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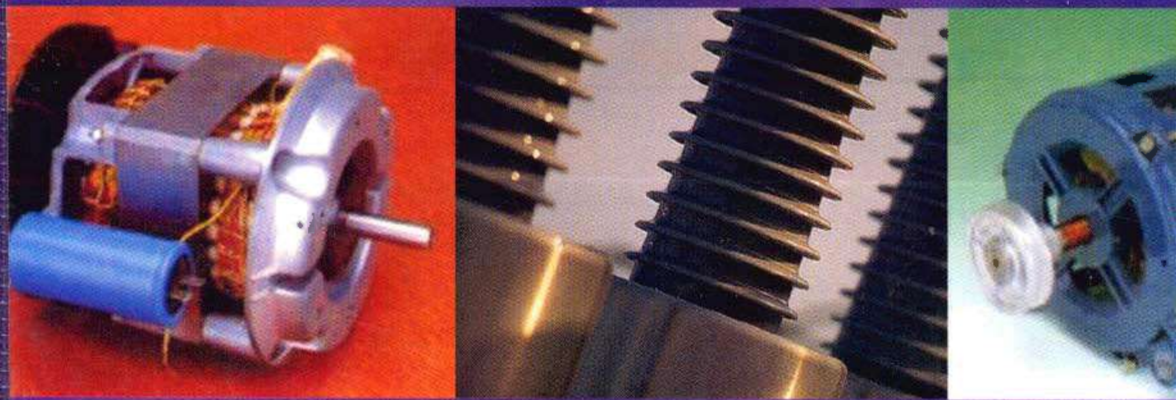
A TEXTBOOK OF

ELECTRICAL TECHNOLOGY

IN S.I. UNITS

Volume II

AC & DC MACHINES



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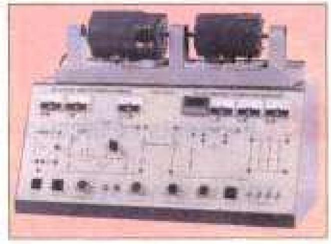
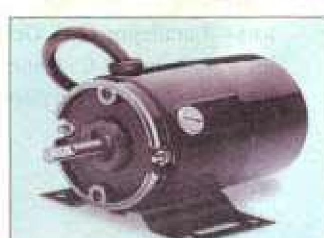
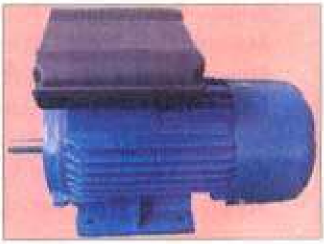






C H A P T E R 25

ELEMENTS OF ELECTRO-MECHANICAL ENERGY CONVERSION

Learning Objectives

- Introduction
- Salient aspects of conversions
- Energy-balance
- Magnetic-field system: Energy and Co-energy
- Linear System
- A Simple Electro-mechanical System
- In terms of Field Energy
- In terms of field Co-energy
- Energy in terms of Electrical parameters
- Rotary Motion
- Description of Simple System
- Energy stored in the coils
- Different Categories
- One coil each on Stator and on Rotor
- Vital Role of Air-gap
- Statically induced emf and Dynamically induced emf.

An electric motor is a machine which converts electrical energy into mechanical (rotational or kinetic) energy

25.1. Introduction

"Energy can ***neither*** be created ***nor*** be destroyed". We can only change its forms, using appropriate energy-conversion processes.

An interesting aspect about the energy in "Electrical form" is that neither it is so available ***directly from nature*** nor it is required to be finally consumed in that form. Still, it is the most popular form of Energy, since it can be transported at remote Load-locations, for optimum utilization of resources. Further, technological progress has now made it possible to device Electrical-Power-modulation systems so flexible and controllable that modern systems tend to be energy-efficient, with increase in life-span of main equipment and the associated auxiliary components (like switches, connecting cables, contactors, etc.) since it is now possible to avoid overstrain (= over-currents or over-voltage) on the system. This means a lot for the total production process (for which electrical energy is being used) since the quality of the production improves, and plant-maintenance is minimal. Energy-conversion systems then assume still higher importance.

Generator

Energy conversion takes place between well known pairs of forms of Energy: Electrical ↔ Chemical, Electrical ↔ Thermal, Electrical ↔ Optical, Electrical ↔ Sound, and Electrical ↔ Mechanical are the common forms with numerous varieties of engineering - applications. Electrical ↔ mechanical conversion is the focus of discussion in this chapter.

The elements of electro-mechanical energy conversion shall deal with basic principles and systems dealing with this aspect. Purpose of the study is to have a general approach to understand to design, and later to modify the system with the help of modern technologies, for overall improvisation.

It is necessary to be aware about:

(*a*) basic conditions to be fulfilled by the conversion system.

(*b*) methods for innovating the conversion systems.

Electromechanical energy-conversion finds applications in following categories of systems:

(*a*) transducers: Devices for obtaining signals for measurement / control,

(*b*) force-producing devices: Solenoid-actuators, relays, electromagnets,

(*c*) devices for continuous-energy-conversion: Motors / Generators

These systems have different configurations. But the principles of their working are common. Understanding these principles enables us to analyze / design / improvise / innovate such systems. As a result of such development, newer types of motors and the associated modern power controllers have recently been manufactured and become popular. Controllers using power-electronics switching devices offer energy-efficient, user-friendly, and high-performance drives. Their initial investment may be larger but two important parameters justify their use: (*i*) Considerable energy is saved, resulting into payback periods as short as 18-24 months, (*ii*) The controllers ensure to limit the currents to pre-set values under conditions of starting /overload/ unbalanced supply. Hence, the entire system enjoys longer life. Both these effects lead to better production-process and hence these are readily acceptable by industries.

25.2 Salient Aspects of Conversions

Purpose of electro-mechanical conversion device is to change the form of energy. Here, for simpler discussion, only rotary systems will be dealt with. When it is converting mechanical input to electrical output the device is "generating". With electrical input, when mechanical output is obtained, the device is motoring.

Some simple aspects of an electrical machine (motor / generator) have to be noted at this place:

(1) Electrical machine has a Stator, a Rotor, and an air-gap in between the two. For a flux path, the magnetic circuit has these three parts in series. In general, magnetic poles are established in Stator and in Rotor.

(2) Magnetic effects of following types can be categorized:

(*a*) **Electromagnetic:** Due to currents passed through windings on Stator and / or Rotor, producing certain number of poles on these members.

(*b*) **Permanent Magnets:** One side (Stator or Rotor) can have permanent magnets.

(*c*) **Reluctance variation:** Surface of Rotor near the air-gap can be suitably shaped to have a particular pattern of Reluctance-variation so as to control the machine behaviour as per requirements.

Stator and Rotor

(3) Basic conditions which must be satisfied by such devices are:

(*a*) Equal number of poles must be created on the two sides.

(*b*) In some cases, reluctance-variation is primarily used for machine-action. The Stator side must accommodate a winding carrying current for the electromagnetic effect, when rotor surface is shaped so as to have the desired pattern of reluctance variation. Or, non-cylindrical rotor cannot have the current -carrying winding for machine action.

(4) Out of stator, rotor and air-gap, maximum energy-storage at any angular position takes place in the air-gap, since its reluctance is highest out of the three members.

(5) Stored energy must depend on rotor-position and the device tends to occupy that angular position which corresponds to maximum stored energy. If this position varies as a function of time, the device produces continuous torque.

(6) Ideal output of a motor is a constant unidirectional torque with given currents through its windings. In some cases, the output torque (as a compromise) is an average value of a cyclically varying torque.

(7) Where current-switching is done for motor-control, as in modern controllers, instantaneous effect has to be understood to conclude on any of the points mentioned above.

(8) A device can work either as a generator or as a motor, provided pertinent conditions are satisfied for the concerned mode of operation.

25.3. Energy – Balance

For an electro-mechanical system, following terms are important:

(*i*) Electrical port (= armature terminals): receiving / delivering electrical energy.

(*ii*) Mechanical port (= shaft): delivering / receiving mechanical energy.

(*iii*) Coupling field: Magnetic field *or* Electric field.

Even though, theoretically, both the types of fields mentioned above are able to convert the energy, the magnetic medium is most popular since the voltage levels required are not very high, and the devices of given power rating are smaller in size and are economical. Hence, only those will be dealt with.

Conversion of electrical energy into mechanical energy

It is obvious that an electrical motor receives energy at the electrical port and delivers it at the mechanical port. While an electric generator receives the energy at the mechanical port and delivers it at the electrical port. It is also known that the following losses take place in such systems and are dissipated away as heat: (*i*) i^2r losses in the windings of the machines, (*ii*) friction and windage losses, (*iii*) core-losses.

These can be either neglected or attached to electrical port, mechanical port and coupling magnetic field respectively, for simpler analysis. With this, the simple energy balance equation can be written as:

Change in Electrical Energy = Change in Mechanical Energy + Change in Field-Energy

$$dW_{elec} = dW_{mech} + dW_{fld} \qquad \ldots(25.1)$$

It is natural that this equation has +ve signs for electrical and mechanical-energy-terms when the device is motoring. For generating mode, however, both the terms assume –ve signs.

In case no mechanical work is done, eqn. (25.1) reduces to eqn. (25.2) below indicating that Electrical energy - input is stored in the magnetic field.

$$dW_{elec} = dW_{fld} \qquad \ldots(25.2)$$

25.4. Magnetic-field System: Energy and Co-energy

25.4.1 Linear System

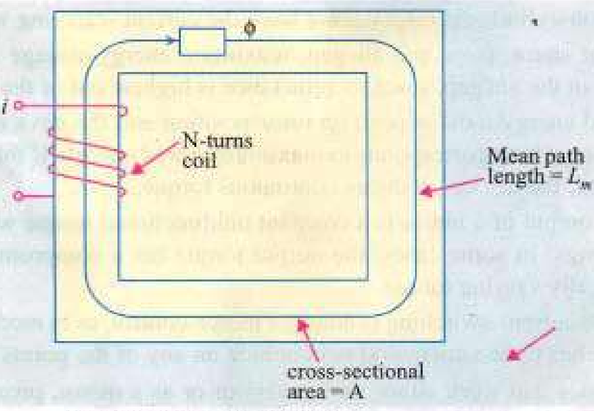


Fig. 25.1 (*a*) Magnetic circuit

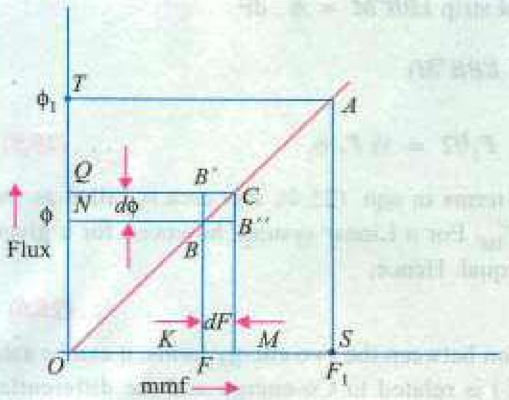


Fig. 25.1 (*b*) Characteristic of a magnetic circuit

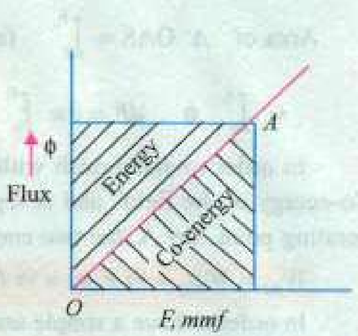


Fig. 25.1 (*c*) Energy and co-energy

A simple magnetic current is shown in Fig 25.1 (*a*), with assumptions that air-gap length at the joints is negligible, and the magnetic medium is not saturated. With A as the cross-sectional area of the core and L_m as the mean length of the path, a coil with N turns carrying a current of i amp has an mmf of F, establishing a flux of ϕ, related by

$$\phi = F \times \mathcal{P}_m$$

where $\mathcal{P}_m$ = Permeance of the Magnetic circuit

$$= \mu_0 \mu_r \; A/L_m$$

with μ_r = relative permeability of the magnetic medium,

This corresponds to the following relationships:

Coil Inductance, $L = N^2 \mathcal{P}_m = N\phi \,/\, i = \lambda \,/\, i$

where λ = flux-linkage of the coil, in weber-turns

$$W_{fld} = \text{Energy stored in the coil} = \tfrac{1}{2}\; Li^2 = \tfrac{1}{2}\, N^2 \mathcal{P}_m\, i^2 = \tfrac{1}{2}\, F^2\, \mathcal{P}_m$$

$$= \tfrac{1}{2}\, F(F\mathcal{P}_m) = \tfrac{1}{2}\, F\phi \qquad \text{. . . (25.3)}$$

In this eqn., $\mathcal{P}_m$ is the slope of the characteristic in Fig 25.1 (*b*). Hence, the inductance is proportional to the slope of F-ϕ plot. In Fig., 25.1 (*b*), for the operating point A, the mmf is F_1 and the flux ϕ_1. At the point A, the energy stored in the field is given by eqn. below:

$$W_{fld} = \tfrac{1}{2}\, F_1\, \phi_1$$

F_1 is due to the current i_1. W_{fld} is given by area *OATO* in Fig 25.1(*b*).

In Fig 25.1(*b*), the origin refers to the system without magnetization.

The system can reach the point A, starting from O as the current in the coil is increased from O to i_1.

Let us understand the intermediate events.

At point B, the flux is ϕ due to the mmf F.

An increment in coil current results into increase in mmf by dF. This increases the core-flux by $d\phi$. New operating point is C.

Eqn. (25.3) is to be suitably re-written in terms of these incremental values.

$$W_{fld} = \int dW_{fld} = \int_0^{\phi_1} F d\phi = \int_0^{\phi_1} \text{(area of the strip } NBB'Q\text{)} = \text{area of triangle } OAT$$

$$= (1/\mathcal{P}_m) \int_0^{\phi_1} \phi .\, d\phi = \tfrac{1}{2} .\phi_1 \,.\, F_1 \qquad \text{. . . (25.4)}$$

Alternatively, we have the area of elemental strip $kBB''M = \phi \,.\, dF$

$$\text{Area of } \Delta\ OAS = \int_0^{F_1} \quad \text{(area of the strip } KBB''M)$$

$$= \int_0^{F_1} \phi \,.\, dF = \mathcal{P}_m \int_0^{F_1} F\, dF = \mathcal{P}_m \; F_1^2/2 = \tfrac{1}{2} F_1\, \phi_1 \qquad \ldots (25.5)$$

In order to distinguish with respect to the terms in eqn. (25.4), this area is called as the "Co-energy of the field" and is represented by W'_{fld}. For a Linear system, however, for a given operating point, say A, the two energy-terms are equal. Hence,

$$W_{fld} = W'_{fld} = \tfrac{1}{2} F_1 \phi_1 = \tfrac{1}{2} L i_1^2 \qquad \ldots (25.6)$$

In order to have a simple and clear distinction between the two energy terms, it can be said that the differential variable ***"Current"*** (or mmf) is related to Co-energy and the differential variable "flux - linkage" (or flux) is related to Energy.

This energy stored in the magnetic field comes from the electrical source connected to the coil in Fig. 25.1(*a*).

25.4.2. A Simple Electromechanical System

A simple electro-mechanical system is shown in Fig. 25.2(*a*).

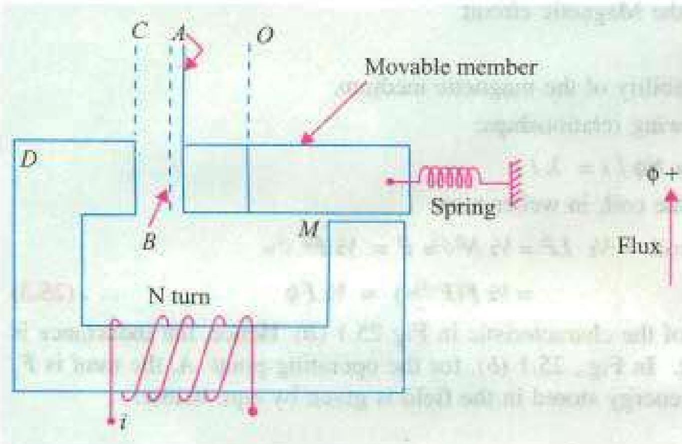


Fig. 25.2 (*a*)

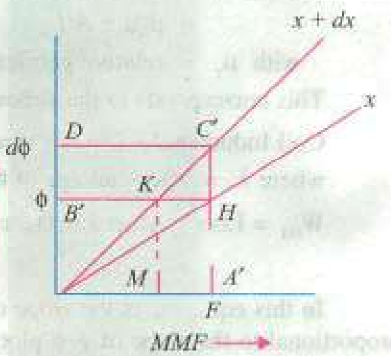
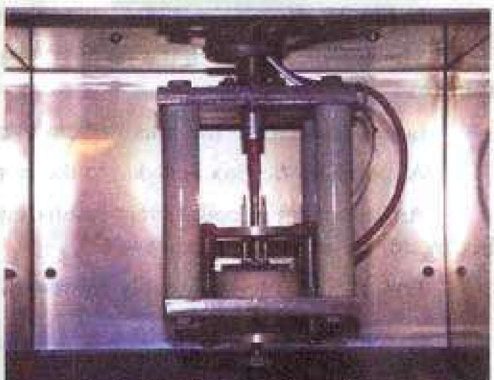


Fig. 25.2 (*b*)

Reference point O corresponds to the unstretched spring. Energy stored in the spring is then Zero. In position A of the movable member, the spring is elongated by x, and the corresponding energy stored in the spring is $\tfrac{1}{2} K_x x^2$, where K_x is "spring-constant" of the linear system in Nw / m. In Fig 25.2 (*a*), the distance OA is x. The elemental distance AB is dx, so that OB is $x + dx$. For simpler analysis, it is assumed that magnetic material is highly permeable and that the clearance at point M (for movement of the member) is negligible. So that the mmf of the coil is required to drive the flux in the region $OABC$ only. The flux-mmf



A simple electromechanical system

relationships are plotted for these two positions in Fig 25.2 (*b*). In position A, the movable member has moved a distance of x from its unstretched position or reference point. Let the operating point be H, so that the coil-mmf OA $(=F)$ establishes a flux OB $(=\phi)$. In this position, the movable member experiences a force in such a direction that the energy stored in the field tends to increase. It tends to reach B, so that an additional displacement of dx shifts the characteristic upwards and final operating point in position B is C'. From H to C', the operating point can move in any one of the following ways:

(*a*) HC' vertically, if the mechanical movement is too slow so that change of flux is slow and induced emf in the coil is negligible. This corresponds to the coil-mmf remaining constant at F during the transition. Constant mmf means vertical travel of the operating point from H to C'.

(*b*) H to K horizontally and then K to C' along the characteristic corresponding to $(x + dx)$ as the displacement of the movable part. This is possible when the motion is very fast, resulting into flux remaining constant till the operating point traverses from H to K. Then, from K to C', the flux increases, an emf in induced the coil and the mmf finally reaches its value of F, at the point C'.

(*c*) In reality, the transition from H to C' will be somewhere in between these two extremes mentioned above.

However, for simplicity, one of these extreme conditions has to be accepted. In (*a*) above, the mmf remains constant. In (*b*) above, the flux (and hence the flux-linkage) remains constant. Let us take the case of constant-mmf. If the process has taken a time of dt,

Electrical-energy input during the process $= dW_{elec}$

$=$ (voltage applied to the coil) $\times$ current $\times$ $dt = eidt$

$= d\lambda/dt \times i \times dt = id\lambda = i\,N\,d\phi = Fd\phi =$ area of rectangle $B'HC'D$

In this case, coil-resistance has been neglected.

25.4.2.1. In terms of Field Energy

At the previous operating point H, the energy stored in the magnetic field,

W_{fld1} = area of $\Delta\ OHB'$

At the new position corresponding to the operating point C', the field energy stored is given by W_{fld2} = area of the $\Delta\ OC'D$

The difference of these two is the change in the energy stored in the

magnetic field $= dW_{fld} = \frac{1}{2}\,[OA' \times A'C' - OA' \times A'H]$

$= \frac{1}{2}\,OA'\ [A'C' \times A'H] = \frac{1}{2}\,OA'\ \ HC'$

$= \frac{1}{2}\,.\,F\,.\,d\phi = \frac{1}{2}\ dW_{elec}$

Out of the energy delivered by the source, half is stored in the magnetic field. Where has the remaining half been utilized? Obviously, this must have been transformed into the mechanical work done. In this case, neglecting losses, it is finally stored in the stretched spring due to its elongation by dx.

Comparing this with the equation (25.1),

$$dW_{elec} = dW_{mech} + dW_{fld}$$

$$= dW_{mech} + \tfrac{1}{2}\ dW_{elec}$$

or $$dW_{mech} = dW_{fld} = \tfrac{1}{2}\ dW_{elec}$$

Consider that a force F is operative at the displacement of x. This force is in such a direction that x increases or the movable member is attracted towards D. In the same direction, a displacement by dx results into the increase in the energy stored by the spring. Relating the concerned terms,

$$F = k_s x$$

dW_{mech} = mechanical work done against the force of the stretched spring

$$= -F\,dx = dW_{fld}$$

or $F = -dW_{fld} / dx$, in this case

$$= -\delta W_{fld} / \delta x, \text{ in general}$$

Alternatively, the difference in the energy stored in the spring also gives a very useful relationship.

In the position corresponding to $x + dx$, the energy stored in the spring $= \frac{1}{2}\ k_s\ (x + dx)^2$. Similarly, at x, the energy $= \frac{1}{2}\ k_s\ x^2$

$$\text{Difference} = \tfrac{1}{2}\ k_s\ [(x + dx)^2 - (x)^2]$$

$$= \tfrac{1}{2}\ k_s\ [x^2 + 2.x.dx + (dx)^2 - x^2]$$

$$= \tfrac{1}{2}\ k_s\ [2.\ x.\ dx], \quad \text{neglecting } (dx)^2$$

$$= k_s.\ x.\ dx = F\ dx$$

This difference is nothing but dW_{mech}, which is equal in magnitude to dW_{fld} and confirms the relationship obtained earlier.

25.4.2.2. In terms of field Co-energy

Proceeding along lines similar to those while dealing with field-energy above, following relationships exist. For simpler discussion, the transition is assumed to be along HKC'. Neglecting area of the small triangle HKC', we have

at x, W'_{fld1} = area of $\Delta\ OA'H$

at $x + dx$, W'_{fld2} = area of $\Delta\ OA'C'$ (neglecting $\Delta\ HKC'$)

$$dW'_{fld} = W'_{fld2} - W'_{fld1} = \phi.\ dF \quad \text{where } dF = MA'$$

$$= \text{Co-energy in the field} \quad F\ dx$$

Hence, $F = +\ dW'_{fld} / dx$, in this case

$$= +\ \delta W'_{fld} / \delta x, \text{ in general}$$

25.5. Energy in Terms of Electrical Parameters

In the preceding article, the energy and force were related in terms of magnetic-system parameters, namely flux and mmf, through the third parameter, the permeance.

It is at times convenient to relate these things in terms of electrical-system-parameters, namely, the inductances and currents. That is being dealt with here only for linear systems. Let $\mathcal{P}$ be the permeance of the magnetic circuit and L be the coil-inductance.

$$W_{fld} = \text{Field-energy} = \tfrac{1}{2}\ F\,\phi = \tfrac{1}{2} Ni\ (Ni\ \mathcal{P}) = \tfrac{1}{2}\ (N^2 \mathcal{P}) i^2$$

$$= \tfrac{1}{2}\ Li^2$$

$$F = -\,dW_{fld} / dx = -\tfrac{1}{2}\,.\,i^2.\ dL/dx$$

Thus, a force exists if the coil-inductance is dependent on x. Such analysis is more suitable when the system has more than one coils coupled through the magnetic circuit. If two such coils are considered, following data should be known for evaluation of the force, in case of linear displacement:

L_{11} = self inductance of coil - 1

L_{22} = self inductance of coil - 2

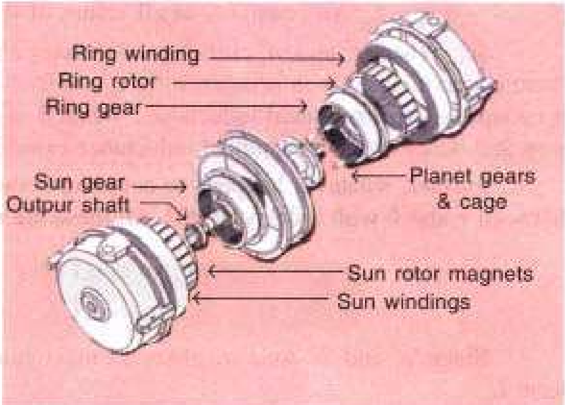
L_{12} = Mutual inductance between two coils, 1 and 2

i_1, i_2 = currents through the two coils.

W_{fld} = Total energy stored in the field $= ½ L_{11} i_1^2 + ½ L_{22} i_2^2 + L_{12} i_1 i_2$

Magnitude of Force $F = dW_{fld}/dx = ½ i_1^2 \, dL_{11}/dx + ½ i_2^2 \, dL_{22}/dx + i_1 i_2 \, dL_{12}/dx$

From the right-hand side of this equation, it is noted that the inductance-term which is dependent on x contributes to the force.



25.6. Rotary Motion

Most popular systems for electro-mechanical energy are Generators and Motors. The preceding discussion dealt with the Linear motions, wherein x represented the displacement parameter, and force was being calculated.

Now we shall deal with the rotary systems, wherein angular displacement parameters (such as θ) and corresponding torque developed by the system will be correlated, through a systematic procedure for a typical rotary machine

25.6.1. Description of Simple System

A simple rotary system has a 'stator' and a 'rotor'. Air-gap separates these two. Stator has two similar coils 'a' and 'b' located at 90^0 electrical, with respect to each other. Inner surface of the stator is cylindrical. Outer surface of the rotor is also cylindrical resulting into uniform air-gap length for the machine.

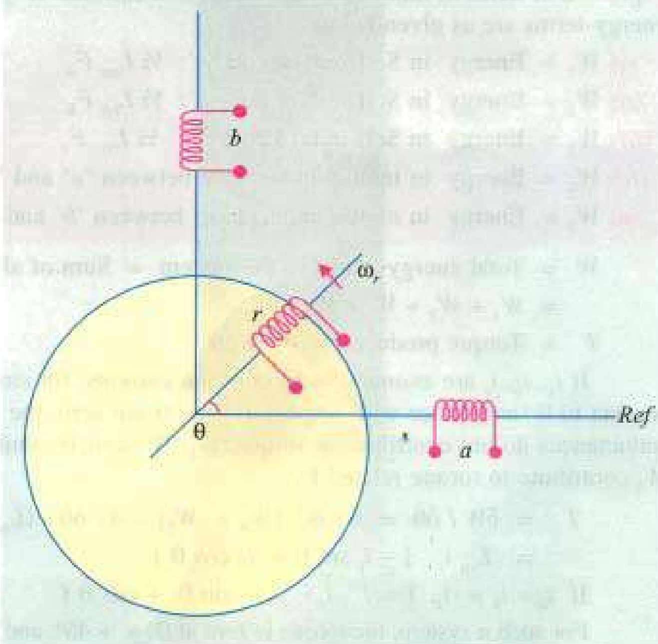


Fig. 25.3. Simple rotary system

The diagram represents a two-pole machine. Axis of coil 'a' may be taken as reference, with respect to which the rotor-coil axis makes an angle of θ, at a particular instant of time. For a continuous rotation of the rotor at ω radians / sec, θ = ωt. Coil-'b'-axis is perpendicular to the reference, as shown. Due to the uniform air-gap length, and due to the perpendi-cularity between coils 'a' and 'b', inductance-parameters exhibit the following patterns:

Let x represents the inductance parameter as a function of θ. The subscripts indicate the particular parameter. x_{aa} = self-inductance of coil 'a', x_{ab} = mutual inductance between coils a and b and so on. x represents value of the particular inductance parameter, which will help in knowing the variation of inductance with θ.

(a) Self inductances of coils 'a' and 'b' are not dependent on rotor position.

$x_{aa} = L_{aa}$ and $x_{bb} = L_{bb}$, at all values of θ.

(b) Mutual inductance between stator coils 'a' and 'b' is zero, due to perpendicularity.

x_{ab} = zero, for all values of θ.

(c) Self-inductance of rotor-coil is constant and not dependent on θ.

$x_{rr} = L_{rr}$ (= constant), at all values of θ.

(d) When 'r' and 'a' coils have their axes aligned, at $\theta = 0$, the mutual inductance between them is maximum, which is denoted by x_{ar}. At $\theta = 90^0$, their axes are perpendicular resulting into no coupling or zero mutual inductance. When $\theta = 180^0$, r and a coils are aligned in anti-parallel way and hence maximum mutual inductance exists between them with negative sign.

Further, whatever happens to coupling between r and 'a' at a value of θ happens to that between r and b with a delay of 90^0. All these are mathematically represented as :

$$x_{ra} = L_{ra} \cos\theta = L_{ra} \cos(\omega_r t)$$

$$x_{rb} = L_{ra} \sin\theta = L_{ra} \sin(\omega_r t) = L_{ra} \cos(\omega_r t - 90^0)$$

Since 'a' and 'b' coils are alike, the maximum mutual inductance is represented by the same term L_{ra}.

25.6.2. Energy stored in the coils

Energy stored in the magnetic field can *either* be expressed in terms of mmf and flux *or* be expressed in terms of inductance-terms and coil currents. If i_a, i_b and i_r are the coil-currents, stored-energy-terms are as given below:

(i) W_1 = Energy in Self-ind. of coil 'a': $\frac{1}{2} L_{aa} i^2_a$

(ii) W_2 = Energy in Self-ind. of coil 'b': $\frac{1}{2} L_{bb} i^2_b$

(iii) W_3 = Energy in Self-ind. of coil 'r': $\frac{1}{2} L_{rr} i^2_r$

(iv) W_4 = Energy in mutual inductance between 'a' and 'r': $x_{ra}\, i_r i_a = x_{ra}\, i_r i_a = x_{ra} \cos\theta . i_r . i_a$

(v) W_5 = Energy in mutual inductance between 'b' and 'r' : $x_{rb}\, i_r . i_b = x_{ra} \sin\theta . i_r . i_b$

W = Total energy stored in the system = Sum of all the energy-terms cited above

$= W_1 + W_2 + W_3 + W_4 + W_5$

T = Torque produced = $\delta W / \delta\theta$

If i_a, i_b, i_r are assumed to be constant currents, for simplicity, so that their derivatives with respect to θ (and hence with respect to time t) are zero, the energy-terms which include constant inductances do not contribute to torque. W_1, W_2, and W_3 thus cannot contribute to torque. W_4 and W_5 contribute to torque related by:

$$T = \delta W / \delta\theta = \delta / \delta\theta\ [W_4 + W_5] = \delta / \delta\theta\ [L_{ra}\, i_r i_a \cos\theta + L_{ra}\, i_r i_b \sin\theta]$$

$$= L_{ra}\, i_r\ [-i_a \sin\theta + i_b \cos\theta]$$

If $i_a = i_b = i_s$, $T = L_{rs}\, i_r\, i_s\ [-\sin\theta + \cos\theta]$

For such a system, the torque is zero at $\theta = +45°$, and the torque is maximum at $\theta = -45°$.

If one of the stator currents is reversed, the result differs. For this, let $i_a = -i_s$ and $i_b = +i_s$

$$T = L_{ra}\, i_r\, i_s\, [\sin\theta + \cos\theta]$$

And the maximum torque occurs at $\theta = 45^0$. This is a position for rotor, which is midway between the two stator coils.

25.6.2.1. Different Categories

From the torque expressions above, it is clear that the torque exists only when stator and rotor-coils carry currents. When only stator-coils (or only rotor coil) carry current, torque cannot be produced.

(a) One coil each on Stator and on Rotor

In the above mentioned case, let us excite only one stator-coil. Let $i_a = i_s$, $i_b = 0$, and i_r maintained as before.

$$T = -L_{ra}\; i_r i_s\; \sin\theta$$

Following observations are made for such a case:

(i) At $\theta = 0$, $T = 0$. Mutual inductance x_{ra} is maximum ($= L_{ra}$) and hence the stored energy in mutual inductance is maximum, but torque is zero.

(ii) At $\theta = 90^0$, T is maximum. x_{ra} is zero, hence the concerned stored energy is zero.

(iii) As seen earlier, $W_4 = L_{ra}\, i_r i_a \cos\theta = L_{ra}\, i_r i_s \cos\theta$

Torque $= dW_4 / d\theta = i_r i_s\; d\,(L_{ra}\cos\theta) / d\theta$

Contributed by W_4, the power is related as follows:

Power = rate of change of energy with time

$$= dW_4 / dt = dW_4 / d\theta .\ d\theta / dt = T.\ \omega_r = i_r i_s\, L_{ra}\,(-\sin\theta)\,\omega_r$$

Magnitudes of these terms are maximum for $\theta = 90^0$. If θ can be set at 90^0, at all instants of time, torque obtained is maximum. Such a situation does exist in a d. c. machine in which rotor carries an armature winding which is a lap- or wave-connected commutator winding. The brushes are so placed on the commutator that rotor-coil-axis satisfies the above-mentioned condition of $\theta = 90^0$, irrespective of the rotor-position or rotor speed. Such an equivalence of a **rotating** armature coil with such an **effectively stationary** coil is referred to as a **quasi-stationary** coil. It means that a rotating coil is being analyzed as a stationary coil due to its typical behaviour for electro-mechanical energy conversion purposes.

Permanent magnet synchronous motor for washing machine

(b) Two stator coils carrying two-phase currents and rotor-coil carrying d. c.: When two stator coils carry two-phase alternating currents, a synchronously rotating mmf is established. If the rotor-coil carries direct current, and the rotor is run at same synchronous speed, a unidirectional constant torque is developed. Mathematically, similar picture can be visualized, with a difference that the total system is imagined to rotate at synchronous speed. Such a machine is Synchronous machine, (to be discussed in Later chapters). It can be understood through the simple system described here.

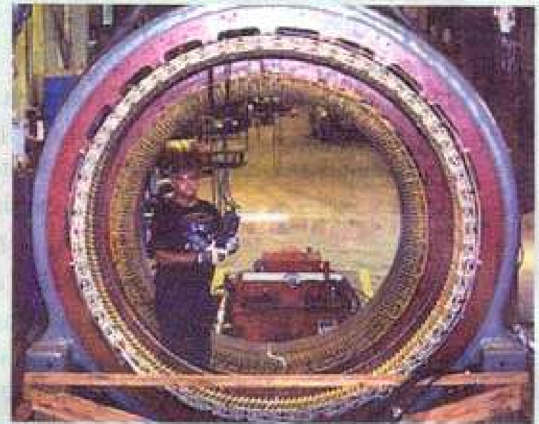
Permanent magnet

(c) Machines with Permanent Magnets.

With suitable interpretation, the field side of the simple system can be imagined to be with permanent magnets in place of coil-excited electromagnets. All the interpretations made above are valid, except for the difference that in this case there is no scope for controlling the rotor-coil-current-magnitude.

(d) Machines with no rotor coil, but with premeance variation.

Smooth cylindrical rotor surfaces do not exist in such cases. There are no rotor-coils. Due to geometry of the rotor surface, stator-coil- self inductances vary with rotor position. Thinking on lines of relating energy terms and their derivatives for torque-calculations, the working principles can be understood. With simple construction, Reluctance motors belong to this category.



Current in stator coils

(e) Switched currents in Stator Coils.

In yet another type, stator coils are distributed and properly grouped. One group carries currents during certain time interval. Then, this current is switched off. Another group carries current in the next time interval and so on. The rotor surface is so shaped that it responds to this current switching and torque is produced. Even though stator-coil-inductances are complicated functions of rotor position, the method of analysis for such machines is same. Prominent types of machines of this type are: switched reluctance motors, stepper motors, etc.

25.6.2.2. Vital Role of Air-gap

Magnetic circuit of an electrical machine has a flux established due to coil-mmfs. This flux is associated with stator core, rotor core and air-gap. An important point for understanding is to know which out of these three stores major portion of the field energy. Through an illustrative case, it will be clear below, in example 25.1.

Example 25.1. *Let a machine with following data be considered.*

Calculate the energy stored in the air-gap and compare the same with that stored in the cores.

Stator-core outer diameter = 15 cm

Stator-core inner diameter = 10.05 cm

Rotor-core outer diameter = 10.00 cm

Rotor-core inner diameter = 5 cm

Axial length of the machine = 8 cm

Effect of slotting is neglected. The core volumes and air-gap volume for the machine shown in Fig 25.4 have to be calculated.

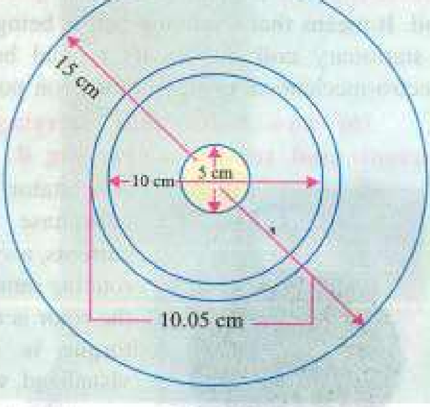


Fig. 25.4

Solution.

Volume of Stator-core $= (\pi / 4) \times (15^2 - 10.05^2) \times 8 \text{ cm}^3 = 779 \text{ cm}^3$

Volume of Rotor-core $= (\pi / 4) \times (10^2 - 5^2) \times 8 \text{ cm}^3 = 471 \text{ cm}^3$

Volume of air-gap in the machine $= (\pi / 4) \times (10.05^2 - 10^2) \times 8 \text{ cm}^3$

$= 6.3 \text{ cm}^3$

Let the relative permeability of the core material be 1000. If the flux density is B Wb / m^2, and μ is the permeability, the energy-density is ½ x B^2 / μ Joules / m^3. Let the flux density be 1.20 Wb / m^2. Energy density in air-gap = ½ × 1.20^2 / (4 π × 10^{-7}) = 572350 Joules / m^3

= 0.573 Joules / cm^3

Energy stored in air-gap = 0.573 × 6.3 = 3.6 Joules

Energy-density in Magnetic medium = ½ × 1.20^2 / (4π × 10^{-7} × 1000) = 573 J / m^3

It is assumed only for simplicity that the flux density is same for the entire core of stator and of rotor.

Energy stored in stator-core = 573 × 779 × 10^{-6} = 0.45 Joule

Energy stored in rotor-core = 573 × 471 × 10^{-6} = 0.27 Joule

It is worth noting that even though the ratio of volumes is 198, the ratio of energies is 0.2, since, for the present case,

$$K_v = \frac{\text{volume of (Stator - core + Rotor - core)}}{\text{Volume of air - gap}} = \frac{779+471}{6.3} = 198$$

k_E = Energy stored in cores / Energy stored in air-gap = (0.45 + 0.27) / 3.6 = 0.2

The ratio are like this due to μ_r being 1000, and k_v / k_E = 198 /0.2 = 1000

Alternatively, an air-gap of volume 6.3 cm^3, [surrounded by the magnetic medium of μ_r = 1000] is equivalent to the magnetic medium of volume 6.3 × 1000 cm^3.

$$\frac{\text{Converted equivalent volume of air-gap}}{\text{Volume of (Stator + Rotor)}} = \frac{6300}{779+471}$$

$$\frac{\text{Energy stored in air-gap}}{\text{Energy stored in (Stator + Rotor)}} = \frac{3.6}{(0.45+0.27)} = 5$$

This correlates the various parameters and confirms that the stored energy is maximum in the air-gap.

Or, one can now say that in the process of electro-mechanical energy-conversion, the air-gap plays a very vital role.

However, the stator-core and rotor-core help in completing the flux-path in a well defined manner for effective and efficient working of a rotary machine.

Example 25.2. *An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross-sectional area of 5 cm × 5 cm. Neglect reluctance of the magnetic circuit and fringing.*

(a) (i) Find the coil inductance if the air-gap length is 0.5 cm.

(ii) Find the field energy stored for a coil current of 1.25 amp.

(b) Coil-current remaining constant at 1.25 A, find the mechanical energy output based on field-energy changes when the armature moves to a position for which x = 0.25 cm. Assume slow movement of armature.

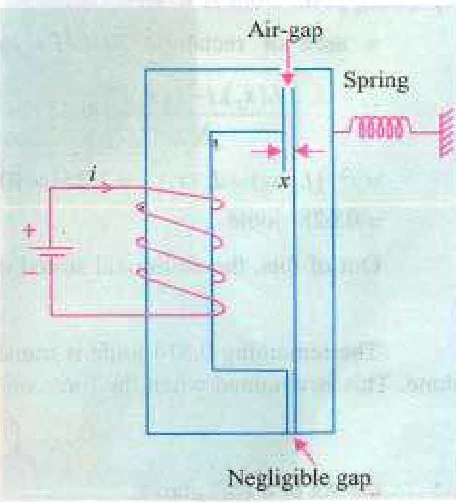


Fig. 25.5 Electro-magnetic relay

(c) Repeat (b) above based on force-calculations and mechanical displacement.

(d) What will be change in above results of mechanical work done, if the mechanical movement is fast, keeping the flux initially constant ?

Solution.

(a) (i) Permeance at air-gap $= \dfrac{\mu_0 \, 5\times5\times10^{-4}}{0.5\times10^{-2}} = 4\pi\times10^{-7}\times10^{-2} = 6.28\times10^{-7}$

Coil Inductance $= N_2 \, \mathcal{P}_m = 800 \times 800 \times 6.28 \times 10^{-7} = 0.402$ H

(ii) Energy stored in magnetic field $= 1/2 \; Li^2 = 1/2 \times 0.402 \times 1.25^2$

$= 0.314$ joule

(iii) $W'_{fld} = 1/2\,L(x)i^2 = 1/2\left[\dfrac{H^2\mu_0 A}{l_s}\right] = \dfrac{1/2\times800\times800\times4\pi\times10^{-7}\times5\times5\times10^{-4}}{x}\times i^2$

$$= \frac{1.005\times10^{-3}}{x}\times i^2$$

$$F_f = \frac{\delta}{\delta_x}\frac{[i^2\times(1.005\times10^{-3})]}{x} = [1.005\times10^{-3}]\times i^2\times\frac{-1}{x^2}$$

This is to be evaluated at $\quad x = 0.5\times10^{-2}$

$$= \frac{-1.005\times10^{-3}\times1.25\times1.25}{(0.5\times10^{-2})^2} = -62.8\,\text{NW}$$

This force has to be balanced by the spring-tension.

(b) Energy-computations : Inductance for $x = 0.25$ cm is first calculated. $L(x_2) = N^2 \mathcal{P}_{m_2}$

$= 800 \times 800 \times 2 \times 6.28 \times 10^{-7} = 0.804$ Henry

If the mechanical movement is slow, net mmf remains unchanged and the operating point moves along *HC* vertically upwards and settles at *C*. Added Electrical Energy input during change-over of the operating point from *H* to *C*,

$=$ area of rectangle $BDCH = (\phi_2 - \phi_1)\,F_1$

$$\left[\frac{L(x_2).i - L(x_1)i}{N}\right] Ni$$

$= i^2 \;[L(x_2) - L(x_1)] = 1.25^2 \times [0.804 - 0.402]$

$= 0.628$ joule

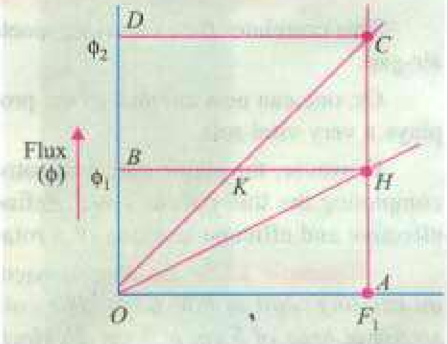


Fig. 25.6 Graphical correlation of energy-terms for the relay

Out of this, the additional stored energy in field, $\quad dW_{fld} = ½\,[L(x_2) - L(x_1)]$

$= 0.314$ joule

The remaining 0.314 joule is transformed into mechanical form and is related to the work-done. This is obtained when the force on moving member is multiplied by the displacement.

$$\therefore \int_{x_1}^{x_2} dW_{mech} = 0.314 \text{ joule}$$

(c) As in *a (iii)* above,

$F(x) = [1.005 \times 10^{-3} \times 1.25^2]\;[-1/x^2]$

$$dW_{mech} = F(x).dx$$

$$W_{mech} = \int_{x_1}^{x_2} F(x)dx = k.\ [\int_{x_1}^{x_2} -1/x^2 d_x] = -k.\int_{x_1}^{x_2} x^{-2}dx$$

$$= 1.005\times10^{-3}\times1.25^2\times\left[\frac{1}{0.25}-\frac{1}{0.5}\right]\times10^2 = \frac{1.005\times1.25^2}{10}\times2 = 0.314 \text{ joule}$$

This agrees with answer obtained in (*b*) above.

With fast movement of armature, the operating point will move from H to K first, then follow the path *KC*.

This means that the energy represented by the area of the triangle KHC corresponds to the reduced consumption of energy.

OC has a slope of $\mathcal{P}_{m_2}$ = 12.56×10^{-7}

OH has a slope of $\mathcal{P}_{m_1}$ = 6.28×10^{-7}

BK = mmf required for establishing a flux of ϕ_1 with an air-gap of 0.25 cm

BK = ½ *OA* = ½ × (800 × 1.25) = 500 amp-turns = *HK* in the present case.

Area of the triangle *KHC* = ½ × *KH* × *HC*

= ½ × 500 × $[\phi_2 - \phi_1]$

= ¼th of area of rectangle *BDCH*, in this case

= ¼ × 0.628 = 0.157 joule.

Hence, Electrical energy fed during this process = area *BKCD*

= area *BDCH* – area *KHC* = 0.628 – 0.157 = 0.471 joule

Increase in field energy stored ΔW_{fld} = area *OKH*

= area OHC – area *KHC* = 0.314 – 0.157 = 0.157

Mechanical Energy output = 0.471 – 0.157 = 0.314 Joule

It indicates that with fast movement, the electrical energy-input and the field-stored energy have decreased by 0.157 J each but the mechanical-energy-term remains unaffected by fast or slow movements of armature.

25.7. Dynamic Equations and System-model of a Simple System

It is quite necessary to analyze electro-mechanical conversion system for predicting the performance and/or for monitoring the system. A simple system is being taken up here to deal with dynamic equations and a simple model with its components is being related to the system. The details will vary from system to system, and accordingly the equations will vary.

Fig. 25.7 shows different components of such a system meant for electrical to mechanical conversion. On one side, an electrical source feeds the device at the 'electrical port'. On the other side, a force f_e is developed at the 'mechanical port'. Mechanical load is connected to this port.

