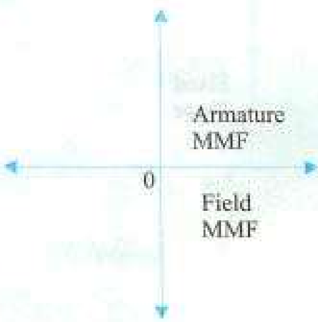
DC servo motor

39.23. DC Servomotors

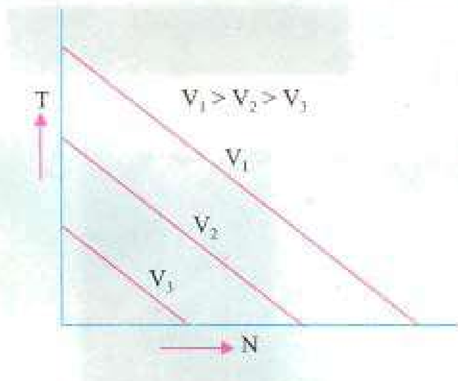
These motors are either separately-excited dc motors or permanent-magnet dc motors. The schematic diagram of a separately-excited d.c. motor alongwith its armature and field MMFs and torque/speed characteristics is shown in Fig. 39.26. The speed of d.c. servomotors is normally controlled by varying the armature voltage. Their armature is deliberately designed to have large resistance so that torque-speed characteristics are linear and have a large negative slope as shown in Fig. 39.26 (*c*). The negative slope serves the purpose of providing the viscous damping for the servo drive system.



(a)



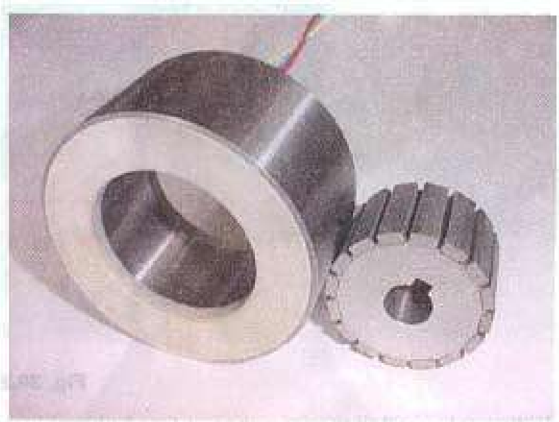
(b)



(c)

Fig. 39.26

As shown in Fig. 39.26 (b), the armature m.m.f. and excitation field mmf are in quadrature. This fact provides a fast torque response because torque and flux become decoupled. Accordingly, a step change in the armature voltage or current produces a quick change in the position or speed of the rotor.



Permanent magnet stepper motor

39.24. AC Servomotors

Presently, most of the ac servomotors are of the two-phase squirrel-cage induction type and are used for low power applications. However, recently three-phase induction motors have been modified for high power servo systems which had so far been using high power d.c. servomotors.

(a) Two-phase AC Servomotor

Such motors normally run on a frequency of 60 Hz or 400 Hz (for airborne systems). The stator has two distributed windings which are displaced from each other by 90° (electrical). The main