(a) At Electrical Port : A voltage source is shown to feed the device, r is its effective internal resistance. At the electrical port, the inputs are λ (= flux linkage with the coil) and *i*. From λ , the voltage induced in the coil can always be evaluated.

(b) Role of the Conversion device: With these inputs, the device converts the energy into mechanical form, and is available as a force f_e (in case of linear motions), and, displacement *x* measured from a suitable reference.

***(c) At the Mechanical Port*:** The possible items are: spring, damper, mass and an applied mechanical force. Their natural and simple dependence on displacement x and its derivatives are indicated below:

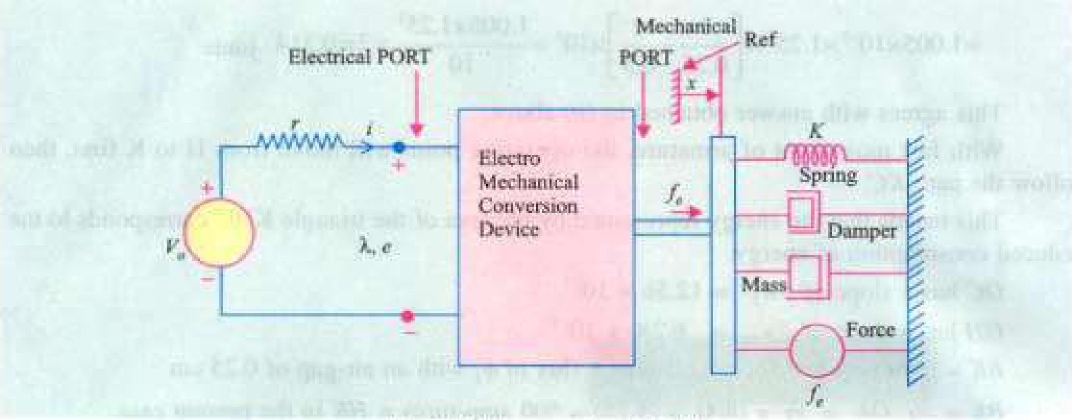


Fig. 25.7 Linear motion: MODEL

(i) Spring: Force required to overcome spring elongation is proportional to the displacement x.

(ii) Damper: Force required to overcome damping action in the system is proportional to derivative of x.

(iii) Mass: Force required to overcome acceleration of mass is proportional to second derivative of x.

(iv) Applied force, f_o: This has to be overcome by f_e . In terms of an equation, these terms are related as follows:

$$f_e = k_s\ (x - x_o) + B\dot{x} + M\ddot{x} + f_o$$

where

k_s = spring constant

x_o = value of x for unstretched spring

B = damping constant

M = Mass to be accelerated

f_o = External mechanical force applied to the system.

25.8. Statically induced emf and Dynamically induced emf :

In Fig. 25.7 source voltage is. v_o. Let $L\ (x)$ be the coil inductance as a function of displacement x. In a very general case,

$$\begin{aligned} v_o(t) &= ri + d\lambda/dt \\ &= ri + d/dt\ [L(x) \times i] \\ &= ri + L(x).\ di/dt + i.\ dL(x)/dx.\ dx/dt \end{aligned}$$

The second term on the right-hand side is statically induced emf (or transformer-emf), since change of current with time is responsible for it. This cannot produce any force (or torque) and hence cannot convert energy from electrical to mechanical form (or vice-versa).

The third term on the right hand side includes the speed (= dx/dt) and dependence of L (x) on x. Any of these, if non-existent, will mean that third term reduces to zero. This term relates dynamically induced emf (= speed emf) and is the main indicator of the process of electro-mechanical energy conversion. So, for conversion, there must be an inductance which varies with the system position, and a motion must be there. In addition, coil must carry a current.

Having understood the linear-motion-system, it is easier to understand the system with rotary motion, with due modifications.

Example 25.3. *A doubly excited rotating machine has the following self and mutual inductances.*

$$r_s = 40\,\Omega, \quad L_s = 0.16\ H$$
$$r_r = 2\ \Omega, \quad L_r = 0.04 + 0.02 \cos 2\theta$$
$$M_{sr} = 0.08 \cos \theta$$

where θ is the space-angle between axes of rotor-coil and of stator-coil. The rotor is revolving at a speed of 100 radians/sec. For i_s = 10 Amp d. c., and i_r = 2 Amp d. c., obtain an expression for torque and corresponding electrical power.

[Rajiv Gandhi Technical University, Bhopal, Summer 2001]

Solution. W_{fld} = Total energy stored

$$= \tfrac{1}{2}\ L_s\ i^2_s + 1/2\, L_r\, i^2_r + M_{sr}\ i_s i_r$$
$$= \tfrac{1}{2}\ (0.16)\ i^2_s + \tfrac{1}{2}\ [0.04 + 0.02\ \cos 2\theta] i^2_r$$
$$+ \ [0.08 \cos \theta]\ i_s i_r$$

since i_s and i_r are direct currents of constant magnitudes, there is no variation with φ or with t.

Relating torque with W_{fld} and substituting current-magnitudes,

$$\text{Torque, } T = \frac{-dW_{fld}}{d\theta} = -\,[0 + \tfrac{1}{2} \times 0.02 \times 2^2\ (-\,2\sin 2\,\theta) + 0.08\ (-\sin \theta\,)\ (10 \times 2)]$$
$$= 0.08\ \sin 2\,\theta + 1.6\ \sin \theta$$
$$= 1.6 \sin \theta\ + 0.08 \sin 2\,\theta\ \text{Nw-m}$$

On the right hand side, the first term is electromagnetic Torque which is dependent on both the currents. Second term is dependent only on one current, and is of the type categorized as Reluctance-torque which depends on non-cylindrical shape,in this case, on the stator side, as shown in Fig. 25.8

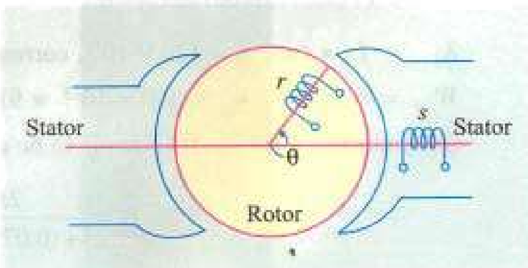


Fig. 25.8

Starting from W_{fld}, electrical power can be expressed, since it is well -known that

Power = time rate of change of energy

$$\text{Electrical power,} \quad p = \frac{dW_{fld}}{dt}$$

$$= \frac{d}{dt}\left[\tfrac{1}{2} L_s i^2{}_s + \tfrac{1}{2} L_r\, i^2{}_r + M_{ST}\, i_s i_r\right]$$

$$= \tfrac{1}{2} i^2{}_s \frac{d}{dt}(L_s) + \tfrac{1}{2} i^2{}_r \frac{d}{dt}(L_r) + i_s i_r \frac{d}{dt}(M_{sr})$$

$$= \tfrac{1}{2} i_s^2 \left[\frac{d}{d\theta}(L_s)\right]\frac{d\theta}{dt} + \tfrac{1}{2} i_r^2 \left[\frac{d}{d\theta}+(L_r)\right]\frac{d\theta}{dt} + i_s i_r \left[\frac{d}{d\theta}(M_{sr})\right]\frac{d\theta}{dt}$$

$$= \tfrac{1}{2} i_s^2 \,[\text{zero}]\times 100 + \tfrac{1}{2}\, i_r^2 \,[-0.02\times 2\times \sin 2\theta]\times 100 + i_s i_r \,[-0.08\sin\theta]\times 100$$

Substituting numerical values of currents,the electrical power is expressed as a function of θ, as below :

$$p = [\text{zero} + (-8) \sin 2\theta + (-160) \sin \theta] \text{ watts}$$
$$= [-160 \sin\theta - 8 \sin 2\theta] \text{ watts}$$

Proper interpretation of sign of power (as dependent on θ) is important. Positive power is received by the coils, while negative power is received by the source.

Example 25.4. *An inductor has an inductance which varies with displacement x as*

$$L = 2\, L_o / [1 + (x / x_o)]$$

Where L_o = 50 mh, x_o = 0.05cm, x = displacement in cm.

The coil –resistance is 0.5 ohm.

(a) The displacement x is held constant at 0.075 cm, and the current is increased from 0 to 3 amp. Find the resultant magnetic stored energy in the inductor.
(b) The current is then held constant at 3 amp and the displacement is increased to 0.15 cm. Find the corresponding change in the magnetic stored energy.

Note. Assume that all electrical transients are negligible.

Solution. *(a)* Inductance at x = 0.075 cm is calculated first.

$$L_1 = \frac{2L_0}{1+(0.075/0.05)} = 40\,\text{mH}$$

$\lambda_1 = L_1 \times$ current $= 120 \times 10^{-3}$, corresponding to point A in Fig. 25.9

$W_{fld} = \frac{1}{2} L_1 (3)^2 = \frac{1}{2} (40 \times 10^{-3} \times 9) = 0.18$ joule

(b) Inductance for x = 0.15 cm is to be calculated now.

$$L_2 = \frac{2L_0}{1+(0.075/0.05)} = 25\,\text{mH}$$

With current held constant at 3amp, the flux-Linkage is now $\lambda_2 = (25 \times 10^{-3}) \times 3 = 75 \times 10^{-3}$

Since the current is constant at 3amp, magnetic stored energy is reduced by the area of triangle *OAB*, in Fig. 25.9

Area of triangle *OAB* $= \frac{1}{2} \times 3 \times (120 - 75)\, 10^{-3} = 0.0675$ joule

Check : Stored-energy at *B* in terms of L_2 and *i*, is given by

$W_{fld} = \frac{1}{2} (25 \times 10^{-3}) \times 3^2 = 0.1125$ joule

Alternatively, $W_{fld2} = W_{fld1}$ – area of Δ OAB
$= 0.18 - 0.0675 = 0.1125$ Joule

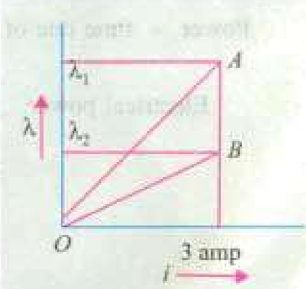


Fig. 25.9

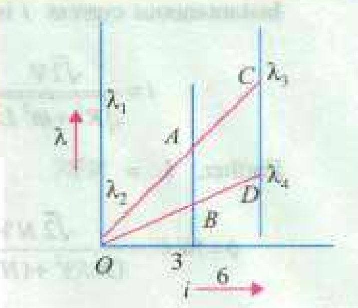


Fig. 25.10

Example 25.5. *If the inductor in the previous case is connected to a voltage source which increases from 0 to 3 V [part (a)] and then is held constant at 3 V [part (b)], repeat the problem, assuming that electrical transients are negligible.*

Solution. Coil resistance is 0.5 ohm. When the voltage reaches 3 V, the coil current is 6 amp. In part (*a*), L_1 = 40 mH. Hence, W_{fld3} = energy stored = $\frac{1}{2}\ L_1\ i_1^2 = 0.72$ joule, at point *C* in Fig. 25.10. In part (*b*), $L_2 = 25\ mH$. The current is held constant at 6 amp. Working on similar lines,

$\Delta\ W_{fld}$ = change in the field energy stored = area of triangle *ODC* or $\Delta W_{fld} = W_{fld3} - W_{fld4}$

$W_{fld} = \frac{1}{2} \times 25 \times 10^{-3} \times 36 = 0.45$ Joule, at point *D*

Change in energy stored in the field $= W_{fld3} - W_{fld4} = 0.72 - 0.45 = 0.27$ joule

Or $\Delta\ W_{fld}$ = area of $\Delta ODC = \frac{1}{2} \times 6 \times (\lambda_3 - \lambda_4)$

Here $\lambda_3 = 40 \times 10^{-3} \times 6$, and $\lambda_4 = 25 \times 10^{-3} \times 6$

$\Delta\ W_{fld} = \frac{1}{2} \times 6 \times 6 \times 10^{-3}\ (40 - 25) = 0.27$ joule

Example 25.6. *A coil of an electromagnetic relay is associated with a magnetic circuit whose reluctance is given by*

$= a + bx$

where a and b are positive constants decided by the details of the magnetic circuit, in which x is the length of the air-gap between fixed and movable members. If the coil is connected to an A.C. source where voltage is described by

$v = V_m\ \sin \omega t,$

find the expression for the average force on armature, with air-gap held constant at x.

Solution. If ϕ = flux established, in Webers

N = number of turns on the coil, λ = flux-linkage in Weber-turns

$W_{fld} = \frac{1}{2}\ \mathcal{R}\ \phi^2$

And force $F = \dfrac{\delta W_{fld}}{\delta x} = \frac{1}{2}\ \phi^2\ \dfrac{\delta \mathcal{R}}{\delta x} = -\frac{1}{2} b \phi^2$

The current in the coil is given by

$$v = R + L\frac{di}{dt}$$

for which, the steady-state solution for current with an a.c. voltage applied to the coil is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle -\theta \text{ where } \theta = \tan^{-1}\frac{(\omega L)}{R}$$

RMS voltage, $V = (V_m)\ /\mathcal{R}\sqrt{2}$

Instantaneous current i is expressed as

$$i = \frac{\sqrt{2}V}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

Further, L = N²/ℛ

$$\phi = Ni/\mathcal{R} = \frac{\sqrt{2}\,NV}{(R\mathcal{R})^2 + (N^2\omega)^2} \sin(\omega t - \theta)$$

Force, $$F_f = \frac{\mathcal{R} - bN^2V^2}{(R\quad)^2 + (N^2\omega)^2} \sin^2(\omega t - \theta)$$

The last term $\sin^2(\omega t - \theta)$ has a time average (over a cycle) of ½.

Hence, average force, $$F_f(\text{av}) = -\tfrac{1}{2} \frac{\mathcal{R}bN^2V^2}{(R\quad)^2 + (N^2\omega)^2}$$

Force is in such a direction that x will be reduced, or that the energy stored tends to increase.

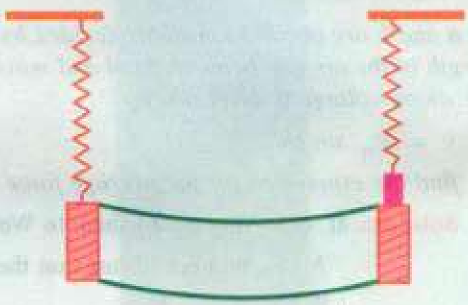
Example 25.7. *Two coupled coils have self- and mutual-inductances as expressed below:*

$$L_{11} = 1 + (1/x),$$
$$L_{22} = 0.5 + (1/x),$$
$$L_{12} = L_{21} = 1/x$$

These expressions are valid over a certain range of linear displacement x, in cms. The first coil is excited by a constant current of 20 A and the second one, by a constant current of – 10 A. Find

(a) mechanical work done if x changes from 0.5 to 1.0 cm

(b) energy supplied by the two electrical sources in (a) above.



Two coupled coils

Solution. With data given, substituting the values of currents,

$$W_{fld} = \tfrac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \tfrac{1}{2} L_{22} i_2^2$$
$$= 225 + (50/x)$$

$$F_f = -\delta W_{fld}/\delta x = 50/x^2$$

(a) $$\Delta W_{mech} = \int_{0.5}^{1.0} (50/x^2)\,dx$$

$$= 50[x^{-1}/-1]_{0.5}^{1.0} = +50 \text{ joules}$$

At $x = 0.5$, $W_{fld} = 325$ joules, and

at $x = 1.0$, $W_{fld} = 275$ joules

Thus, increase in x from 0.50 to 1.0 cm decreases the stored energy in the field from 325 to 275 joules. The field-system, thus, releases an energy of 50 joules.

(b) Calculations of Energy input from electrical sources –

$$\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2$$

$$= 20\ [1 + (1/x)] - 10\,(1/x) = 20 + (10/x)$$

At $x = 0.5$, $\lambda_1 = 20 + 20 = 40$ Wb-turns

$x = 1.0$, $\lambda_1 = 20 + 10 = 30$ Wb-turns

$\Delta_{Welec1} = i_1$ [change in λ_1 due to displacement]

$= 20 \times (-10) = -200$ Joules

Similarly, $\lambda_2 = L_{12}\, i_1 + L_{22}\, i_2 = -5 + (10/x)$

At $x = 0.5$, $\lambda_2 = -5 + 20 = +15$ Wb-turns

$x = 1.0$, $\lambda_2 = -5 + 10 = +5$ Wb-turns

$\Delta W_{elec2} = (-10)(-10) = +100$ Joules.

Seeing the signs and numerical values, it can be seen that Source 1 receives an energy of 200 Joules, which comes from three constituents:

100 J from source 2,

50 J from field energy stored,

and 50 J from mechanical system.

Tutorial Problems 25.1

(a) A magnetic circuit has a coil with 1000 turns. Its reluctance is expressed as

$\mathcal{R} = [\,8.5 + 40\ g\,] \times 10^{+3}$ MKS units

where g = air- gap length in mm, between fixed and movable parts. For a coil current of 2.0 amp held constant and with slow movement, calculate the change in the field energy stored, if the length of the air-gap changes from 0.20 to 0.15 cm. Calculate the mechanical force experienced by the system.

Hint: $\Delta W_e = i\,(\lambda_2 - \lambda_1)$, $\Delta W_{fld} = ½\ \Delta W_e$

Force, $F = -\Delta W_{fld} / \Delta x$

[**Ans.** $\Delta W_{fld} = 6.60$ J, Force = 13200 Nw]

(b) An electro-magnetic relay with an air-gap of x cm has the current and flux-linkage relationship as

$i = \lambda^2 + \lambda\,(0.5 - x)^2$ amp, for $x < 0.5$ cm

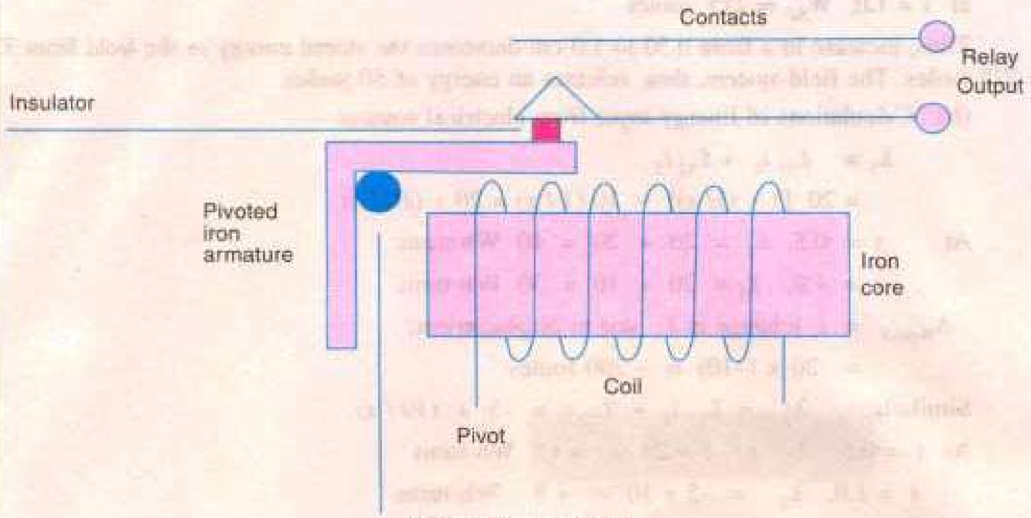
Find the force on armature as a function of λ and x.

Hint: $W_f(\lambda_1\, x) = \int_{0.5}^{1.0} i d\lambda$

And $F_f = -\delta W_f / \delta x$

[**Ans.** $W_f\,(\lambda, x) = (\lambda^3/3) + (\lambda^2/2)(0.5 - x)^2$

$F_f = \lambda^2\ (0.5 - x)$



Electromagnetic Relay

(c) For a rotary system, the stator-coil and the rotor-coil have self and mutual-inductances as described below, with suffix 1 for stator and 2 for rotor:

$L_{11} = L_{22} = 4 - (6\theta / \pi)$ for $0 < \theta < \pi / 2$

$= 1 + (6 / \pi)(\theta - 0.5\pi)$ for $\pi / 2 < \theta < \pi$

(Note: Self inductances cannot be negative.)

$L_{12} = L_{21} = 6(1 - 2\theta/\pi)$ for $0 < \theta < \pi$

Evaluate the inductances and the torque for $\theta = \pi / 4$ and the two coil currents of 5 amp constant in magnitude.

Hint: $\delta L / \delta\theta$ contributes to torque.

[**Ans.** $L_{11} = L_{22} = 2.5\ H$

$L_{12} = +3\ H$

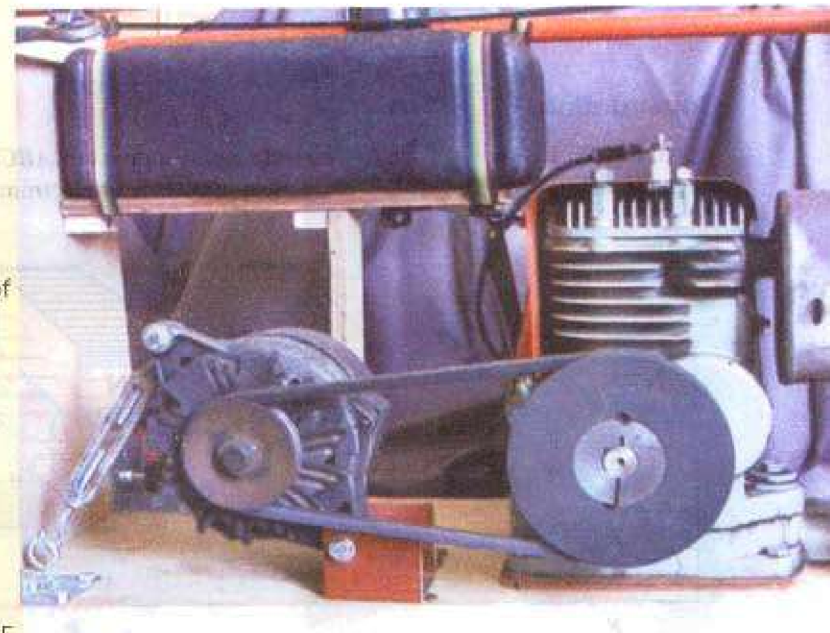
$T = 450 / \pi$ Nw]

C H A P T E R 26

D.C. GENERATORS

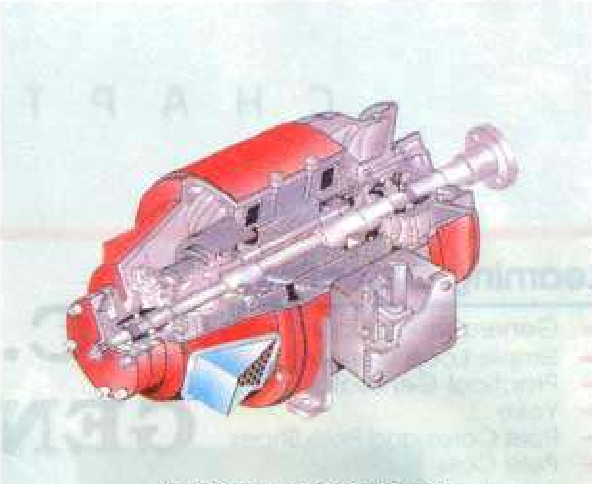
Learning Objectives

- Generator Principal
- Simple Loop Generator
- Practical Generator
- Yoke
- Pole Cores and Pole Shoes
- Pole Coils
- Armature Core
- Armature Windings
- Bushes and Bearings
- Pole-pitch
- Conductor-Coil and Winding Element
- Coil-span or Coil-pitch
- Pitch of a Winding
- Back Pitch
- Front Pitch
- Resultant Pitch
- Commutator Pitch
- Single-layer Winding
- Two-layer Winding
- Degree of Re-entrancy of an Armature Winding
- Multiplex Winding
- Lap and Wave Winding
- Simplex-lap Winding
- Numbering of Coils and Commutator Segments
- Simplex Wave Winding
- Dummy or Idle Coils
- Uses of Lap and Wave Windings
- Types of Generators
- Brush Contact Drop
- Generated E.M.F. or E.M.F. Equation of a Generator
- Iron Loss in Armature
- Total loss in a D.C. Generator
- Stray Losses
- Constant or Standing Losses
- Power Stages
- Condition for Maximum Efficiency



Generator converts mechanical energy into electrical energy using electromagnetic induction

26.1. Generator Principle



Cut-away view of dc generator

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power).

The energy conversion is based on the principle of the production of dynamically (or motionally) induced e.m.f. As seen from Fig. 26.1, whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic Induction. This e.m.f. causes a current to flow if the conductor circuit is closed.

Hence, two basic essential parts of an electrical generator are *(i)* a magnetic field and *(ii)* a conductor or conductors which can so move as to cut the flux.

26.2. Simple Loop Generator

Construction

In Fig. 26.1 is shown a single-turn rectangular copper coil *ABCD* rotating about its own axis in a magnetic field provided by either permanent magnet is or electromagnets. The two ends of the coil

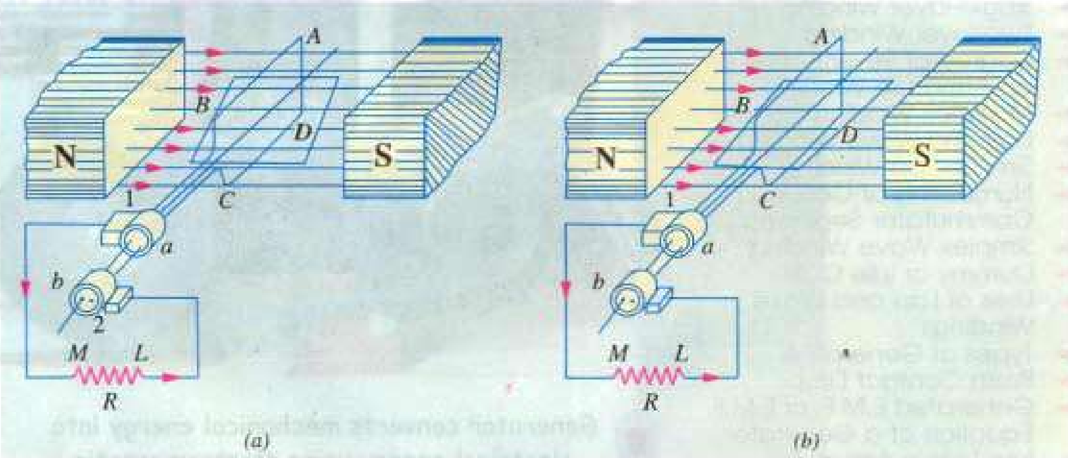


Fig. 26.1

are joined to two slip-rings '*a*' and '*b*' which are insulated from each other and from the central shaft. Two collecting brushes (of carbon or copper) press against the slip-rings. Their function is to collect the current induced in the coil and to convey it to the external load resistance *R*.

The rotating coil may be called 'armature' and the magnets as 'field magnets'.

Working

Imagine the coil to be rotating in clock-wise direction (Fig. 26.2). As the coil assumes successive positions in the field, the flux linked with it changes. Hence, an e.m.f. is induced in it which is proportional to the rate of change of flux linkages ($e = Nd\Phi dt$). When the plane of the coil is at right angles to lines of flux *i.e.* when it is in position, 1, then flux linked with the coil is maximum but ***rate of change of flux linkages is minimum.***

It is so because in this position, the coil sides *AB* and *CD* do not cut or shear the flux, rather they slide along them *i.e.* they move parallel to them. Hence, there is no induced e.m.f. in the coil. Let us take this no-e.m.f. or vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

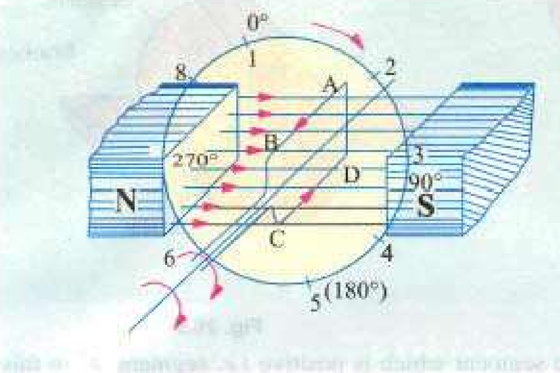


Fig. 26.2

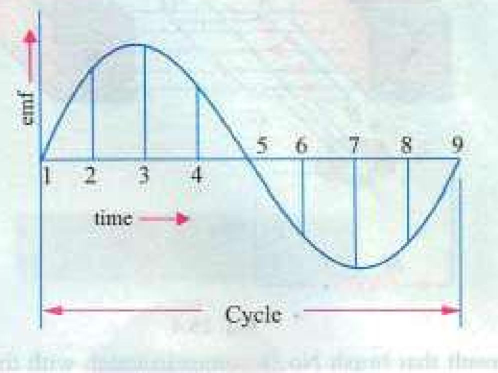


Fig. 26.3

As the coil continues rotating further, the rate of change of flux linkages (and hence induced e.m.f. in it) increases, till position 3 is reached where $\theta = 90°$. Here, the coil plane is horizontal *i.e.* parallel to the lines of flux. As seen, the flux linked with the coil is minimum but ***rate of change of flux linkages is maximum.*** Hence, maximum e.m.f. is induced in the coil when in this position (Fig. 26.3).

In the next quarter revolution *i.e.* from 90° to 180°, the flux linked with the coil gradually ***increases*** but the rate of change of flux linkages ***decreases.*** Hence, the induced e.m.f. decreases gradually till in position 5 of the coil, it is reduced to zero value.

So, we find that in the first half revolution of the coil, no (or minimum) e.m.f. is induced in it when in position 1, maximum when in position 3 and no e.m.f. when in position 5. The direction of this induced e.m.f. can be found by applying Fleming's Right-hand rule which gives its direction from *A* to *B* and *C* to *D*. Hence, the direction of current flow is ***ABMLCD*** (Fig. 26.1). The current through the load resistance *R* flows from *M* to *L* during the first half revolution of the coil.

In the next half revolution *i.e.* from 180° to 360°, the variations in the magnitude of e.m.f. are similar to those in the first half revolution. Its value is maximum when coil is in position 7 and minimum when in position 1. But it will be found that the direction of the induced current is from *D* to *C* and *B* to *A* as shown in Fig. 26.1 (*b*). Hence, the path of current flow is along ***DCLMBA*** which is just the reverse of the previous direction of flow.

Therefore, we find that the current which we obtain from such a simple generator reverses its direction after every half revolution. Such a current undergoing periodic reversals is known as alternating current. It is, obviously, different from a direct current which continuously flows in one and the same direction. It should be noted that alternating current not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two half-cycles may be called positive and negative half-cycles respectively (Fig. 26.3).

For making the flow of current unidirectional in the ***external*** circuit, the slip-rings are replaced by split-rings (Fig. 26.4). The split-rings are made out of a conducting cylinder which is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material (Fig. 26.5).

As before, the coil ends are joined to these segments on which rest the carbon or copper brushes.

It is seen [Fig. 26.6 (*a*)] that in the first half revolution current flows along **(ABMNLCD)** *i.e.* the brush No. 1 in contact with segment '*a*' acts as the positive end of the supply and '*b*' as the negative end. In the next half revolution [Fig. 26.6 (*b*)], the direction of the induced current in the coil has reversed. But at the same time, the positions of segments '*a*' and '*b*' have also reversed with the result that brush No. 1 comes in touch with the segment which is positive *i.e.* segment '*b*' in this case. Hence, current in the load resistance again flows from **M** to **L**. The waveform of the current through the external circuit is as shown in Fig. 26.7. ***This current is unidirectional but not continuous like pure direct current.***

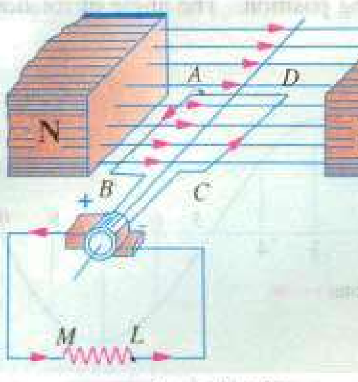


Fig. 26.4

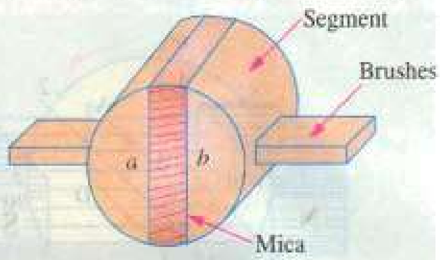


Fig. 26.5

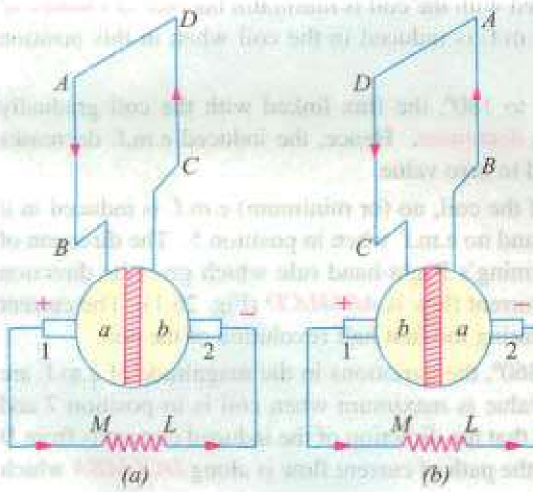


Fig. 26.6

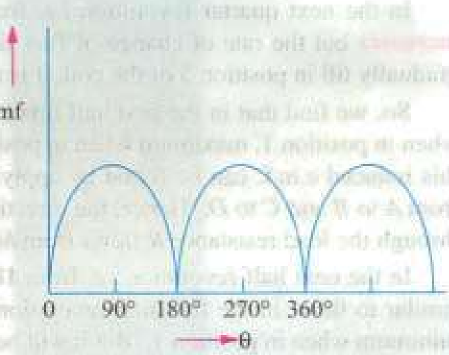


Fig. 26.7

It should be noted that the position of brushes is so arranged that the change over of segments '*a*' and '*b*' from one brush to the other takes place when the plane of the rotating coil is at right angles to the plane of the lines of flux. It is so because in that position, the induced e.m.f. in the coil is zero.

Another important point worth remembering is that even now the current induced in the coil sides is alternating as before. It is only due to the rectifying action of the split-rings (also called commutator) that it becomes unidirectional in the external circuit. Hence, it should be clearly understood that even in the armature of a d.c. generator, the induced voltage is alternating.

26.3. Practical Generator

The simple loop generator has been considered in detail merely to bring out the basic principle underlying construction and working of an actual generator illustrated in Fig. 26.8 which consists of the following essential parts :

1. Magnetic Frame or Yoke
2. Pole-Cores and Pole-Shoes
3. Pole Coils or Field Coils
4. Armature Core
5. Armature Windings or Conductors
6. Commutator
7. Brushes and Bearings

Of these, the yoke, the pole cores, the armature core and air gaps between the poles and the armature core or the magnetic circuit whereas the rest form the electrical circuit.

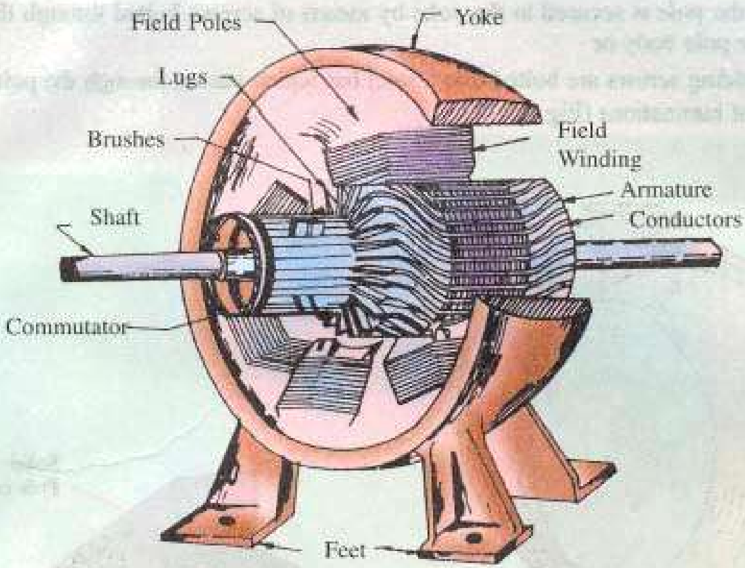


Fig. 26.8

26.4. Yoke

Yoke

The outer frame or yoke serves double purpose :

(*i*) It provides mechanical support for the poles and acts as a protecting cover for the whole machine and

(*ii*) It carries the magnetic flux produced by the poles.

In small generators where cheapness rather than weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed. The modern process of forming the yoke consists of rolling a steel slab round a cylindrical mandrel and then welding it at the bottom. The feet and the terminal box etc. are welded to the frame afterwards. Such yokes possess sufficient mechanical strength and have high permeability.

26.5. Pole Cores and Pole Shoes

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes (*i*) they spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path (*ii*) they support the exciting coils (or field coils) as shown in Fig. 26.14.

There are two main types of pole construction.

(*a*) The pole core itself may be a solid piece made out of either cast iron or cast steel but the pole shoe is laminated and is fastened to the pole face by means of counter sunk screws as shown in Fig. 24.10.

(*b*) In modern design, the complete pole cores and pole shoes are built of thin laminations of annealed steel which are rivetted together under hydraulic pressure (Fig. 26.11). The thickness of laminations varies from 1 mm to 0.25 mm. The laminated poles may be secured to the yoke in any of the following two ways :

(*i*) Either the pole is secured to the yoke by means of screws bolted through the yoke and into the pole body or

(*ii*) The holding screws are bolted into a steel bar which passes through the pole across the plane of laminations (Fig. 26.12).

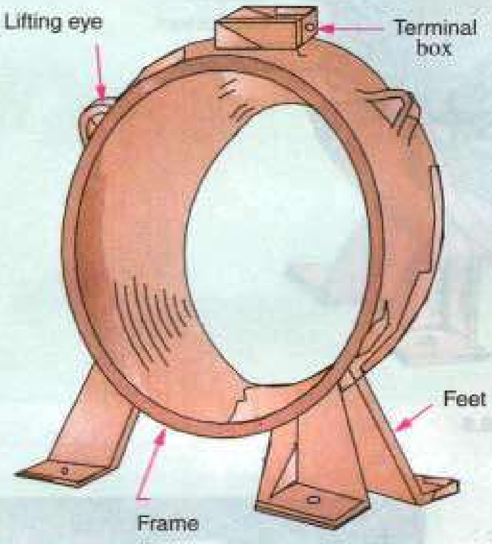


Fig. 26.9

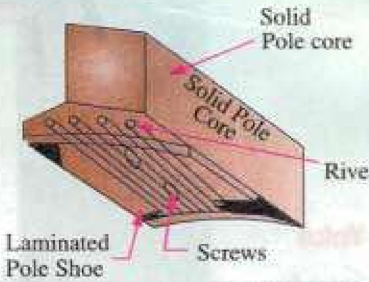


Fig. 26.10

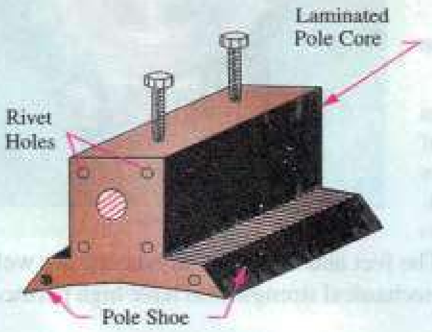


Fig. 26.11

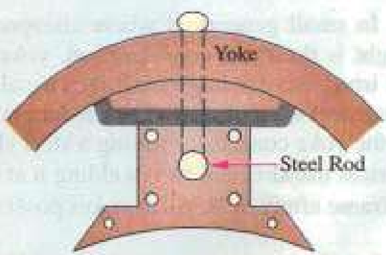


Fig. 26.12

26.6. Pole Coils

The field coils or pole coils, which consist of copper wire or strip, are former-wound for the correct dimension (Fig. 26.13). Then, the former is removed and wound coil is put into place over the core as shown in Fig. 26.14.

When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by revolving armature conductors.

26.7. Armature Core

It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux through the armature from a *N*-pole to a *S*-pole.

It is cylindrical or drum-shaped and is built up of usually circular sheet steel discs or laminations approximately 0.5 mm thick (Fig. 26.15). It is keyed to the shaft.

The slots are either die-cut or punched on the outer periphery of the disc and the keyway is located on the inner diameter as shown. In small machines, the armature stampings are keyed directly to the shaft. Usually, these laminations are perforated for air ducts which permits axial flow of air through the armature for cooling purposes. Such ventilating channels are clearly visible in the laminations shown in Fig. 26.16 and Fig. 26.17.

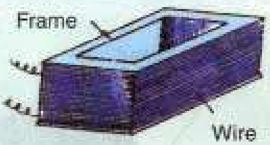


Fig. 26.13

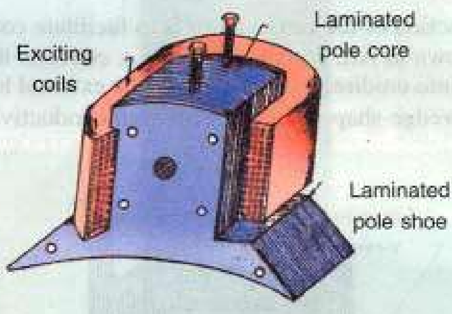


Fig. 26.14

Up to armature diameters of about one metre, the circular stampings are cut out in one piece as shown in Fig. 26.16. But above this size, these circles, especially of such thin sections, are difficult to handle because they tend to distort and become wavy when assembled together. Hence, the circular laminations, instead of being cut out in one piece, are cut in a number of suitable sections or segments which form part of a complete ring (Fig. 26.17).

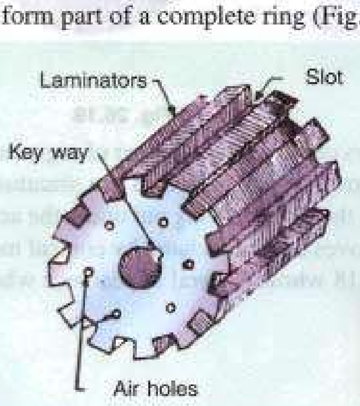


Fig. 26.15

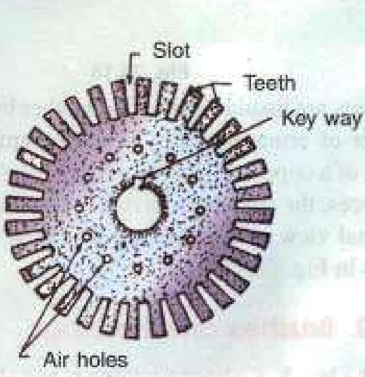


Fig. 26.16

A complete circular lamination is made up of four or six or even eight segmental laminations. Usually, two keyways are notched in each segment and are dove-tailed or wedge-shaped to make the laminations self-locking in position.

The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced e.m.f., smaller the current and hence lesser the $I^2 R$ loss in the core.

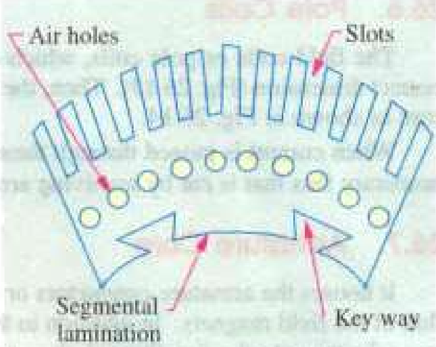


Fig. 26.17

26.8. Armature Windings

The armature windings are usually former-wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fibre wedges.

26.9. Commutator

The function of the commutator is to facilitate collection of current from the armature conductors. As shown in Art. 26.2, it rectified *i.e.* converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of cylindrical structure and is built up of wedge-shaped segments of high-conductivity hard-drawn or drop forged copper. These

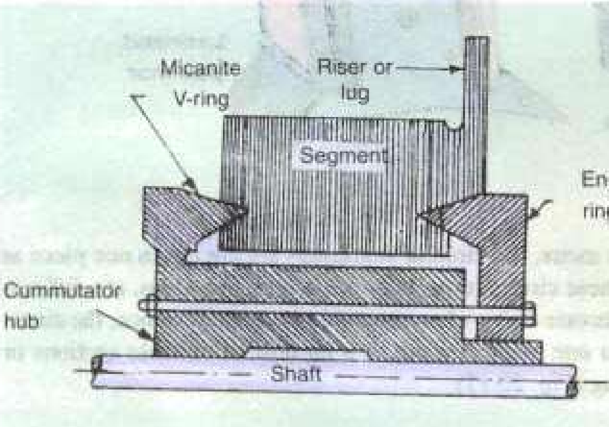


Fig. 26.18

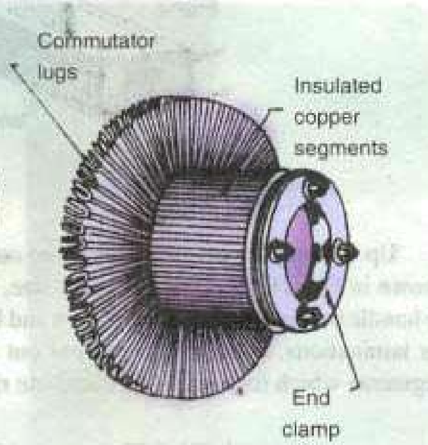


Fig. 26.19

segments are insulated from each other by thin layers of mica. The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor by means of a copper lug or strip (or riser). To prevent them from flying out under the action of centrifugal forces, the segments have *V*-grooves, these grooves being insulated by conical micanite rings. A sectional view of commutator is shown in Fig. 26.18 whose general appearance when completed is shown in Fig. 26.19.

26.10. Brushes and Bearings

The brushes whose function is to collect current from commutator, are usually made of carbon or graphite and are in the shape of a rectangular block. These brushes are housed in brush-holders usually of the box-type variety. As shown in Fig. 26.20, the brush-holder is mounted on a spindle and the brushes can slide in the rectangular box open at both ends. The brushes are made to bear down on the commutator by a spring whose tension can be adjusted by changing the position of lever in the notches. A flexible copper pigtail mounted at the top of the brush conveys current from the brushes to the holder. The number of brushes per spindle depends on the magnitude of the current to be collected from the commutator.

Bearing

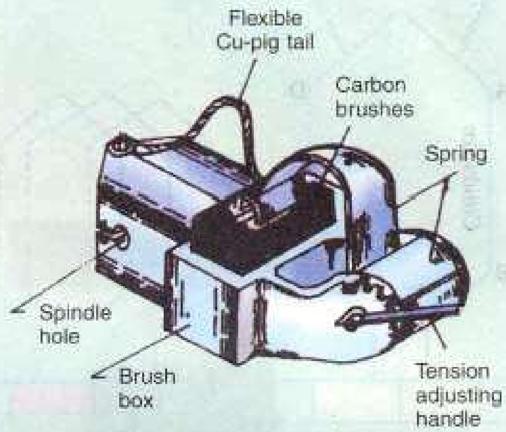


Fig. 26.20

Because of their reliability, ball-bearings are frequently employed, though for heavy duties, roller bearings are preferable. The ball and rollers are generally packed in hard oil for quieter operation and for reduced bearing wear, sleeve bearings are used which are lubricated by ring oilers fed from oil reservoir in the bearing bracket.

26.11. Armature Windings

Now, we will discuss the winding of an actual armature. But before doing this, the meaning of the following terms used in connection with armature winding should be clearly kept in mind.

Armature winding

26.12. Pole-pitch

It may be variously defined as :

(*i*) The periphery of the armature divided by the number of poles of the generator *i.e.* the distance between two adjacent poles.

(*ii*) It is equal to the number of armature conductors (or armature slots) per pole. If there are 48 conductors and 4 poles, the pole pitch is 48/4 = 12.

26.13. Conductor

The length of a wire lying in the magnetic field and in which an e.m.f. is induced, is called a conductor (or inductor) as, for example, length *AB* or *CD* in Fig. 26.21.

26.14. Coil and Winding Element

With reference to Fig. 26.21, the two conductors *AB* and *CD* along with their end connections constitute one coil of the armature winding. The coil may be single-turn coil (Fig. 26.21) or multi-turn coil (Fig. 26.22). A single-turn coil will have two conductors. But a multi-turn coil may have many conductors per coil side. In Fig. 26.22, for example, each coil side has 3 conductors. The

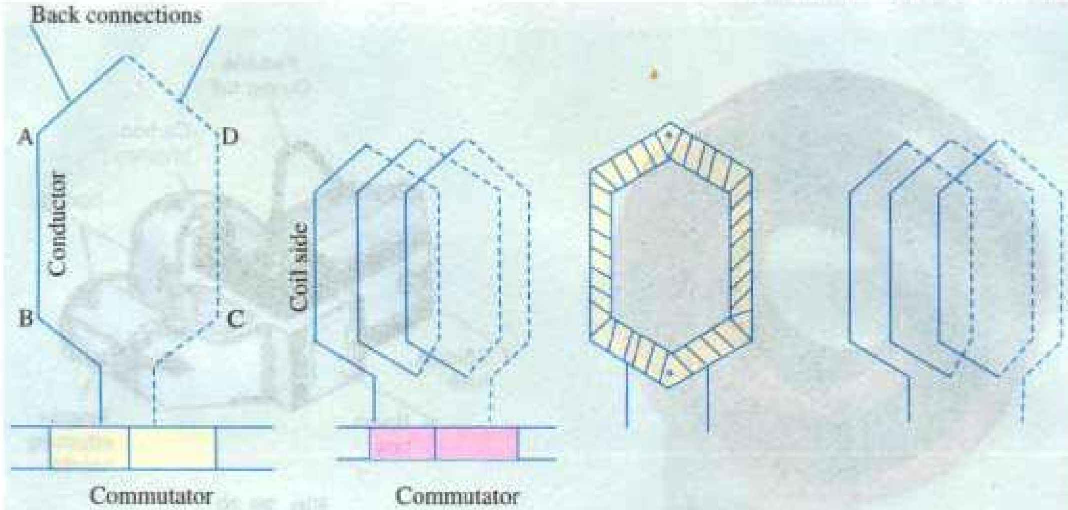


Fig. 26.21 Fig. 26.22 Fig. 26.23

group of wires or conductors constituting a coil side of a multi-turn coil is wrapped with a tape as a unit (Fig. 26.23) and is placed in the armature slot. It may be noted that since the beginning and the end of each coil must be connected to a commutator bar, there are as many commutator bars as coils for both the lap and wave windings (see Example 26.1).

The side of a coil (1-turn or multiturn) is called a winding element. Obviously, the number of winding elements is twice the number of coils.

26.15. Coil-span or Coil-pitch (Y_S)

It is the distance, measured in terms of armature slots (or armature conductors) between two sides of a coil. It is, in fact, the periphery of the armature spanned by the two sides of the coil.

If the pole span or coil pitch is equal to the pole pitch (as in the case of coil *A* in Fig. 26.24 where pole-pitch of 4 has been assumed), then winding is called ***full-pitched***. It means that coil span is 180 electrical degrees. In this case, the coil sides lie under opposite poles, hence the induced e.m.fs. in them are additive. Therefore, maximum e.m.f. is induced in the coil as a whole, it being the sum of the e.m.f.s induced in the two coil sides. For example, if there are 36 slots and 4 poles, then coil span is 36/4 = 9 slots. If number of slots is 35, then Y_S = 35/4 = 8 because it is customary to drop fractions.

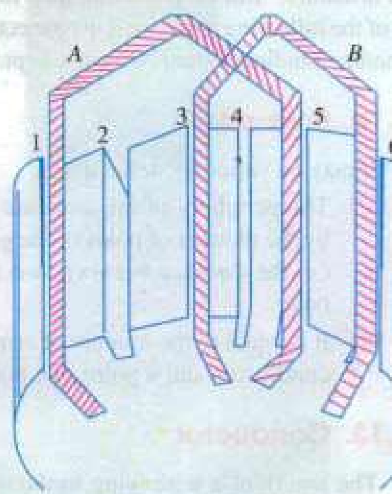


Fig. 26.24

If the coil span is less than the pole pitch (as in coil *B* where coil pitch is 3/4th of the pole pitch), then the winding is fractional-pitched. In this case, there is a phase difference between the e.m.fs. in the two sides of the coil. Hence, the total e.m.f. round the coil which is the vector sum of e.m.fs. in the two coil sides, is less in this case as compared to that in the first case.

26.16. Pitch of a Winding (Y)

In general, it may be defined as the distance round the armature between two successive conductors which are directly connected together. Or, it is the distance between the beginnings of two consecutive turns.

$$Y = Y_B - Y_F \quad \text{........for lap winding}$$
$$= Y_B + Y_F \quad \text{........for wave winding}$$

In practice, coil-pitches as low as eight-tenths of a pole pitch are employed without much serious reduction in the e.m.f. Fractional-pitched windings are purposely used to effect substantial saving in the copper of the end connections and for improving commutation.

26.17. Back Pitch (Y_B)

The distance, measured in terms of the armature conductors, which a coil advances on the back of the armature is called back pitch and is denoted by Y_B.

As seen from Fig. 26.28, element 1 is connected on the back of the armature to element 8. Hence, $Y_B = (8 - 1) = 7$.

26.18. Front Pitch (Y_F)

The number of armature conductors or elements spanned by a coil on the front (or commutator end of an armature) is called the front pitch and is designated by Y_F. Again in Fig. 26.28, element 8 is connected to element 3 on the front of the armature, the connections being made at the commutator segment. Hence, $Y_F = 8 - 3 = 5$.

Alternatively, the front pitch may be defined as the distance (in terms of armature conductors) between the second conductor of one coil and the first conductor of the next coil which are connected together at the front *i.e.* commutator end of the armature. Both front and back pitches for lap and wave-winding are shown in Fig. 26.25 and 26.26.

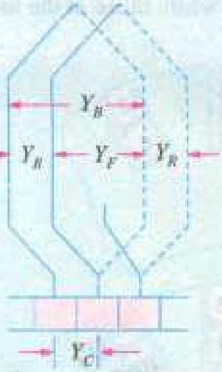


Fig. 26.25

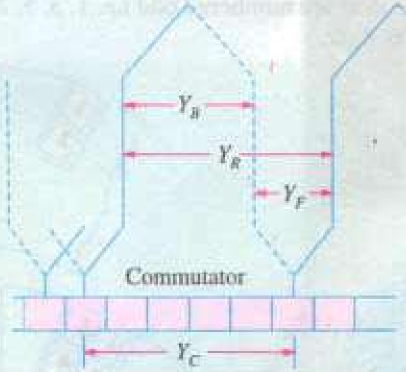


Fig. 26.26

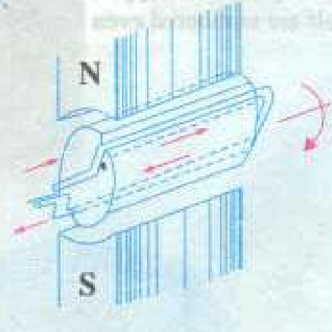


Fig. 26.27

26.19. Resultant Pitch (Y_R)

It is the distance between the beginning of one coil and the beginning of the next coil to which it is connected (Fig. 26.25 and 26.26).

As a matter of precaution, it should be kept in mind that all these pitches, though normally stated in terms of armature conductors, are also sometimes given in terms of armature slots or commutator bars because commutator is, after all, an image of the winding.

26.20. Commutator Pitch (Y_C)

It is the distance (measured in commutator bars or segments) between the segments to which the two ends of a coil are connected. From Fig. 26.25 and 26.26 it is clear that for lap winding, Y_C is the *difference* of Y_B and Y_F whereas for wavewinding it is the *sum* of Y_B and Y_F. Obviously, commutator pitch is equal to the number of bars between coil leads. In general, Y_C equals the 'plex' of the lap-wound armature. Hence, it is equal to 1, 2, 3, 4 etc. for simplex-, duplex, triplex–and quadruplex etc. lap-windings.

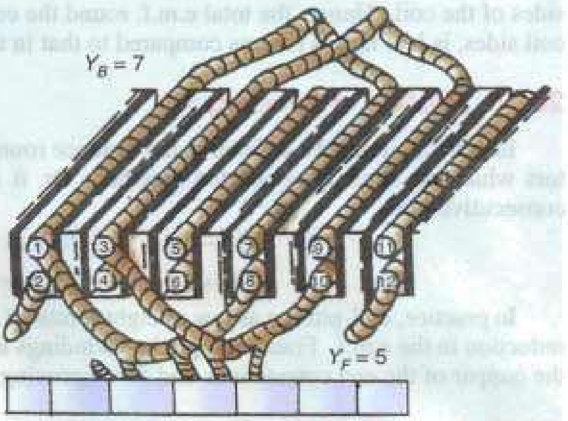


Fig. 26.28

26.21. Single-layer Winding

It is that winding in which one conductor or one coil side is placed in each armature slot as shown in Fig. 26.27. Such a winding is not much used.

26.22. Two-layer Winding

In this type of winding, there are two conductors or coil sides per slot arranged in two layers. Usually, one side of every coil lies in the upper half of one slot and other side lies in the lower half of some other slot at a distance of approximately one pitch away (Fig. 26.28). The transfer of the coil from one slot to another is usually made in a radial plane by means of a peculiar bend or twist at the back end as shown in Fig. 26.29. Such windings in which two coil sides occupy each slot are most commonly used for all medium-sized machines. Sometimes 4 or 6 or 8 coil sides are used in each slot in several layers because it is not practicable to have too many slots (Fig. 26.30). The coil sides lying at the upper half of the slots are numbered odd *i.e.* 1, 3, 5, 7 etc. while those at the lower half are numbered even *i.e.* 2, 4, 6, 8 etc.

Fig. 26.29

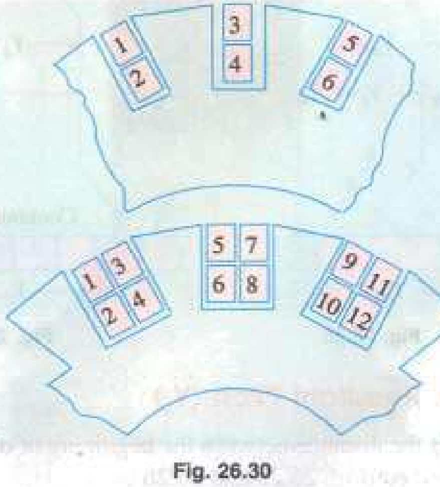


Fig. 26.30

26.23. Degree of Re-entrant of an Armature Winding

A winding is said to be single re-entrant if on tracing through it once, all armature conductors are included on returning to the starting point. It is double re-entrant if only half the conductors are included in tracing through the winding once and so on.

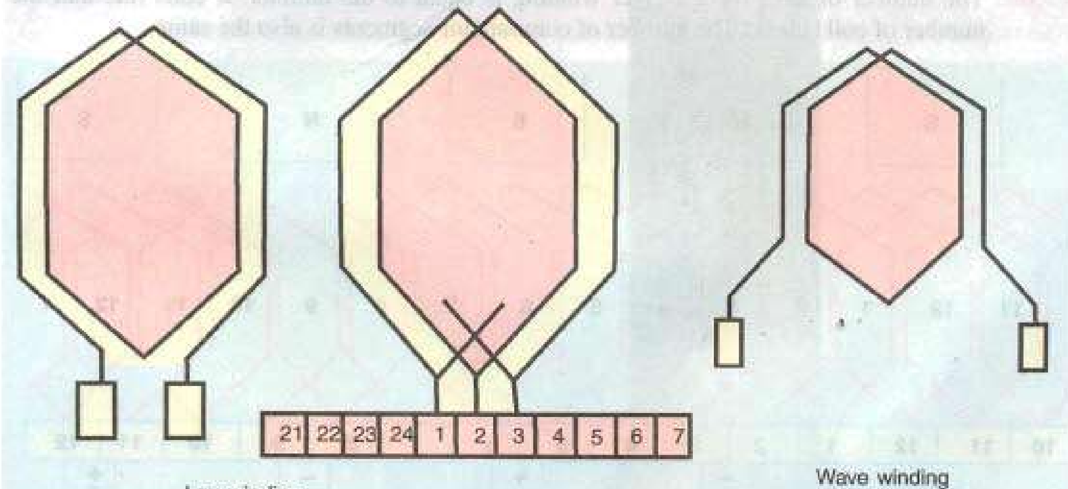
Multiplex Winding

26.24. Multiplex Winding

In such windings, there are several sets of completely closed and independent windings. If there is only one set of closed winding, it is called simplex wave winding. If there are two such windings on the same armature, it is called duplex winding and so on. The multiplicity affects a number of parallel paths in the armature. For a given number of armature slots and coils, as the multiplicity increases, the number of parallel paths in the armature increases thereby increasing the current rating but decreasing the voltage rating.

26.25. Lap and Wave Windings

Two types of windings mostly employed for drum-type armatures are known as Lap Winding and Wave Winding. The difference between the two is merely due to the different arrangement of the end connections at the front or commutator end of armature. Each winding can be arranged progressively or retrogressively and connected in simplex, duplex and triplex. The following rules, however, apply to both types of the windings :



Lap winding

Wave winding

(*i*) The front pitch and back pitch are each approximately equal to the pole-pitch *i.e.* windings should be full-pitched. This results in increased e.m.f. round the coils. For special purposes, fractional-pitched windings are deliberately used (Art. 26.15).

(*ii*) Both pitches should be odd, otherwise it would be difficult to place the coils (which are former-wound) properly on the armature. For exmaple, if Y_B and Y_F were both even, the *all* the coil sides and conductors would lie either in the upper half of the slots or in the lower half. Hence, it would become impossible for one side of the coil to lie in the upper half. Hence, it would become impossible for one side of the coil to lie in the upper half of one slot and the other side of the same coil to lie in the lower half of some other slot.

(iii) The number of commutator segments is equal to the number of slots or coils (or half the number of conductors) because the front ends of conductors are joined to the segments in pairs.

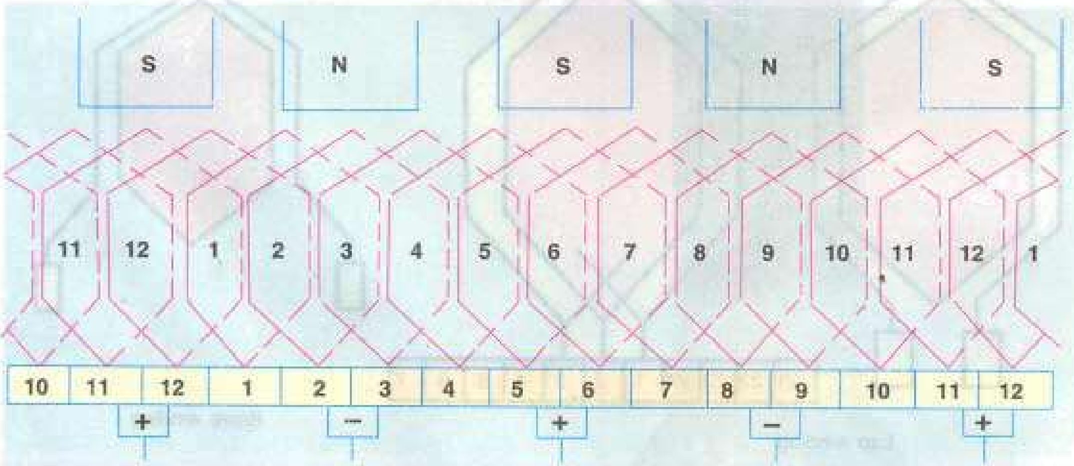
(iv) The winding must close upon itself *i.e.* if we start from a given point and move from one coil to another, then all conductors should be traversed and we should reach the same point again without a break or discontinuity in between.

26.26. Simplex Lap-winding*

It is shown in Fig. 26.25 which employs single-turn coils. In lap winding, the finishing end of one coil is connected to a commutator segment and to the starting end of the adjacent coil situated under the same pole and so on, till and the coils have been connected. This type of winding derives its name from the fact it doubles or laps back with its succeeding coils.

Following points regarding simplex lap winding should be carefully noted :

1. The back and front pitches are odd and of opposite sign. But they cannot be equal. They differ by 2 or some multiple thereof.
2. Both Y_B and Y_F should be nearly equal to a pole pitch.
3. The average pitch $Y_A = \dfrac{Y_B + Y_F}{2}$. It equals pole pitch $= \dfrac{Z}{P}$.
4. Commutator pitch $Y_C = \pm 1$. (In general, $Y_C = \pm\, m$)
5. Resultant pitch Y_R is even, being the arithmetical difference of two odd numbers, *i.e.*, $Y_R = Y_B - Y_F$.
6. The number of slots for a 2-layer winding is equal to the number of coils (*i.e.* half the number of coil sides). The number of commutator segments is also the same.



Simplex lap winding

* However, where heavy currents are necessary, duplex or triplex lap windings are used. The duplex lap winding is obtained by placing two similar windings on the same armature and connecting the even-numbered commutator bars to one winding and the odd-numbered ones to the second winding. Similarly, in triplex lap winding, there would be three windings, each connected to one third of the commutator bars.

7. The number of parallel paths in the armature = mP where m is the multiplicity of the winding and P the number of poles.

Taking the first condition, we have $Y_B = Y_F \pm 2$.**

(a) If $Y_B > Y_F$ *i.e.* $Y_B = Y_F + 2$, then we get a progressive or right-handed winding *i.e.* a winding which progresses in the clockwise direction as seen from the commutator end. In this case, obviously, $Y_C = +1$.

(b) If $Y_B < Y_F$ *i.e.* $Y_B = Y_F - 2$, then we get a retrogressive or left-handed winding *i.e.* one which advances in the anti-clockwise direction when seen from the commutator side. In this case, $Y_C = -1$.

(c) Hence, it is obvious that

$$\left.\begin{aligned} Y_F &= \frac{Z}{P} - 1 \\ Y_B &= \frac{Z}{P} + 1 \end{aligned}\right] \text{ for progressive winding and } \quad \left.\begin{aligned} Y_F &= \frac{Z}{P} + 1 \\ Y_B &= \frac{Z}{P} - 1 \end{aligned}\right] \text{ for retrogressive winding}$$

Obviously, Z/P must be even to make the winding possible.

26.27. Numbering of Coils and Commutator Segments

In the d.c. winding diagrams to follow, we will number the coils only (not individual turns). The upper side of the coil will be shown by a firm continuous line whereas the lower side will be shown by a broken line. The numbering of coil sides will be consecutive *i.e.* 1, 2, 3 ..... etc. and such that odd numbers are assigned to the top conductors and even numbers to the lower sides for a two-layer winding. The commutator segments will also be numbered consecutively, the number of the segments will be the same as that of the upper side connected to it.

Example 26.1. *Draw a developed diagram of a simple 2-layer lap-winding for a 4-pole generator with 16 coils. Hence, point out the characteristics of a lap-winding.*

(Elect. Engineering, Madras Univ. 1981)

Solution. The number of commutator segments = 16

Number of conductors or coil sides $16 \times 2 = 32$; pole pitch = 32/4 = 8

Now remembering that (*i*) Y_B and Y_F have to be odd and (*ii*) have to differ by 2, we get for a progressive winding $Y_B = 9$; $Y_F = -7$ (retrogressive winding will result if $Y_B = 7$ and $Y_F = -9$). Obviously, commutator pitch $Y_C = -1$.

[Otherwise, as shown in Art. 26.26, for progressive winding

$$Y_F = \frac{Z}{P} - 1 = \frac{32}{4} - 1 = 7 \text{ and } Y_B = \frac{Z}{P} - 1 = \frac{32}{4} + 1 = 9]$$

The simple winding table is given as under :

Back Connections		*Front Connections*
1 to (1 + 9) = 10	⟶	10 to (10 − 7) = 3
3 to (3 + 9) = 12	⟶	12 to (12 − 7) = 5
5 to (5 + 9) = 14	⟶	14 to (14 − 7) = 7
7 to (7 + 9) = 16	⟶	16 to (16 − 7) = 9
9 to (9 + 9) = 18	⟶	18 to (18 − 7) = 11
11 to (11 + 9) = 20	⟶	20 to (20 − 7) = 13
13 to (13 + 9) = 22	⟶	22 to (22 − 7) = 15
15 to (15 + 9) = 24	⟶	24 to (24 − 7) = 17
17 to (17 + 9) = 26	⟶	26 to (26 − 7) = 19
19 to (19 + 9) = 28	⟶	28 to (28 − 7) = 21

** In general, $Y_B = Y_F \pm 2m$ where m = 1 for simplex lap winding and m = 2 for duplex lap winding etc.

$21 \text{ to } (21 + 9) = 30 \longrightarrow 30 \text{ to } (20 - 7) = 23$

$23 \text{ to } (23 + 9) = 32 \longrightarrow 32 \text{ to } (32 - 7) = 25$

$25 \text{ to } (25 + 9) = 34 = (34 - 32) = 2 \longrightarrow 2 \text{ to } (34 - 7) = 27$

$27 \text{ to } (27 + 9) = 36 = (36 - 32) = 4 \longrightarrow 4 \text{ to } (36 - 7) = 29$

$29 \text{ to } (29 + 9) = 38 = (38 - 32) = 6 \longrightarrow 6 \text{ to } (38 - 7) = 31$

$31 \text{ to } (31 + 9) = 40 = (40 - 32) = 8 \longrightarrow 8 \text{ to } (40 - 7) = 33 = (33 - 32) = 1$

The winding ends here because we come back to the conductor from where we started.

We will now discuss the developed diagram which is one that is obtained by imagining the armature surface to be removed and then laid out flat so that the slots and conductors can be viewed without the necessity of turning round the armature in order to trace out the armature windings. Such a developed diagram is shown in Fig. 26.31.

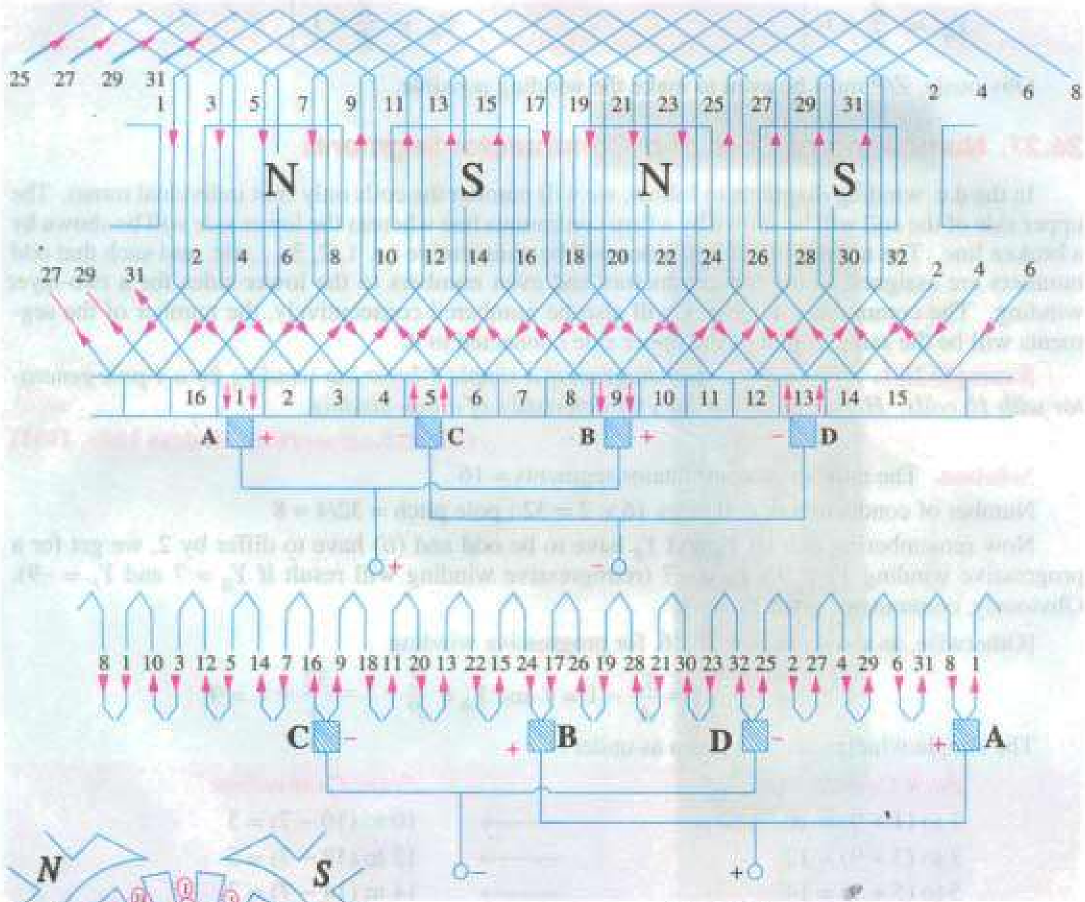


Fig. 26.31

Fig. 26.32

The procedure of developing the winding is this :

Front end of the upper side of coil No. 1 is connected to a commutator segment (whose number is also 1). The back end is joined at the back to the 1 + 9 = 10th coil side in the *lower* half of 5th slot. The front end of coil side 10 is joined to commutator segment 2 to which is connected the front end of 10 − 7 = 3 *i.e.* 3rd coil side lying in the upper half of second armature slot. In this way, by travelling 9 coil sides to the right at the back and 7 to the left at the front we complete the winding, thus including every coil side once till we reach the coil side 1 from where we started. Incidentally, it should be noted that all upper coil sides have been given odd numbers, whereas lower ones have been given even numbers as shown in the polar diagram (Fig. 26.32) of the winding of Fig. 26.31.

Brush positions can be located by finding the direction of currents flowing in the various conductors. If currents in the conductors under the influence of a *N*-pole are assumed to flow downwards (as shown), then these will flow upwards in conductors under the influence of *S*-pole. By putting proper arrows on the conductors (shown separately in the equivalent ring diagram), it is found that commutator bars No. 1 and 9 are the meeting points of e.m.fs. and hence currents are flowing out of these conductors. The positive brushes should, therefore, be placed at these commutator bars. Similarly, commutator bars No. 5 and 13 are the separating points of e.m.fs. hence negative brushes are placed there. In all, there are four brushes, two positive and two negative. If brushes of the same polarity are connected together, then all the armature conductors are divided into four parallel paths.

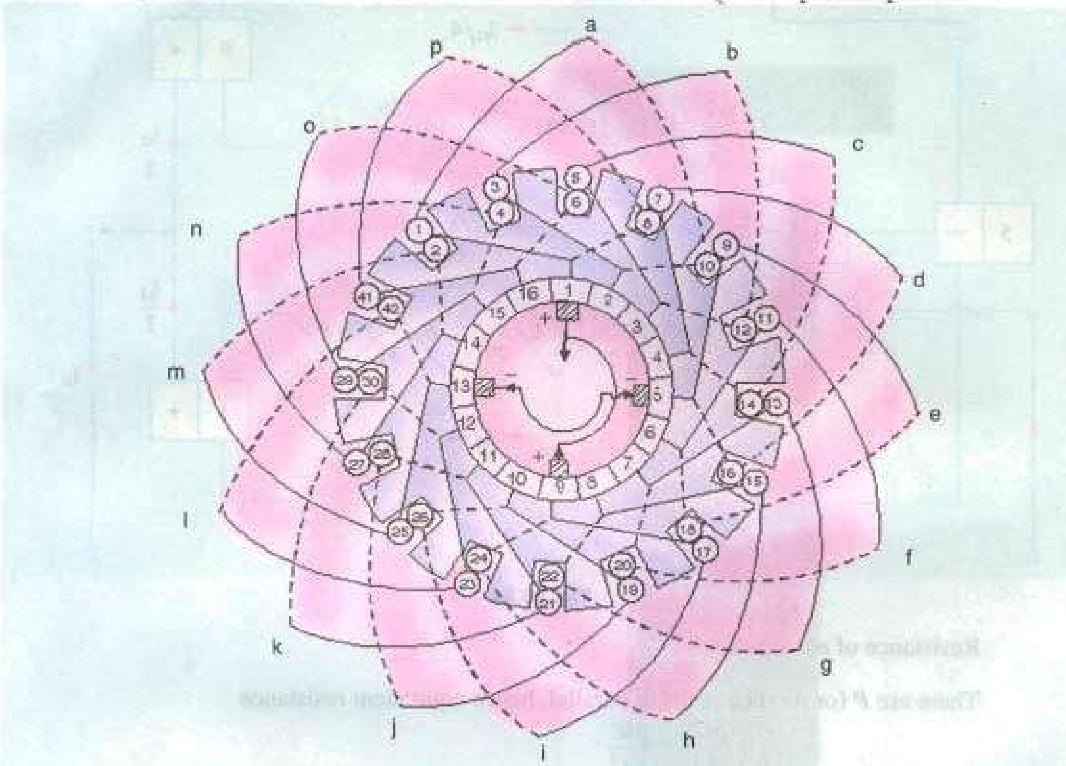


Fig. 26.33

Division of conductors into parallel paths is shown separately in the schematic diagram of Fig. 26.34. Obviously, if I_a is the total current supplied by the generator, then current carried by each parallel path is $I_a/4$.

Summarizing these conclusions, we have

1. The total number of brushes is equal to the number of poles.
2. There are as many parallel paths in the armature as the number of poles. That is why such a winding is sometimes known as 'multiple circuit' or 'parallel' winding. In general, number of parallel paths in armature = *mP* where *m* is the multiplicity (plex) of the lap winding. For example, a 6-pole duplex lap winding has (6 × 2) = 12 parallel paths in its armature.
3. The e.m.f. between the +ve and –ve brushes is equal to the e.m.f. generated in any one of the parallel paths. If *Z* is the total number of armature conductors and *P* the number of poles, then the number of armature conductors (connected in series) in any parallel path is *Z/P*.

$\therefore$ Generated e.m.f. E_g = (Average e.m.f./conductor) $\times \dfrac{Z}{P} = e_{av} \times \dfrac{Z}{P}$

4. The total or equivalent armature resistance can be found as follows :

Let l = length of each armature conductor; S = its cross-section

A = No. of parallel paths in armature = P – for simplex lap winding

R = resistance of the whole winding then $R = \dfrac{\rho l}{S} \times Z$

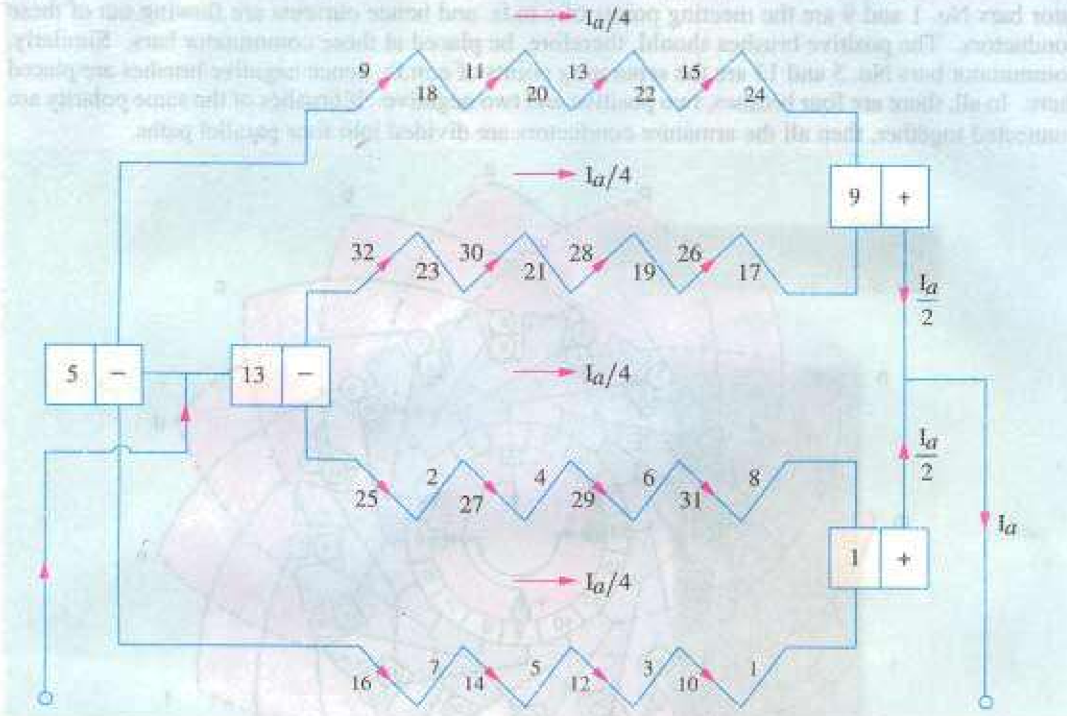


Fig. 26.34

Resistance of each path $= \dfrac{\rho l Z}{S \times A}$

There are P (or A) such paths in parallel, hence equivalent resistance

$$= \frac{1}{A} \times \frac{\rho l Z}{SA} = \frac{\rho l Z}{SA^2}$$

5. If I_a is the total armature current, then current per parallel path (or carried by each conductor) is I_a/P.

26.28. Simplex Wave Winding*

From Fig. 26.31, it is clear that in lap winding, a conductor (or coil side) under one pole is connected at the back to a conductor which occupies an almost corresponding position under the *next* pole of *opposite* polarity (as conductors 3 and 12). Conductor No. 12 is then connected to conductor No. 5 under the ***original*** pole but which is a little removed from the initial conductor No. 3. If, instead of returning to the same *N*-pole, the conductor No. 12 were taken *forward* to the next *N*-pole, it would make no difference so far as the direction and magnitude of the e.m.f. induced in the circuit are concerned.

* Like lap winding, a wave winding may be duplex, triplex or may have any degree of multiplicity. A simplex wave winding has two paths, a duplex wave winding four paths and a triplex one six paths etc.

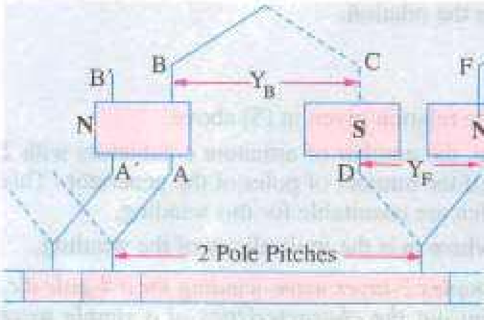


Fig. 26.35

As shown in Fig. 26.35, conductor *AB* is connected to *CD* lying under *S*-pole and then to *EF* under the next *N*-pole. In this way, the winding progresses, passing successively under every *N*-pole and *S*-pole till it returns to a conductor *A′B′* lying under the original pole. Because the winding progresses in one direction round the armature in a series of 'waves', it is known as wave winding.

If, after passing once round the armature, the winding falls in a slot to the left of its starting point (as *A′B′* in Fig. 26.35) then the winding is said to be retrogressive. If, however, it falls one slot to the right, then it is progressive.

Assuming a 2-layer winding and supposing that conductor *AB* lies in the upper half of the slot, then going once round the armature, the winding ends at *A′B′* which must be at the upper half of the slot at the left or right. Counting in terms of ***conductors***, it means that *AB* and *A′B′* differ by two conductors (although they differ by one slot).

From the above, we can deduce the following relations. If P = No. of poles, then

$$\left.\begin{aligned} Y_B &= \text{back pitch} \\ Y_F &= \text{front pitch} \end{aligned}\right\} \text{nearly equal to pole pitch}$$

then $$Y_A = \frac{Y_B + Y_F}{2} = \text{average pitch ; } Z = \text{total No. of conductors or coil sides}$$

Then, $$Y_A \times P = Z \pm 2 \qquad Y_A = \frac{Z \pm 2}{P}$$

Since P is always even and $Z = PY_A \pm 2$, hence Z must always be even. Put in another way, it means that $\dfrac{Z \pm 2}{P}$ must be an even integer.

The plus sign will give a progressive winding and the negative sign a retrogressive winding.

Points to Note :

1. Both pitches Y_B and Y_F are odd and of the same sign.
2. Back and front pitches are nearly equal to the pole pitch and may be equal or differ by 2, in which case, they are respectively one more or one less than the average pitch.
3. Resultant pitch $Y_R = Y_F + Y_B$.
4. Commutator pitch, $Y_C = Y_A$ (in lap winding $Y_C = \pm 1$).

   Also, $$Y_C = \frac{\text{No. of Commutator bars } \pm 1}{\text{No. of pair of poles}}$$
5. The average pitch which must be an integer is given by

$$Y_A = \frac{Z \pm 2}{P} = \frac{\frac{Z}{2} \pm 1}{P/2} = \frac{\text{No. of Commutator bars } \pm 1}{\text{No. of pair of poles}}$$

It is clear that for Y_A to be an integer, there is a restriction on the value of Z. With $Z = 32$, this winding is impossible for a 4-pole machine (though lap winding is possible). Values of $Z = 30$ or 34 would be perfectly alright.

6. The number of *coils i.e.* N_C can be found from the relation.

$$N_C = \frac{PY_A \pm 2}{2}$$

This relation has been found by rearranging the relation given in (5) above.

7. It is obvious from (5) that for a wave winding, the number of armature conductors with 2 either added or subtracted must be a multiple of the number of poles of the generator. This restriction eliminates many even numbers which are unsuitable for this winding.

8. The number of armature parallel paths = $2m$ where m is the multiplicity of the winding.

Example 26.2. *Draw a developed diagram of a simplex 2-layer wave-winding for a 4-pole d.c. generator with 30 armature conductors. Hence, point out the characteristics of a simple wave winding.* **(Elect. Engg-I, Nagpur Univ. 1991)**

Solution. Here, $Y_A = \dfrac{30 \pm 2}{4}$ = 8* or 7. Taking $Y_A = 7$, we have $Y_B = Y_F = 7$

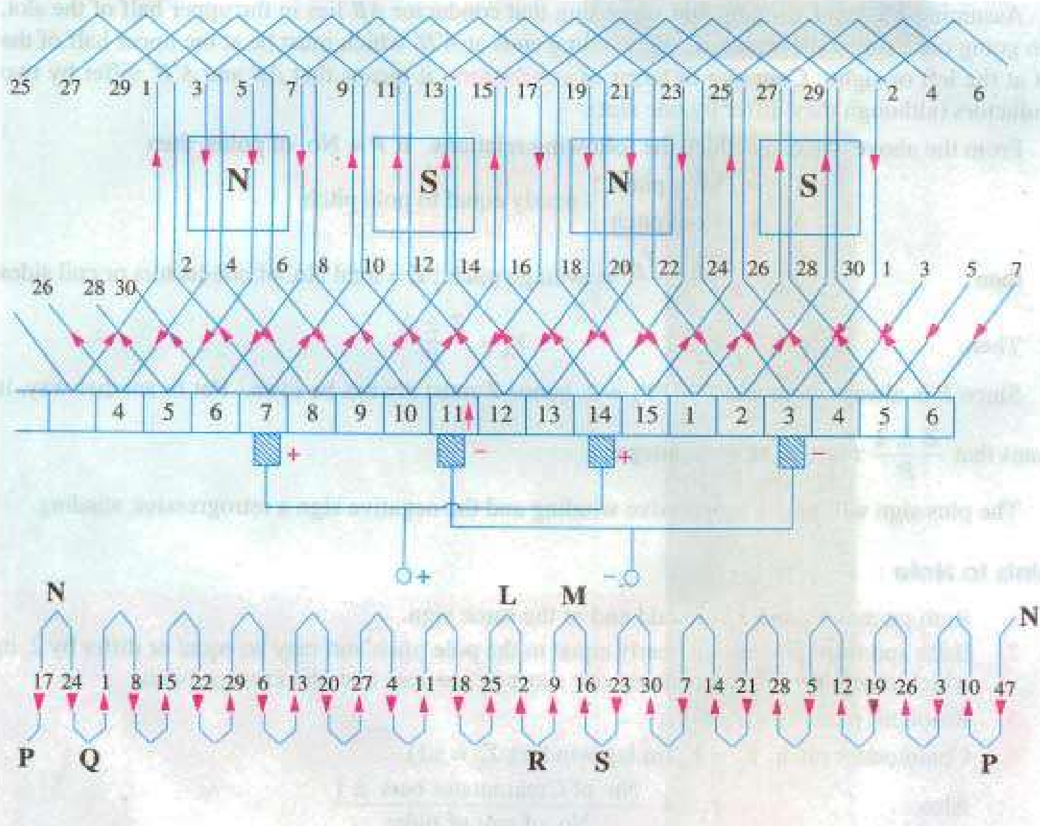


Fig. 26.36

As shown in Fig. 26.36 and 26.37, conductor No. 5 is taken to conductor No. 5 + 7 = 12 at the back and is joined to commutator segment 5 at the front. Next, the conductor No. 12 is joined to commutator segment 5 + 7 = 12 ($\because$ $Y_C = 7$) to which is joined conductor No. 12 + 7 = 19. Continuing this way, we come back to conductor No. 5 from where we started. Hence, the winding closes upon itself.

* If we take 8, then the pitches would be : $Y_B = 9$ and $Y_F = 7$ or $Y_B = 7$ and $Y_F = 9$. Incidentally, if $Y_A = Y_C$ is taken as 7, armature will rotate in one direction and if $Y_C = 8$, it will rotate in the opposite direction.

The simple winding table is as under :

Back Connections		*Front Connections*
1 to (1 + 7) = 8	⟶	8 to (8 + 7) = 15
15 to (15 + 7) = 22	⟶	22 to (22 + 7) = 29
29 to (29 + 7) = 36 = (36 − 30) = 6	⟶	6 to (6 + 7) = 13
13 to (13 + 7) = 20	⟶	20 to (20 + 7) = 27
27 to (27 + 7) = 34 = (34 − 30) = 4	⟶	4 to (4 + 7) = 11
11 to (11 + 7) = 18	⟶	18 to (18 + 7) = 25
25 to (25 + 7) = 32 = (32 − 30) = 2	⟶	2 to (2 + 7) = 9
9 to (9 + 7) = 16	⟶	16 to (16 + 7) = 23
23 to (23 + 7) = 30	⟶	30 to (30 + 7) = 37 = (37 − 30) = 7
7 to (7 + 7) = 14	⟶	14 to (14 + 7) = 21
21 to (21 + 7) = 28	⟶	28 to (28 + 7) = 35 = (35 − 30) = 5
5 to (5 + 7) = 12	⟶	12 to (12 + 7) = 19
19 to (19 + 7) = 26	⟶	26 to (26 + 7) = 33 = (33 − 30) = 3
3 to (3 + 7) = 10	⟶	10 to (10 + 7) = 17
17 to (17 + 7) = 24	⟶	24 to (24 + 7) = 31 = (31 − 30) = 1

Since we come back to the conductor No. 1 from where we started, the winding gets closed at this stage.

Brush Position

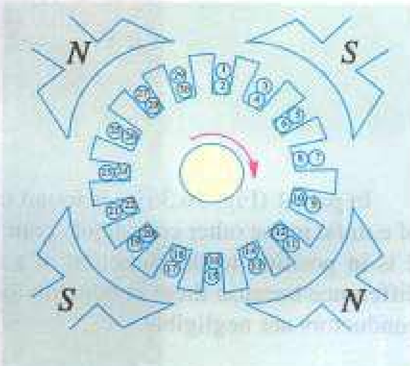


Fig. 26.37

Location of brush position in wave-winding is slightly difficult. In Fig. 26.36 conductors are supposed to be moving from left to right over the poles. By applying Fleming's Right-hand rule, the directions of the induced e.m.fs in various armature conductors can be found. The directions shown in the figure have been found in this manner. In the lower part of Fig. 26.36 is shown the equivalent ring or spiral diagram which is very helpful in understanding the formation of various parallel paths in the armature. It is seen that the winding is electrically divided into two portions. One portion consists of conductors lying between points N and L and the other of conductors lying between N and M. In the first portion, the general trend of the induced e.m.fs. is from left to right whereas in the second portion it is from right to left. Hence, in general, there are only two parallel paths through the winding, so that two brushes are required, one positive and one negative.

From the equivalent ring diagram, it is seen that point N is the separating point of the e.m.fs. induced in the two portions of the winding. Hence, this fixes the position of the negative brush. But as it is at the back and not at the commutator end of the armature, the negative brush has two alternative positions *i.e.* either at point P or Q. These points on the equivalent diagram correspond to commutator segments No. 3 and 11.

Now, we will find the position of the positive brush. It is found that there are two meeting points of the induced e.m.fs. *i.e.* points L and M but both these points are at the back or non-commutator end of the armature. These two points are separated by one loop only, namely, the loop composed of conductors 2 and 9, hence the middle point R of this loop fixes the position of the positive brush, which should be placed in touch with commutator segment No. 7. We find that for one position of the +ve brush, there are two alternative positions for the −ve brush.

Taking the +ve brush at point R and negative brush at point P, the winding is seen to be divided into the following two paths.