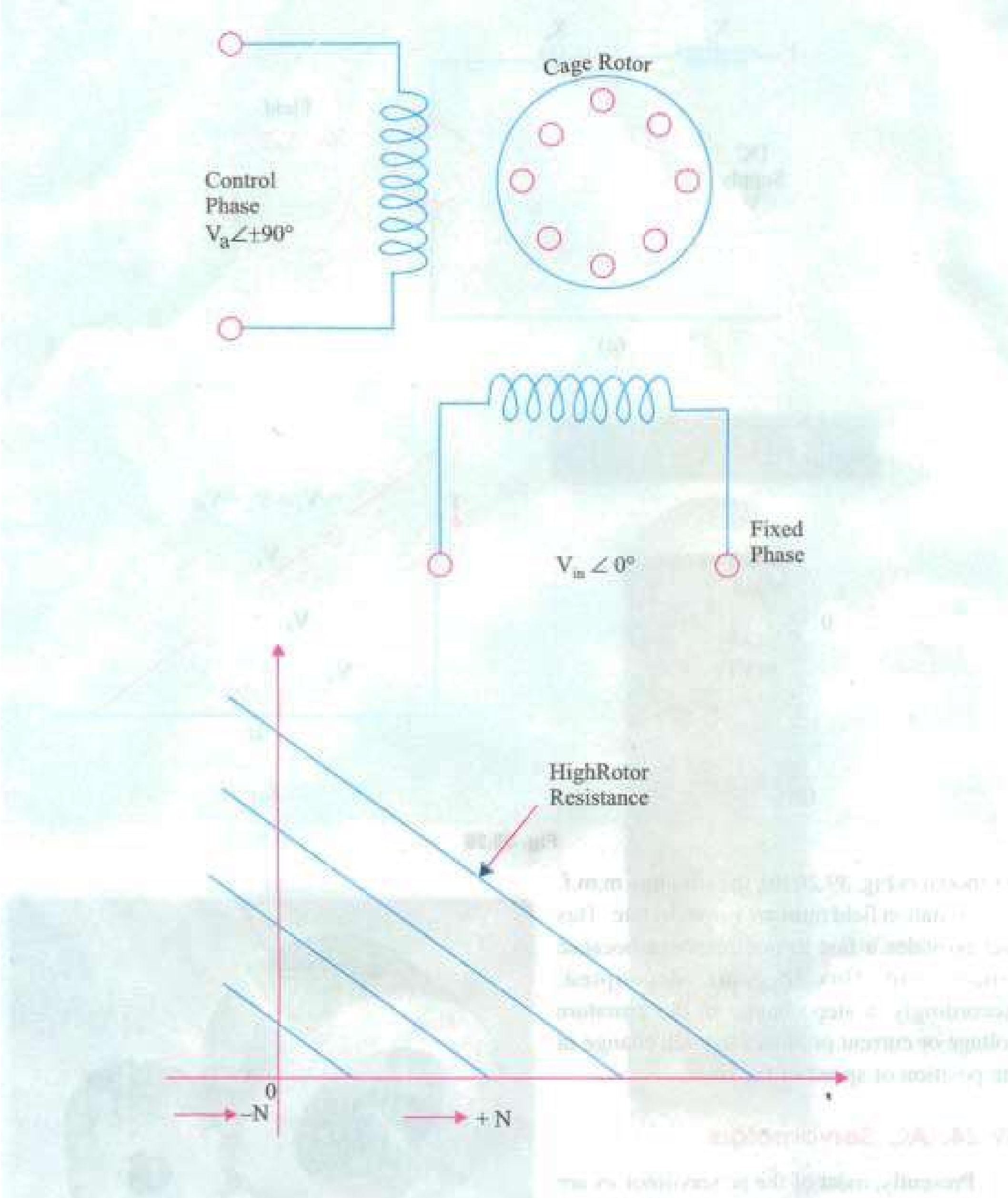


Fig. 39.27

winding (also called the reference or fixed phase) is supplied from a constant voltage source, $V_m \angle 0°$ (Fig. 39.27). The other winding (also called the control phase) is supplied with a variable voltage of the same frequency as the reference phase but is phase-displaced by 90° (electrical). The control-phase voltage is controlled by an electronic controller. The speed and torque of the rotor are controlled by the phase difference between the main and control windings. Reversing the phase difference from leading to lagging (or *vice-versa*) reverses the motor direction.

Since the rotor bars have high resistance, the torque-speed characteristics for various armature voltages are almost linear over a wide speed range particularly near the zero speed. The motor operation can be controlled by varying the voltage of the main phase while keeping that of the reference phase constant.