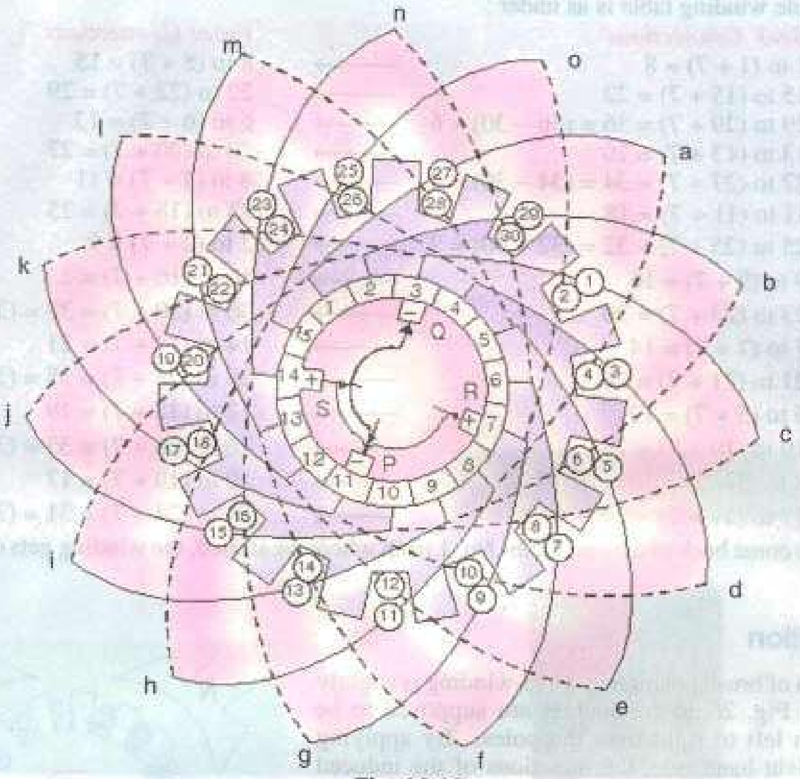


Fig. 26.38

In path 1 (Fig. 26.36) it is found that e.m.f. in conductor 9 is in opposition to the general trend of e.m.fs. in the other conductors comprising this path. Similarly, in path 2, the e.m.f. in conductor 2 is in position to the direction of e.m.fs. in the path as a whole. However, this will make no difference because these conductors lie almost in the interpolar gap and, therefore e.m.fs. in these conductors are negligible.

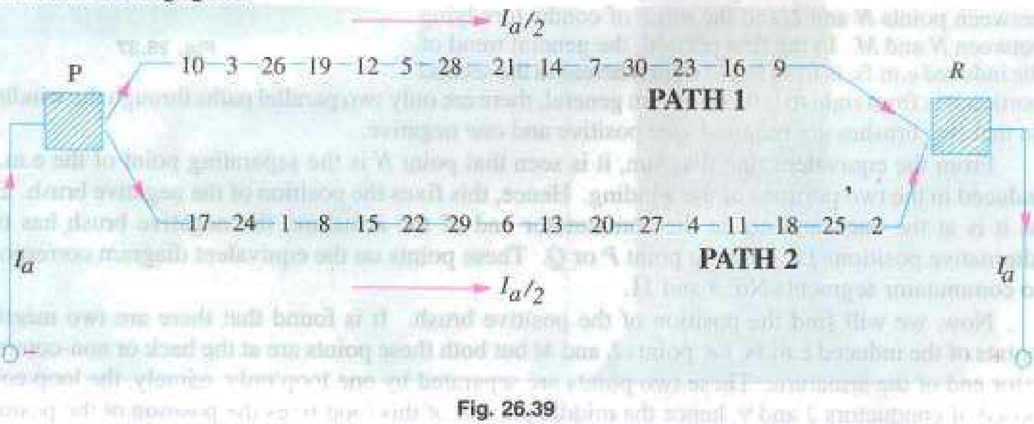


Fig. 26.39

Again, take the case of conductors 2 and 9 situated between points L and M. Since the armature conductors are in continuous motion over the pole faces, their positions as shown in the figure are only instantaneous. Keeping in this mind, it is obvious that conductor 2 is about to move from the influence of S-pole to that of the next N-pole. Hence, the e.m.f. in it is at the point of reversing. However, conductor 9 has already passed the position of reversal, hence its e.m.f. will not reverse, rather it will increase in magnitude gradually. It means that in a very short interval, point M will

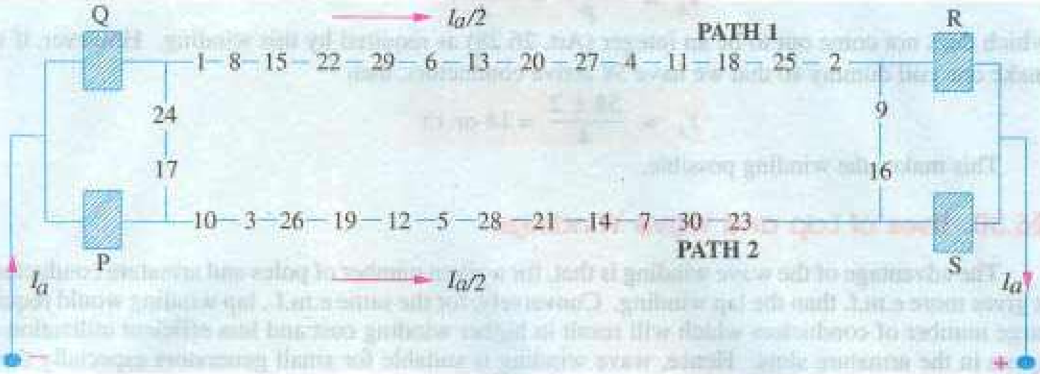


Fig. 26.40

become the meeting point of the e.m.fs. But as it lies at the back of the armature, there are two alternative positions for the +ve brush *i.e.* either point R which has already been considered or point S which corresponds to commutator segment 14. This is the second alternative position of the positive brush. Arguing in the same way, it can be shown that after another short interval of time, the alternative position of the positive brush will shift from segment 14 to segment 15. Therefore, if one positive brush is in the contact with segment 7, then the second positive brush if used, should be in touch with both segments 14 and 15.

It may be noted that if brushes are placed in both alternative positions for both positive and negative (*i.e.* if in all, 4 brushes are used, two +ve and two −ve), then the effect is merely to short-circuit the loop lying between brushes of the same polarity. This is shown in Fig. 26.40 where it will also be noted that irrespective of whether only two or four brushes are used, the number of parallel paths through the armature winding is still two.

Summarizing the above facts, we get

1. Only two brushes are necessary, though their number may be equal to the number of poles.
2. The number of parallel paths through the armature winding is two irrespective of the number of generator poles. That is why this winding is sometimes called 'two-circuit' or 'series' winding.
3. The generator e.m.f. is equal to the e.m.f. induced in any one of the two parallel paths. If e_{av} is the e.m.f. induced/conductor, then generator e.m.f. is $E_g = e_{av} \times Z/2$.
4. The equivalent armature resistance is nearly one-fourth of the total resistance of the armature winding.
5. If I_a is the total armature current, then current carried by each path or conductor is obviously $I_a/2$ whatever the number of poles.

26.29. Dummy or Idle Coils

These are used with wave-winding and are resorted to when the requirements of the winding are not met by the standard armature punchings available in armature-winding shops. These dummy coils do not influence the electrical characteristics of the winding because they are not connected to the commutator. They are exactly similar to the other coils except that their ends are cut short and taped. They are there simply to provide mechanical balance for the armature because an armature having some slots without windings would be out of balance mechanically. For example, suppose number of armature slots is 15, each containing 4 sides and the number of poles is 4. For a simplex wave-windings,

Dummy coils

$$Y_A = \frac{Z \pm 2}{P} = \frac{60 \pm 2}{4}$$

which does not come out to be an integer (Art. 26.28) as required by this winding. However, if we make one coil dummy so that we have 58 active conductors, then

$$Y_A = \frac{58 \pm 2}{4} = 14 \text{ or } 15$$

This makes the winding possible.

26.30. Uses of Lap and Wave Windings

The advantage of the wave winding is that, for a given number of poles and armature conductors, it gives more e.m.f. than the lap winding. Conversely, for the same e.m.f., lap winding would require large number of conductors which will result in higher winding cost and less efficient utilization of space in the armature slots. Hence, wave winding is suitable for small generators especially those meant for 500-600 V circuits.

Another advantage is that in wave winding, equalizing connections are not necessary whereas in a lap winding they definitely are. It is so because each of the two paths contains conductors lying under all the poles whereas in lap-wound armatures, each of the *P* parallel paths contains conductors which lie under one pair of poles. Any inequality of pole fluxes affects two paths equally, hence their induced e.m.fs. are equal. In lap-wound armatures, unequal voltages are produced which set up a circulating current that produces sparking at brushes.

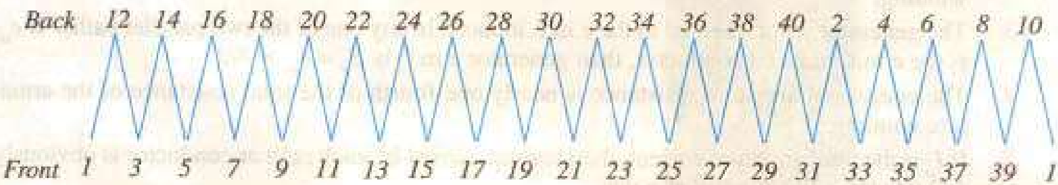
However, when large currents are required, it is necessary to use lap winding, because it gives more parallel paths.

Hence, lap winding is suitable for comparatively low-voltage but high-current generators whereas wave-winding is used for high-voltage, low-current machines.

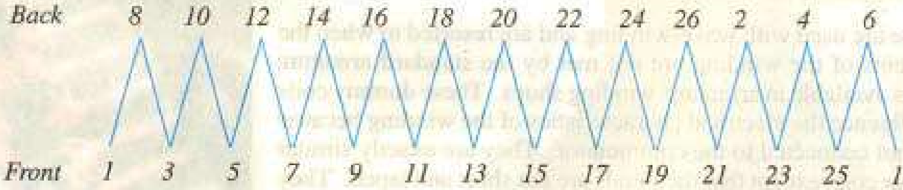
Tutorial Problem No. 26.1

1. Write down the winding table for a 2-layer simplex lap-winding for a 4-pole d.c. generator having (*a*) 20 slots and (*b*) 13 slots. What are the back and front pitches as measured in terms of armature conductors ?

[Hint : (*a*) No. of conductors = 40 ; $Y_B = 11$ and $Y_F = -9$] **(Elect. Engineering, Madras Univ. 1978)**

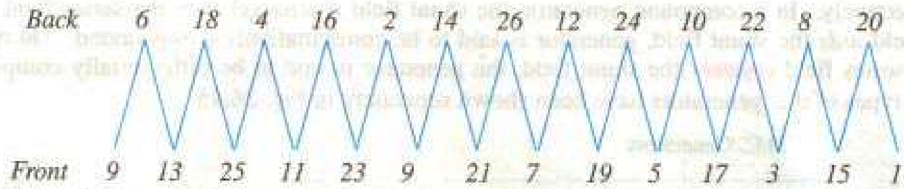


(*b*) No. of conductors = 26 ; $Y_B = 7$; $Y_F = -5$



2. With a simplex 2-layer wave winding having 26 conductors and 4-poles, write down the winding table. What will be the front and back pitches of the winding ?

[Hint : $Y_F = 7$ and $Y_B = 5$] **(Electric Machinery-I, Madras Univ. Nov. 1979)**



3. Is it possible to get simplex wave winding for a 4-pole d.c. machine with 28 conductors ? Explain the reason for your answer. **[No, it would contain only 4 conductors]**

4. State for what type of winding each of the following armatures could be used and whether the winding must be four or six-pole if no dummy coils are to be used (*a*) 33 slots, 165 commutator segments (*b*) 64 slots, 256 commutator segments (*c*) 65 slots, 260 commutator segments.

[(*a*) 4-pole lap with commutator pitch 82 or 83 or 6-pole lap.
(*b*) 4-pole lap or 6-pole wave with commutator pitch 85.
(*c*) 6-pole wave with commutator pitch 87.]

26.31. Types of Generators

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into (*a*) separately-excited generators and (*b*) self-excited generators.

(*a*) **Separately-excited** generators are those whose field magnets are energised from an independent external source of d.c. current. It is shown diagramatically in Fig. 26.41.

(*b*) **Self-excited generators** are those whose field magnets are energised by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

(*i*) **Shunt wound**

The field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them (Fig. 26.42).

(*ii*) **Series Wound**

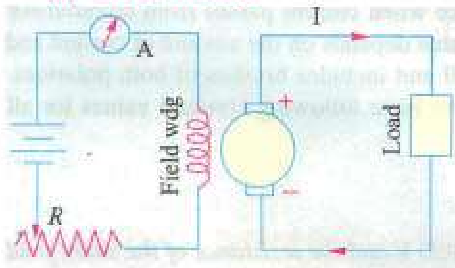


Fig. 26.41

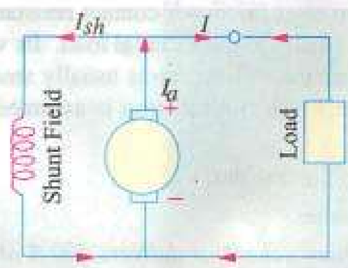


Fig. 26.42

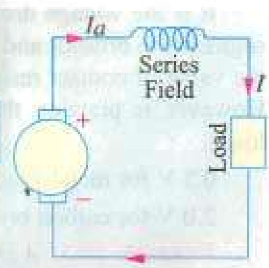


Fig. 26.43

In this case, the field windings are joined in series with the armature conductors (Fig. 26.43). As they carry full load current, they consist of relatively few turns of thick wire or strips. Such generators are rarely used except for special purposes *i.e.* as boosters etc.

(*iii*) **Compound Wound**

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig. 26.44 (*a*) and

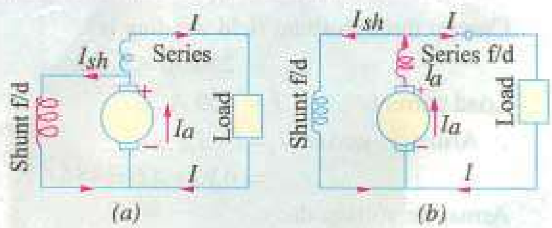


Fig. 26.44

(*b*) respectively. In a compound generator, the shunt field is stronger than the series field. When series field *aids* the shunt field, generator is said to be commutatively-compounded. On the other hand if series field ***opposes*** the shunt field, the generator is said to be differentially compounded. Various types of d.c. generators have been shown separately in Fig. 26.45.

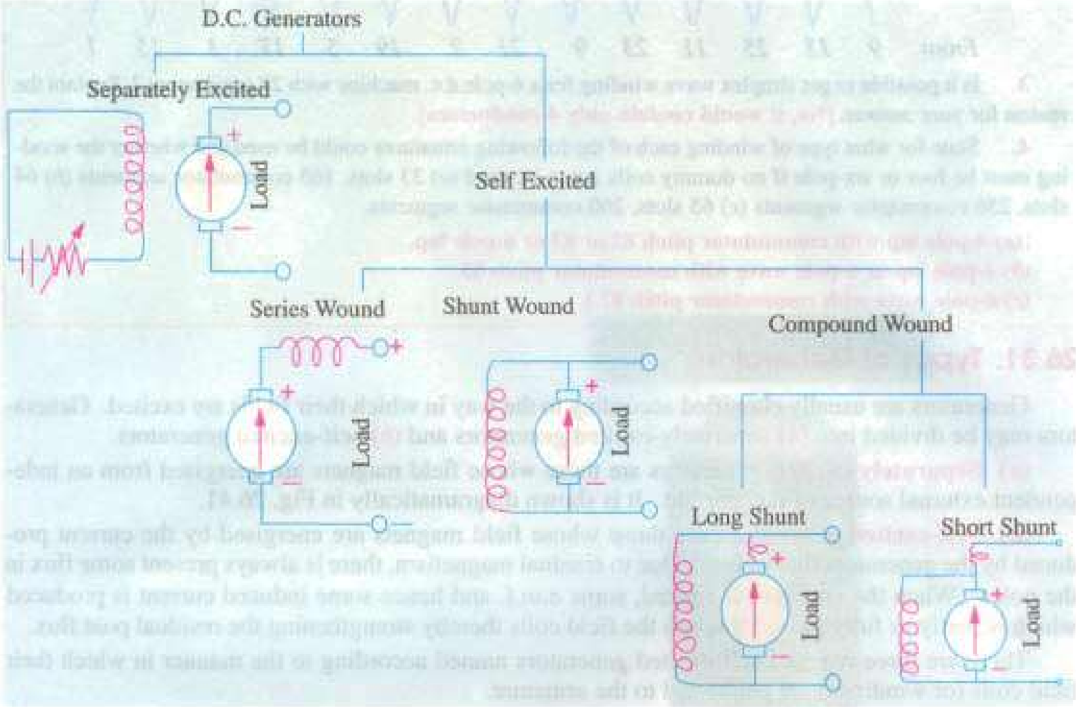


Fig. 26.45

26.32. Brush Contact Drop

It is the voltage drop over the brush contact resistance when current passes from commutator segments to brushes and finally to the external load. Its value depends on the amount of current and the value of contact resistance. This drop is usually small and includes brushes of both polarities. However, in practice, the brush contact drop is assumed to have following constant values for all loads.

0.5 V for metal-graphite brushes.

2.0 V for carbon brushes.

Example 26.3. *A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.*

Solution. Generator circuit is shown in Fig. 26.46.

Current through shunt field winding is

$$I_{sh} = 230/50 = 4.6 \text{ A}$$

Load current $\quad I = 450$ A

$\therefore$ Armature current $I_a = I + I_{sh}$

$$= 450 + 4.6 = 454.6 \text{ A}$$

Armature voltage drop

$$I_a R_a = 454.6 \times 0.03 = \mathbf{13.6\ V}$$

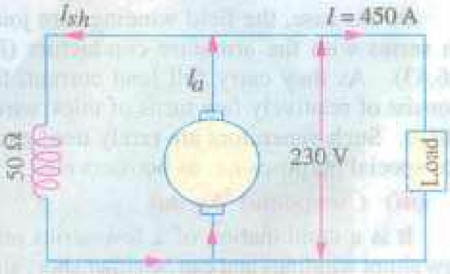


Fig. 26.46

Now E_g = terminal voltage + armature drop

$= V + I_a R_a$

∴ e.m.f. generated in the armature

$E_g = 230 + 13.6 =$ **243.6 V**

Example 26.4. *A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.*

(Elect. Science 1, Allahabad Univ. 1992)

Solution. Generator circuit is shown in Fig. 26.47.

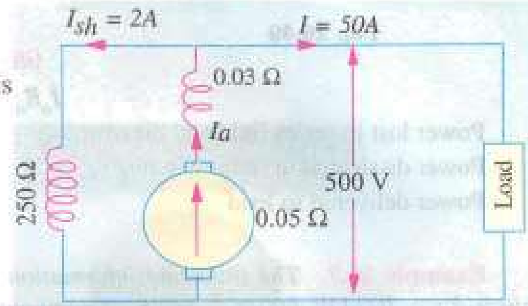


Fig. 26.47

$I_{sh} = 500/250 = 2$ A

Current through armature and series winding is

$= 50 + 2 = 52$ A

Voltage drop on series field winding

$= 52 \times 0.03 = 1.56$ V

Armature voltage drop

$I_a R_a = 52 \times 0.05 = 2.6$ V

Drop at brushes $= 2 \times 1 = 2$ V

Now, $E_g = V + I_a R_a$ + series drop + brush drop

$= 500 + 2.6 + 1.56 + 2 =$ **506.16 V**

Example 26.5. *A short-shunt compound generator delivers a load current of 30 A at 220 V, and has armature, series-field and shunt-field resistances of 0.05 Ω, 0.30 Ω and 200 Ω respectively. Calculate the induced e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.*

(AMIE Sec. B. Elect. Machines 1991)

Solution. Generator circuit diagram is shown in Fig. 26.48.

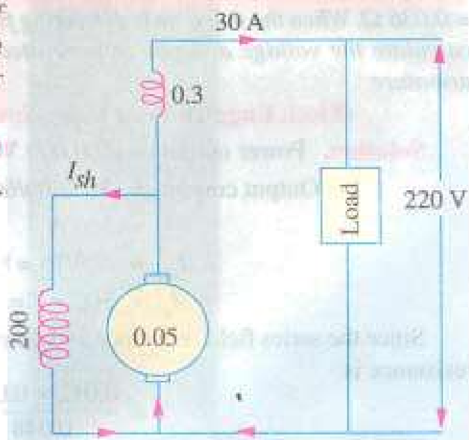


Fig. 26.48

Voltage drop in series winding $= 30 \times 0.3 = 9$ V

Voltage across shunt winding $= 220 + 9 = 229$ V

$I_{sh} = 229/200 = 1.145$ A

$I_a = 30 + 1.145 = 31.145$ A

$I_a R_a = 31.145 \times 0.05 = 1.56$ V

Brush drop $= 2 \times 1 = 2$ V

$E_g = V$ + series drop + brush drop + $I_a R_a$

$= 220 + 9 + 2 + 1.56 = 232.56$ V

Example 26.6. *In a long-shunt compound generator, the terminal voltage is 230 V when generator delivers 150 A. Determine (i) induced e.m.f. (ii) total power generated and (iii) distribution of this power. Given that shunt field, series field, divertor and armature resistance are 92 Ω, 0.015 Ω, 0.03 Ω and 0.032 Ω respectively.*

(Elect. Technology-II, Gwalior Univ. 1987)

Solution. $I_{sh} = 230/92 = 2.5$ A

$I_a = 150 + 2.5 = 152.5$ A

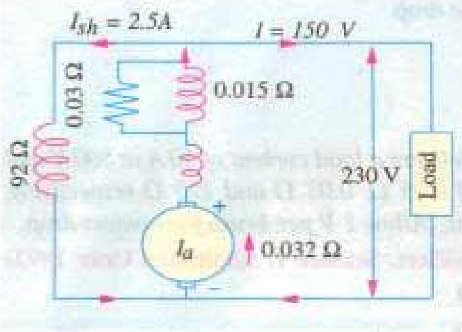


Fig. 26.49

Since series field resistance and divertor resistances are in parallel (Fig. 26.49) their combined resistance is

$$= 0.03 \times 0.015/0.045 = 0.01\ \Omega$$

Total armature circuit resistance is

$$= 0.032 + 0.01 = 0.042\ \Omega$$

Voltage drop $= 152.5 \times 0.042 = 6.4$ V

(i) Voltage generated by armature

$$E_g = 230 + 6.4 = \mathbf{236.4\ V}$$

(ii) Total power generated in armature

$$E_g I_a = 236.4 \times 152.5 = \mathbf{36{,}051\ W}$$

(iii) Power lost in armature

	$I_a R_a$ = $152.5^2 \times 0.032$	=	744 W
Power lost in series field and divertor	= $152.5^2 \times 0.01$	=	232 W
Power dissipated in shunt winding	= $VI_{sh} = 230 \times 0.01$	=	575 W
Power delivered to load	= 230×150	=	34500 W
	Total	=	36,051 W.

Example 26.7. *The following information is given for a 300-kW, 600-V, long-shunt compound generator : Shunt field resistance = 75 Ω, armature resistance including brush resistance = 0.03 Ω, commutating field winding resistance = 0.011 Ω, series field resistance = 0.012 Ω, divertor resistance = 0.036 Ω. When the machine is delivering full load, calculate the voltage and power generated by the armature.*

(Elect. Engg-II, Pune Univ. Nov. 1989)

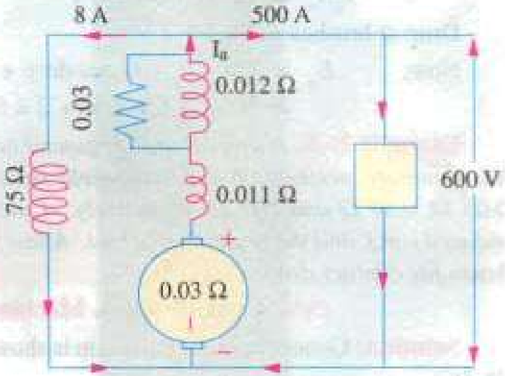


Fig. 26.50

Solution. Power output = 300,000 W

Output current = 300,000/600

= 500 A

I_{sh} = 600/75 = 8 A,

I_a = 500 + 8 = 508 A

Since the series field resistance and divertor resistance are in parallel (Fig. 26.50) their combined resistance is

$$= \frac{0.012 \times 0.036}{0.048} = 0.009\ \Omega$$

Total armature circuit resistance

$$= 0.03 - 0.011 + 0.009 = 0.05\ \Omega$$

Voltage drop = $508 \times 0.05 = 25.4$ V

Voltage generated by armature

$$= 600 + 25.4 = 625.4\ \text{V}$$

Power generated = $625.4 \times 508 = 317{,}700$

W = **317.7 kW**

26.33. Generated E.M.F. or E.M.F. Equation of a Generator

Let Φ = flux/pole in weber

Z = total number of armature conductors

= No. of slots × No. of conductors/slot

P = No. of generator poles

A = No. of parallel paths in armature

N = armature rotation in revolutions per minute (r.p.m.)

E = e.m.f. induced in any parallel path in armature

Generated e.m.f. E_g = e.m.f. generated in any one of the parallel paths *i.e.* E.

Average e.m.f. generated/conductor $= \dfrac{d\Phi}{dt}$ volt $(\because n = 1)$

Now, flux cut/conductor in one revolution $d\Phi = \Phi P$ Wb

No. of revolutions/second = $N/60$ $\therefore$ Time for one revolution, $dt = 60/N$ second

Hence, according to Faraday's Laws of Electromagnetic Induction,

E.M.F. generated/conductor $= \dfrac{d\Phi}{dt} = \dfrac{\Phi PN}{60}$ volt

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors (in series) in one path = $Z/2$

$\therefore$ E.M.F. generated/path $= \dfrac{\Phi PN}{60} \times \dfrac{Z}{2} = \dfrac{\Phi ZPN}{120}$ volt

For a simplex lap-wound generator

No. of parallel paths = P

No. of conductors (in series) in one path = Z/P

$\therefore$ E.M.F. generated/path $= \dfrac{\Phi PN}{60} \times \dfrac{Z}{P} = \dfrac{\Phi ZN}{60}$ volt

In general generated e.m.f. $E_g = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

where A = 2-for simplex wave-winding

= P-for simplex lap-winding

Also, $$E_g = \frac{1}{2\pi}\cdot\left(\frac{2\pi N}{60}\right)\Phi Z\left(\frac{P}{A}\right) = \frac{\omega\Phi Z}{2\pi}\left(\frac{P}{A}\right) \text{ volt} - \omega \text{ in rad/s}$$

For a given d.c. machine, Z, P and A are constant. Hence, putting $K_a = ZP/A$, we get

$E_g = K_a \Phi N$ volts—where N is in r.p.s.

Example 26.8. *A four-pole generator, having wave-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rpm assuming the flux per pole to be 7.0 mWb ?* **(Elect. Machines-I, Allahabad Univ. 1993)**

Solution. $E_g = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right)$ volts

Here, $\Phi = 7 \times 10^{-3}$ Wb, $Z = 51 \times 20 = 1020$, $A = P = 4$, $N = 1500$ r.p.m.

$\therefore$ $E_g = \dfrac{7 \times 10^{-3} \times 1020 \times 1500}{60}\left(\dfrac{4}{2}\right) =$ **178.5 V**

Example 26.9. *An 8-pole d.c. generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1200 rpm ? What must be the speed at which it is to be driven produce the same e.m.f. if it is wave-wound?*

(U.P. Technical Univ. 2001)

Solution. With lap-winding, $P = a = 8$

$$E = \phi\ (N/60)\ (P/a)$$
$$= 0.05 \times 500 \times 20 \times 1,$$
$$= 500 \text{ volts}$$

for lap-winding

If it is wave-wound, $P = 8,\ a = 2,\ P/a = 4$

and $E = 0.05 \times 500 \times (N/60) \times 4$

For $E = 500$ volts, $N = 300$ rpm

Hence, with wave-winding, it must be driven at 300 rpm to generate 500 volts.

Additional Explanation. Assume 1 amp as the current per conductor.

(*a*) Lap-wound, 1200 rpm : 500 V per coil-group, 8 groups in parallel

Net output current = 8 amp as in Fig. 26.51 (*a*).

Power output = 4 kW

(*b*) Wave-wound, 300 rpm : 2 groups in parallel, one group has four coils in series, as shown in Fig. 26.51 (*b*).

Total power-output is now

$$500 \times 2 = 1000 \text{ W}.$$

It is reduced to one fourth, being proportional to the speed.

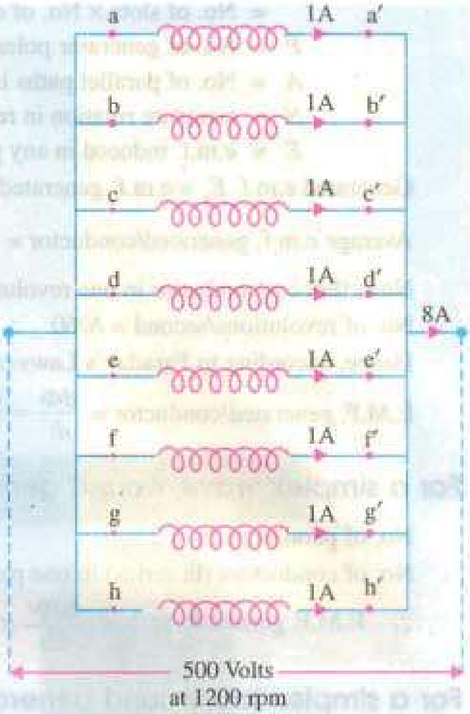


Fig. 26.51(*a*)

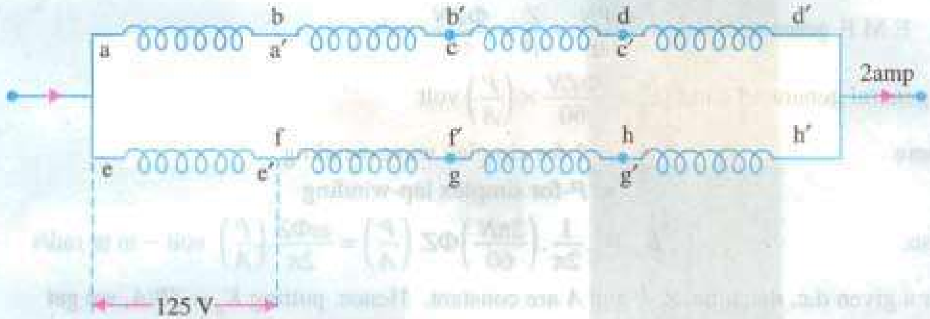


Fig. 26.51(*b*)

Example 26.10. *A d.c. shunt generator has an induced voltage on open-circuit of 127 volts. When the machine is on load, the terminal voltage is 120 volts. Find the load current if the field-circuit resistance is 15 ohms and the armature-resistance is 0.02 ohm. Ignore armature reaction.*

(Madras University April 1997, Bharathiar University Nov. 1997)

Solution.

Note : Even though the question does not specify some conditions, the solution given here is based on correct approach to deal with the case.

Generator on no load :

As shown in Fig. 26.52 (*a*), the machine is run at N_1 rpm.

$$E_g = 127 + 8.47 \times 0.02 = 127.17 \text{ volts}$$

As in Fig. 26.52 (*b*), $i_f = 8$ amp

E_g can be 127.17 volts, if the speed is increased to N_2 rpm, such that

$$8.47\, N_1 = 8N_2, \text{ or } N_2 = \frac{8.47}{8}\, N_1 = 1.05875\, N_1$$

Thus the effect due to 5.875% decrease in flux is compensated by 5.875% increase in speed.

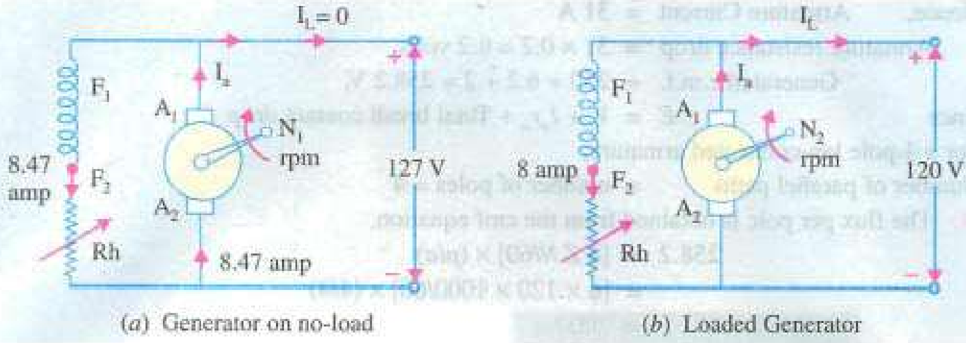


Fig. 26.52

If E_g is assumed to remain unaltered at 127.17 V,

$$I_a = \frac{127.17 - 120}{0.02} = 358.5 \text{ amp}$$

Hence, $\quad I_L = 358.5 - 8 = 350.5$ amp.

Example 26.11(a). *An 8-pole d.c. shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of 12.5 Ω resistance at terminal voltage of 50 V. The armature resistance is 0.24 Ω and the field resistance is 250 Ω. Find the armature current, the induced e.m.f. and the flux per pole.*

(Electrical Engg-I, Bombay Univ. 1988)

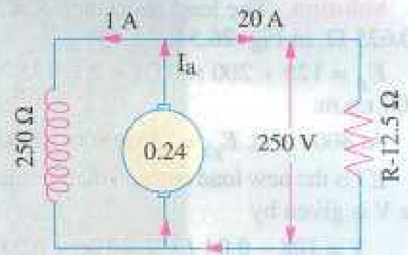


Fig. 26.53

Solution. The circuit is shown in Fig. 26.53

Load current $= V/R = 250/12.5 = 20$ A

Shunt current $= 250/250 = 1$ A

Armature current $= 20 + 1 =$ **21 A**

Induced e.m.f. $= 250 + (21 \times 0.24) =$ **255.04 V**

Now, $\quad E_g = \dfrac{\Phi Z N}{60} \times \left(\dfrac{P}{A}\right)$

$\therefore \quad 255.04 = \dfrac{\Phi \times 778 \times 500}{60}\left(\dfrac{8}{2}\right)$

$\therefore \quad \Phi =$ **9.83 mWb**

Example 26.11(b). *A 4-pole lap-connected armature of a d.c. shunt generator is required to supply the loads connected in parallel :*

(1) 5 kW Geyser at 250 V, and

(2) 2.5 kW Lighting load also at 250 V.

The Generator has an armature resistance of 0.2 ohm and a field resistance of 250 ohms. The armature has 120 conductors in the slots and runs at 1000 rpm. Allowing 1 V per brush for contact drops and neglecting friction, find

(1) Flux per pole, (2) Armature-current per parallel path. **(Nagpur University Nov. 1998)**

Solution.

$$\text{Geyser current} = 5000/250 = 20 \text{ A}$$
$$\text{Current for Lighting} = 2500/250 = 10 \text{ A}$$
$$\text{Total current} = 30 \text{ A}$$
$$\text{Field Current for Generator} = 1 \text{ A}$$

Hence, Armature Current = 31 A

$$\text{Armature resistance drop} = 31 \times 0.2 = 6.2 \text{ volts}$$
$$\text{Generated e.m.f.} = 250 + 6.2 + 2 = 258.2 \text{ V},$$

since $E = V_t + I_a r_a + \text{Total brush contact drop}$

For a 4-pole lap-connected armature,

Number of parallel paths = number of poles = 4

(1) The flux per pole is obtained from the emf equation

$$\begin{aligned} 258.2 &= [\phi\, Z\, N/60] \times (p/a) \\ &= [\phi \times 120 \times 1000/60] \times (4/4) \\ &= 2000\,\phi \\ \phi &= 129.1 \text{ mWb} \end{aligned}$$

(2) Armature current per parallel path = 31/4 = 7.75 A.

Example 26.12. *A separately excited generator, when running at 1000 r.p.m. supplied 200 A at 125 V. What will be the load current when the speed drops to 800 r.p.m. if I_f is unchanged ? Given that the armature resistance = 0.04 ohm and brush drop = 2 V.* **(Elect. Machines Nagpur Univ. 1993)**

Solution. The load resistance $R = 125/200 = 0.625\ \Omega$, in Fig. 26.54.

$E_{g1} = 125 + 200 \times 0.04 + 2 = 135$ V ; $N_1 = 1000$ r.p.m.

At 800 r.p.m. $E_{g2} = 135 \times 800/1000 = 108$ V

If I is the new load current, then terminal voltage V is given by

$$V = 108 - 0.04\,I - 2 = 106 - 0.04\,I$$

$$\therefore\ I = V/R = (106 - 0.04\,I)/0.635\ ;\ I = 159.4 \text{ A}$$

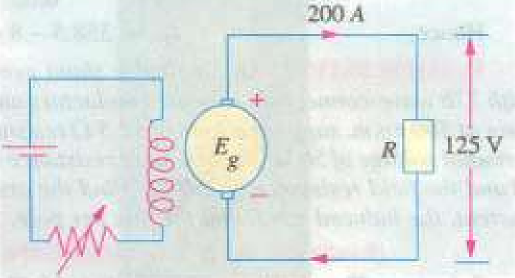


Fig. 26.54

Example 26.13. *A 4-pole, 900 r.p.m. d.c. machine has a terminal voltage of 220 V and an induced voltage of 240 V at rated speed. The armature circuit resistance is 0.2 Ω. Is the machine operating as a generator or a motor ? Compute the armature current and the number of armature coils if the air-gap flux/pole is 10 mWb and the armature turns per coil are 8. The armature is wave-wound.* **(Elect. Machines AMIE Sec. B 1990)**

Solution. Since the induced voltage E is more than the terminal voltage v, the machine is working as a generator.

$$E - V = I_a R_a \quad \text{or} \quad 240 - 220 = I_a \times 0.2\ ;\ I_a = 100 \text{ A}$$

Now, $E_b = ZN\,(P/A)$ or $240 = 10 \times 10^{-3} \times z \times (900/600)\ (4/2)\ ;\ Z = 8000$

Since there are 8 turns in a coil, it means there are 16 active conductors/coil. Hence, the number of coils = 8000/16 = 500.

Example 26.14. *In a 120 V compound generator, the resistances of the armature, shunt and series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load current is 100 A at 120 V. Find the induced e.m.f. and the armature current when the machine is connected as (i) long-shunt and as (ii) short-shunt. How will the ampere-turns of the series field be changed in (i) if a diverter of 0.1 ohm be connected in parallel with the series winding ? Neglect brush contact drop and ignore armature reaction.* **(Elect. Machines AMIE Sec. B, 1992)**

Solution. (*i*) Long Shunt [Fig. 26.55 (*a*)]

$$I_{sh} = 120/125 = 4.8 \text{ A} ; I = 100 \text{ A} ; I_a = 104.8 \text{ A}$$

Voltage drop in series winding $= 104.8 \times 0.04 = 4.19$ V

Armature voltage drop $= 104.8 \times 0.06 = 6.29$ V

$$\therefore \quad E_g = 120 + 3.19 + 6.29 = 130.5 \text{ V}$$

(*ii*) Short Shunt [Fig. 26.55 (*c*)]

Voltage drop in series winding $= 100 \times 0.04 = 4$ V

Voltage across shunt winding $= 120 + 4 = 124$ V

$$\therefore \quad I_{sh} = 124/25 = 5 \text{ A} ; \quad \therefore \quad I_a = 100 + 5 = 105 \text{ A}$$

Armature voltage drop $= 105 \times 0.06 = 6.3$ V

$$E_g = 120 + 5 + 4 = 129 \text{ V}$$

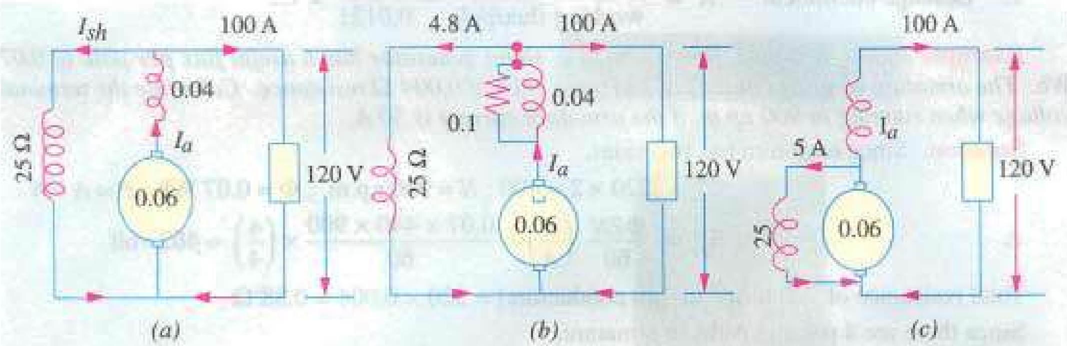


Fig. 26.55

When a diverter of 0.1 Ω is connected in parallel with the series winding, the diagram becomes as shown in Fig. 26.55 (*b*). As per current-divider rule, the current through the series winding is = $104.8 \times 0.1/(0.1 + 0.04) = 74.86$ A. It means that the series field current has decreased from an original value of 104.8 A to 74.86 A. Since No. of turns in the series winding remains the same, the change in series field ampere-turns would be the same as the change in the field current. Hence, the percentage decrease in the series field ampere-turns = $(74.86 - 104.8) \times 100/104.8 = -28.6\%$.

Example 26.15. *A 4-pole, long-shunt lap-wound generator supplies 25 kW at a terminal voltage of 500 V. The armature resistance is 0.03 ohm, series field resistance is 0.04 ohm and shunt field resistance is 200 ohm. The brush drop may be taken as 1.0 V. Determine the e.m.f. generated.*

Calculate also the No. of conductors if the speed is 1200 r.p.m. and flux per pole is 0.02 weber. Neglect armature reaction.

(Elec. Engineering-I, St. Patel Univ. 1986)

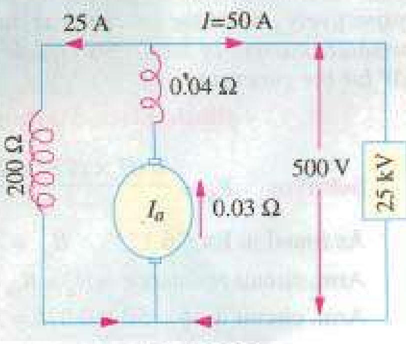


Fig. 26.56

Solution. $I = 25{,}000/500 = 50$ A, $I_{sh} = 500/200 = 2.5$ A (Fig. 26.56)

$$I_a = I + I_{sh} = 50 + 2.5 = 52.5 \text{ A}$$

Series field drop $= 52.5 \times 0.04 = 2.1$ V

Armature drop $= 52.5 \times 0.03 = 1.575$ V

Brush drop $= 2 \times 1 = 2$ V

Generated e.m.f., $E_g = 500 + 2.1 + 1.575 + 2 =$ **505.67 V**

Now, $$E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right)$$

or $$505.67 = \frac{0.02 \times Z \times 1200}{60}\left(\frac{4}{4}\right), Z = \mathbf{1264}$$

Example 26.16. *A 4-pole d.c. generator runs at 750 r.p.m. and generates an e.m.f. of 240 V. The armature is wave-wound and has 792 conductors. If the total flux from each pole is 0.0145 Wb, what is the leakage coefficient ?*

Solution. Formula used :

$$E = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ volt} \quad \therefore \quad 240 = \frac{\Phi \times 750 \times 792}{60} \times \frac{4}{2}$$

$\therefore$ Working flux/pole, $\Phi = 0.0121$ Wb ; Total flux/pole = 0.0145 Wb

$\therefore$ Leakage coefficient $$\lambda = \frac{\text{total flux/pole}}{\text{working flux/pole}} = \frac{0.0145}{0.0121} = \mathbf{1.2}$$

Example 26.17. *A 4-pole, lap-wound, d.c. shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004 Ω resistance. Calculate the terminal voltage when running at 900 r.p.m. if the armature current is 50 A.*

Solution. Since each turn has two sides,

$$Z = 220 \times 2 = 400 \text{ ; } N = 900 \text{ r.p.m. ; } \Phi = 0.07 \text{ Wb ; } P = A = 4$$

$$\therefore \quad E_g = \frac{\Phi ZN}{60}.\left(\frac{P}{A}\right) = \frac{0.07 \times 440 \times 900}{60} \times \left(\frac{4}{4}\right) = 462 \text{ volt}$$

Total resistance of 220 turns (or 440 conductors) = $220 \times 0.004 = 0.88$ Ω

Since there are 4 parallel paths in armature,

$\therefore$ Resistance of each path = 0.88/4 = 0.22 Ω

Now, there are four such resistances in parallel each of value 0.22 Ω

$\therefore$ Armature resistance, $R_a = 0.22/4 = 0.055$ Ω

Armature drop $= I_aR_a = 50 \times 0.055 = 2.75$ Ω

Now, terminal voltage $V = E_g - I_aR_a = 462 - 2.75 =$ **459.25 volt.**

Example 26.18. *A 4-pole, lap-wound, long-shunt, d.c. compound generator has useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns and the resistance per turn is 0.004 ohms. Calculate the terminal voltage if the resistance of shunt and series field are 100 ohms and 0.02 ohms respectively ; when the generator is running at 900 r.p.m. with armature current of 50 A. Also calculate the power output in kW for the generator.*

(Basic Elect. Machine Nagpur Univ. 1993)

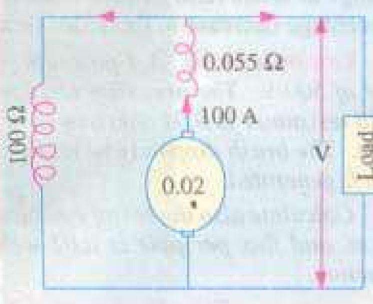


Fig. 26.57

Solution. $$E_b = \frac{0.07 \times (220 \times 2) \times 900}{60} \times \left(\frac{4}{4}\right) = 462 \text{ V}$$

As found in Ex. 26.17, $R_a = 0.055$ Ω

Arm. circuit resistance = $R_a + R_{se} = 0.055 + 0.02 = 0.075$ Ω

Arm. circuit drop = $50 \times 0.075 = 3.75$ V

$V = 462 - 3.75 = 458.25$ V, in Fig. 26.57.

$I_{sh} = 458.25/100 = 4.58$ A ; $I = 50 - 4.58 = 45.42$ A

Output = $VI = 458.25 \times 45.42 = 20{,}814$ W = 20.814 kW

Example 26.19. *A separately excited d.c. generator, when running at 1200 r.p.m. supplies 200 A at 125 V to a circuit of constant resistance. What will be the current when the speed is dropped to 1000 r.p.m. and the field current is reduced to 80% ? Armature resistance, 0.04 Ω and total drop at brushes, 2 V. Ignore saturation and armature reaction.*

(Elect. Machines AMIE Sec. B, 1991)

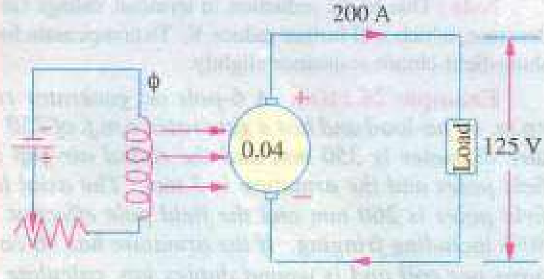


Fig. 26.58

Solution. We will find the generated e.m.f. when the load current is 200 A.

$E_{g1} = V +$ brush drop $+ I_aR_a = 125 + 200 \times 0.04 = 135$ V, in Fig. 26.58.

Now, $E_{g1} \propto \Phi_1 N_1$ and $E_{g2} \propto \Phi_2 N_2$

$$\therefore \quad \frac{E_{g2}}{E_{g1}} = \frac{\Phi_2 N_2}{\Phi_1 N_1}$$

$$\text{or} \quad \frac{E_{g2}}{135} = 0.8 \times \frac{1000}{1200} = 90 \text{ V}$$

Example 26.20(*a*). *A 4-pole, d.c. shunt generator with a shunt field resistance of 100 Ω and an armature resistance of 1 Ω has 378 wave-connected conductors in its armature. The flux per pole is 0.02 Wb. If a load resistance of 10 Ω is connected across the armature terminals and the generator is driven at 1000 r.p.m., calculate the power absorbed by the load.*

(Elect. Technology, Hyderabad Univ. 1991)

Solution. Induced e.m.f. in the generator is

$$E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ volt}$$

$$= \frac{0.02 \times 378 \times 1000}{60}\left(\frac{4}{2}\right) = 252 \text{ volt}$$

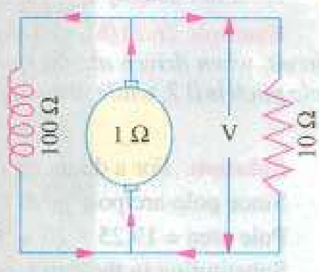


Fig. 26.59

Now, let V be the terminal voltage *i.e.* the voltage available across the load as well as the shunt resistance (Fig. 26.59).

Load current = $V/10$ A and Shunt current = $V/100$ A

Armature current $= \frac{V}{10} + \frac{V}{100} = \frac{11\,V}{100}$

Now, $V = E_g -$ armature drop

$$\therefore \quad V = 252 - 1 \times \frac{11\,V}{100} \quad \therefore \quad V = 227 \text{ volt}$$

Load current = 227/10 = 22.7 A, Power absorbed by the load is = 227 × 22.7 = **5,153 W**

Example 26.20(*b*). *A four-pole, lap-wound shunt generator has 300 armature conductors and a flux/pole of 0.1 Wb. It runs at 1000 r.p.m. The armature and field-resistances are 0.2 ohm and 125 ohms respectively. Calculate the terminal voltage when it is loaded to take a load current of 90 A. Ignore armature reaction.* **(Nagpur University, April 1999)**

Solution. First, the e.m.f .should be calculated

$$E = 0.1 \times 300 \times (1000/60) \times (4/4) = 500 \text{ volts}$$

The field current 500/125 = 4 amp

For the load current of 90 amp, armature current = 94 amp

$$I_a r_a = 94 \times 0.20 = 18.8 \text{ volts}$$

Terminal voltage, $\quad V = 500 - 18.8 = 481.2$ volts

Note : Due to the reduction in terminal voltage (as an effect of loading), the shunt field current tends to decrease, which will further reduce V. To compensate for this, either increase the speed slightly or decrease the shunt-field-circuit resistance slightly.

Example 26.21(a). *A 6-pole dc generator runs at 1200 r.p.m. on no-load and has a generated e.m.f. of 250 V. Its armature diameter is 350 mm and the radial air-gap between the field poles and the armature is 3 mm. The axial length of the field poles is 260 mm and the field pole effective coverage is 80% including fringing. If the armature has 96 coils having 3 turns per coil and is wound duplex lap, calculate (a) flux per pole (b) effective pole arc length and (c) average air-gap flux density.*

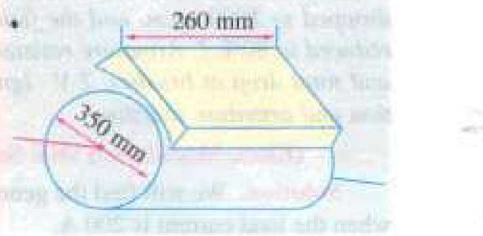


Fig. 26.60

Solution. (*a*) $Z = (96 \times 3) \times 2 = 576$, $P = 6$, $A = P \times$ plex $= 6 \times 2 = 12$, $N = 1200$ r.p.m.

$$\therefore \quad 250 = \frac{\Phi \times 576 \times 1200}{6}\left(\frac{6}{12}\right); \quad \therefore \quad \Phi = 0.0434 \text{ Wb}$$

(*b*) Inner diameter of the pole shoe circle is $= 350 + 6 = 356$ mm.

Since there are 6 poles, the net field pole flux coverage is 80% of one-sixth of the pole shoe circle. Hence, the effective pole arc length is

$$= \frac{1}{6} \times \pi d \times 0.8 = \frac{1}{6} \times \pi \times 356 \times 0.8 = 149 \text{ mm} = 0.149 \text{ m.}$$

(*c*) Pole surface area = pole shoe arc × axial length of the pole (Fig. 26.60).

$$= 0.149 \times 0.260 = 0.03874 \text{ mm}^2$$

$\therefore$ Flux density $B = 0.0434/0.03874 = 1.12$ T

Example 26.21(b). *A 4-pole d.c. Generator with 1200 conductors generates 250 volts on open circuit, when driven at 500 rpm. The pole-shoes have a bore of 35 cm and the ratio of pole-arc to pole pitch is 0.7, while, the length of the pole shoe is 20 cm. Find the mean flux density in the airgap.* **(Bharthiar Univ. Nov. 1972 & April 1998)**

Solution. For a diameter of 35, 4-pole machine has a pole-pitch of $(35\pi/4) = 27.5$ cm

Since pole-arc/pole pitch is 0.7, Pole-arc $= 0.7 \times 27.5 = 19.25$ cm

Pole area $= 19.25 \times 20 = 385$ sq. cm.

Substituting in the e.m.f. equation, $250 = (\phi\, ZN/60)\,(p/a)$

For Lap-winding, in the case, $p = a = 4$

Hence, flux/pole $= (250 \times 60)/(1200 \times 500) = 0.025$ Wb

This flux is uniformly distributed over the pole-area.

Mean flux density in the air-gap $= (0.025)/(385 \times 10^{-4}) = 0.65 \text{ Wb/m}^2$

Example 26.21(c). *A four-pole lap-wound dc shunt generator having 80 slots with 10 conductors per slot generates at no-load an e.m.f. of 400 V, when run at 1000 rpm. How will you obtain a generated open-circuit voltage of 220 V ?* **(Nagpur University November 1996)**

Solution. (*i*) Keeping operating speed at 1000 rpm only, **change the flux per pole**

The O.C. e.m.f. is given by $E = (\phi\, ZN/60) \times (P/a)$

For the given operating conditions,

$$400 = \phi \times (80 \times 10) \times (1000/60) \times (4/4)$$

which gives $\phi = 30$ mWb

When speed is kept constant at 1000 rpm only,

$$E \propto \phi$$

Or to get 220 V on O.C., $\phi_2 = (220/400) \times 30 \text{ mWb} = 16.5$ mWb

Thus, by increasing the shunt-field-circuit resistance with the help of adding external rheostatic, the current in the field-circuit is decreased so as to decrease the flux to 16.5 mWb.

(*ii*) Keep same flux per pole, change the speed.

If ϕ is held constant at 30 mWb, an O.C. e.m.f. of 220 V is obtained at a speed of N r.p.m., given by

$$220 = 30 \times 10^{-3} \times 800 \times N/60, \quad N = 550 \text{ rpm}$$

At 220 V, the flux can be maintained at 30 mWb provided the field **current is unchanged.**

$$400/R_{f1} = 200/R_{f2}$$

or $$R_{f2} = 0.55\, R_{f1}$$

Thus, the field circuit resistance must be reduced to the new value of $0.55\, R_{f1}$ in order to obtain 30 mWb of flux per pole from a voltage of 220 V.

(*iii*) Any other combination of proper speed and flux/pole can be chosen and worked out on similar lines.

Example 26.21(*d*). *A short-shunt d.c. compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 ohms respectively. Find the emf generated. Also find the emf generated if same machine is connected as a long-shunt machine.* **(Nagpur University, April 1998)**

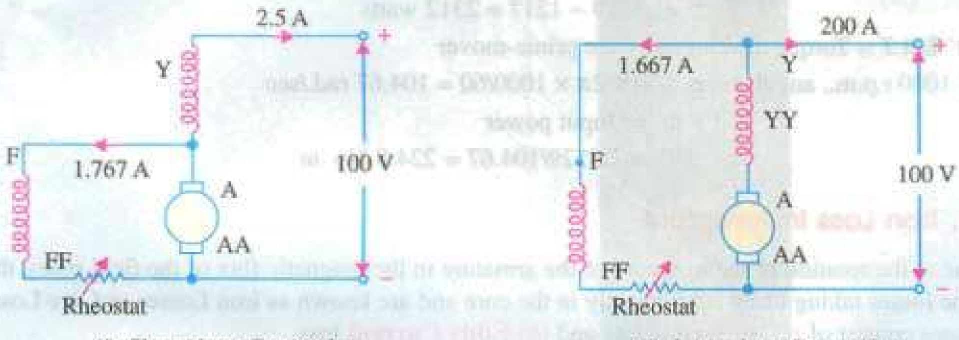
Solution. With short-shunt connection, shown in Fig. 26.61 (*a*).

V_a = armature terminal voltage = 100 (200 × 0.03) = 106 V

Shunt field current = 106/60 = 1.767 amp

Armature current = I_a = 200 + 1.767 = 201.767 amp

Armature induced e.m.f. = 106 + (201.767 × 0.04) = 114.07 volts



(*i*) Short-shunt Connection (*ii*) Long-shunt Connection

Fig. 26.61(*a*)

Now, with long-shunt connection shown in Fig. 26.61 (*b*),

Shunt field current = 100/60 = 1.667 amp

Armature current = 201.667 amp

Total voltage drop in armature and series field winding

= 201.667 (0.04 + 0.03) = 14.12 volts

Armature induced e.m.f. = 100 + 14.12 = 114.12 volts

Note : In case of long shunt connection, the generator has to develop the e.m.f. with shunt field current slightly reduced, compared to the case of short shunt connection. However, the series field winding carries a slightly higher current in latter case. Still, in practice, slight speed adjustment (or shunt field rheostatic variation) may be required to get this e.m.f., as per calculations done above.

Example 26.22. *A long shunt dynamo running at 1000 r.p.m. supplies 20 kW at a terminal voltage of 220 V. The resistance of armature, shunt field, and series field are 0.04, 110 and 0.05 ohm respectively. Overall efficiency at the above load is 85%. Find :*

(i) Copper loss,

(ii) Iron and friction loss,

(iii) Torque developed by the prime mover.

(Amravati University 1999)

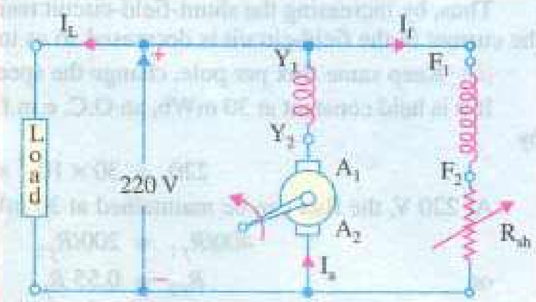


Fig. 26.61(*b*)

Solution. I_L, Load current $= \dfrac{20,000}{220} = 90.91$ amp

Shunt field current, $I_f = \dfrac{220}{110} = 2$ amp

Armature current, $I_a = 92.91$ amp

Input power $= 20,000/0.85 = 23529$ watts

Total losses in the machine $=$ Input $-$ Output $= 23529 - 20,000 = 3529$ watts

(i) ***Copper losses :***

Power loss in series field-winding + armature winding $= 92.91^2 \times 0.09$ watts $= 777$ watts

Power-loss in shunt field circuit : $2^2 \times 110 = 440$ watts

Total copper losses $= 777 + 400 = 1217$ watts

(ii) Iron and friction losses $=$ Total losses $-$ Copper losses

$= 3529 - 1217 = 2312$ watts

(iii) Let T = Torque developed by the prime-mover

At 1000 r.p.m., angular speed, $\omega = 2\pi \times 1000/60 = 104.67$ rad./sec

$T \times \omega =$ Input power

$\therefore \quad T = 23529/104.67 = 224.8$ Nw-m

26.34. Iron Loss in Armature

Due to the rotation of the iron core of the armature in the magnetic flux of the field poles, there are some losses taking place continuously in the core and are known as Iron Losses or Core Losses. Iron losses consist of *(i)* **Hysteresis** loss and *(ii)* **Eddy Current** loss.

(i) **Hysteresis Loss (W_h)**

This loss is due to the reversal of magnetisation of the armature core. Every portion of the rotating core passes under N and S pole alternately, thereby attaining S and N polarity respectively. The core undergoes one complete cycle of magnetic reversal after passing under one *pair* of poles. If P is the number of poles and N, the armature speed in r.p.m., then frequency of magnetic reversals is $f = PN/120$.

The loss depends upon the volume and grade of iron, maximum value of flux density B_{max} and frequency of magnetic reversals. For normal flux densities (*i.e.* upto 1.5 Wb/m²), hysteresis loss is given by **Steinmetz** formula. According to this formula,

$$W_h = \eta B_{max}^{1.6} f V \text{ watts}$$

where V = volume of the core in m³

η = Steinmetz hysteresis coefficient.

Value of η for :

Good dynamo sheet steel = 502 J/m^3, Silicon steel = 191 J/m^2, Hard Cast steel = 7040 J/m^3, Cast steel = 750 – 3000 J/m^3 and Cast iron = 2700 – 4000 J/m^3.

(ii) Eddy Current Loss (W_e)

When the armature core rotates, it also cuts the magnetic flux. Hence, an e.m.f. is induced in the body of the core according to the laws of electromagnetic induction. This e.m.f. though small, sets up large current in the body of the core due to its small resistance. This current is known as eddy current. The power loss due to the flow of this current is known as eddy current loss. This loss would be considerable if solid iron core were used. In order to reduce this loss and the consequent heating of the core to a small value, the core is built up of thin laminations, which are stacked and then riveted at right angles to the path of the eddy currents. These core laminations are insulated from each other by a thin coating of varnish. The effect of laminations is shown in Fig. 26.62. Due to the core body being one continuous solid iron piece [Fig. 26.62 (*a*)], the magnitude of eddy currents is large. As armature cross-sectional area is large, its resistance is very small, hence eddy current loss is large. In Fig. 26.62 (*b*), the same core has been split up into thin circular discs insulated from each other. It is seen that now each current path, being of much less cross-section, has a very high resistance. Hence, magnitude of eddy currents is reduced considerably thereby drastically reducing eddy current loss.

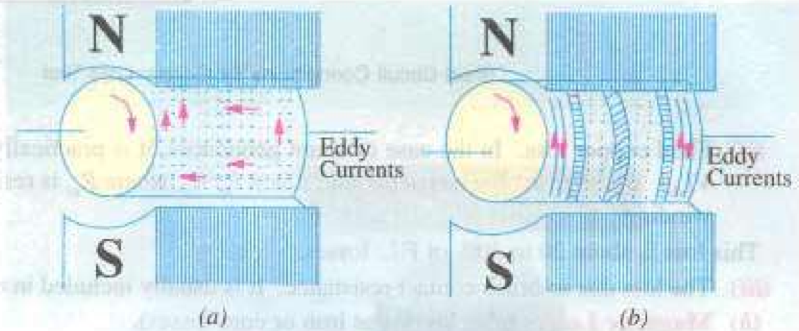


Fig. 26.62

It is found that eddy current loss W_e is given by the following relation :

$$W_e = KB^2_{max} f^2 t^2 V^2 \text{ watt}$$

where B_{max} = maximum flux density f = frequency of magnetic reversals
t = thickness of each lamination V = volume of armature core.

It is seen from above that this loss varies directly as the square of the thickness of laminations, hence it should be kept as small as possible. Another point to note is that $W_h \propto f$ but $W_e \propto f^2$. This fact makes it possible to separate the two losses experimentally if so desired.

As said earlier, these iron losses if allowed to take place unchecked not only reduce the efficiency of the generator but also raise the temperature of the core. As the output of the machines is limited, in most cases, by the temperature rise, these losses have to be kept as small as is economically possible.

Eddy current loss is reduced by using laminated core but hysteresis loss cannot be reduced this way. For reducing the hysteresis loss, those metals are chosen for the armature core which have a low hysteresis coefficient. Generally, special silicon steels such as stalloys are used which not only have a low hysteresis coefficient but which also possess high electrical resistivity.

26.35. Total Loss in a D.C. Generator

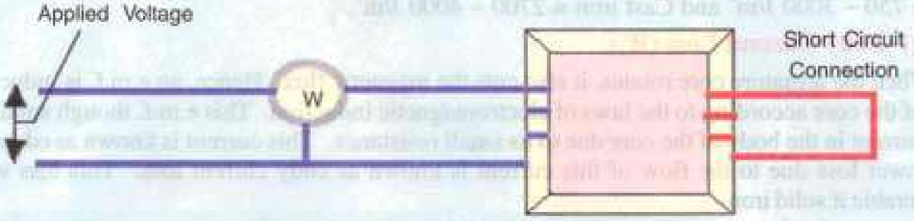
The various losses occurring in a generator can be sub-divided as follows :

(a) Copper Losses

(i) Armature copper loss = $I_a^2 R_a$ [Note : $E_g I_a$ is the power output from armature.]

where R_a = resistance of armature and interpoles and series field winding etc.

This loss is about 30 to 40% of full-load losses.



Short Circuit Connections for Copper Loss Test

(*ii*) Field copper loss. In the case of shunt generators, it is practically constant and $I_{sh}^2 R_{sh}$ (or VI_{sh}). In the case of series generator, it is $= I_{se}{}^2 R_{se}$ where R_{se} is resistance of the series field winding.

This loss is about 20 to 30% of F.L. losses.

(*iii*) The loss due to brush contact resistance. It is usually included in the armature copper loss.

(*b*) **Magnetic Losses** (also known as iron or core losses),

(*i*) hysteresis loss, $W_h \propto B_{max}^{1.6} f$ and (*ii*) eddy current loss, $W_e \propto B_{max}^2 f^2$

These losses are practically constant for shunt and compound-wound generators, because in their case, field current is approximately constant.

Both these losses total up to about 20 to 30% of F.L. losses.

(*c*) **Mechanical Losses.** These consist of :

(*i*) friction loss at bearings and commutator.

(*ii*) air-friction or windage loss of rotating armature.

These are about 10 to 20% of F.L. Losses.

The total losses in a d.c. generator are summarized below :

- Total losses
  - Copper losses
    - Armature Cu loss
    - Shunt Cu loss
    - Series Cu loss
  - Iron losses
    - Hysteresis
    - Eddy current
  - Mechanical losses
    - Friction
    - Windage

26.36. Stray Losses

Usually, magnetic and mechanical losses are collectively known as *Stray Losses*. These are also known as rotational losses for obvious reasons.

26.37. Constant or Standing Losses

As said above, field Cu loss is constant for shunt and compound generators. Hence, stray losses and shunt Cu loss are constant in their case. These losses are together known as standing or constant losses W_c.

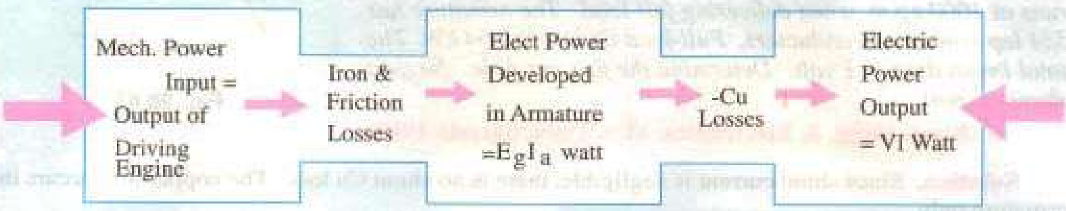
Hence, for shunt and compound generators, Total loss = armature copper loss + $W_c = I_a^2R_a + W_c = (I + I_{sh})^2R_a + W_c$.

Armature Cu loss $I_a^2R_a$ is known as variable loss because it varies with the load current.

Total loss = variable loss + constant losses W_c

26.38. Power Stages

Various power stages in the case of a d.c. generator are shown below :



Following are the three generator efficiencies :

1. Mechanical Efficiency

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}} = \frac{E_gI_a}{\text{output of driving engine}}$$

2. Electrical Efficiency

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{E_gI_a}$$

3. Overall or Commercial Efficiency

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}}$$

It is obvious that overall efficiency $\eta_c = \eta_m \times \eta_e$. For good generators, its value may be as high as 95%.

Note. Unless specified otherwise, commercial efficiency is always to be understood.

26.39. Condition for Maximum Efficiency

Generator output = VI

Generator input = output + losses

$$= VI + I_a^2R_a + W_c = VI + (I + I_{sh})^2R_a + W_c \qquad (\because I_a = I + I_{sh})$$

However, if I_{sh} is negligible as compared to load current, then $I_a = I$ (approx.)

$$\therefore \quad \eta = \frac{\text{output}}{\text{input}} = \frac{VI}{VI + I_a^2R_a + W_c} = \frac{VI}{VI + I^2R_a + W_c} \qquad (\because I_a = I)$$

$$= \frac{1}{1 + \left(\frac{IR_a}{V} + \frac{W_c}{VI}\right)}$$

Now, efficiency is maximum when denominator is minimum *i.e.* when

$$\frac{d}{dI}\left(\frac{IR_a}{V} + \frac{W_c}{VI}\right) = 0 \text{ or } \frac{R_a}{V} - \frac{W_c}{VI^2} = \quad \text{or} \quad I^2R_a = W_c$$

Hence, generator efficiency is maximum when

Variable loss = constant loss.

The load current corresponding to maximum efficiency is given by the relation.

$$I^2R_a = W_c \quad \text{or} \quad I = \sqrt{\frac{W_c}{R_a}}.$$

Variation of η with load current is shown in Fig. 26.63.

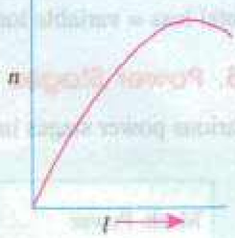


Fig. 26.63

Example 26.23. *A 10 kW, 250 V, d.c., 6-pole shunt generator runs at 1000 r.p.m. when delivering full-load. The armature has 534 lap-connected conductors. Full-load Cu loss is 0.64 kW. The total brush drop is 1 volt. Determine the flux per pole. Neglect shunt current.*

(Elect. Engg. & Electronics, M.S. Univ./Baroda 1987)

Solution. Since shunt current is negligible, there is no shunt Cu loss. The copper loss occurs in armature only.

$I = I_a = 10{,}000/250 = 40$ A ; $I_a^2R_a$ = Arm. Cu loss or $40^2 \times R_a = 0.64 \times 10^3$; $R_a = 0.4\ \Omega$

I_aR_a drop $= 0.4 \times 40 = 16$ V ; Brush drop $= 2 \times 1 = 2$ V

$\therefore$ Generated e.m.f. $E_g = 250 + 16 + 1 = 267$ V

Now, $E_g = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right)$ volt $\quad \therefore 267 = \dfrac{\Phi \times 534 \times 1000}{60}\left(\dfrac{6}{6}\right) \quad \therefore \quad \Phi = 30 \times 10^{-3}$ Wb = **30 mWb**

Example 26.24(*a*). *A shunt generator delivers 195 A at terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0.02 Ω and 50 Ω respectively. The iron and friction losses equal 950 W. Find*

(a) E.M.F. generated (b) Cu losses (c) output of the prime motor

(d) commercial, mechanical and electrical efficiencies.

(Elect. Machines-I, Nagpur Univ. 1991)

Solution. (*a*) $I_{sh} = 250/50 = 5$ A ; $I_a = 195 + 5 = 200$ A

Armature voltage drop $= I_aR_a = 200 \times 0.02 = 4$ V

$\therefore$ Generated e.m.f. $= 250 + 4 =$ **254 V**

(*b*) Armature Cu loss $= I_a^2R_a = 200^2 \times 0.02 = 800$ W

Shunt Cu loss $= V.I_{sh} = 250 \times 5 = 1250$ W

$\therefore$ Total Cu loss $= 1250 + 800 =$ **2050 W**

(*c*) Stray losses $= 950$ W ; Total losses $= 2050 + 950 = 3000$ W

Output $= 250 \times 195 = 48{,}750$ W ; Input $= 48{,}750 + 3000 = 51750$ W

$\therefore$ Output of prime mover = **51,750 W**

(*d*) Generator input $= 51{,}750$ W ; Stray losses $= 950$ W

Electrical power produced in armature $= 51{,}750 - 950 = 50{,}800$

$$\eta_m = (50{,}800/51{,}750) \times 100 = \mathbf{98.2\%}$$

Electrical or Cu losses = 2050 W

$$\therefore \quad \eta_e = \frac{48{,}750}{48{,}750 + 2{,}050} \times 100 = \mathbf{95.9\%}$$

and $\quad \eta_c = (48{,}750/51{,}750) \times 100 = \mathbf{94.2\%}$

Example 26.24(*b*). *A 500 V, D.C. shunt motor draws a line current of 5 amps, on light load. If armature resistance is 0.15 ohm, and field resistance is 200 ohms, determine the efficiency of the machine running as a generator, delivering a load current of 40 Amp.*

(Bharathiar Univ. Nov. 1997)

Solution. (*i*) As a motor, on Light load, out of 5 Amps of line current, 2.5 Amps are required for field circuit and 2.5 Amps are required for field circuit and 2.5 Amps are required for armature. Neglecting copper-loss in armature at no load (since it works out to be just one watt), the armature-power goes towards armature-core-loss and no-load mechanical loss at the rated speed. This amounts to $(500 \times 2.5) = 1250$ watts.

(*ii*) As a generator, for a line current of 40 Amp, the total current for the armature is 42.5 amp. Output of generator $= 500 \times 40 \times 10^{-3} = 20$ kW

Total losses as a generator = 1250 + field copper-loss + arm. copper-loss

$$= (1250 + 1250 + 42.5^2 \times 0.15) \text{ watts} = 2.771 \text{ kW}$$

$$\text{Efficiency} = \frac{20}{20 + 2.771} \times 100 = 87.83\%$$

Example 26.25. *A shunt generator has a F.L. current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55 Ω. If it has a F.L. efficiency of 88%, find the armature resistance. Also, find the load current corresponding to maximum efficiency.*

(Electrical Technology Punjab Univ. Nov. 1988)

Solution. Output $= 220 \times 196 = 43{,}120$ W ; $\eta = 88\%$ (overall efficiency)

Electrical input $= 43{,}120/0.88 = 49{,}000$ W

Total losses $= 49{,}000 - 43{,}120 = 5{,}880$ W

Shunt field current $= 220/55 = 4$ A $\qquad \therefore I_a = 196 + 4 = 200$ A

$\therefore$ Shunt Cu loss $= 220 \times 4 = 880$ W ; Stray losses = 720 W

Constant losses $= 880 + 720 = 1{,}600$

$\therefore$ Armature Cu loss $= 5{,}880 - 1{,}600 = 4{,}280$ W

$\therefore$ $I_a^2 R_a = 4{,}280$ W

$200^2 R_a = 4{,}280$ or $R_a = 4{,}280/200 \times 200 =$ **0.107 Ω**

For maximum efficiency,

$$I^2 R_a = \text{constant losses} = 1{,}600 \text{ W} ; I = \sqrt{1{,}600/0.107} = \mathbf{122.34 \text{ A}}$$

Example 26.26. *A long-shunt dynamo running at 1000 r.p.m. supplies 22 kW at a terminal voltage of 220 V. The resistances of armature, shunt field and the series field are 0.05, 110 and 0.06 Ω respectively. The overall efficiency at the above load is 88%. Find (a) Cu losses (b) iron and friction losses (c) the torque exerted by the prime mover.*

(Elect. Machinery-I, Bangalore Univ. 1987)

Solution. The generator is shown in Fig. 26.64.

$I_{sh} = 220/110 = 2$ A

$I = 22{,}000/220 = 100$ A,

$I_a = 102$ A

Drop in series field winding $= 102 \times 0.06 = 6.12$ V

(*a*) $I_a^2 R_a = 102^2 \times 0.05 = 520.2$ W

Series field loss $= 102^2 \times 0.06 = 624.3$ W

Shunt field loss $= 4 \times 110 = 440$ W

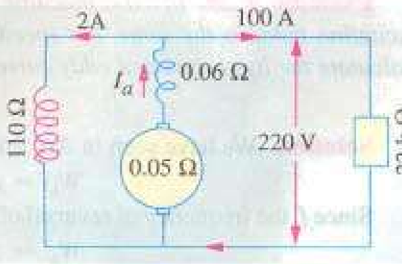


Fig. 26.64

$$\text{Total Cu losses} = 520.2 + 624.3 + 440 = \mathbf{1584.5\ W}$$

(*b*) Output = 22,000 W ; Input = 22,000/0.88 = 25,000 W

$\therefore$ Total losses = 25,000 − 22,000 = 3,000 W

$\therefore$ Iron and friction losses = 3,000 − 1,584.5 = **1,415.5 W**

$$\text{Now,}\quad T \times \frac{2\pi N}{60} = 25{,}000\ ;\qquad T = \frac{25{,}000 \times 60}{1{,}000 \times 6.284} = \mathbf{238.74\ N\text{-}m}$$

Example 26.27. *A 4-pole d.c. generator is delivering 20 A to a load of 10 Ω. If the armature resistance is 0.5 Ω and the shunt field resistance is 50 Ω, calculate the induced e.m.f. and the efficiency of the machine. Allow a drop of 1 V per brush.*

(Electrical Technology-I, Osmania Univ., 1990)

Solution. Terminal voltage = 20 × 10 = 200 V

I_{sh} = 200/50 = 4 A ; I_a = 20 + 4 = 24 A

$I_a R_a$ = 24 × 0.5 = 12 V ; Brush drop = 2 × 1 = 2 V

$\therefore$ E_g = 200 + 12 + 2 = **214 V**, as in Fig. 26.65.

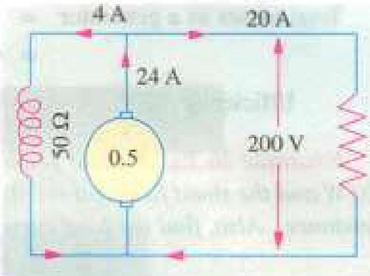


Fig. 26.65

Since iron and friction losses are not given, only electrical efficiency of the machine can be found out.

Total power generated in the armature

= 214 × 24 = 5,136 W

Useful output = 200 × 20 = 4,000 W

$\therefore$ η_e = 4,000/5,136 = 0.779 or **77.9%**

Example 26.28. *A long-shunt compound-wound generator gives 240 volts at F.L. output of 100 A. The resistances of various windings of the machine are : armature (including brush contact) 0.1 Ω, series field 0.02 Ω, interpole field 0.025 Ω, shunt field (including regulating resistance) 100 Ω. The iron loss at F.L. is 1000 W ; windage and friction losses total 500 W. Calculate F.L. efficiency of the machine.* **(Electrical Machinery-I, Indore Univ. 1989)**

Solution. Output = 240 × 100 = 24,000 W

Total armature circuit resistance = 0.1 + 0.02 + 0.025 = 0.145 Ω

I_{sh} = 240/100 = 2.4 A $\quad\therefore I_a$ = 100 + 2.4 = 102.4 A

$\therefore$ Armature circuit copper loss = $102.4^2 \times 0.145$ = 1,521 W

Shunt field copper loss = 2.4 × 240 = 576 W

Iron loss = 1000 W ; Friction loss = 500 W

$$\text{Total loss} = 1{,}521 + 1{,}500 + 576 = 3{,}597\ \text{W}\ ;\ \eta = \frac{24{,}000}{24{,}000 + 3{,}597} = 0.87 = \mathbf{87\%}$$

Example 26.29. *In a d.c. machine the total iron loss is 8 kW at its rated speed and excitation. If excitation remains the same, but speed is reduced by 25%, the total iron loss is found to be 5 kW. Calculate the hysteresis and eddy current losses at (i) full speed (ii) half the rated speed.*

(Similar Example, JNTU, Hyderabad, 2000)

Solution. We have seen in Art. 26.32 that

$$W_h \propto f \quad \text{and} \quad W_e \propto f^2$$

Since *f*, the frequency of reversal of magnetization, is directly proportional to the armature speed,

$$W_h \propto N \quad \text{and} \quad W_e \propto N^2$$

$\therefore \quad W_h = A \times N$ and $W_e = BN^2$, where A and B are constants.

Total loss $\quad W = W_h + W_e = AN + BN^2$

Let the full rated speed be 1.

Then $8 = A \times 1 + B \times 1^2$ or $8 = A + B$ ...(*i*)

Now, when speed is 75% of full rated speed, then

$5 = A \times (0.75) + B(0.75)^2$ ...(*ii*)

Multiplying (*i*) by 0.75 and subtracting (*ii*) from it, we get

$0.1875\,B = 1 \quad \therefore B = 1/0.1875 =$ **5.33 kW**

Substituting this value in (*i*) above

$8 = 5.33 + A \quad \therefore \quad A =$ **2.67 kW**

(*i*) W_h at rated speed = 2.67 kW, $\quad W_e$ at rated speed = **5.33 kW**

(*ii*) W_h at half the rated speed = $2.67 \times 0.5 =$ **1.335 kW**

W_e at half the rated speed $= 5.33 \times 0.5^2 =$ **1.3325 kW**

Example 26.30. *The hysteresis and eddy current losses in a d.c. machine running at 1000 r.p.m. are 250 W and 100 W respectively. If the flux remains constant, at what speed will be total iron losses be halved ?* **(Electrical Machines-I, Gujarat Univ. 1989)**

Solution. Total loss $\quad W = W_h + W_e = AN + BN^2$

Now, $\quad W_h = 250\text{ W} \quad \therefore \quad A \times (1000/60) = 250\,; \quad A = 15$

$W_e = 100\text{ W} \quad \therefore \quad B \times (1000/60)^2 = 100\,; \quad B = 9/25$

Let N be the new speed in r.p.s. at which total loss is one half of the loss at 1000 r.p.m. New loss = (250 + 100)/2 = 175 W

$\therefore \quad 175 = 15\,N + (9/25)N^2 \quad$ or $\quad 9N^2 + 375\,N - 4{,}375 = 0$

$$\therefore \quad N = \frac{-375 \pm \sqrt{375^2 + 36 \times 4{,}375}}{2 \times 9} = \frac{-375 \pm 546}{18} = 9.5 \text{ r.p.s} = \textbf{570 r.p.m.*}$$

Note. It may be noted that at the new speed, $W_h = 250 \times (570/100) = 142.5$ W and $W_e = 100 \times (570/1000)^2 = 32.5$ W. Total loss = 142.5 + 32.5 = 175 W.

Example 26.31. *A d.c. shunt generator has a full load output of 10 kW at a terminal voltage of 240 V. The armature and the shunt field winding resistances are 0.6 and 160 ohms respectively. The sum of the mechanical and core-losses is 500 W. Calculate the power required, in kW, at the driving shaft at full load, and the corresponding efficiency.* **(Nagpur University November 99)**

Solution. $\quad$ Field current $= \dfrac{240}{160} = 1.5$ amp, Load current $= \dfrac{10{,}000}{240} = 41.67$ amp

Armature current $= 41.67 + 1.5 = 43.17$ amp

Field copper losses = 360 W, Armature copper losses $= 43.17^2 \times 0.6 = 1118$ W

Total losses in kW $= 0.36 + 1{,}118 + 0.50 = 1{,}978$ kW

Hence, Power input at the shaft = 11.978 kW

$$\text{Efficiency} = \frac{10}{11.978} \times 100\% = 83.5\%$$

* The negative value has been rejected–being mathematically absurd.

Example 26.32. *A long shunt d.c. compound generator delivers 110 kW at 220 V.*

If r_a = 0.01 ohm, r_{se} = 0.002 ohm, and shunt field has a resistance of 110 ohms, calculate the value of the induced e.m.f. **(Bharathithasan University Nov. 1997)**

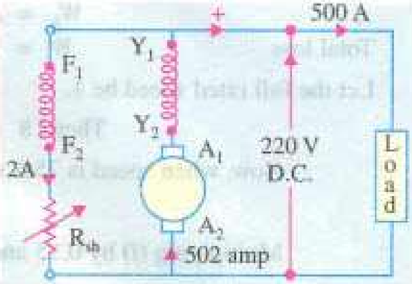


Fig. 26.66. Long-shunt d.c. compound generator

Solution.

$$\text{Load current} = 110 \times 1000/220 = 500 \text{ A}$$

$$\text{Shunt field current} = 220/110 = 2 \text{ A}$$

$$\text{Armature current} = 502 \text{ A}$$

$$r_a + r_{se} = 0.012 \text{ ohm}$$

$$E_a = 220 + [502 \times (0.012)] = 226.024 \text{ V}$$

Example 26.33. *The armature of a four-pole d.c. shunt generator is lap-wound and generates 216 V when running at 600 r.p.m. Armature has 144 slots, with 6 conductors per slot. If this armature is rewound, wave-connected, find the e.m.f. generated with the same flux per pole but running at 500 r.p.m.* **(Bharathithasan University April 1997)**

Solution. Total number of armature conductors = $Z = 144 \times 6 = 864$

For a Lap winding, number of parallel paths in armature = number of poles

In the e.m.f. equation, $E = (\phi\, ZN/60)\,(P/a)$

Since $P = a$

$$E = \phi\, ZN/60$$

$$216 = \phi \times 864 \times 600/60 = 8640\phi$$

Hence ϕ = 25 milli-webers

If the armature is rewound with wave-connection, number of parallel paths = 2.

Hence, at 500 r.p.m., with 25 mWb as the flux per pole,

$$\text{the armature emf} = (25 \times 10^{-3} \times 864 \times 500/60) \times 4/2$$

$$= 25 \times 864 \times 0.50 \times 2/60$$

$$= 360 \text{ volts}$$

Additional note :

Extension to Que : *Comment on the armature output power in the two cases.*

Solution. Assumption is that field side is suitably modified in the two cases.

Case (*i*) : Lap-wound Machine at 600 r.p.m.

Armature e.m.f.= 216 V

Let each armature-conductor be rated to carry a current of 10 amp.

In simple lap-wound machines, since a four-pole machine has four parallel paths in armature, the total armature output-current is 40 amp.

Hence, armature-output-power = $216 \times 40 \times 10^{-3}$ = 8.64 kW

Case (*ii*) : Wave-wound machine, at 500 r.p.m.

Armature e.m.f. = 360 V

Due to wave-winding, number of parallel paths in armature = 2

Hence, the total armature output current = 20 amp

Thus, Armature Electrical output-power = $360 \times 20 \times 10^{-3} = 7.2$ kW

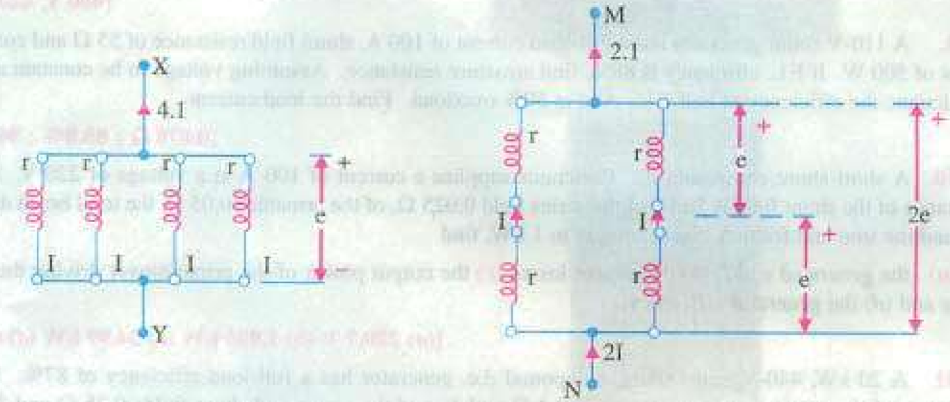
Observation. With same flux per pole, the armature power outputs will be in the proportion of the speeds, as (7.2/8.64) = 5/6).

Further Conclusion. In case of common speed for comparing Electrical Outputs with same machine once lap-wound and next wave-wound, there is no difference in the two cases. Lap-wound machine has lower voltage and higher current while the wave-wound machine has higher voltage and lower current.

Example 26.34. *A 4-pole, Lap-connected d.c. machine has an armature resistance of 0.15 ohm. Find the armature resistance of the machine is rewound for wave-connection.*

(Bharthiar Univ. Nov. 1997)

Solution. A 4-pole lap-winding has 4 parallel paths in armature. If it is rewound for wave-connection, the resistance across the terminal becomes $(4 \times 0.15) = 0.6$ ohm, as it obvious from Fig. 26.67.



4 parallel paths (Lap-winding)
Power rating = $eI \times 4$
Resistance Between X and $Y = r/4$
(*a*) Lap-winding

2 parallel paths (wave-winding)
Power rating = $(2e) \times (2I) = 4eI$
Resistance Between M and $N = r$
(*b*) Wave-winding

Fig. 26.67. Resistances for different methods

Tutorial Problem No. 26.2

1. A 4-pole, d.c. generator has a wave-wound armature with 792 conductors. The flux per pole is 0.0121 Wb. Determine the speed at which it should be run to generate 240 V on no-load. **[751.3 r.p.m.]**

2. A 20 kW compound generator works on full-load with a terminal voltage of 230 V. The armature, series and shunt field resistances are 0.1, 0.05 and 115 Ω respectively. Calculate the generated e.m.f. when the generator is connected short-shunt. **[243.25 V]** *(Elect. Engg. Madras Univ. April, 1978)*

3. A d.c. generator generates an e.m.f. of 520 V. It has 2,000 armature conductors, flux per pole of 0.013 Wb, speed of 1200 r.p.m. and the armature winding has four parallel paths. Find the number of poles.

[4] *(Elect. Technology, Aligarh Univ. 1978)*

4. When driven at 1000 r.p.m. with a flux per pole of 0.02 Wb, a d.c. generator has an e.m.f. of 200 V. If the speed is increased to 1100 r.p.m. and at the same time the flux per pole is reduced to 0.019 Wb per pole, what is then the induced e.m.f. ? **[209 V]**

5. Calculate the flux per pole required on full-load for a 50 kW, 400 V, 8-pole, 600 r.p.m. d.c. shunt generator with 256 conductors arranged in a lap-connected winding. The armature winding resistances is 0.1 Ω, the shunt field resistance is 200 Ω and there is a brush contact voltage drop of 1 V at each brush on full-load. **[0.162 Wb]**

6. Calculate the flux in a 4-pole dynamo with 722 armature conductors generating 500 V when running at 1000 r.p.m. when the armature is (*a*) lap connected (*b*) wave connected.

[(*a*) 41.56 mWb (*b*) 20.78 mWb] ***(City & Guilds, London)***

7. A 4-pole machine running at 1500 r.p.m. has an armature with 90 slots and 6 conductors per slot. The flux per pole is 10 mWb. Determine the terminal e.m.f. as d.c. Generator if the coils are lap-connected. If the current per conductor is 100 A, determine the electrical power.

[810 V, 324 kW] ***(London Univ.)***

8. An 8-pole lap-wound d.c. generator has 120 slots having 4 conductors per slot. If each conductor can carry 250 A and if flux/pole is 0.05 Wb, calculate the speed of the generator for giving 240 V on open circuit. If the voltage drops to 220 V on full load, find the rated output of the machine.

[600 V, 440 kW]

9. A 110-V shunt generator has a full-load current of 100 A, shunt field resistance of 55 Ω and constant losses of 500 W. If F.L. efficiency is 88%, find armature resistance. Assuming voltage to be constant at 110 V, calculate the efficiency at half F.L. And at 50% overload. Find the load current.

[0.078 Ω ; 85.8% ; 96.2 A]

10. A short-shunt compound d.c. Generator supplies a current of 100 A at a voltage of 220 V. If the resistance of the shunt field is 50 Ω, of the series field 0.025 Ω, of the armature 0.05 Ω, the total brush drop is 2 V and the iron and friction losses amount to 1 kW, find

(*a*) the generated e.m.f. (*b*) the copper losses (*c*) the output power of the prime-mover driving the generator and (*d*) the generator efficiency.

[(*a*) 229.7 V (*b*) 1.995 kW (*c*) 24.99 kW (*d*) 88%]

11. A 20 kW, 440-V, short-shunt, compound d.c. generator has a full-load efficiency of 87%. If the resistance of the armature and interpoles is 0.4 Ω and that of the series and shunt fields 0.25 Ω and 240 Ω respectively, calculate the combined bearing friction, windage and core-loss of the machine.

[725 W]

12. A long-shunt, compound generator delivers a load current of 50 A at 500 V and the resistances of armature, series field and shunt field are 0.05 ohm and 250 ohm respectively. Calculate the generated electromotive force and the armature current. Allow 1.0 V per brush for contact drop.

[506.2 V ; 52 A] ***(Elect. Engg. Banaras Hindu Univ. 1977)***

13. In a 110-V compound generator, the resistances of the armature, shunt and the series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load consists of 200 lamps each rated at 55 W, 110 V.

Find the total electromotive force and armature current when the machine is connected (*i*) long shunt (*ii*) short shunt. Ignore armature reaction and brush drop.

[(*a*) 1200.4, 104.4 A (*b*) 120.3 V, 104.6 A] ***(Electrical Machines-I, Bombay Univ. 1979)***

14. Armature of a 2-pole, 200-V generator has 400 conductors and runs at 300 r.p.m. Calculate the useful flux per pole. If the number of turns in each field coil is 1200, what is the average value of e.m.f induced in each coil on breaking the field if the flux dies away completely in 0.1 sec ?