(*b*) Three-phase AC Servomotors

A great deal of research has been to modify a three-phase squirrel-cage induction motor for use in high power servo systems. Normally, such a motor is a highly non-linear coupled-circuit device. Recently, this machine has been operated successfully as a linear decoupled machine (like a d.c. machine) by using a control method called vector control or field oriented control. In this method, the currents fed to the machine are controlled in such a way that its torque and flux become decoupled as in a dc machine. This results in a high speed and a high torque response.

OBJECTIVE TESTS – 39

1. A single-stack, 4-phase, 6-pole VR stepper motor will have a step angle of
   (*a*) 15°
   (*b*) 30°
   (*c*) 45°
   (*d*) 90°
2. In a three-stack 12/8-pole VR motor, the rotor pole pitch is
   (*a*) 15°
   (*b*) 30°
   (*c*) 45°
   (*d*) 60°
3. A three-stack VR stepper motor has a step angle of 10°. What is the number of rotor teeth in each stack ?
   (*a*) 36
   (*b*) 24
   (*c*) 18
   (*d*) 12
4. If a hybrid stepper motor has a rotor pitch of 36° and a step angle of 9°, the number of its phases must be
   (*a*) 4
   (*b*) 2
   (*c*) 3
   (*d*) 6
5. What is the step angle of a permanent-magnet stepper motor having 8 stator poles and 4 rotor poles ?
   (*a*) 60°
   (*b*) 45°
   (*c*) 30°
   (*d*) 15°
6. A stepping motor is a .... device.
   (*a*) mechanical
   (*b*) electrical
   (*c*) analogue
   (*d*) incremental
7. Operation of stepping motors at high speeds is referred to as
   (*a*) fast forward
   (*b*) slewing
   (*c*) inching
   (*d*) jogging
8. Which of the following phase switching sequence represents half-step operation of a VR stepper motor ?
   (*a*) A, B, C, A ....
   (*b*) A, C, B, A ....
   (*c*) AB, BC, CA, AB ....
   (*d*) A, AB, B, BC ....
9. The rotational speed of a given stepper motor is determined solely by the
   (*a*) shaft load
   (*b*) step pulse frequency
   (*c*) polarity of stator current
   (*d*) magnitude of stator current.
10. A stepper motor may be considered as a .... converter.
    (*a*) dc to dc
    (*b*) ac to ac
    (*c*) dc to ac
    (*d*) digital-to-analogue
11. The rotor of a stepper motor has no

(a) windings
(b) commutator
(c) brushes
(d) all of the above.

12. Wave excitation of a stepper motor results in
(a) microstepping
(b) half-stepping
(c) increased step angle
(d) reduced resolution.

13. A stepper motor having a resolution of 300 steps/rev and running at 2400 rpm has a pulse rate of— pps.
(a) 4000
(b) 8000
(c) 6000
(d) 10,000

14. The torque exerted by the rotor magnetic field of a PM stepping motor with unexcited stator is called ....torque.
(a) reluctance
(b) detent
(c) holding
(d) either (b) or (c)

15. A variable reluctance stepper motor is constructed of .... material with salient poles.
(a) paramagnetic
(b) ferromagnetic
(c) diamagnetic
(d) non-magnetic

16. Though structurally similar to a control transmitter, a control receiver differs from it in the following way :
(a) it has three-phase stator winding
(b) it has a rotor of dumbell construction
(c) it has a mechanical damper on its shaft
(d) it has single-phase rotor excitation.

17. The control .... synchro has three-phase winding both on its stator and rotor.
(a) differential
(b) transformer
(c) receiver
(d) transmitter

18. Regarding voltages induced in the three stator windings of a synchro, which statement is false ?
(a) they depend on rotor position.
(b) they are in phase.
(c) they differ in magnitude.
(d) they are polyphase voltages.

19. The low-torque synchros cannot be used for
(a) torque transmission
(b) error detection
(c) instrument servos
(d) robot arm positioning.

20. Which of the following synchros are used for error detection in a servo control system ?
(a) control transmitter
(b) control transformer
(c) control receiver
(d) both (a) and (b).

21. For torque transmission over a long distance with the help of electrical wires only, which of the following two synchros are used ?
(a) *CX* and *CT*
(b) *CX* and *CR*
(c) *CX* and *CD*
(d) *CT* and *CD*.