(JNTU, Hyderabad, 2000)

Hint: Calculate the flux per pole generating 200 V at 300 rpm. Calculate the e.m.f. induced in 1200-turn field coil due to this flux reducing to zero in 0.1 sec, from the rate of change of flux-linkage.

[ϕ = 0.1 Wb, e = 1200 V]

15. A 1500 kW, 550-V, 16 pole generator runs at 150 rev. per min. What must be the useful flux if there are 2500 conductors lap-connected and the full-load copper losses are 25 kW? Calculate the area of the pole shoe if the gap density has a uniform value of 0.9 wb/m^2 and find the no-load terminal voltage, neglecting armature reaction and change in speed.

(Rajiv Gandhi Techn. Univ., Bhopal, 2000) **[0.09944 m^2, 559.17 V]**

OBJECTIVE TESTS – 26

1. The basic requirement of a d.c. armature winding is that it must be
(*a*) a closed one
(*b*) a lap winding
(*c*) a wave winding
(*d*) either (*b*) or (*c*)

2. A wave winding must go at least .... around the armature before it closes back where it started.
(*a*) once
(*b*) twice
(*c*) thrice
(*d*) four times

3. The d.c. armature winding in which coil sides are a pole pitch apart is called .... winding.
(*a*) multiplex
(*b*) fractional-pitch
(*c*) full-pitch
(*d*) pole-pitch

4. For making coil span equal to a pole pitch in the armature winding of a d.c. generator, the back pitch of the winding must equal the number of
(*a*) commutator bars per pole
(*b*) winding elements
(*c*) armature conductors per path
(*d*) armature parallel paths.

5. The primary reason for making the coil span of a d.c. armature winding equal to a pole pitch is to
(*a*) obtain a coil span of 180° (electrical)
(*b*) ensure the addition of e.m.fs. of consecutive turns
(*c*) distribute the winding uniformly under different poles
(*d*) obtain a full-pitch winding.

6. In a 4-pole, 35 slot d.c. armature, 180 electrical-degree coil span will be obtained when coils occupy .... slots.
(*a*) 1 and 10
(*b*) 1 and 9
(*c*) 2 and 11
(*d*) 3 and 12

7. The armature of a d.c. generator has a 2-layer lap-winding housed in 72 slots with six conductors/slot. What is the minimum number of commutator bars required for the armature?
(*a*) 72
(*b*) 432
(*c*) 216
(*d*) 36

8. The sole purpose of a commutator in a d.c. Generator is to
(*a*) increase output voltage
(*b*) reduce sparking at brushes
(*c*) provide smoother output
(*d*) convert the induced a.c. into d.c.

9. For a 4-pole, 2-layer, d.c., lap-winding with 20 slots and one conductor per layer, the number of commutator bars is
(*a*) 80
(*b*) 20
(*c*) 40
(*d*) 160

10. A 4-pole, 12-slot lap-wound d.c. armature has two coil-sides/slot. Assuming single turn coils and progressive winding, the back pitch would be
(*a*) 5
(*b*) 7
(*c*) 3
(*d*) 6

11. If in the case of a certain d.c. armature, the number of commutator segments is found either one less or more than the number of slots, the armature must be having a simplex .... winding.
(*a*) wave
(*b*) lap
(*c*) frog leg
(*d*) multielement

12. Lap winding is suitable for .... current, ... voltage d.c. generators.
(*a*) high, low
(*b*) low, high
(*c*) low, low
(*d*) high, high

13. The series field of a short-shunt d.c. generator is excited by .... currents.
(*a*) shunt
(*b*) armature
(*c*) load
(*d*) external

14. In a d.c. generator, the generated e.m.f. is directly proportional to the
(*a*) field current
(*b*) pole flux
(*c*) number of armature parallel paths
(*d*) number of dummy coils

15. In a 12-pole triplex lap-wound d.c. armature, each conductor can carry a current of 100 A. The rated current of this armature is .... ampere.
(*a*) 600
(*b*) 1200
(*c*) 2400
(*d*) 3600

16. The commercial efficiency of a shunt generator is maximum when its variable loss equals .... loss.
(*a*) constant
(*b*) stray
(*c*) iron
(*d*) friction and windage

17. In small d.c. machines, armature slots are sometimes not made axial but are skewed. Though skewing makes winding a little more difficult, yet it results in
(*a*) quieter operation
(*b*) slight decrease in losses
(*c*) saving of copper
(*d*) both (*a*) and (*b*)

18. The critical resistance of the d.c. generator is the resistance of
(*a*) armature
(*b*) field
(*c*) load
(*d*) brushes **(Grad. I.E.T.E Dec. 1985)**

ANSWERS

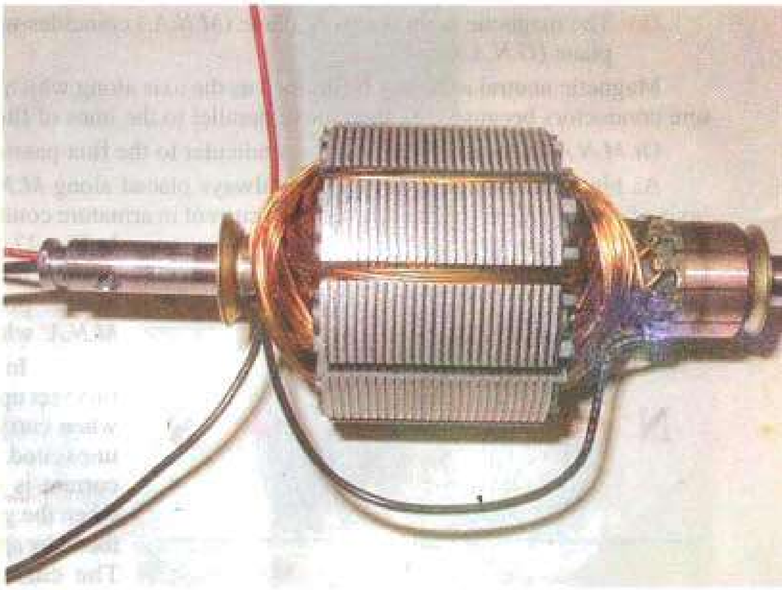
1. (*a*)	2. (*b*)	3. (*c*)	4. (*a*)	5. (*b*)	6. (*b*)	7. (*c*)	8. (*d*)	9. (*b*)	10. (*b*)
11. (*a*)	12. (*a*)	13. (*c*)	14. (*b*)	15. (*d*)	16. (*a*)	17. (*d*)	18. (*b*)		

CHAPTER 27

ARMATURE REACTION AND COMMUTATION

Learning Objectives

- Armature Reaction
- Demagnetising and Cross-magnetising Conductors
- Demagnetising AT per Pole
- Cross-magnetising AT per pole
- Compensating Windings
- No. of Compensating Windings
- Commutation
- Value of Reactance Voltage
- Methods of Improving Commutation
- Resistance Commutation
- E.M.F. Commutation
- Interpoles or Compoles
- Equalising Connections
- Parallel Operation of Shunt Generators
- Paralleling D.C. Generator
- Load Sharing
- Procedure for Paralleling D.C. Generators
- Compound Generators in Parallel
- Series Generators in Parallel



Armature reaction is the change in the neutral plane and the reaction of the magnetic field

27.1 Armature Reaction

By armature reaction is meant the effect of magnetic field set up by armature current on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :

(*i*) ***It demagnetises or weakens the main flux*** and

(*ii*) ***It cross-magnetises or distorts it.***

The first effect leads to reduced generated voltage and the second to the sparking at the brushes.

These effects are well illustrated in Fig. 27.1 which shows the flux distribution of a bipolar generator when there is no current in the armature conductors. For convenience, only two poles have been considered, though the following remarks apply to multipolar fields as well. Moreover, the brushes are shown touching the armature conductors directly, although in practice, they touch commutator segments. It is seen that

(*a*) the flux is distributed symmetrically with respect to the polar axis, which is the line joining the centres of *NS* poles.

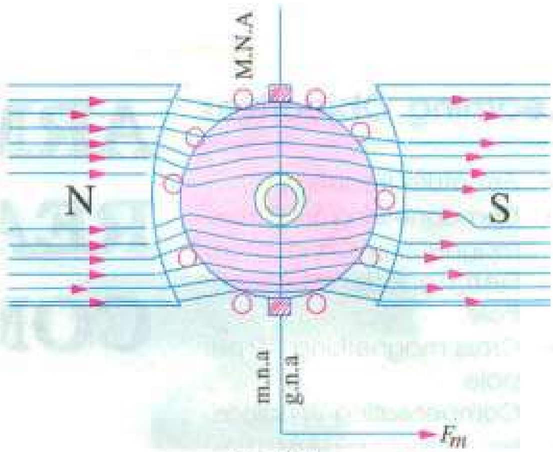


Fig. 27.1

(*b*) The magnetic neutral axis or plane (*M.N.A.*) coincides with the geometrical neutral axis or plane (*G.N.A.*)

Magnetic neutral axis may be defined as the axis along which no e.m.f. is produced in the armature conductors because they then move parallel to the lines of flux.

Or *M.N.A.* is the axis which is perpendicular to the flux passing through the armature.

As hinted in Art. 27.2, brushes are always placed along *M.N.A.* Hence, *M.N.A.* is also called 'axis of commutation' because reversal of current in armature conductors takes place across this axis.

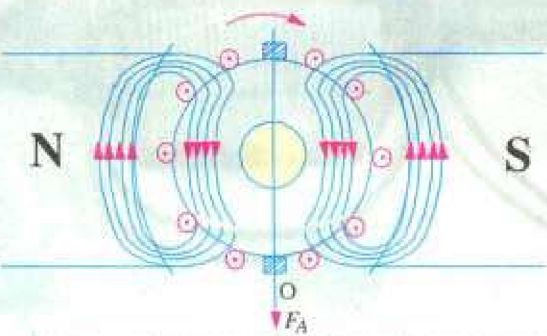


Fig. 27.2

In Fig. 27.1 is shown vector OF_m which represents, both in magnitude and direction, the m.m.f. producing the main flux and also *M.N.A.* which is perpendicular to OF_m.

In Fig. 27.2 is shown the field (or flux) set up by the armature conductors *alone* when carrying current, the field coils being unexcited. The direction of the armature current is the same as it would actually be when the generator is loaded. It may even be found by applying Fleming's Right-hand Rule. The current direction is downwards in conductors under *N*-pole and upwards in those under *S*-pole. The downward flow is represented by crosses and upward flow by dots.

As shown in Fig. 27.2, the m.m.fs. of the armature conductors combine to send flux downwards through the armature. The direction of the lines of force can be found by applying cork-screw rule. The armature m.m.f. (depending on the strength of the armature current) is shown separately both in magnitude and direction by the vector OF_A which is parallel to the brush axis.

So far, we considered the main m.m.f. and armature m.m.f. separately as if they existed independently, which is not the case in practice. Under actual load conditions, the two exist simultaneously in the generator as shown in Fig. 27.3.

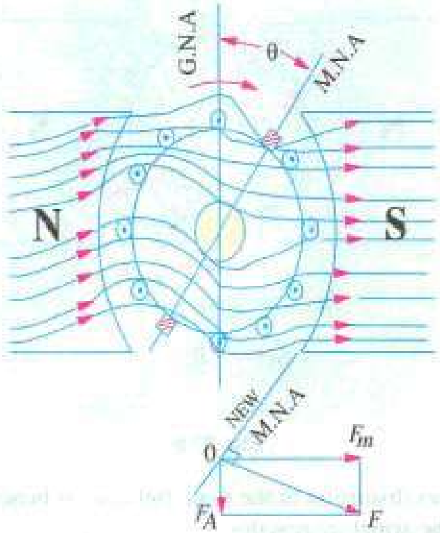


Fig. 27.3

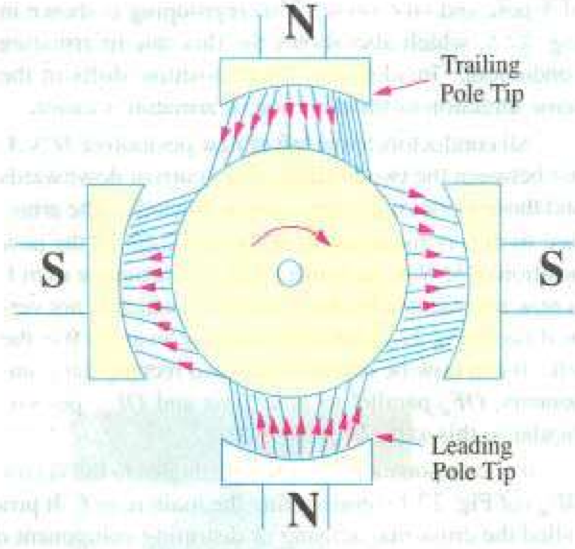
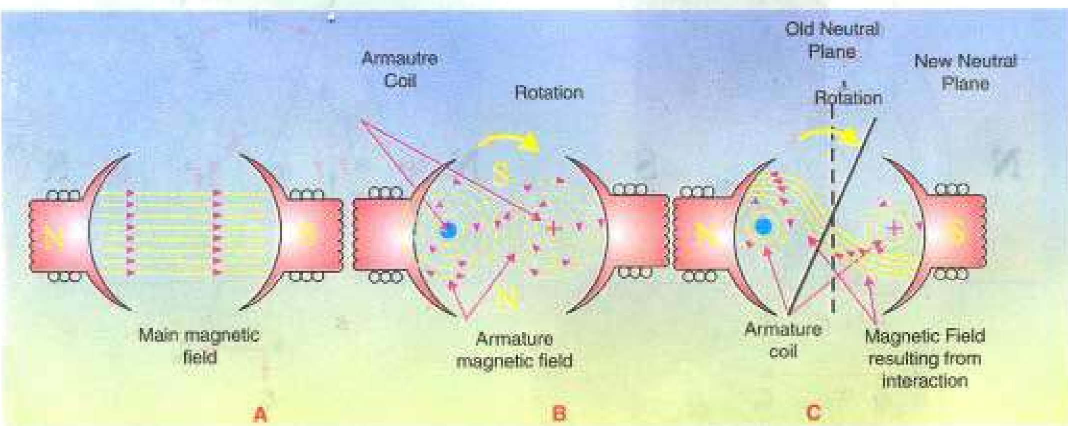


Fig. 27.4

It is seen that the flux through the armature is no longer uniform and symmetrical about the pole axis, rather it has been distorted. The flux is seen to be crowded at the trailing pole tips but weakened or thinned out at the leading pole tips (the pole tip which is first met during rotation by armature conductors is known as the leading pole tip and the other as trailing pole tip). The strengthening and weakening of flux is separately shown for a four-pole machine in Fig. 27.4. As seen, air-gap flux density under one pole half is greater than that under the other pole half.

If Fig. 27.3 is shown the resultant m.m.f. OF which is found by vectorially combining OF_m and OF_A.

The new position of *M.N.A.*, which is always perpendicular to the resultant m.m.f. vector OF, is also shown in the figure. With the shift of *M.N.A.*, say through an angle θ, brushes are also shifted so as to lie along the new position of *M.N.A.* Due to this brush shift (or forward lead), the armature conductors and hence armature current is redistributed. Some armature conductors which were earlier



Armature reaction

under the influence of N-pole come under the influence of S-pole and *vice-versa*. This regrouping is shown in Fig. 27.5, which also shows the flux due to armature conductors. Incidentally, brush position shifts in the *same* direction as the direction of armature rotation.

All conductors to the left of new position of *M.N.A.* but between the two brushes, carry current downwards and those to the right carry current upwards. The armature m.m.f. is found to lie in the direction of the new position of *M.N.A.* (or brush axis). The armature m.m.f. is now represented by the vector OF_A which is not vertical (as in Fig 27.2) but is inclined by an angle θ to the left. It can now be resolved into two rectangular components, OF_d parallel to polar axis and OF_C perpendicular to this axis. We find that

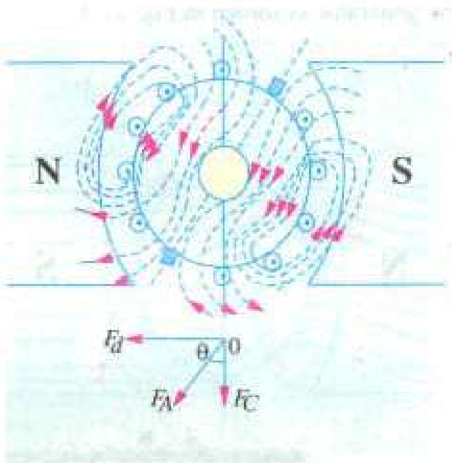


Fig. 27.5

(*i*) component OF_C is at right angles to the vector OF_m (of Fig. 27.1) representing the main m.m.f. It produces distortion in the main field and is hence called the cross-magnetising or distorting component of the armature reaction.

(*ii*) The component OF_d is in direct opposition of OF_m which represents the main m.m.f. It exerts a demagnetising influence on the main pole flux. Hence, it is called the demagnetising or weakening component of the armature reaction.

It should be noted that both distorting and demagnetising effects will increase with increase in the armature current.

27.2. Demagnetising and Cross-magnetising Conductors

The exact conductors which produce these distorting and demagnetising effects are shown in Fig. 27.6 where the brush axis has been given a forward lead of θ so as to lie along the new position of *M.N.A.* All conductors lying within angles $AOC = BOD = 2\theta$ at the top and bottom of the armature, are carrying current in such a direction as to send the flux *through* the armature from right to left. This fact may be checked by applying crockscrew rule. It is these conductors which act in direct opposition to the main field and are hence called the demagnetising armature conductors.*

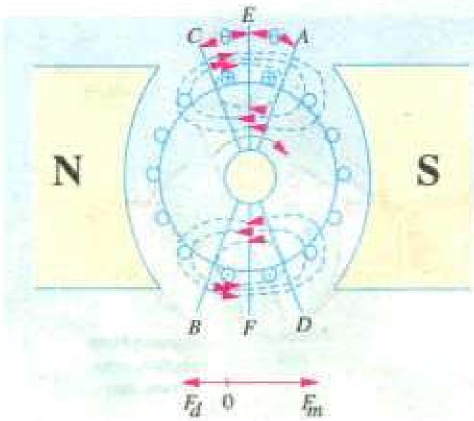


Fig. 27.6

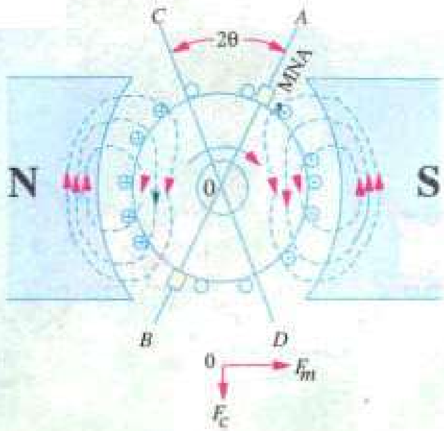


Fig. 27.7

Now consider the remaining armature conductors lying between angles AOD and COB. As shown in Fig. 27.7, these conductors carry current in such a direction as to produce a combined flux pointing vertically downwards *i.e.* at right angles to the main flux. This results in distortion of the main field. Hence, these conductors are known as cross-magnetising conductors and constitute distorting ampere-conductors.

27.3. Demagnetising AT per Pole

Since armature demagnetising ampere-turns are neutralized by adding extra ampere-turns to the main field winding, it is essential to calculate their number. But before proceeding further, it should be remembered that the number of turns is equal to half the number of conductors because two conductors-constitute one turn.

Let Z = total number of armature conductors

I = current in each armature conductor

$= I_a/2$ ... for simplex wave winding

$= I_a/P$ ... for simplex lap winding

θ_m = forward lead in mechanical or geometrical or angular degrees.

Total number of armature conductors in angles AOC and BOD is $\frac{4\theta_m}{360} \times Z$

As two conductors constitute one turn,

$$\therefore \quad \text{Total number of turns in these angles} = \frac{2\theta_m}{360} \times Z\,I$$

$$\therefore \quad \text{Demagnetising amp-turns per pair of poles} = \frac{2\theta_m}{360} \times Z\,I$$

$$\therefore \quad \text{Demagnetising amp - turns/pole} = \frac{\theta_m}{360} \times ZI \qquad \therefore AT_d \text{ per pole} = ZI \times \frac{\theta_m}{360}$$

27.4. Cross-magnetising AT per pole

The conductors lying between angles AOD and BOC constitute what are known as distorting or cross-magnetising conductors. Their number is found as under :

Total armature-*conductors*/pole both cross and demagnetising = Z / P

$$\text{Demagnetising } \textit{conductors}\text{/pole} = Z.\frac{2\theta_m}{360} \qquad \text{(found above)}$$

$$\therefore \quad \text{Corss-magnetising } \textit{conductors}\text{/pole} = \frac{Z}{P} - Z \times \frac{2\theta_m}{360} = Z\left(\frac{1}{P} - \frac{2\theta_m}{360}\right)$$

$$\therefore \quad \text{Cross-magnetising } \textit{amp-conductors}\text{/pole} = ZI\left(\frac{1}{P} - \frac{2\theta_m}{360}\right)$$

$$\therefore \quad \text{Corss-magnetising } \textit{amp-turns}\text{/pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

(Remembering that two conductors make one turn)

$$\therefore \quad AT_c\text{/pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

Note. (*i*) For neutralizing the demagnetising effect of armature-reaction, an extra number of turns may be put on each pole.

$$\text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \qquad \text{–for shunt generator}$$

$$= \frac{AT_d}{I_a} \quad \text{–for series generator}$$

If the leakage coefficient λ is given, then multiply each of the above expressions by it.

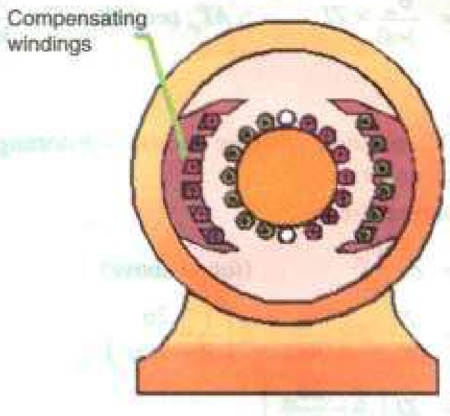
(*ii*) If lead angle is given in electrical degrees, it should be converted into mechanical degrees by the following relation.

$$\theta\,(\text{mechanical}) = \frac{\theta\,(\text{electrical})}{\text{pair of poles}} \text{ or } \theta_m = \frac{\theta_e}{P/2} = \frac{2\theta_e}{P}$$

27.5. Compensating Windings

These are used for large direct current machines which are subjected to large fluctuations in load *i.e.* rolling mill motors and turbo-generators etc. Their function is to neutralize the cross magnetizing effect of armature reaction. In the absence of compensating windings, the flux will be suddenly shifting backward and forward with every change in load. This shifting of flux will induce ***statically*** induced e.m.f. in the armature coils. The magnitude of this e.m.f. will depend upon the rapidity of changes in load and the amount of change. It may be so high as to strike an arc between the consecutive commutator segments across the top of the mica sheets separating them. This may further develop into a flash-over around the whole commutator thereby short-circuiting the whole armature.

Fig. 27.8



Compensating windings

These windings are embedded in slots in the pole shoes and are connected in series with armature in such a way that the current in them flows in opposite direction to that flowing in armature conductors directly below the pole shoes. An elementary scheme of compensating winding is shown in Fig. 27.8.

It should be carefully noted that compensating winding must provide sufficient m.m.f so as to counterbalance the armature m.m.f. Let

Z_c = No. of compensating conductos/pole face

Z_a = No. of active armature conductors/pole,

I_a = Total armature current

I_a/A = current/armature conductor

$\therefore\ Z_c I_a = Z_a\,(I_a/A)$ or $Z_c = Z_a/A$

Owing to their cost and the room taken up by them, the compensating windings are used in the case of large machines which are subject to violent fluctuations in load and also for generators which have to deliver their full-load output at considerable low induced voltage as in the Ward-Leonard set.

27.6. No. of Compensating Windings

No. of armature *conductors*/pole = $\frac{Z}{P}$ No. of armature *turns*/pole = $\frac{Z}{2P}$

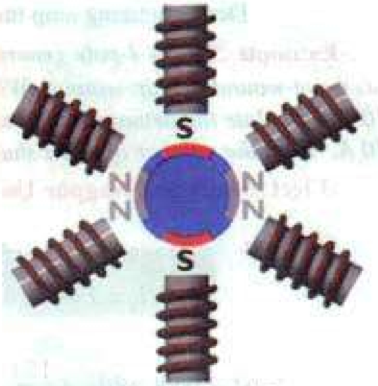
∴ No. of armature-turns immediately under one pole

$$= \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7 \times \frac{Z}{2P} \text{ (approx.)}$$

∴ No. of armature amp-*turns*/pole for compensating winding

$$= 0.7 \times \frac{Z}{2P} = 0.7 \times \text{armature amp-turns/pole}$$

Example 27.1. *A 4-pole generator has a wave-wound armature with 722 conductors, and it delivers 100 A on full load. If the brush lead is 8°, calculate the armature demagnetising and cross-magnetising ampere turns per pole.* **(Advanced Elect. Machines AMIE Sec. B 1991)**



4-pole generator

Solution. $I = I_a/2 = 100/2 = 50\text{A}$; $Z = 722$; $\theta_m = 8°$

$$\text{AT}_d/\text{ pole} = ZI.\frac{\theta_m}{360} = 722 \times 50 \times \frac{8}{360} = 802$$

$$\text{AT}_c/\text{ pole} = ZI.\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$= 722 \times 50\left(\frac{1}{2 \times 4} - \frac{8}{360}\right) = 37/8$$

Example 27.2 *An 8-pole generator has an output of 200 A at 500 V, the lap-connected armature has 1280 conductors, 160 commutator segments. If the brushes are advanced 4-segments from the no-load neutral axis, estimate the armature demagnetizing and cross-magnetizing ampere-turns per pole.* **(Electrical Machines-I, South Gujarat Univ. 1986)**

Solution. $I = 200/8 = 25$ A, $Z = 1280$, $\theta_m = 4 \times 360/160 = 9°$; $P = 8$

$$\text{AT}_d/\text{ pole} = ZI\theta_m/360 = 1280 \times 25 \times 9/360 = \mathbf{800}$$

$$\text{AT}_c/\text{ pole} = ZI\left(\frac{1}{2p} - \frac{\theta_m}{360}\right) = 1280 \times 25\left(\frac{1}{2\times8} - \frac{9}{360}\right) = \mathbf{1200}$$

Note. No. of coils = 160, No. of conductors = 1280. Hence, each coil side contains 1280/160 = 8 conductors.

Example 27.3(*a*). *A 4-pole wave-wound motor armature has 880 conductors and delivers 120 A. The brushes have been displaced through 3 angular degrees from the geometrical axis. Calculate (a) demagnetising amp-turns/pole (b) cross- magnetising amp-turns/pole (c) the additional field current for neutralizing the demagnetisation of the field winding has 1100 turns/pole.*

Solution. $Z = 880$; $I = 120/2 = 60$ A ; $\theta = 3°$ angular

(*a*) $\therefore \text{AT}_d = 880 \times 60 \times \frac{3}{360} = \mathbf{440\ AT}$

(*b*) $\therefore \text{AT}_c = 880 \times 60\left(\frac{1}{8} - \frac{3}{360}\right) = 880 \times \frac{7}{60} \times 60 = \mathbf{6{,}160}$

or Total AT/pole = 440 × 60/4 = 6600

Hence, $\text{AT}_C/\text{pole} = \text{Total AT/pole} - \text{AT}_d/\text{ pole} = 6600 - 440 = 6160$

(*c*) Additional field current = 440/1100 = **0.4 A.**

Example 27.3(*b*). *A 4-pole lap-wound Generator having 480 armature conductors supplies a current of 150 Amps. If the brushes are given an actual lead of 10°, calculate the demagnetizing and cross-magnetizing amp-turns per pole.* **(Bharathiar Univ. April 1998)**

Solution. 10° mechanical (or actual) shift = 20° electrical shift for a 4-pole machine.

Armature current = 150 amp

For 4-pole lap-wound armature, number of parallel paths = 4. Hence, conductor-current = 150/4 = 37.5 amps.

$$\text{Total armature amp-turns/pole} = \frac{1}{2} \times \frac{(480 \times 37.5)}{14} = 2250$$

$$\text{Cross} - \text{magnetizing amp turns/pole} = 2250 \times \left(1 - \frac{2 \times 20^\circ}{180^\circ}\right) = 1750$$

$$\text{Demagnetizing amp turns/pole} = 2250 \times (2 \times 20^\circ/180^\circ) = 500$$

Example 27.4. *A 4-pole generator supplies a current of 143 A. It has 492 armature conductors (a) wave-wound (b) lap-wound. When delivering full load, the brushes are given an actual lead of 10°. Calculate the demagnetising amp-turns/pole. This field winding is shunt connected and takes 10 A. Find the number of extra shunt field turns necessary to neutralize this demagnetisation.*

(Elect. Machines, Nagpur Univ. 1993 & JNTU Hyderabad, 2000 & RGPU, Bhopal, 2000)

Solution. $Z = 492$; $\theta_m = 10°$; $AT_d/\text{pole} = ZI \times \frac{\theta_m}{360}$

$I_a = 143 + 10 = 153$ A ; $I = 153/2$ ... when wave-wound

$= 153/4$ ... when lap-wound

(*a*) $\therefore AT_d/\text{pole} = 492 \times \frac{153}{2} \times \frac{10}{360} =$ **1046 AT**

Extra shunt field turns = 1046/10 = **105 (approx.)**

(*b*) $AT_d/\text{pole} = 492 \times \frac{153}{2} \times \frac{10}{360} =$ **523**

Extra shunt field turns = 523/10 = **52 (approx.)**

Example 27.5. *A 4-pole, 50-kW, 250-V wave-wound shunt generator has 400 armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetisation amp-turns/pole if shunt field resistance is 50 Ω. Also, calculate extra shunt field turns/pole to neutralize the demagnetisation.*

Solution. Load current supplied = 50,000/250 = 200 A

$I_{sh} = 250/50 = 5$ A $\therefore I_a = 200 + 5 = 205$ A

Current in each conductor $I = 205/2$ A

No. of commutator segments $= N/A$ where $A = 2$ ... for wave-winding

$\therefore$ No. of segments $= \frac{400}{2} = 200$; $\theta = \frac{4}{200} \times 360 = \frac{36}{5}$ degrees

$\therefore AT_d/\text{pole} = 400 \times \frac{205}{2} \times \frac{36}{5 \times 360} =$ **820 AT**

Extra shunt turns/poles $= \frac{AT_d}{I_{sh}} = \frac{820}{5} =$ **164**

Example 27.6. *Determine per pole the number (i) of cross-magnetising ampere-turns (ii) of back ampere-turns and (iii) of series turns to balance the back ampere-turns in the case of a d.c. generator having the following data.*

500 conductors, total current 200 A, 6 poles, 2-circuit wave winding, angle of lead = 10°, leakage coefficient = 1.3 **(Electrical Machines-I, Bombay University, 1986)**

Solution. Current/path, $I = 200/2 = 100$ A, $\theta = 10°$ (mech), $Z = 500$

(*a*) $AT_c/\text{pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right) = 500 \times 100\left(\frac{1}{2 \times 6} - \frac{10}{360}\right) =$ **2,778**

(*b*) $AT_d/\text{pole} = 500 \times 100 \times 10/360 =$ **1390**

(c) Series turns required to balance the demagnetising ampere-turns are

$$= \lambda \times \frac{AT_d}{I_a} = 1.3 \times \frac{1390}{200} = 9$$

Example 27.7. *A 22.38 kW, 440-V, 4-pole wave-wound d.c. shunt motor has 840 armature conductors and 140 commutator segments. Its full-load efficiency is 88% and the shunt field current is 1.8 A. If brushes are shifted backward through 1.5 segments from the geometrical neutral axis, find the demagnetising and distorting amp-turns/pole.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. The shunt motor is shown diagrammatically in Fig. 27.9.

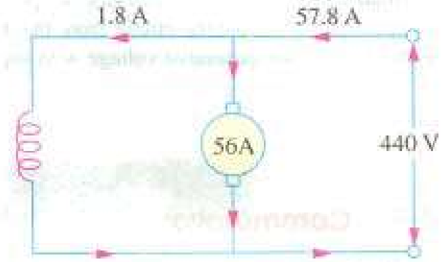


Fig. 27.9

Motor output $= 22{,}380$ W ; $\eta = 0.88$

$\therefore$ Motor input $= 22{,}380/0.88$ W

Motor input current $= \dfrac{22{,}380}{0.88 \times 440} = 57.8$ A

$I_{sh} = 1.8$ A ; $I_a = 57.8 - 1.8 = 56$ A

Current in each conductor $= 56/2 = 28$ A

$\theta = 1.5 \times 360/140$

$= 27/7$ degrees

$$\therefore \quad AT_d/\text{pole} = 840 \times 28 \times \frac{27}{7 \times 360} = 252$$

$$AT_c/\text{pole} = Z\,I\left(\frac{1}{2P} - \frac{\theta_m}{360}\right) = 840 \times 28\left(\frac{1}{8} - \frac{27}{7 \times 360}\right) = 2{,}688$$

Example 27.8. *A 400-V, 1000-A, lap-wound d.c. machine has 10 poles and 860 armature conductors. Calculate the number of conductors in the pole face to give full compensation if the pole face covers 70 % of pole span.*

Solution. AT/pole for compensating winding

$$= \text{armature amp-turn/pole} \times \frac{\text{pole arc}}{\text{pole pitch}} = 0.7 \times \frac{ZI}{2P}$$

Here I = current in each armature conductor $= 1{,}000/10 = 100$ A

$Z = 860$; $P = 10$

$\therefore$ AT/pole for compensating winding $= 0.7 \times 860 \times 100/2 \times 10 = 3{,}010$

Tutorial Problem No. 27.1

1. Calculate the demagnetising amp-turns of a 4-pole, lap-wound generator with 720 turns, giving 50 A, if the brush lead is 10° (mechanical). **(250 AT/pole)**

2. A 250-V, 25-kW, 4-pole d.c. generator has 328 wave-connected armature conductors. When the machine is delivering full load, the brushes are given a lead of 7.2 electrical degrees. Calculate the cross-magnetising amp-turns/pole. **(1886, 164)**

3. An 8-pole lap-connected d.c. shunt generator delivers an output of 240 A at 500 V. The armature has 1408 conductors and 160 commutator segments. If the brushes are given a lead of 4 segments from the no-load neutral axis, estimate the demagnetising and cross-magnetising AT/pole.
(1056, 1584) *(Electrical Engineering, Bombay Univ. 1978)*

4. A 500-V, wave-wound, 750 r.p.m. shunt generator supplies a load of 195 A. The armature has 720 conductors and shunt field resistance is 100 Ω. Find the demagnetising amp-turns/pole if brushes are advanced through 3 commutator segments at this load. Also, calculate the extra shunt field turns required to neutralize this demagnetisation. **(600, 4800, 120)**

5. A 4-pole, wave-wound generator has 320 armature conductors and carries an armature current of 400 A. If the pole arc/pole pitch ratio is 0.68, calculate the AT/pole for a compensating winding to give uniform flux density in the air gap. **(5440)**

6. A 500-kW, 500-V, 10 pole d.c. generator has a lap-wound armature with 800 conductors. Calculate the number of pole-face conductors in each pole of a compensating winding if the pole face covers 75 percent of the pitch.

(6 conductors/pole)

7. Three shunt generators, each having an armature resistance of 0.1 ohm are connected across a common bus feeding a two ohms load. Their generated voltages are 127 V, 120 V, and 119 V. Neglecting field currents, calculate the bus voltage and modes of operations of the three machines. **(JNTU, Hyderabad, 200)**

Hint : Solve the circuit from the data given. Since the voltages differ considerably, first machine with 127 V as the generated voltage with supply the largest current.

(I_1 = 70 amp, Generating mode, I_2 = 0,
Floating (= neither generating nor motoring).
I_3 = −10 amp, motoring mode I_L = 60 amp.)

27.7. Commutation

It was shown in Art 26.2 that currents induced in armature conductors of a d.c. generator are alternating. To make their flow unidirectional in the external circuit, we need a commutator. Moreover, these currents flow in one direction when armature conductors are under *N*-pole and in the opposite direction when they are under *S*-pole. As conductors pass out of the influence of a *N*-pole and enter that of *S*-pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis or brush axis *i.e.* when the brush spans and hence short-circuits that particular coil undergoing reversal of current through it. This process by which current in the short-circuited coil is reversed while it crosses the *M.N.A.* is called commutation. The brief period during which coil remains short-circuited is known as commutation period T_c.


Commutation

If the current reversal *i.e.* the change from + *I* to zero and then to − *I* is completed by the end of short circuit or commutation period, then the commutation is ideal. If current reversal is not complete by that time, then sparking is produced between the brush and the commutator which results in progressive damage to both.

Let us discuss the process of commutation or current reversal in more detail with the help of Fig. 27.10 where ring winding has been used for simplicity. The brush width is equal to the width of one commutator segment and one mica insulation. In Fig. 27.10 (*a*) coil *B* is about to be short circuited because brush is about to come in touch with commutator segment '*a*'. It is assumed that each coil carries 20 A, so that brush current is 40 A. It is so because every coil meeting at the brush supplies half the brush current lap wound or wave wound. Prior to the beginning of short circuit, coil *B* belongs to the group of coils lying to the left of the brush and carries 20 A from left to right. In Fig. 27.10 (*b*) coil *B* has entered its period of short-circuit and is approximately at one-third of this period. The current through coil *B* has reduced down from 20 A to 10 A because the other 10 A flows *via* segment '*a*'. As area of contact of the brush is more with segment '*b*' than with segment '*a*', it receives 30 A from the former, the total again being 40 A.

Fig. 27.10 (c) shows the coil B in the middle of its short-circuit period. The current through it has decreased to zero. The two currents of value 20 A each, pass to the brush directly from coil A and C as shown. The brush contact areas with the two segments 'b' and 'a' are equal.

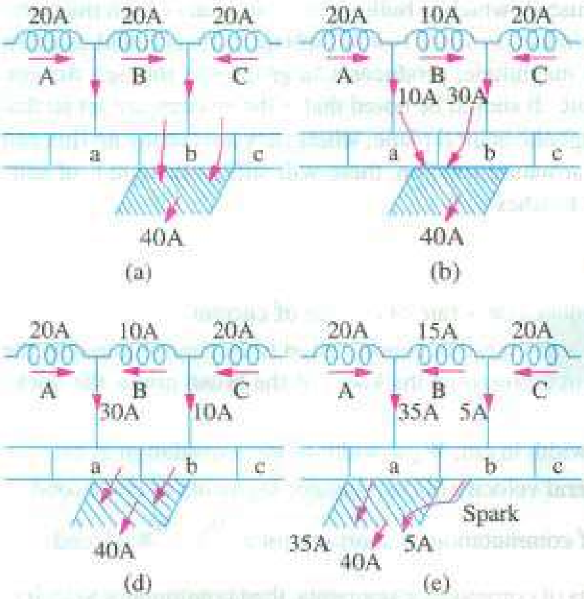


Fig. 27.10

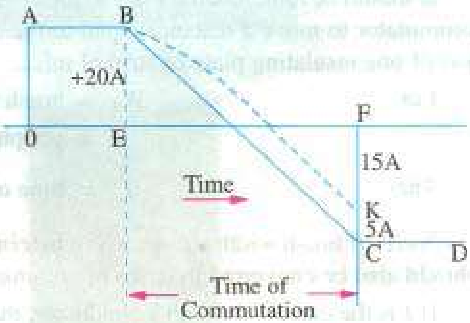


Fig. 27.11

In Fig. 27.10 (d), coil B has become part of the group of coils lying to the right of the brush. It is seen that brush contact area with segment 'b' is decreasing rapidly whereas that with segment 'a' is increasing. Coil B now carries 10 A in the reverse direction which combines with 20 A supplied by coil A to make up 30 A that passes from segment 'a' to the brush. The other 10 A is supplied by coil C and passes from segment 'b' to the brush, again giving a total of 40 A at the brush.

Fig. 27.10 (e) depicts the moment when coil B is almost at the end of commutation or short-circuit period. For ideal commutation, current through it should have reversed by now but, as shown, it is carrying 15 A only (instead of 20 A). The difference of current between coils C and B *i.e.* 20.15 = 5 A in this case, jumps directly from segment b to the brush through air thus producing spark.

If the changes of current through coil B are plotted on a time base (as in Fig. 27.11) it will be represented by a horizontal line AB *i.e.* a constant current of 20 A up to the time of beginning of commutation. From the finish of commutation, the current will be represented by another horizontal line CD. Now, again the current value is FC = 20 A, although in the reversed direction. The way in which current changes from its positive value of 20 A (= BE) to zero and then to its negative value of 20 A (= CF) depends on the conditions under which the coil B undergoes commutation. If the current varies at a uniform rate *i.e.* if BC is a straight line, then it is referred to as linear commutation. However, due to the production of self-induced e.m.f. in the coil (discussed below) the variations follow the dotted curve. It is seen that, in that case, current in coil B has reached only a value of KF = 15 A in the reversed direction, hence the difference of 5 A (20 A − 15 A) passes as a spark.

So, we conclude that sparking at the brushes, which results in poor commutation is due to the inability of the current in the short-circuited coil to reverse completely by the end of short-circuit period (which is usually of the order of 1/500 second).

At this stage, the reader might ask for the reasons which make this current reversal impossibly in the specified period *i.e.* what factors stand in the way of our achieving ideal commutation. The main cause which retards or delays this quick reversal is the production of self-induced e.m.f. in the coil undergoing commutation. It may be pointed out that the coil possesses appreciable amount of self inductance because it lies embedded in the armature which is built up of a material of high magnetic permeability. This self-induced e.m.f. is known as ***reactance voltage*** whose value is found as given below. This voltage, even though of a small magnitude, produces a large current through the coil whose resistance is very low due to short circuit. It should be noted that if the brushes are set so that the coils undergoing short-circuit are in the magnetic neutral plane, where they are cutting no flux and hence have no e.m.f. induced in them due to armature rotation, there will still be the e.m.f. of self-induction which causes severe sparking at the brushes.

27.8. Value of Reactance Voltage

Reactance voltage = coefficient of self-inductance × rate of change of current.

It should be remembered that the time of short-circuit or commutation is the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating plate of strip of mica.

Let W_b = brush width in cm; W_m = width of mica insulation in cm

v = peripheral velocity of commutator segments in cm/second

Then T_c = time of commutation or short-circuit = $\dfrac{W_b - W_m}{v}$ second

Note. If brush width etc. are given in terms of commutator segments, then commutator velocity should also be converted in terms of commutator segments per second.

If I is the current through a conductor, then total change during commutation = $I - (-I) = 2I$.

∴ Self-induced or reactance voltage

$$= L \times \frac{2I}{T_c} \quad \text{– if commutation is linear}$$

$$= 1.11\ L \times \frac{2I}{T_c} \quad \text{– if commutation is sinusodial}$$

As said earlier, the reactance e.m.f. hinders the reversal of current. This means that there would be sparking at the brushes due to the failure of the current in short-circuited coil to reach its full value in the reversed direction by the end of short-circuit. This sparking will not only damage the brush and the commutator but this being a cumulative process, it may worsen and eventually lead to the short-circuit of the whole machine by the formation of an arc round the commutator from brush to brush.

Example 27.9. *The armature of a certain dynamo runs at 800 r.p.m. The commutator consists of 123 segments and the thickness of each brush is such that the brush spans three segments. Find the time during which the coil of an armature remains short-circuited.*

Solution. As W_m is not given, it is considered negligible.

W_b = 3 segments and v = (800/60) × 123 segments/second

∴ commutation time = $\dfrac{3 \times 60}{800 \times 123}$ = 0.00183 second = **1.83 millisecond**

Example 27.10. *A 4-pole, wave-wound, d.c. machine running at 1500 r.p.m. has a commutator of 30 cm diameter. If armature current is 150 A, thickness of brush 1.25 cm and the self-inductance of each armature coil is 0.07 mH, calculate the average e.m.f. induced in each coil during commutation. Assume linear commutation.*

Solution. Formula : $E = L\,\dfrac{2I}{T_c}$

Here $L = 0.07 \times 10^{-3}$ H, $I = 150/2 = 75$ A (It is wave-wound)

$W_b = 1.25$ cm, $W_m = 0$ ...considered negligible

$v = \pi \times 30 \times (1500/60) = 2356$ cm/s ; $T_c = W_b/v = 1.25/2356 = 5.3 \times 10^{-4}$ second

$E = L \times 2I/T_c = 0.07 \times 10^{-3} \times 2 \times 75/5.3 \times 10^{-4} =$ **19.8 V**

Example 27.11. *Calculate the reactance voltage for a machine having the following particulars. Number of commutator segments = 55, Revolutions per minute = 900, Brush width in commutator segments = 1.74, Coefficient of self-induction = 153×10^{-6} henry, Current per coil = 27 A.*

(Advanced Elect. Machines. AMIE Sec. Winter 1991)

Solution. Current per coil, $I = 27$ A ; $L = 153 \times 10^{-6}$ H

$v = 55 \times (900/60) = 825$ segments/second; $T_c = W_b/v = 1.74/825 = 2.11 \times 10^{-3}$ second

Assuming linear commutation, $E = L \times 2I/T_C$

$\therefore \quad E = 153 \times 10^{-6} \times 2 \times 27/2.11 \times 10^{-3} =$ **3.91 V**

Example 27.12. *A 4-pole, lap-wound armature running at 1500 r.p.m. delivers a current of 150 A and has 64 commutator segments. The brush spans 1.2 segments and inductance of each armature coil is 0.05 mH. Calculate the value of reactance voltage assuming (i) linear commutation (ii) sinusoidal commutation. Neglect mica thickness.*

Solution. Formula : $E = L\,.\,\dfrac{2I}{T_c}$ Now, $L = 0.05 \times 10^{-3}$ H; $W_b = 1.2$ segments

$$v = \frac{1500}{60} \times 64 = 1600 \text{ segments/second}$$

$$\therefore \quad T_c = \frac{1.2}{1600} = 7.5 \times 10^{-4} \text{ second} ; I = \frac{150}{4} \text{ A} = 37.5 \text{ A}$$

$$\therefore \quad \frac{2I}{T_c} = \frac{2 \times 37.5}{7.5 \times 10^{-4}} = 10^5 \text{ A/s}$$

For linear commutation, $E = 0.05 \times 10^{-3} \times 10^5 =$ **5 V**

For sinusoidal commutation, $E = 1.11 \times 5 =$ **5.55 V**

27.9. Methods of Improving Commutation

There are two practical ways of improving commutation *i.e.* of making current reversal in the short-circuited coil as sparkless as possible. These methods are known as *(i)* resistance commutation and *(ii)* e.m.f. commutation (which is done with the help of either brush lead or interpoles, usually the later).

27.10. Resistance Commutation

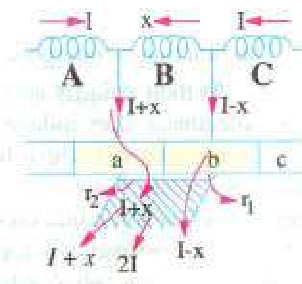


Fig. 27.12

This method of improving commutation consists of replacing low-resistance Cu brushes by comparatively high-resistance carbon brushes.

From Fig. 27.12, it is seen that when current I from coil' C reaches the commutator segment b, it has two parallel paths open to it. The first part is straight from bar 'b' to the brush and the other parallel path is *via* the short-circuited coil B to bar 'a' and then to the brush. If the Cu brushes (which have low contact resistance) are used, then there is no inducement for the current to follow the second longer path, it would preferably follow the first path. But when carbon brushes having high resistance are used, then current I coming from C will prefer to pass through the second path because (*i*) the resistance r_1 of the first path will increase due to the diminishing area of contact of bar '*b*' with the brush and because (*ii*) resistance r_2 of second path will decrease due to rapidly increasing contact area of bar '*a*' with the brush.

Hence, carbon brushes have, usually, replaced Cu brushes. However, it should be clearly understood that the main cause of sparking commutation is the self-induced e.m.f. (*i.e.* reactance voltage), so brushes alone do not give a sparkless commutation; though they do help in obtaining it.

The additional advantages of carbon brushes are that (*i*) they are to some degree self-lubricating and polish the commutator and (*ii*) should sparking occur, they would damage the commutator less than when Cu brushes are used.

But some of their minor disadvantages are : (*i*) Due to their high contact resistance (which is beneficial to sparkless commutation) a loss of approximately 2 volt is caused. Hence, they are not much suitable for small machines where this voltage forms an appreciable percentage loss. (*ii*) Owing to this large loss, the commutator has to be made some what larger than with Cu brushes in order to dissipate heat efficiently without greater rise of temperature. (*iii*) because of their lower current density (about 7-8 A/cm^2 as compared to 25-30 A/cm^2 for Cu brushes) they need larger brush holders.

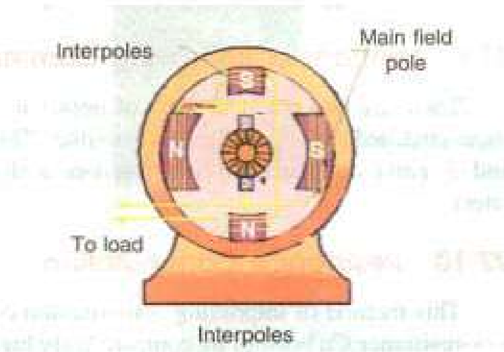
27.11. E.M.F. Commutation

In this method, arrangement is made to neutralize the reactance voltage by producing a reversing e.m.f. in the short-circuited coil under commutation. This reversing e.m.f., as the name shows, is an e.m.f. in opposition to the reactance voltage and if its value is made equal to the latter, it will completely wipe it off, thereby producing quick reversal of current in the short-circuited coil which will result in sparkless commutation. The reversing e.m.f. may be produced in two ways : (*i*) either by giving the brushes a forward lead sufficient enough to bring the short-circuited coil under the influence of next pole of opposite polarity or (*ii*) by using interpoles.

The first method was used in the early machines but has now been abandoned due to many other difficulties it brings along with.

27.12. Interpoles of Compoles

These are small poles fixed to the yoke and spaced in between the main poles. They are wound with comparatively few heavy gauge Cu wire turns and are connected in series with the armature so that they carry full armature current. Their polarity, in the case of a generator, is the same as that of the main pole *ahead* in the direction of rotation (Fig. 25.13).



The function of interpoles is two-fold :

(*i*) As their polarity is the same as that of the main pole ahead, they induce an e.m.f. in the coil (under commutation) which helps the reversal of current. The e.m.f. induced by the compoles is known as ***commutating or reversing e.m.f.*** The commutating e.m.f. neutralizes the reactance e.m.f. thereby making commutation sparkless. With interpoles, sparkless commutation can be obtained up to 20 to 30% overload with fixed brush position. In fact, interpoles raise sparking limit of a machine to *almost* the same value as heating limit. Hence, for a given output, an interpole machine can be made smaller and, therefore, cheaper than a non-interpolar machine.

As interpoles carry armature current, their commutating e.m.f. is proportional to the armature current. ***This ensures automatic neutralization of reactance voltage which is also due to armature current.*** Connections for a shunt generator with interpoles are shown in Fig. 27.14.

(*ii*) Another function of the interpoles is to neutralize the cross-magnetising effect of armature reaction. Hence, brushes are not to be shifted from the original position. In Fig 27.15, *OF* as before, represents the m.m.f. due to main poles. *OA* represents the cross-magnetising m.m.f. due to armature. *BC* which represents m.m.f. due to interpoles, is obviously in opposition to *OA*, hence they cancel each other out. ***This cancellation of cross-magnetisation is automatic and for all loads because both are produced by the same armature current.***

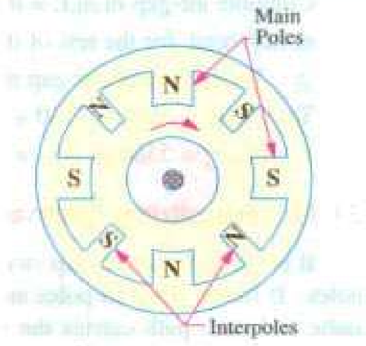


Fig. 27.13

The distinction between the interpoles and compensating windings should be clearly understood. Both are connected in series and thier m.m.fs. are such as to neutralize armature reaction. But compoles additionally supply m.m.f. for counteracting the reactance voltage induced in the coil undergoing commutation. Moreover, the action of the compoles is localized, they have negligible effect on the armature reaction occurring on the remainder of the armature periphery.

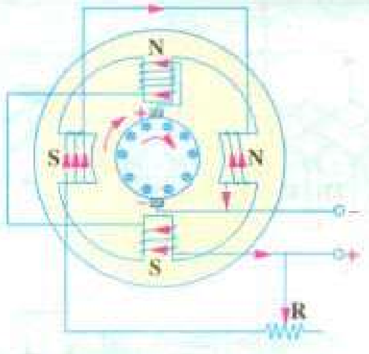


Fig. 27.14

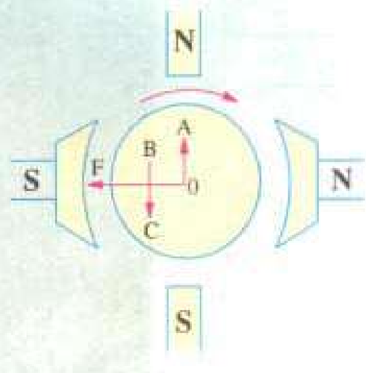


Fig. 27.15

Example 27.13. *Determine the number of turns on each commutating pole of a 6-pole machine, if the flux density in the air-gap of the commutating pole = 0.5 Wb/m^2 at full load and the effective length of the air-gap is 4 mm. The full-load current is 500 A and the armature is lap-wound with 540 conductors. Assume the ampere turns required for the remainder of the magnetic circuit to be one-tenth of that the air gap.* **(Advanced Elect. Machines AMIE Sec.B, 1991)**

Solution. It should be kept in mind that compole winding must be sufficient to oppose the armature m.m.f. (which is directed along compole axis) and to provide the m.m.f. for compole air-gap and its magnetic circuit.

$\therefore \quad N_{CP}\, I_a = ZI_C/2P + B_g\, l_g/\mu_0$

where N_{CP} = No. of turns on the compole ; I_a = Armature current

Z = No. of armature conductors ; I_c = Coil current

P = No. of poles ; l_g = Air-gap length under the compole

Z = 540 ; I_a = 500 A; I_c = 500/6 A; P = 6

$\therefore$ Arm. m.m.f. = 540 × (500/6)/2 × 6 = 3750

Compole air-gap m.m.f. $= B_g \times l_g/\mu_0 = 0.5 \times 4 \times 10^{-3}/4\pi \times 10^{-7} = 1591$

m.m.f. reqd. for the rest of the magnetic circuit = 10% of 1591 = 159

$\therefore$ Total compole air-gap m.m.f. = 1591 + 159 = 1750

Total m.m.f. reqd. = 3750 + 1750 = 5500

$\therefore$ $N_{cp} I_a = 5500$ *or* $N_{cp} = 5500/500 = \mathbf{11}$

27.13. Equalizing Connections

It is characteristic of lap -winding that all conductors in any parallel path lie under one pair of poles. If fluxes from all poles are exactly the same, then e.m.f. induced in each parallel path is the same and each path carries the same current. But despite best efforts, some inequalities in flux inevitably occur due either to slight variations in air-gap length or in the magnetic properties of steel. Hence, there is always a slight imbalance of e.m.f. in the various parallel paths. The result is that conductors under stronger poles generate greater e.m.f. and hence carry larger current. The current distribution at the brushes becomes unequal. Some brushes are overloaded *i.e.* carry more than their normal current whereas others carry less. Overloaded brushes spark badly whatever their position may be. This results in poor commutation and may even limit the output of the machine.

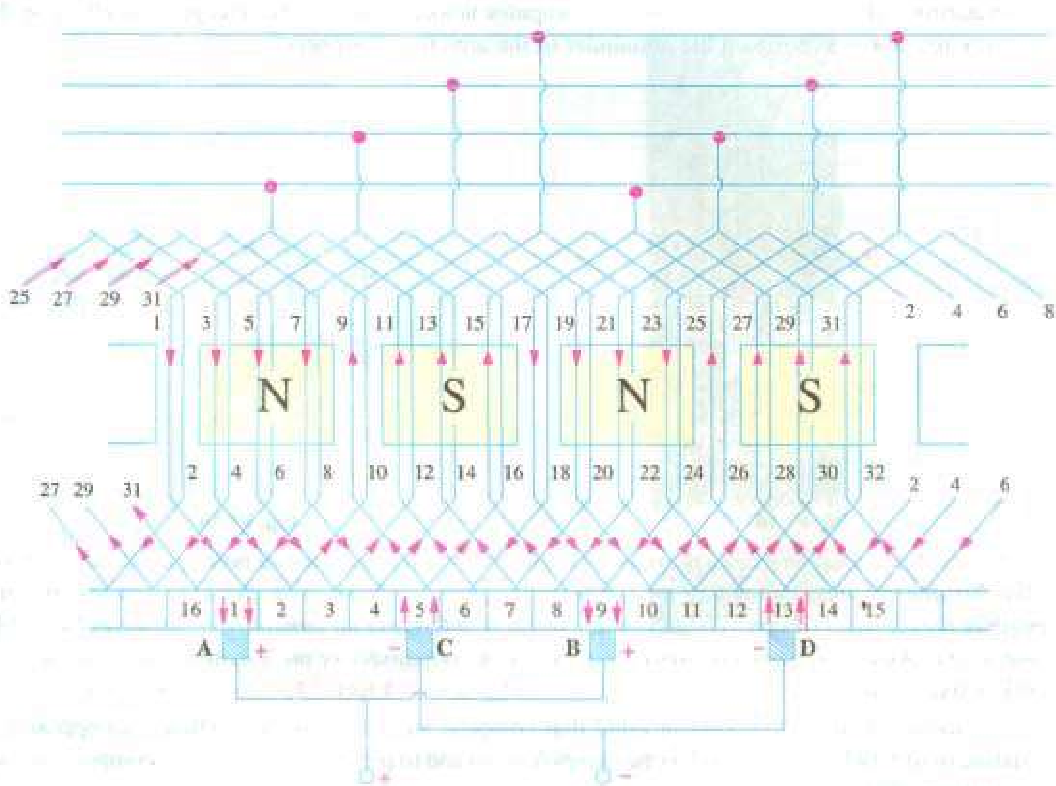


Fig. 27.16

By connecting together a number of symmetrical points on armature winding which would be at equal potential if the pole fluxes were equal, the difference in brush currents is diminished. This requires that there should be a whole number of slots per pair of poles so that, for example, if there is a slot under the centre of a *N*-pole, at some instant, then there would be one slot under the centre of every other *N*-pole. The equalizing conductors, which are in the form of Cu rings at the armature back and which connect such points are called *Equalizer Rings*. The circulating current due to the

Equalizer rings

slight difference in the e.m.fs. of various parallel paths, passes through these *equalizer* rings instead of passing through the brushes.

Hence, the function of equalizer rings is to avoid unequal distribution of current at the brushes thereby helping to get sparkless commutation.

One equalizer ring is connected to all conductors in the armature which are two poles apart (Fig. 27.17). For example, if the number of poles is 6, then the number of connections for each equalizer ring is 3 *i.e.* equal to the number of pair of poles. Maximum number of equalizer rings is equal to the number of conductors under one pair of poles. Hence, number of rings is

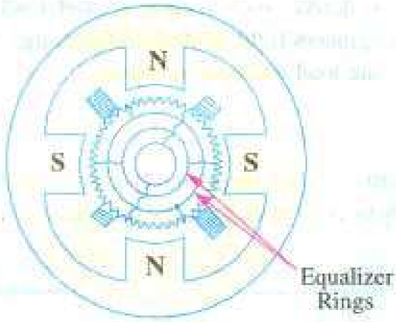


Fig. 27.17

$$= \frac{\text{No. of conductors}}{\text{No. of pair of poles}}$$

In practice, however, the number of rings is limited to 20 on the largest machines and less on smaller machines. In Fig. 27.16 is shown a developed armature winding. Here, only 4 equalizing bars have been used. It will be seen that the number of equalizing connections to each bar is two *i.e.* half the number of poles. Each alternate coil has been connected to the bar. In this case, the winding is said to be 50% equalized. If all conductors were connected to the equalizer rings, then the winding would have been 100% equalized.

Equalizer rings are not used in wave-wound armatures, because there is no imbalance in the e.m.fs. of the two parallel paths. This is due to the fact that armature conductors in either parallel path are not confined under one pair of poles as in lap-winding but are distributed under all poles. Hence, even if there are inequalities in the pole flux, they will affect each path equally.

27.14. Parallel Operation of Shunt Generators

Power plants, whether in d.c. or a.c. stations, will be generally found to have several smaller generators running in parallel rather than large single units capable of supplying the maximum peak load. These smaller units can be run single or in various parallel combinations to suit the actual load demand. Such practice is considered extremely desirable for the following reasons :

(*i*) Continuity of Service

Continuity of service is one of the most important requirements of any electrical apparatus. This would be impossible if the power plant consisted only of a single unit, because in the event of breakdown of the prime mover or the generator itself, the entire station will be shut down. In recent years, the requirement of uninterrupted service has become so important especially in factories etc. that it is now recognized as an economic necessity.

(*ii*) Efficiency

Usually, the load on the electrical power plant fluctuates between its peak value sometimes during the day and its minimum value during the late night hours. Since generators operate most efficiently when delivering full load, it is economical to use a single small unit when the load is light. Then, as the load demand increases, a larger generator can be substituted for the smaller one or another smaller unit can be connected to run in parallel with the one already in operation.

(iii) Maintenance and Repair

It is considered a good practice to inspect generators carefully and periodically to forestall any possibility of failure or breakdown. This is possible only when the generator is at rest which means that there must be other generators to take care of the load. Moreover, when the generator does actually breakdown, it can be repaired with more care and not in a rush, provided there are other generators available to maintain service.

(iv) Additions to Plant

Additions to power plants are frequently made in order to deliver increasingly greater loads. Provision for future extension is, in fact, made by the design engineers fight from the beginning. It becomes easy to add other generators for parallel operation as the load demand increases.

27.15. Paralleling DC Generator

Whenever generators are in parallel, their +ve and −ve terminals are respectively connected to the +ve and −ve sides of the bus-bars. These bus-bars are heavy thick copper bars and they act as +ve and −ve terminals for the whole power station. If polarity of the incoming generator is not the same as the line polarity, as serious short-circuit will occur when S_1, is closed.

Moreover, paralleling a generator with reverse polarity effectively short-circuits it and results in damaged brushes, a damaged commutator and a blacked-out plant. Generators that have been tripped off the bus-because of a heavy fault current should always be checked for reversed polarity before paralleling.

In Fig. 27.18 is shown a shunt generator No. 1 connected across the bus-bars *BB* and supplying some of the load. For putting generator No. 2 in parallel with it the following procedure is adopted.

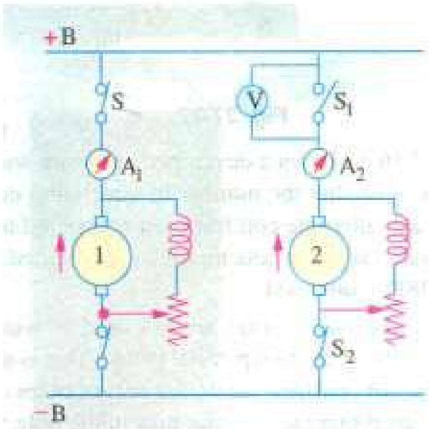


Fig. 27.18

The armature of generator No. 2 is speeded by the prime-mover up to its rated value and then switch S_2 is closed and circuit is completed by putting a voltmeter *V* across the open switch S_1. The excitation of the incoming generator No. 2 is changed till *V* reads zero. Then it means that its terminal voltage is the same as that of generator No. 1 or bus-bar voltage. After this, switch S_1 is closed and so the incoming machine is paralleled to the system. Under these conditions, however, generator No. 2 is not taking any load, because its ***induced*** e.m.f. is the same as bus-bar voltage and there can be no flow of current between two points at the same potential. The generator is said to be 'floating' on the bus-bar. If generator No. 2 is to deliver any current, then its induced e.m.f. *E* should be greater than the bus-bar voltage *V*. In that case, current supplied by it is $I = (E - V)/R_a$ where R_a is the resistance of the armature circuit. The induced e.m.f. of the incoming generator is increased by strengthening its field till it takes its proper share of load. At the same time, it may be found necessary to weaken the field of generator No. 1 to maintain the bus-bar voltage *V* constant.

27.16. Load Sharing

Because of their slightly drooping voltage characteristics, shunt generators are most suited for stable parallel operation. Their satisfactory operation is due to the fact that any tendency on the part of a generator to take more or less than its proper share of load results in certain changes of voltage in the system which immediately oppose this tendency thereby restoring the original division of load. Hence, once paralleled, they are automatically held in parallel.

Similarly, for taking a generator out of service, its field is weakened and that of the other generator is increased till the ammeter of the generator to be cleared reads zero. After that, its breaker and then the switch are opened thus removing the generator out of service. This method of connecting in and removing a generator from service helps in preventing any shock or sudden disturbance to the prime-mover or to the system itself.

It is obvious that if the field of one generator is weakened too much, then power will be delivered to it and it will run in its original direction as a motor, thus driving its prime-mover.

In Fig. 27.19. and 27.20 are shown the voltage characteristics of two shunt generators. It is seen that for a common terminal voltage V, the generator No. 1 delivers I_1 amperes and generator No. 2, I_2 amperes. It is seen that generator No. 1, having more drooping characteristic, delivers less current. It is found that two shunt generators will divide the load properly at all points if their characteristics are similar in form and each has the same voltage drop from no-load to full-load.

If it desired that two generators of different kW ratings automatically share a load in proportion to their ratings, then their external characteristics when plotted in terms of their percentage full-load currents (not actual currents) must be identical as shown in Fig. 27.21. If, for example, a 100-kW generator is working in parallel with a 200-kW generator to supply a total of 240-kW, then first generator will supply 80 kW and the other 160 kW.

When the individual characteristics of the generators are known, their combined characteristics can be drawn by adding the separate currents at a number of equal voltage (because generators are running in parallel). From this combined characteristic, the voltage for any combined load can be read off and from there, the current supplies by each generator can be found (Fig. 27.20).

If the generators have straight line characteristics, then the above result can be obtained by simple calculations instead of graphically.

Let us discuss the load sharing of two generators which have unequal no-load voltages.

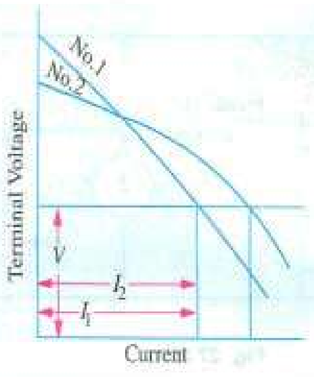


Fig. 27.19

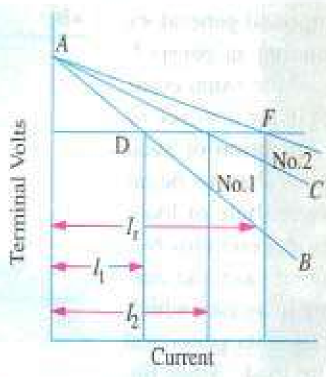


Fig. 27.20

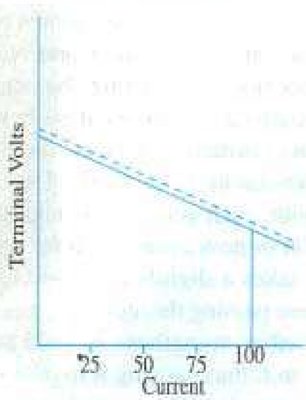


Fig. 27.21

Let

$$E_1, E_2 = \text{no-load voltages of the two generators}$$
$$R_1, R_2 = \text{their armature resistances}$$
$$V = \text{common terminal voltage}$$

Then

$$I_1 = \frac{E_1 - V}{R_1} \text{ and } I_2 = \frac{E_2 - V}{R_2}$$

$\therefore$

$$\frac{I_2}{I_1} = \frac{E_2 - V}{E_1 - V} \cdot \frac{R_1}{R_2} = \frac{K_2 N_2 \Phi_2 - V}{K_1 N_1 \Phi_1 - V} \cdot \frac{R_1}{R_2}$$

From the above equation, it is clear that bus-bar voltage can be kept constant (and load can be transferred from 1 to 2) by increasing Φ_2 or N_2 or by reducing N_1 and Φ_1.N_2 and N_1 are changed by changing the speed of driving engines and Φ_1 and Φ_2 are changed with the help of regulating shunt field resistances.

It should be kept in mind that

(i) Two parallel shunt generators having equal no-load voltages share the load in such a ratio that the load current of each machine produces the same drop in each generator.

(ii) In the case of two parallel generators having unequal no-load voltages, the load currents produce sufficient voltage drops in each so as to keep their terminal voltage the same.

(iii) The generator with the least drop assumes greater share of the change in bus load.

(iv) Paralleled generators with different power ratings but the same voltage regulation will divide any oncoming bus load in direct proportion to their respective power ratings (Ex. 27.14).

27.17. Procedure for Paralleling D.C. Generators

(i) Close the disconnect switch of the incoming generator

(ii) Start the prime-mover and adjust it to the rated speed of the machine

(iii) Adjust the voltage of the incoming machine a few volts higher than the bus voltage

(iv) Close the breaker of the incoming generator

(v) Turn the shunt field rheostat of the incoming machine in the raise-voltage direction and that of the other machine(s) already connected to the bus in the lower-voltage direction till the desired load distribution (as indicated by the ammeters) is achieved.

27.18. Compound Generators in Parallel

In Fig. 27.22 are shown two compound generators (designated as No. 1 and No. 2) running in parallel. Because of the rising characteristics of the usual compounded generators, it is obvious that in the absence of any corrective devices, the parallel operation of such generators is unstable. Let us suppose that, to begin with, each generator is taking its proper share of load. Let us now assume that for some reason, generator No. 1 takes a slightly increased load. In that case, the current passing through its series winding increases which further strengthens its field and so raises its generated e.m.f. thus causing it to take still more load. Since the system load is assumed to be constant, generator No. 2 will drop some of its load, thereby weakening its series field which will result in its further dropping off its load. Since this effect is cumulative. Generator No. 1 will, therefore, tend to take the entire load and finally drive generator No. 2 as a motor. The circuit breaker of at least one of the two generators will open, thus stopping their parallel operation.

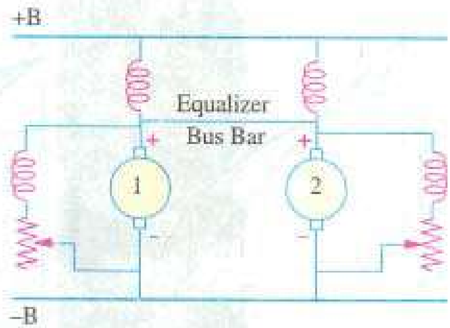


Fig. 27.22

For making the parallel operation of over-compound and level-compound generators stable,* they are always used with an equalizer bar (Fig. 27.22) connected to the armature ends of the series coils of the generators. The equalizer bar is a conductor of low resistance and its operation is as follows :

* Like shunt generators, the under-compound generators also do not need equalizers for satisfactory parallel operation.

Suppose that generator No. 1 starts taking more than its proper share of load. Its series field current is increased. But now this increased current passes partly through the series field coil of generator No. 1 and partly it flows via the equalizer bar through the series field winding of generator No. 2. Hence, the generators are affected in a similar manner with the result the generator No. 1 cannot take the entire load. For maintaining proper division of load from no-load to full-load, it is essential that

(*i*) the regulation of each generator is the same.

(*ii*) the series field resistances are inversely proportional to the generator rating.

27.19. Series Generators in Parallel

Fig 27.23 shows two identical series generators connected in parallel. Suppose E_1 and E_2 are initially equal, generators supply equal currents and have equal shunt resistances. Suppose E_1 increases slightly so that $E_1 > E_2$. In that case, I_1 becomes greater than I_2. Consequently, field of machine 1 is strengthened thus increasing E_1 further whilst the field of machine 2 is weakened thus decreasing E_2 further. A final stage is reached when machine 1 supplies not only the whole load but also supplies power to machine 2 which starts running as a motor. Obviously, the two machines will form a short-circuited loop and the current will rise indefinitely. This condition can be prevented by using equalizing bar because of which two similar machines pass approximately equal currents to the load, the slight difference between the two currents being confined to the loop made by the armatures and the equalizer bar.

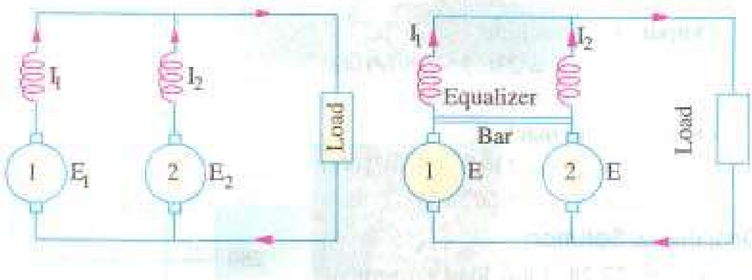


Fig. 27.23

Example 27.14. *A 100-kW, 250-V generator is paralleled with a 300 kW, 250-V generator. Both generators have the same voltage regulation. The first generator is supplying a current of 200 A and the other 500 A. What would be the current supplied by each generator if an additional load of 600 A is connected to the bus ?*

Solution. As explained in Art. 27.17, this additional load would be divided in direct proportion to the respective power ratings of the two generators.

$$\Delta I_1 = \left(\frac{100}{100+300}\right)\times 600 = 150 \text{ A};$$

$$\Delta I_2 = \left(\frac{300}{100+300}\right)\times 600 = 450 \text{ A}$$

Example 27.15. *Two 220–V, d.c. generators, each having linear external characteristics, operate in parallel. One machine has a terminal voltage of 270 V on no-load and 220 V at a load current of 35 A, while the other has a voltage of 280 V at no-load and 220 V at 50 A. Calculate the output current of each machine and the bus-bar voltage when the total load is 60 A. What is the kW output of each machine under this condition ?*

Solution. Generator No. 1.

Voltage drop for 35 A = 270–220 = 50 V

$\therefore$ Voltage drop/ampere = 50/35 = 10/7 V/A

Generator No. 2

Voltage drop/ampere $= (280-220)/50 = 1.2$ V/A

Let V = bus-bar voltage

I_1 = current output of generator No.1

I_2 = current output of generator No.2

then $V = 270-(10/7)\,I_1$ ...for generator No. 1

$= 280-1.2\,I_2$ ...for generator No. 2

Since bus-bar voltage is the same.

$\therefore \quad 270-10\,I_1/7 = 280-1.2\,I_2$ or $4.2\,I_2 - 5\,I_1 = 35$ ...(*i*)

Also $I_1 + I_2 = 60$ ...(*ii*)

Solving the two equations, we get $I_1 =$ **23.6 A**; $I_2 =$ **36.4 A**

Now $V = 280-1.2\,I_2 = 280-1.2 \times 36.4$

$=$ **236.3 V**

Output of 1st machine

$= 236.3 \times 23.6/1000$

$=$ **5.577 kW**

Output of 2nd machine

$= 236.3 \times 36.4/1000$

$=$ **8.602 kW**

Graphical Solution.

In Fig. 27.24, total load current of 60 A has been plotted along X-axis and the terminal voltage along Y-axis. The linear characteristics of the two generators are drawn from the given data. The common bus-bar voltage is given by the point of intersection of the two graphs. From the graph, it is seen that $V = 236.3$ V ; $I_1 =$ 23.6 A ; $I_2 = 36.4$ A.

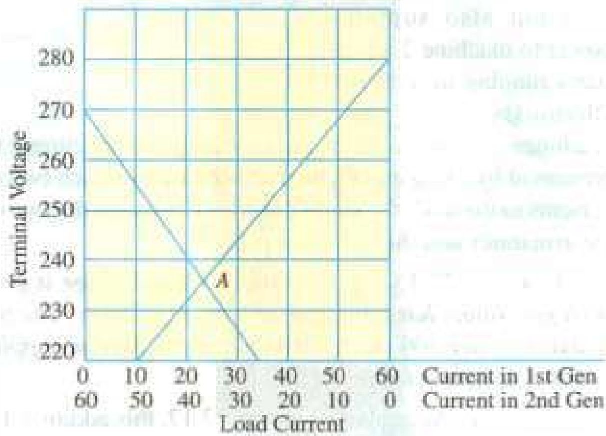


Fig. 27.24

Example 27.16. *Two shunt generators each with an armature resistance of 0.01 Ω and field resistance of 20 Ω run in parallel and supply a total load of 4000 A. The e.m.f.s are respectively 210 V and 220 V. Calculate the bus-bar voltage and output of each machine.* **(Electrical Machines-1, South Gujarat Univ. 1988)**

Solution. Generators are shown in Fig. 27.25.

Let V = bus-bar voltage

I_1 = output current of G_1

I_2 = output current of G_2

Now, $I_1 + I_2 = 4000$ A. $I_{sh} = V/20$.

$I_{a1} = (I_1 + V/20)$; $I_{a2} = (I_2 + V/20)$

In each machine,

V + armature drop = induced e.m.f.

$\therefore \quad V + I_{a1}\,R_a = E_1$

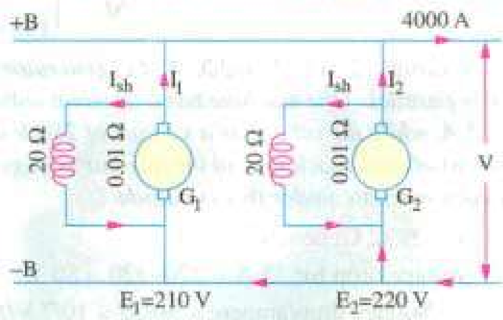


Fig. 27.25

or $V + (I_1 + V/20) \times 0.01$

$= 210$ ...1st machine

Also $V + I_{a2} R_a = E_2$

or $V + (I_2 + V/20) \times 0.01 = 220$ ...2nd machine

Subtracting, we have $0.01\ (I_1 - I_2) = 10$ or $I_1 - I_2 = 1000$

Also, $I_1 + I_2 = 4000$ A $\therefore$ I_1 = **2500 A ; I_2 = 1500 A**

Substituting the value of I_1 above, we get

$V + (2500 + V/20) \times 0.01 = 210$ $\therefore$ V = **184.9 V**

Output of Ist generator = $184.9 \times 2500/1000$ = **462.25 kW**

Output of 2nd generator = $184.49 \times 1500/1000$ = **277.35 kW**

Example 27.17. *Two shunt generators operating in parallel deliver a total current of 250 A. One of the generators is rated 50 kW and the other 100 kW. The voltage rating of both machine is 500 V and have regulations of 6 per cent (smaller one) and 4 percent. Assuming linear characteristics, determine* **(a)** *the current delivered by each machine* **(b)** *terminal voltage.*

(Elect. Machines, Nagpur Univ. 1991)

Solution. 50 kW generator

F.L. voltage drop = $500 \times 0.06 = 30$ V ; F.L. current = 50,000/500 = 100 A

Drop per ampere = 30/100 = 3/10 V/A

100 kW generator

F.L. drop = $500 \times 0.04 = 20$ V ; F.L. current = 100,000/5000 = 200 A

Drop per ampere = 20/200 = 1/10 V/A

If I_1 and I_2 are currents supplied by the two generators and V the terminal voltage, then

$V = 500 - (3I_1/10)$ –1st generator

$= 500 - (I_2/10)$ –2nd generator

$\therefore$ $3I_1/10 = I_2/10$ or $3I_1 = I_2$; Also $I_1 + I_2 = 250$ –given

(a) Solving the above two equations, we get I_1 = **62.5 A ; I_2 = 187.5 A**

(b) $V = 500 - (3 \times 62.5/10)$ = **481.25 V**

Example 27.18. *Two shunt generators, each with a no-load voltage of 125 V are running parallel. Their external characteristics can be taken as straight lines over this operating ranges. Generator No. 1 is rated at 25 kW and its full-load voltage is 119 V, Generator No. 2 is rated at 200 kW at 116 V. Calculate the bus-bar voltage when the total load is 3500 A. How is the load divided between the two ?*

(Elect. Machinery - I, Mysore Univ. 1988)

Solution. Let V = bus-bar voltage

x_1, x_2 = load carried by each generator in terms of percentage of rated load

P_1, P_2 = load carried by each generator in watts

$V = 125 - [(125 - 119)\ (x_1/100)]$ ...Generator No. 1

$V = 125 - [(125 - 116)\ (x_2/100)]$ ...Generator No. 2

$$\therefore \quad 125 - \frac{6x_1}{100} = 125 - \frac{9x_2}{100} \qquad x_2 = \frac{6x_1}{9} = \frac{2x_1}{3}$$

Since in d.c. circuits, power delivered is given by VI watt, the load on both generators is

$$\left(250\, x_1 \times \frac{1000}{100}\right) + \left(200\, x_2 \times \frac{1000}{100}\right) = V \times 3500$$

Now, replacing V and x_2 by terms involving x_1, we get as a result

$$\left(250\,x_1 \times \frac{1000}{100}\right) + \left(200 \times \frac{2x_1}{3} \times \frac{1000}{100}\right) = \left(125 - \frac{6x_1}{100}\right) \times 3500$$

$$x_1 = 108.2 \text{ per cent}$$

$\therefore$ Bus-bar voltage $V = 125 - (6 \times 108.2/100) = $ **118.5 V**

The division of load between the two generators can be found thus :

$$x_1 = \frac{P_1 \times 1000}{250{,}000} \quad \text{and} \quad x_2 = \frac{P_2 \times 1000}{200{,}000}$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{P_1 \times 200{,}000}{P_2 \times 250{,}000} = \frac{4P_1}{5P_2} = \frac{3}{2} \quad \therefore \quad \frac{3}{2} = \frac{4P_1}{5P_2} = \frac{4\,VI_1}{5\,VI_2} = \frac{4I_1}{5I_2} \qquad ...(i)$$

Since $\quad I_1 + I_2 = 3500 \qquad \therefore \quad I_2 = 3500 - I_1$

Hence (*i*) above becomes, $\dfrac{3}{2} = \dfrac{4I_1}{5(3500 - I_1)}$

$\therefore \quad I_1 = $ **2,283** A and $I_2 = $ **1,217 A**

Example 27.19. *Two shunt generators A & B operate in parallel and their load-characteristics may be taken as straight lines. The voltage of generator A falls from 240 V at no load to 220 V at 200 A, while that of B falls from 245 V at no load to 220 V to 150 A. Determine the currents supplied by each machine to a common load of 300 A and the bus-bar voltage.*

(Bharathithasan Univ. April 1997)

Solution. Two graphs are plotted as shown in Fig. 27.26.

Their equations are :

$$240 - (20/200)\,I_A = 245 - (25/150)\,I_B$$

$$\text{Futher, } I_A + I_B = 300$$

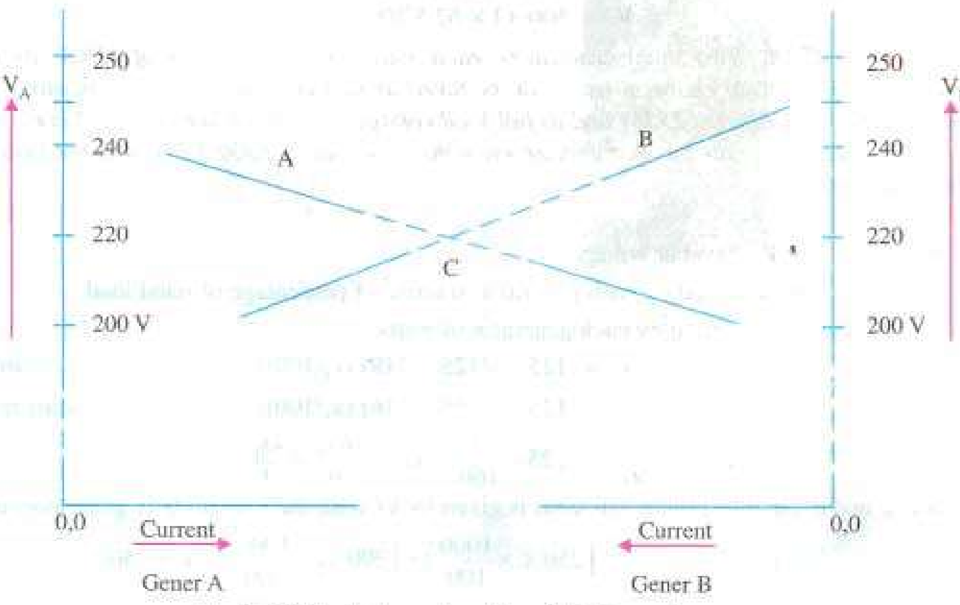


Fig. 27.26. Parallel operation of two D.C. Generators

This gives $I_A = 168.75$ A, $I_B = 131.25$ A

And common voltage of Bus-bar, V_{BUS}

$$= 240 - (20/200) \times 168.75, \text{ or}$$

$$V_{BUS} = 245 - (25/150) \times 131.25 = 223.125 \text{ volts.}$$

It is represented by the point *C*, in graph, as an intersection, satisfying the condition that two currents (I_A and I_B) add up to 300 amp.

Example 27.20. *In a certain sub-station, there are 5 d.c. shunt generators in parallel, each having an armature resistance of 0.1 Ω, running at the same speed and excited to give equal induced e.m.f.s. Each generator supplies an equal share of a total load of 250 kW at a terminal voltage of 500 V into a load of fixed resistance. If the field current of one generator is raised by 4%, the others remaining unchanged, calculate the power output of each machine and their terminal voltages under these conditions. Assume that the speeds remain constant and flux is proportional to field current.*

(Elect. Technology, Allahabad Univ. 1991)

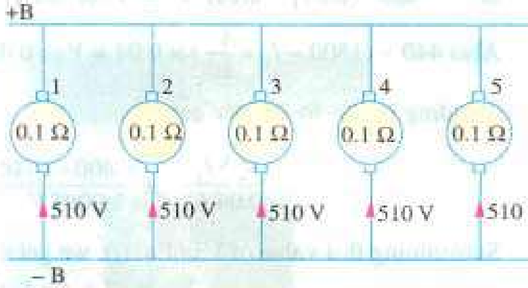


Fig. 27.27

Solution. Generator connections are shown in Fig. 27.27.

Load supplied by each $= 250/5 = 50$kW

$\therefore$ Output of each $= 50{,}000/500 = 100$ A

Terminal voltage of each $= 500$ V

Armature drop of each $= 0.1 \times 100 = 10$ V

Hence, induced e.m.f. of each $= 510$ V

When field current of one is increased, its flux and hence its generated e.m.f. is increased by 4%. Now, 4% of 510 V = 20.4 V

$\therefore$ Induced e.m.f. of one $= 510 + 20.4 = 530.4$ V

Let I_1 = current supplied by one generator after increased excitation

I_2 = current supplied by each of the other 4 generators

V = new terminal or bus-bar voltage

$$\therefore \quad 530.4 - 0.1\, I_1 = V \qquad ...(i)$$

$$510 - 0.1\, I_2 = V \qquad ...(ii)$$

Now, fixed resistance of load $= 500/500 = 1\ \Omega$; Total load current $= I_1 + 4I_2$

$$\therefore \quad 1 \times (I_1 + 4I_2) = V \text{ or } I_1 + 4I_2 = V \qquad ...(iii)$$

Subtracting *(ii)* from *(i)*, we get, $I_1 - I_2 = 204$ ...*(iv)*

Subtracting *(iii)* from *(ii)*, we have $I_1 + 4.1\, I_2 = 510$ ...*(v)*

From *(iv)* and *(v)*, we get $I_2 = 3060/51 = 59/99 = 60$ A (approx.)

From *(iv)* $I_1 = 204 + 60 = 264$ A

From *(iii)* $V = 264 + 240 =$ **504 Volt**

Output of Ist machine $= 504 \times 264$ watt $=$ **133 kW**

Output of *each* of other four generators $= 504 \times 60$ W $=$ **30.24 kW**

Example 27.21. *Two d.c. generators are connected in parallel to supply a load of 1500 A. One generator has an armature resistance of 0.5 Ω and an e.m.f. of 400 V while the other has an armature resistance of 0.04 Ω and an e.m.f. of 440 V. The resistances of shunt fields are 100 Ω and 80 Ω respectively. Calculate the currents I_1 and I_2 supplied by individual generator and terminal voltage V of the combination.* **(Power Apparatus-I, Delhi Univ. Dec. 1987)**

Solution. Generator connection diagram is shown in Fig. 27.28.

Let V = bust–bar voltage

I_1 = output current of one generator

I_2 = output current of other generator

$= (1500 - I_1)$

Now, $I_{sh1} = V/100 \text{ A} ; I_{sh2} = V/80 \text{ A}$

$$I_{a1} = \left(I_1 + \frac{V}{100}\right) \text{ and } I_{a2} = \left(I_2 + \frac{V}{80}\right)$$

or $$I_{a2} = \left(1500 - I_1 + \frac{V}{80}\right)$$

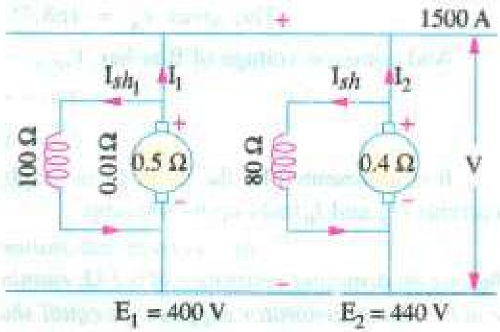


Fig. 27.28

For each machine

$$E - \text{armature drop} = V$$

$$\therefore \; 400 - \left(I_1 + \frac{V}{100}\right) \times 0.5 = V$$

or $400 - 0.5\, I_1 - 0.005\, V = V$ or $0.5\, I_1 = 400 - 1.0005 = V$ ...(*i*)

Also $440 - (1500 - I_1 + \frac{V}{80}) \times 0.04 = V$ or $0.04\, I_1 = 1.0005\, V - 380$ ...(*ii*)

Dividing Eq. (*i*) by (*ii*), we get

$$\frac{0.5\, I_1}{0.04\, I_2} = \frac{400 - 1.005\, V}{1.0005\, V - 380} \quad \therefore V = \mathbf{381.2\ V}$$

Substituting this value of V in Eq. (*i*), we get $0.5\, I_1 = 400 - 1.005 \times 381.2$

$\therefore \quad I_1 = \mathbf{33.8\ A}\ ; I_2 = 1500 - 33.8 = \mathbf{1466.2\ A}$

Output of Ist generator $= 381.2 \times 33.8 \times 10^{-3} = \mathbf{12.88\ kW}$

Output of 2nd generator $= 381.2 \times 1466.2 \times 10^{-3} = \mathbf{558.9\ kW}$

Example 27.22. *Two shunt generators and a battery are working in parallel. The open circuit voltage, armature and field resistances of generators are 250 V, 0.24 Ω, 100 Ω are 248 V, 0.12 Ω and 100 Ω respectively. If the generators supply the same current when the load on the bus-bars is 40 A, calculate the e.m.f. of the battery if its internal resistance is 0.172 Ω.*

Solution. Parallel combination is shown in Fig. 27.29.

Values of currents and induced e.m.fs. are shown in the diagram.

$$V + \left(I + \frac{V}{100}\right) \times 0.24 = 250 \quad \text{...}(i)$$

$$V + \left(I + \frac{V}{100}\right) \times 0.12 = 248 \quad \text{...}(ii)$$

Also $$I + I + I_b = 40$$

$$I_b + 2I = 40 \quad \text{...}(iii)$$

Subtracting (*ii*) from (*i*), we get $\left(I + \frac{V}{100}\right) \times 0.12 = 2$ ...(*iv*)

Putting this value in (*ii*) above, $V = 246$ volt.