22. The arrangement required for producing a rotation equal to the sum or difference of the rotation of two shafts consists of the following coupled synchros.
(a) control transmitter
(b) control receiver
(c) control differential transmitter
(d) all of the above.

23. Which of the following motor would suit applications where constant speed is absolutely essential to ensure a consistent product ?
(a) brushless dc motor
(b) disk motor
(c) permanent-magnet synchronous motor
(d) stepper motor.

24. A switched reluctance motor differs from a *VR* stepper motor in the sense that it
(a) has rotor poles of ferromagnetic material
(b) rotates continuously
(c) is designed for open-loop operation only
(d) has lower efficiency.

25. The electrical displacement between the two stator windings of a resolver is
(a) 120°
(b) 90°
(c) 60°
(d) 45°.

26. Which of the following motor runs from a low dc supply and has permanently magnetized salient poles on its rotor ?
   (*a*) permanent-magnet d.c. motor
   (*b*) disk d.c. motor
   (*c*) permanent-magnet synchronous motor
   (*d*) brushless d.c. motor.
27. A dc servomotor is similar to a regular d.c. motor except that its design is modified to cope with
   (*a*) electronic switching
   (*b*) slow speeds
   (*c*) static conditions
   (*d*) both (*b*) and (*c*).
28. One of the basic requirements of a servomotor is that it must produce high torque at all
   (*a*) loads
   (*b*) frequencies
   (*c*) speeds
   (*d*) voltages.
29. The most common two-phase ac servomotor differs from the standard ac induction motor because it has
   (*a*) higher rotor resistance
   (*b*) higher power rating
   (*c*) motor stator windings
   (*d*) greater inertia.
30. Squirrel-cage induction motor is finding increasing application in high-power servo systems because new methods have been found to
   (*a*) increase its rotor resistance
   (*b*) control its torque
   (*c*) decrease its intertia
   (*d*) decouple its torque and flux.

ANSWERS

1. *a* 2. *c* 3. *d* 4. *a* 5. *b* 6. *d* 7. *b* 8. *d* 9. *b* 10. *d* 11. *d* 12. *b* 13. *c* 14. *d* 15. *b* 16. *c* 17. *a* 18. *d* 19. *d* 20. *d* 21. *b* 22. *d* 23. *c* 24. *b* 25. *b* 26. *a* 27. *d* 28. *c* 29. *a* 30. *d*.

QUESTIONS AND ANSWERS ON SPECIAL MACHINES

Q.1. Do stepper motors have internal or external fans ?

Ans. No. Because the heat generated in the stator winding is conducted through the stator iron to the case which is cooled by natural conduction, convection and radiation.

Q.2. Why do hybrid stepping motors have many phases sometime more than six ?

Ans. In order to obtain smaller step angles.

Q.3. Any disadvantage(s) of having more phases?

Ans. Minor ones are: more leads have to be brought out from the motor, more interconnections are required to the drive circuit and more switching devices are needed.

Q.4. What is the main attraction of a multi-stack *VR* stepper motor ?

Ans. It is well-suited to high stepping rates.

Q.5. You are given a *VR* motor and a hybrid stepper motor which look exactly similar. How would you tell which is which ?

Ans. Spin the rotor after short-circuiting the stator winding. If there is no mechanical resistance to rotation, it is a *VR* motor and if there is resistance, then it is a hybrid motor.

Q.6. How do you explain it ?

Ans. Since *VR* motor has magnetically neutral rotor, it will not induce any e.m.f. in the short-circuited winding *i.e.* the machine will not act as a generator and hence experience no drag on its rotation. However, the rotor of a hybrid motor has magnetic poles, hence it will act as a generator and so experience a drag.

Q.7. Will there be any harm if the rotor of a hybrid stepper motor is pulled out of its stator ?

Ans. Yes. The rotor will probably become partially demagnetized and, on reassembling, will give less holding torque.