Putting this value of V in (*iv*), $\left(I + \frac{246}{100}\right) \times 0.12 = 2$ ...(*v*)

$\therefore \quad I = 50/3 - 2.46 = 14.2 \text{ A}$

From (*iii*), we have $I_b = 40 - (2 \times 14.2) = 11.6 \text{ A}$

Internal voltage drop in battery $= 11.6 \times 0.172 = 2 \text{ V} \quad \therefore E_b = 246 + 2 = \mathbf{248\ V}$

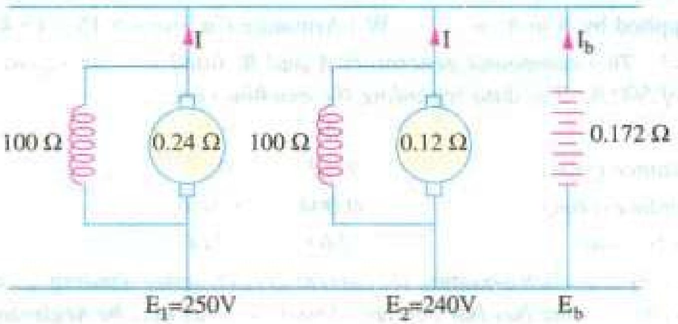


Fig. 27.29

Example 27.23. *Two d.c. generators A and B are connected to a common load. A had a constant e.m.f. of 400 V and internal resistance of 0.25 Ω while B has a constant e.m.f. of 410 V and an internal resistance of 0.4 Ω. Calculate the current and power output from each generator if the load voltage is 390 V. What would be the current and power from each and the terminal voltage if the load was open-circuited ?* **(Elect. Engg; I, Bangalore Univ. 1987)**

Solution. The generator connections are shown in Fig. 27.30 (*a*).

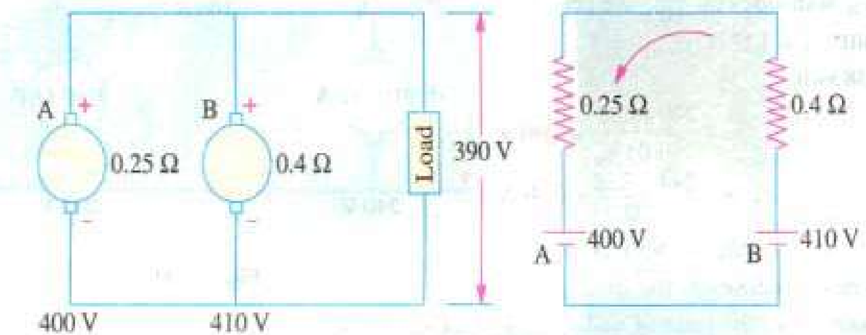


Fig. 27.30

Since the terminal or output voltage is 390 V, hence

Load supplied by $A = (400 - 390)/0.25 =$ **40 A**

Load supplied by $B = (410 - 390)/0.4 =$ **50A**

$\therefore$ Power output from $A = 40 \times 390 =$ **15.6 kW**

Power output from $B = 50 \times 390 =$ **19.5 kW**

If the load is open-circuited as shown in Fig. 27.30.(*b*), then the two generators are put in series with each other and a circulatory current is set up between them.

Net voltage in the circuit $= 410 - 400 = 10$ V

Total resistance $= 0.4 + 0.25 = 0.65\ \Omega$

$\therefore$ circulatory current $= 10/0.65 = 15.4$ A

The terminal voltage $= 400 + (15.4 \times 0.25) =$ **403.8 V**

Obviously, machine *B* with a higher e.m.f. acts as a generator and drives machine *A* as a motor.

Power taken by *A* from *B* $= 403.8 \times 15.4 =$ **6,219 W**

Part of this appears as mechanical output and the rest is dissipated as armature Cu loss.

Mechanical output $= 400 \times 15.4 =$ **6.16 kW** ; Armature Cu loss $= 3.8 \times 15.4 =$ **59 W**

Power supplied by B to A $=$ **6,219** W ; Armature Cu loss $= 6.16 \times 15.4 =$ **95 W**

Example 27.24. *Two compound generators A and B, fitted with an equalizing bar, supply a total load current of 500 A. The data regarding the machines are :*

	A	*B*
Armature resistance (ohm)	*0.01*	*0.02*
Series field winding (ohm)	*0.004*	*0.006*
Generated e.m.fs. (volt)	*240*	*244*

Calculate (a) current in each armature (b) current in each series winding (c) the current flowing in the equalizer bar and (d) the bus-bar voltage. Shunt currents may be neglected.

Solution. The two generators (with their shunt windings omitted) are shown in Fig. 27.31.

Let V = bus-bar voltage ; v = voltage between equalizer bus-bar and the negative

i_1, i_2 = armature currents of the two generators

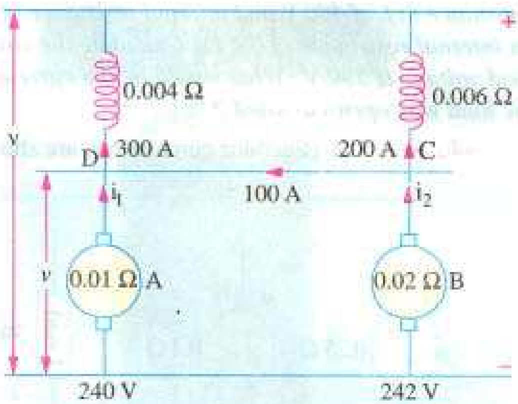


Fig. 27.31

Now, $$i_1 + i_2 = 500$$

or $$\frac{240-v}{0.01} + \frac{244-v}{0.01} = 500$$

Multiplying both sides by $\frac{1}{100}$ we get

$$240 - v + 122 - (v/2) = 5$$

$\therefore \quad v = 238$ volts

(*a*) $\therefore$ $$i_1 = \frac{240-238}{0.01} = \textbf{200 A}$$

$$i_2 = \frac{244-238}{0.02} = \textbf{300 A}$$

(*b*) The total current of 500 A divides between the series windings in the inverse ratio of their resistance *i.e.* in the ratio of $\frac{1}{0.004} : \frac{1}{0.006}$ or in the ratio 3 : 2.

Hence, current in the series winding of generator $A = 500 \times 3/5 =$ **300 A**

Similarly, current in the series winding of generator $B = 500 \times 2/5 =$ **200 A**

(*c*) It is obvious that a current of 100 A flows in the equalizing bar from C to D. It is so because the armature current of generator A is 200 A only. It means that 100 A comes from the armature of generator, B, thus making 300 A for the series field winding of generator A.

(*d*) $V = v -$ voltage drop in one series winding $= 238 - (300 \times 0.004) =$ **236.8 V**

Tutorial Problem No. 27.2

1. Two separately-excited d.c. generators are connected in parallel supply a load of 200 A. The machines have armature circuit resistances of 0.05 Ω and 0.1 Ω and induced e.m.fs. 425 V and 440 V respectively. Determine the terminal voltage, current and power output of each machine. The effect of armature reaction is to be neglected. **(423.3 V ; 33.3 A ; 14.1 kW ; 166.7 A ; 70.6 kW)**

2. Two shunt generators operating in parallel given a total output of 600 A. One machine has an armature resistance of 0.02 Ω and a generated voltage of 455 V and the other an armature resistance of 0.025 Ω and a generated voltage of 460 V. Calculate the terminal voltage and the kilowatt output of each machine. Neglect field currents.

(450.56 V ; 100 kW ; 170.2 kW)

3. The external characteristics of two d.c. shunt generators A and B are straight lines over the working range between no-load and full-load.

	Generator A		*Generator B*	
	No-load	*Full-load*	*No-load*	*Full-load*
Terminal P.D. (V)	400	360	420	370
Load current (A)	0	80	0	70

Determine the common terminal voltage and output current of each generator when sharing a total load of 100 A.

(57.7 A ; 42.3 A ; 378.8 V)

4. Two shunt generators operating in parallel have each an armature resistance of 0.02 Ω. The combined external load current is 2500 A. If the generated e.m.fs. of the machines are 560 V and 550 V respectively, calculate the bus-bar voltage and output in kW of each machine. **(530 V; 795 kW; 530 kW)**

5. Two shunt generators A and B operate in parallel and their load characteristics may be taken as straight lines. The voltage of A falls from 240 V at no-load to 220 V at 200 A, while that of B falls from 245 V at no-load to 220 V at 150 A. Determine the current which each machine supplies to a common load of 300 A and the bus-bar voltage at this load.

(169 A ; 131 A ; 223.1 V)

6. Two shunt-wound d.c. generators are connected in parallel to supply a load of 5,000 A. Each machine has an armature resistane of 0.03 Ω and a field resistance of 60 Ω, but the e.m.f. of one machine is 600 V and that of the other is 640 V. What power does each machine supply ?

(1,004 kW ; 1,730 kW including the fields)

7. Two shunt generators running in parallel share a load of 100 kW equally at a terminal voltage of 230 V. On no-load, their voltages rise to 240 V and 245 V respectively. Assuming that their volt-ampere characteristics are rectilinear, find how would they share the load when the total current is reduced to half its original value ? Also, find the new terminal voltage. **(20 kW ; 30 kW, 236 V)**

8. Two generators, each having no-load voltage of 500 V, are connected in parallel to a constant resistance load consuming 400 kW. The terminal p.d. of one machine falls linearly to 470 V as the load is increased to 850 A while that of the other falls linearly to 460 V when the load is 600 A. Find the load current and voltage of each generator.

If the induced e.m.f. of one machine is increased to share load equally, find the new current and voltage. **(I_1 = 626 A ; I_2 = 313 A ; V = 479 V ; I = 469.5 A ; V = 484.4 V)**

9. Estimate the number of turns needed on each interpole of a 6-pole generator delivering 200 kW at 200 V ; given : number of lap-connected armature conductors = 540 ; interpole air gap = 1.0 cm ; flux-density in interpole air-gap = 0.3 Wb/m^2. Ignore the effect of iron parts of the circuit and of leakage.

[10] *(Electrical Machines, B.H.U. 1980)*

OBJECTIVE TEST – 27

1. In d.c. generators, armature reaction is produced actually by
   (*a*) its field current
   (*b*) armature conductors
   (*c*) field pole winding
   (*d*) load current in armature
2. In a d.c. generator, the effect of armature reaction on the main pole flux is to
   (*a*) reduce it (*b*) distort it
   (*c*) reverse it (*d*) both (*a*) and (*b*)
3. In a clockwise-rotating loaded d.c. generator, brushes have to be shifted
   (*a*) clockwise
   (*b*) counterclockwise
   (*c*) either (*a*) or (*b*)
   (*d*) neither (*a*) nor (*b*).
4. The primary reason for providing compensating windings in a d.c. generator is to
   (*a*) compensate for decrease in main flux
   (*b*) neutralize armature mmf
   (*c*) neutralize cross-magnetising flux
   (*d*) maintain uniform flux distribution.
5. The main function of interpoles is to minimize .... between the brushes and the commutator when the d.c. machine is loaded.
   (*a*) friction (*b*) sparking
   (*c*) current (*d*) wear and tear

6. In a 6-pole d.c. machine, 90 mechanical degrees correspond to .... electrical degrees.
   (a) 30 (b) 180
   (c) 45 (d) 270
7. The most likely cause(s) of sparking at the brushes in a d.c. machine is /are
   (a) open coil in the armature
   (b) defective interpoles
   (c) incorrect brush spring pressure
   (d) all of the above
8. In a 10-pole, lap-wound d.c. generator, the number of active armature conductors per pole is 50. The number of compensating conductors per pole required is
   (a) 5 (b) 50
   (c) 500 (d) 10
9. The commutation process in a d.c. generator basically involves
   (a) passage of current from moving armature to a stationary load
   (b) reversal of current in an armature coil as it crosses MNA
   (c) conversion of a.c. to d.c.
   (d) suppression of reactance voltage
10. Point out the WRONG statement. In d.c. generators, commutation can be improved by
   (a) using interpoles
   (b) using carbon brushes in place of Cu brushes
   (c) shifting brush axis in the direction of armature rotation
   (d) none of the above
11. Each of the following statements regarding interpoles is true **except**
   (a) they are small yoke-fixed poles spaced in between the main poles
   (b) they are connected in parallel with the armature so that they carry part of the armature current
   (c) their polarity, in the case of generators is the same as that of the main pole ahead
   (d) they automatically neutralize not only reactance voltage but cross-magnetisation as well
12. Shunt generators are most suited for stable parallel operation because of their voltage characteristics.
   (a) identical (b) dropping
   (c) linear (d) rising
13. Two parallel shunt generators will divide the total load equally in proportion to their kilowatt output ratings only when they have the same
   (a) rated voltage
   (b) voltage regulation
   (c) internal I_aR_a drops
   (d) boths (a) and (b)
14. The main function of an equalizer bar is to make the parallel operation of two over-compounded d.c. generators
   (a) stable (b) possible
   (c) regular (d) smooth
15. The *essential* condition for stable parallel operation A Two d.c. generators having similar characteristics is that they should have
   (a) same kilowatt ouput ratings
   (b) droping voltage characterisitcs
   (c) same percentage regulation
   (d) same no-load and full-load speed
16. The main factor which loads to unstable parallel operation of flat-and over-compound d.c. generators is
   (a) unequal number of turns in their series field windings
   (b) unequal series field resistances
   (c) their rising voltage characteristics
   (d) unequal speed regulation of their prime movers
17. The simplest way to shift load from one d.c. shunt generator running in parallel with another is to
   (a) adjust their field rheostats
   (b) insert resistance in their armature circuits
   (c) adjust speeds of their prime movers
   (d) use equalizer connections
18. Which one of the following types of generators does NOT need equalizers for satisfactory parallel operation ?
   (a) series (b) over-compound
   (c) flat-compound (d) under-compound.

ANSWERS

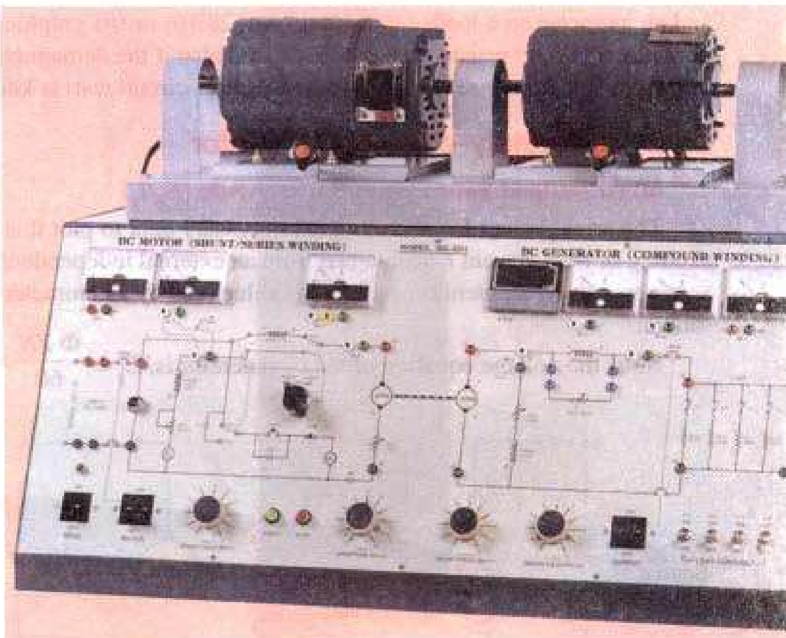
1. (d) 2. (d) 3. (a) 4. (c) 5. (b) 6. (d) 7. (d) 8. (a) 9. (b) 10. (d) 11. (b) 12. (b) 13. (d) 14. (a) 15. (b) 16. (c) 17. (a) 18. (d)

C H A P T E R 28

GENERATOR CHARACTERISTICS

Learning Objectives

- Characteristics of D.C. Generators
- Separately-excited Generator
- No-load Curve for Self-excited Generator
- How to find Critical Resistance R_c?
- How to draw O.C.C. at Different Speeds ?
- Critical Speed
- Voltage Build up of a Shunt Generator
- Conditions for Build-up of a Shunt Generator
- Other factors Affecting Voltage Building of a D.C. Generator
- External Characteristic
- Voltage Regulation
- Internal or Total Characteristic
- Series Generator
- Compound-wound Generator
- How to calculate Required Series Turns?
- Uses of D.C. Generators



Generator characteristics gives the relation between terminal voltage and load current. It is of great importance in judging the suitability of a generator for a particular purpose

28.1. Characteristics of D.C. Generators

Following are the three *most* important characteristics or curves of a d.c. generator :

1. No-load saturation Characteristic (E_0/I_f)

It is also known as Magnetic Characteristic or Open-circuit Characteristic (*O.C.C.*). It shows the relation between the no-load generated e.m.f. in armature, E_0 and the field or exciting current I_f at a given fixed speed. It is just the magnetisation curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately-excited or self-excited.

2. Internal or Total Characteristic (E/I_a)

It gives the relation between the e.m.f. E *actually* induces in the armature (after allowing for the demagnetising effect of armature reaction) and the armature current I_a. This characteristic is of interest mainly to the designer and can be obtained as explained in Art. 28.12.

3. External Characteristic (V/I)

It is also referred to as performance characteristic or sometimes voltage-regulating curve.

It gives relation between that terminal voltage V and the load current I. This curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The values of V are obtained by subtracting I_aR_a from corresponding values of E. This characteristic is of great importance in judging the suitability of a generator for a particular purpose. It may be obtained in two ways (*i*) by making simultaneous measurements with a suitable voltmeter and an ammeter on a loaded generator (Art. 28.10) or (*ii*) graphically from the *O.C.C.* provided the armature and field resistances are known and also if the demagnetising effect (under rated load conditions) or the armature reaction (from the short-circuit test) is known.

28.2. Separately-excited Generator

(*a*) (*i*) No-load Saturation Characteristic (E_0/I_f)

The arrangement for obtaining the necessary data to plot this curve is shown in Fig. 28.1. The exciting or field current I_f is obtained from an external independent d.c. source. It can be varied from zero upwards by a potentiometer and its value read by an ammeter A connected in the field circuit as shown.

Now, the voltage equation of a d.c. generator is, $E_g = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

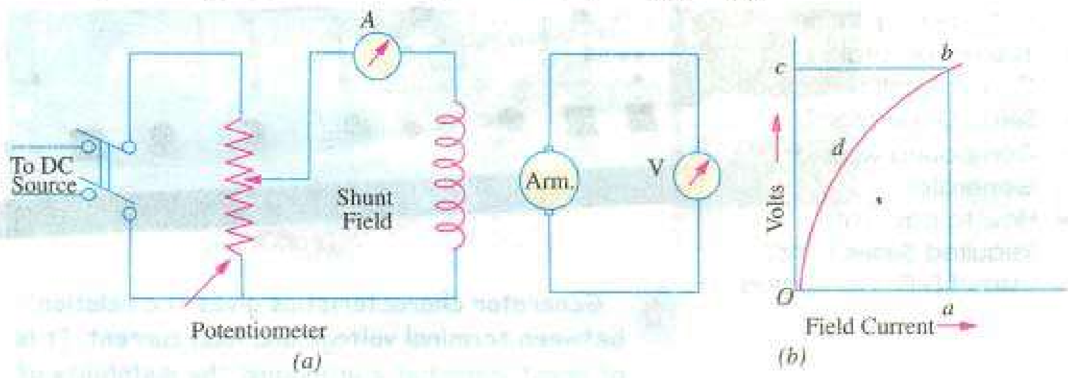


Fig. 28.1

Hence, if speed is constant, the above relation becomes $E = k\Phi$

It is obvious that when I_f is increased from its initial small value, the flux Φ and hence generated e.m.f. E_g increase directly as current so long as the poles are unsaturated. This is represented by the straight portion Od in Fig. 28.1 (*b*). But as the flux density increases, the poles become saturated, so a greater increase in I_f is required to produce a given increase in voltage than on the lower part of the curve. That is why the upper portion db of the curve Odb bends over as shown.

(ii) Load Saturation Curve (V/I_f)

The curve showing relation between the terminal voltage V and field current I_f when the generator is loaded, is known as Load Saturation Curve.

The curve can be deduced from the no-load saturation curve provided the values of armature reaction and armature resistance are known. While considering this curve, account is taken of the demagnetising effect of armature reaction and the voltage drop in armature which are practically absent under no-load conditions. The no-load saturation curve of Fig. 28.1 has been repeated in Fig. 28.2 on a base of field amp-turns (and not current) and it is seen that at no-load, the field amp-turns required for rated no-load voltage are given by Oa. Under load conditions, the voltage will decrease due to demagnetising effect of armature reaction. This decrease can be made up by suitably increasing the field amp-turns. Let ac represent the equivalent demagnetising amp-turns per pole. Then, it means that in order to generate the same e.m.f. on load as at no-load, the field amp-turns/pole must be increased by an amount $ac = bd$. The point d lies on the curve LS which shows relation between the voltage E generated under *load* conditions and the field amp-turns. The curve LS is practically parallel to curve Ob. The terminal voltage V will be less than this generated voltage E by an amount $= I_a R_a$ where R_a is the resistance of the armature circuit. From point d, a vertical line $de = I_a R_a$ is drawn. The point e lies on the full-load saturation curve for the generator. Similarly, other points are obtained in the same manner and the full-load saturation curve Mp is drawn. The right-angled triangle bde is known as **drop reaction triangle**. Load saturation curve for half-load can be obtained by joining the mid-points of such lines as mn and bd etc. In the case of self-excited generators, load saturation curves are obtained in a similar way.

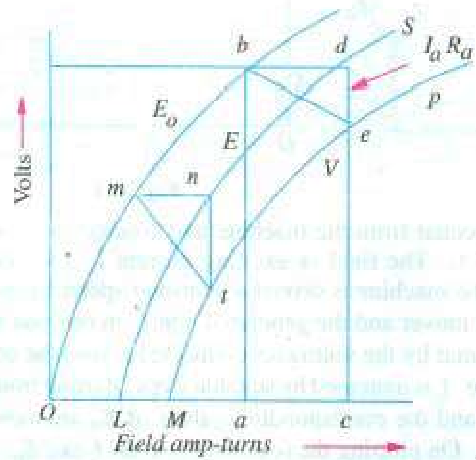


Fig. 28.2

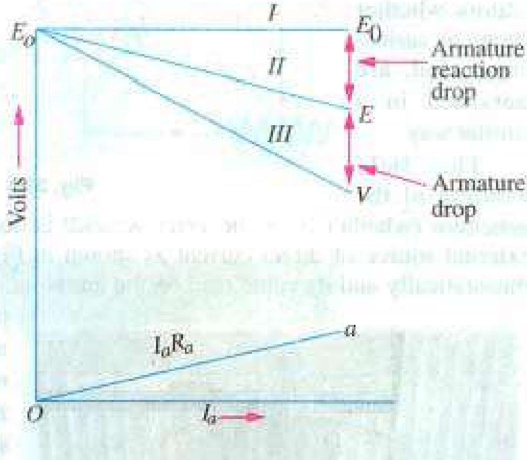


Fig. 28.3

(b) Internal and External Characteristics

Let us consider a separately-excited generator giving its rated no-load voltage of E_0 for a certain constant field current. If there were no armature reaction and armature voltage drop, then this voltage would have remained constant as shown in Fig. 28.3. by the dotted horizontal line I. But when the generator is loaded, the voltage falls due to these two causes, thereby giving slightly dropping characteristics. If we subract from E_0 the values of voltage drops due to armature reaction for different loads, then we get the value of E–the e.m.f. *actually* induced in the armature under load conditions. Curve II is plotted in this way and is known as the *internal characteristic*. The straight line Oa represents the I_aR_a drops corresponding to different armature currents. If we subtract from E the armature drop I_aR_a, we get terminal voltage V. Curve III represents the external characteristic and is obtained by subtracting ordinates the line Oa from those of curve II.

28.3. No-load Curve for Self-excited Generator

The O.C.C. or no-load saturated curves for self-excited generators whether shunt or series-connected, are obtained in a similar way.

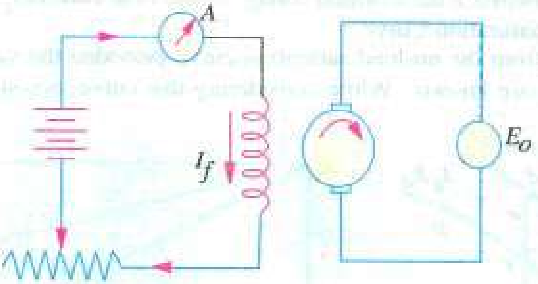


Fig. 28.4

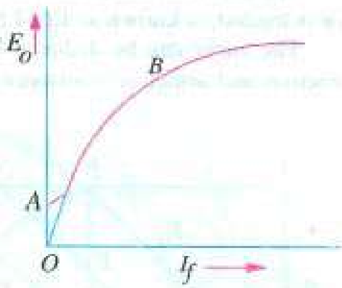
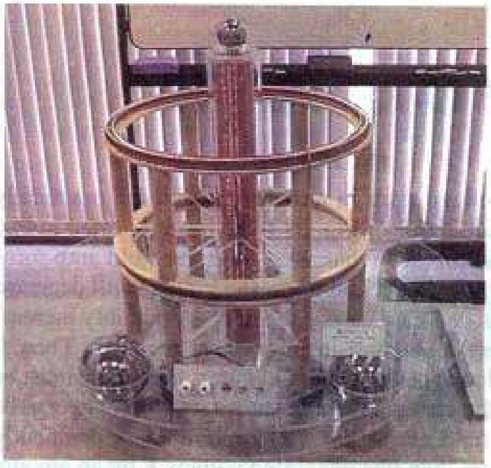


Fig. 28.5

The field winding of the generator (whether *shunt* or *series* wound) is disconnected from the machine and connected to an external source of direct current as shown in Fig. 28.4. The field or exciting current I_f is varied rheostatically and its value read on the ammeter A. The machine is drived at constant speed by the prime mover and the generated e.m.f. on on-load is measured by the voltmeter connected across the armature. I_f is increased by suitable steps (starting from zero) and the corresponding values of E_0 are measured. On plotting the relation between I_f and E_0, a curve of this form shown in Fig. 28.5 is obtained.



Self Excited Generator

Due to residual magnetism in the poles, some e.m.f. (= OA) is generated even when $I_f = 0$. Hence, the curve starts a little way up. The slight curvature at the lower end is due to magnetic inertia. It is seen that the first part of the curve is practically straight. This is due to the fact that at low flux densities, reluctance of iron path being negligible (due to high permeability), total reluctance is given by the air-gap reluctance which is constant. Hence, the flux and consequently, the generated e.m.f. is directly proportional to the exciting current. However, at high flux densities, – where μ is small, iron path reluctance becomes appreciable and straight relation between E and I_f no longer holds good. In other words, after point B, saturation of poles starts. However, the initial slope of the curve is determined by air-gap width.

It should be noted that O.C.C. for a higher speed would lie above this curve and for a lower speed, would lie below it.

It should also be noted and the load-saturation curve for a shunt generator is similar to the one shown in Fig. 28.2.

Critical Resistance for Shunt Generator

Now, connect the field windings back to the armature and run the machine as a shunt generator. Due to residual magnetism in the poles, some e.m.f. and hence current, would be generated. This current while passing through the field coils will strengthen the magnetism of the poles (provided field coils are properly connected as regards polarity). This will increase the pole flux which will further increase the generated e.m.f. Increased e.m.f. means more current which further increases the flux and so on. This mutual reinforcement of e.m.f. and flux proceeds on till equilibrium is reached at some point like P (Fig. 28.6). The point lies on the resistance line OA of the field winding. Let R be the resistance of the field winding. Line OA is drawn such that its slope equals the field winding resistance *i.e.* every point on this curve is such that volt/ampere = R.

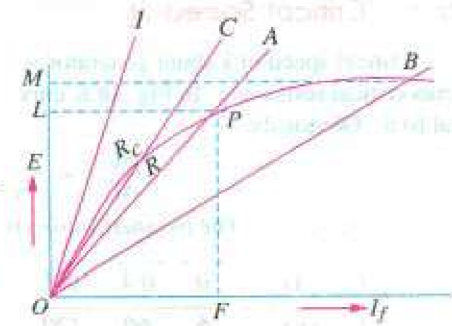


Fig. 28.6

The voltage OL corresponding to point P represents the maximum voltage to which the machine will build up with R as field resistance. OB represents smaller resistance and the corresponding voltage OM is slightly greater than OL. If field resistance is increased, then slope of the resistance line increased, and hence the maximum voltage to which the generator will build up at a given speed, decreases. If R is increased so much that the resistance line does not cut the $O.C.C.$ at all (like OT), then obviously the machine will fail to excite *i.e.* there will be no 'build up' of the voltage. If the resistance line just lies along the slope, then with that value of field resistance, the machine will ***just*** excite. The value of the resistance represented by the tangent to the curve, is known as **critical resistance** R_c for a ***given speed***.

28.4 How to Find Critical Resistance R_c ?

First, $O.C.C.$ is plotted from the given data. Then, tangent is drawn to its initial portion. The slope of this curve gives the critical resistance for the speed at which the data was obtained.

28.5 How to Draw $O.C.C.$ at Different Speeds ?

Suppose we are given the data for $O.C.C.$ of a generator run at a fixed speed, say, N_1. It will be shown that $O.C.C.$ at any other constant speed N_2 can be deduced from the $O.C.C.$ for N_1. In Fig. 28.7 the $O.C.C.$ for speed N_1 is shown.

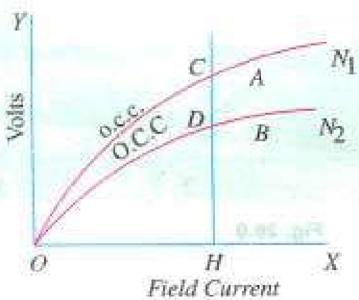


Fig. 28.7

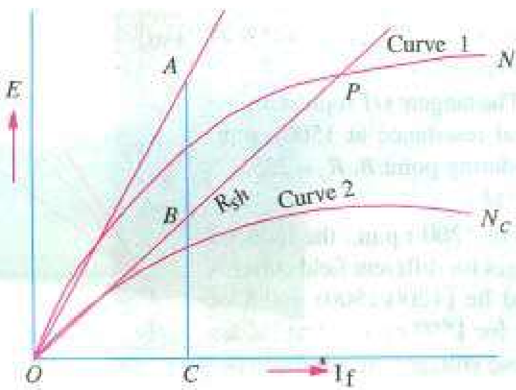


Fig. 28.8

Since $E \propto N$ for any fixed excitation, hence $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$ or $E_2 = E_1 \times \dfrac{N_2}{N_1}$

As seen, for $I_f = OH$, $E_1 = HC$. The value of new voltage for the same I_f but at N_2

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

In this way, point D is located. In a similar way, other such points can be found and the new $O.C.C.$ at N_2 drawn.

28.6. Critical Speed N_c

Critical speed of a shunt generator is that speed for which the given shunt field resistance represents critical resistance. In Fig. 28.8, curve 2 corresponds to critical speed because R_{sh} line is tangential to it. Obviously

$$\frac{BC}{AC} = \frac{N_c}{\text{Full Speed}} = \frac{N_c}{N} \quad \therefore \quad N_c = \frac{BC}{AC} \times \text{Full speed } N$$

Example 28.1. *The magnetization curve of a d.c. shunt generator at 1500 r.p.m. is :*

I_f (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_0 (V) :	6	60	120	172.5	202.5	221	231	237	240

For this generator find (i) no load e.m.f. for a total shunt field resistance of 100 Ω (ii) the critical field resistance at 1500 r.p.m. and (iii) the magnetization curve at 1200 r.p.m. and therefrom the open-circuit voltage for a field resistance of 100 Ω.

(b) A long shunt, compound generator fitted with interpoles is cummutatively-compounded. With the supply terminals unchanged, the machine is now run as compound motor. Is the motor differentially or cumulatively compounded ? Explain. **(Elect, Machines, A.M.I.E. Sec. B, 1990)**

Solution. The magnetisation curve at 1500 r.p.m. is plotted in Fig. 28.9 from the given data. The 100 Ω resistance line *OA* is obtained by joining the origin (0, 0) with the point (1A, 100 V). The voltage corresponding to point *A* is 227.5 V. Hence, no-load voltage to which the generator will build-up is 227.5 V.

The tangent *OT* represents the critical resistance at 1500 r.p.m. considering point *B*, R_c = 225/1.5 = 150 Ω.

For 1200 r.p.m., the induced voltages for different field currents would be (1200/1500) = 0.8 of those for 1500 r.p.m. The values of these voltages are tabulated below :

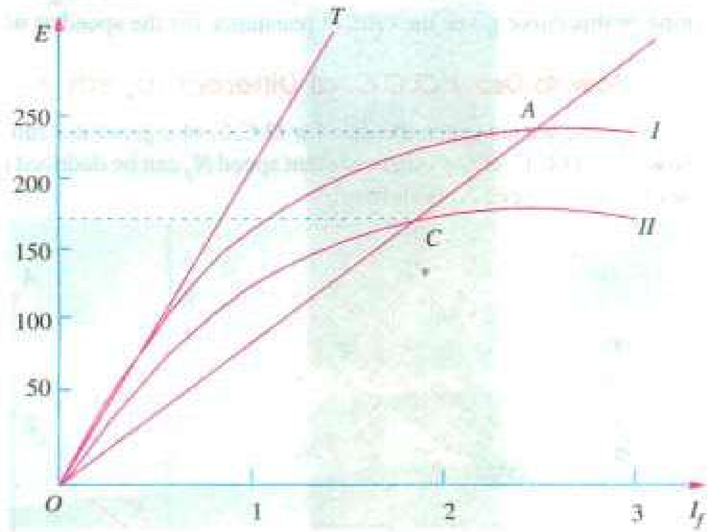


Fig. 28.9

I_f (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_0 (V) :	4.8	48	96	138	162	176.8	184.8	189.6	192

The new magnetisation curve is also plotted in Fig. 28.9. The 100 Ω line cuts the curve at point *C* which corresponds to an induced voltage of 166 V.

Example 28.2. *A shunt generator is to be converted into a level compounded generator by the addition of a series field winding. From a test on the machine with shunt excitation only, it is found that the shunt current is 3.1 A to give 400 V on no load and 4.8 A to give the same voltage when the machine is supplying its full load of 200 A. The shunt winding has 1200 turns per pole. Find the number of series turns required per pole.* **(Elect. Machines, A.M.I.E. Sec. B, 1989)**

Solution. At no-load the ampere turns required to produce 400 V = 3.1 × 1200 = 3720

On full-load ampere turns required to produce the same voltage = 4.8 × 1200 = 5760

Additional ampere turns required due to de-magnetising effect of load current = 5760 − 3720 = 2040.

If N is a number of series turns required when load current is 200 A, then

$$N \times 200 = 2040, N = 10.2$$

Example 28.3. *The open-circuit characteristic of a d.c. shunt generator driven at rated speed is as follows :*

Field Amperes :	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5 A*
Induced Voltage :	*60*	*120*	*138*	*145*	*149*	*151*	*152 V*

If resistance of field circuit is adjusted to 53 Ω, calculate the open circuit voltage and load current when the terminal voltage is 100 V. Neglect armature reaction and assume an armature resistance of 0.1 Ω. **(Electrical Technology Punjab Univ. Dec. 1989)**

Solution. Take I_f = 3 A, R_{sh} = 53 Ω.

Point A is (3A, 159V) point. Line OA is the 53 Ω line in Fig. 28.10. It cuts drop = 3 × 53 = 159 V. $O.C.C.$ at B. Line BM is drawn parallel to the base. OM represents the $O.C.$ voltage which equals **150 V.**

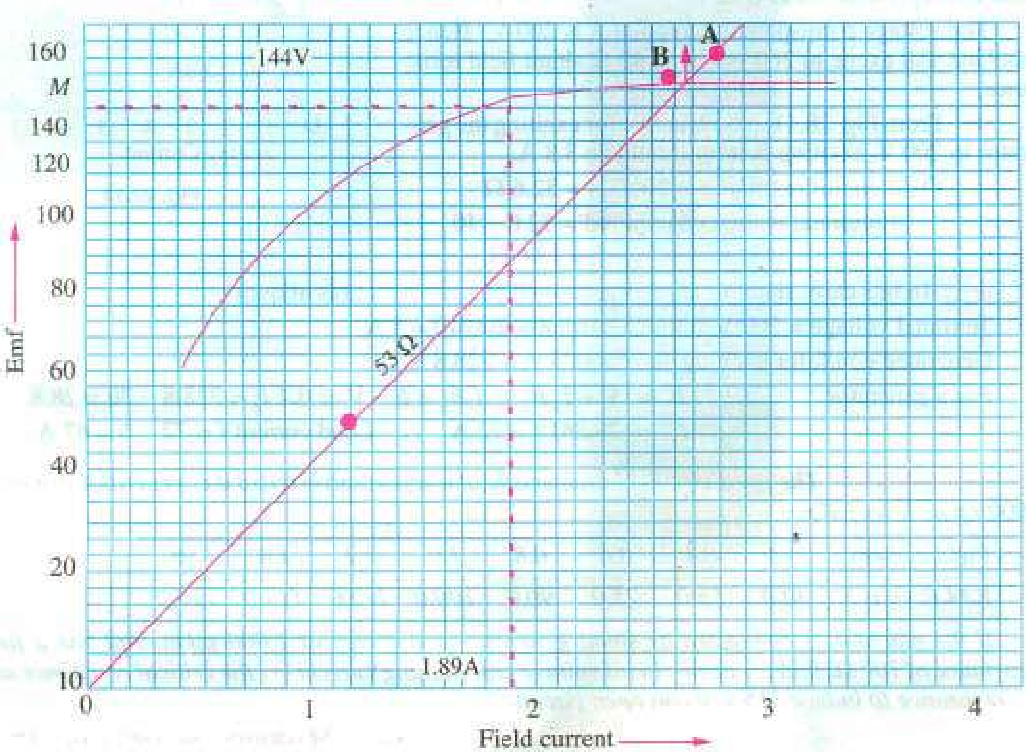


Fig. 28.10

Now, when V = 100 V, $I_{sh} = I_f$ = 100/53 = 1.89 A

Generated or $O.C.$ voltage corresponding to this exciting current as seen from graph of Fig. 26.10 is 144 V.

Now $E = V + I_a R_a$ or $I_a R_a$ = 144 − 100 = 44 V

∴ $0.1\, I_a = 44$ or I_a = 44/0.1 = **440 A**

Example 28.4. *The following figures give the O.C.C. of a d.c. shunt generator at 300 r.p.m.*

Field amperes :	0	2	3	4	5	6	7
Armature volt :	7.5	92	132	162	183	190	212

Plot the O.C.C. for 375 r.p.m. and determine the voltage to which the machine will excite if field circuit resistance is 40 Ω.

(a) What additional resistance would have to be inserted in the field circuit to reduce the voltage to 200 volts at 375 r.p.m.?

(b) Without this additional resistance, determine the load current supplied by the generator, when its terminal voltage is 200 V. Ignore armature reaction and assume speed to be constant. Armature resistance is 0.4 Ω. **(Elect. Machines - I, South Gujarat Univ. 1986)**

Solution. The e.m.f. induced at 375 r.p.m. would be increased in the ratio 375/300 corresponding to different shunt field current values. A new table is given with the voltages multiplied by the above ratio.

Field amperes :	0	2	3	4	5	6	7
Armature volt :	9.4	115	165	202.5	228.8	248.8	265

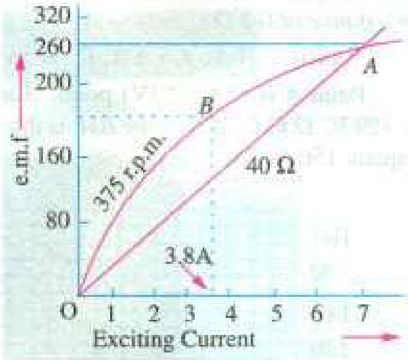


Fig. 28.11

The new *O.C.C.* at 375 r.p.m. is shown in Fig. 26.11. Line *OA* represents 40-Ω line.

The voltage corresponding to point *A* is 260 V. Hence machine will excite to 260 volt with 40 Ω shunt field resistance.

(a) From Fig. 28.11, it is clear that for exciting the generator to 200 V, exciting current should be 3.8 A.

∴ Field circuit resistance = 200/3.8 = 52.6 Ω

∴ Additional resistance required = 52.6 − 40 = **12.6 Ω**

(b) In this case, shunt field resistance = 40 Ω ...(as above)

Terminal voltage = 200 V ∴ Field current = 200/40 = 5 A

Generated e.m.f. for exciting current of 5 A = 228.8 V

For a generator $E = V + I_a R_a$ ∴ $I_a R_a = E - V$ or $0.4\, I_a = 228.8 - 20 = 28.8$

∴ $I_a = 28.8/0.4 = 72$ A ∴ Load current $I = 72 - 5 =$ **67 A**

Example 28.5. *The open-circuit characteristic of a separately-excited d.c. generator driven at 1000 r.p.m. is as follows :*

Field current :	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E.M.F. volts :	30.0	55.0	75.0	90.0	100.0	110.0	115.0	120.0

If the machine is connected as shunt generator and driven at 1,000 r.p.m. and has a field resistance of 100 Ω, find (a) open-circuit voltage and exciting current (b) the critical resistance and (c) resistance to induce 115 volts on open circuit.

(Elect. Machines, Nagpur, Univ. 1993)

Solution. The *O.C.C.* has been plotted in Fig. 28.12. The shunt resistance line *OA* is drawn as usual.

(a) *O.C.* voltage = **100 V**; Exciting current = **1 A**

(b) Line *OT* is tangent to the initial part of the *O.C.C.* It represents critical resistance. As seen from point *C*, value of critical resistance is 90/0.6 = **150 Ω.**

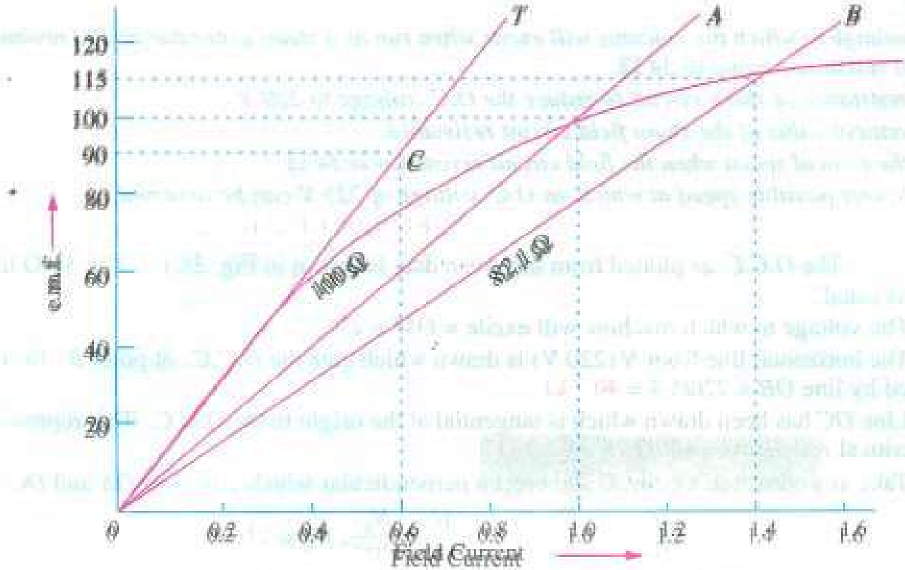


Fig. 28.12

(c) Line *OB* represents shunt resistance for getting 115 V on open-circuit. Its resistance = 115/1.4 = **82.1 Ω.**

Example 28.6. *A d.c. generator has the following magnetisation characteristics.*

Field current (A) :	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Generated e.m.f. (V) :	*23*	*45*	*67*	*85*	*100*	*112*	*121*	*126*

If the generator is shunt excited, determine the load current.

(a) *when terminal p.d. is 120 V, the field resistance is 15 Ω at a speed of 600 r.p.m. And*

(b) *when terminal p.d. is 144 V, the field resistance is 18 Ω at a speed of 700 r.p.m.*

Solution. (a) When terminal p.d. = 120 V, then field current $I_f = V/R_{sh} = 120/15 = 8\ \Omega$

From the given data, it is seen that the generated e.m.f. = 126 V*

54 303

Voltage drop = 126 − 120 = 6 V

Since drop due to armature reaction is neglected, this represents the armature drop.

$\therefore \quad I_a R_a = 6$ or $I_a = 6/0.02 = 300$ A Load current = 300 − 8 = **292 A**

(b) The *O.C.* data at 700 r.p.m. can be obtained by multiplying the given values of generated e.m.f. by a factor of 700/600 = 7/6. Hence, the new data is :

Field current (A)	:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Generated e.m.f. (V)	:	*26.8*	*52.5*	*78.2*	*99.2*	*116.6*	*131*	*141*	*146*

When $V = 144$ V and $R_{sh} = 18\ \Omega$, field current = 144/18 = 8 A

From the given data, the corresponding generated e.m.f. is 146 V.

Voltage drop $\quad I_a R_a = 146 - 144 = 2$V

$\therefore \quad I_a = 2/0.02 = 100$ A $\quad \therefore$ Load current = 100 − 8 = **92 A**

Example 28.7. *The O.C.C. of a d.c. generator driven at 400 rev/min is as follows :*

Field current (A)	:	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Terminal volts	:	*110*	*155*	*186*	*212*	*230*	*246*	*260*	*271*

* We need not plot the *O.C.C.* in this particular case.

Find :

(*a*) *voltage to which the machine will excite when run as a shunt generator at 400 rev/min with shunt field resistance equal to 34 Ω.*

(*b*) *resistance of shunt circuit to reduce the O.C. voltage to 220 V.*

(*c*) *critical value of the shunt field circuit resistance.*

(*d*) *the critical speed when the field circuit resistance is 34 Ω.*

(*e*) *lowest possible speed at which an O.C. voltage of 225 V can be obtained.*

(Electrical Technology, Bombay Univ. 1987)

Solution. The *O.C.C.* as plotted from the given data is shown in Fig. 28.13. The 34-Ω line *OA* is drawn as usual.

(*a*) The voltage to which machine will excite = *OM* = **255 V.**

(*b*) The horizontal line from *N* (220 V) is drawn which cuts the *O.C.C.* at point *B*. Resistance represented by line *OB* = 220/5.4 = **40.7 Ω.**

(*c*) Line *OC* has been drawn which is tangential at the origin to the *O.C.C.* This represents the value of critical resistance = 140/2.25 = **62.2 Ω.**

(*d*) Take any convenient point *D* and erect a perpendicular which cuts both *OA* and *OC*.

$$\frac{DE}{DF} = \frac{N_C}{400} \quad \text{or} \quad \frac{110}{202} = \frac{N_C}{202}, \; N_C = \mathbf{218\ r.p.m.}$$

(*e*) From point *P* (225 V) drawn a horizontal line cutting *OA* at point *G*. From *G*, draw a perpendicular line *GK* cutting the *O.C.C.* at point *H*. If *N'* is the lowest speed possible for getting 225 volt with 34 Ω shunt circuit resistance, then

$$\frac{GK}{HK} = \frac{N'}{400} \quad \text{or} \quad \frac{225}{241} \text{ or } N' = \mathbf{375\ r.p.m.}$$

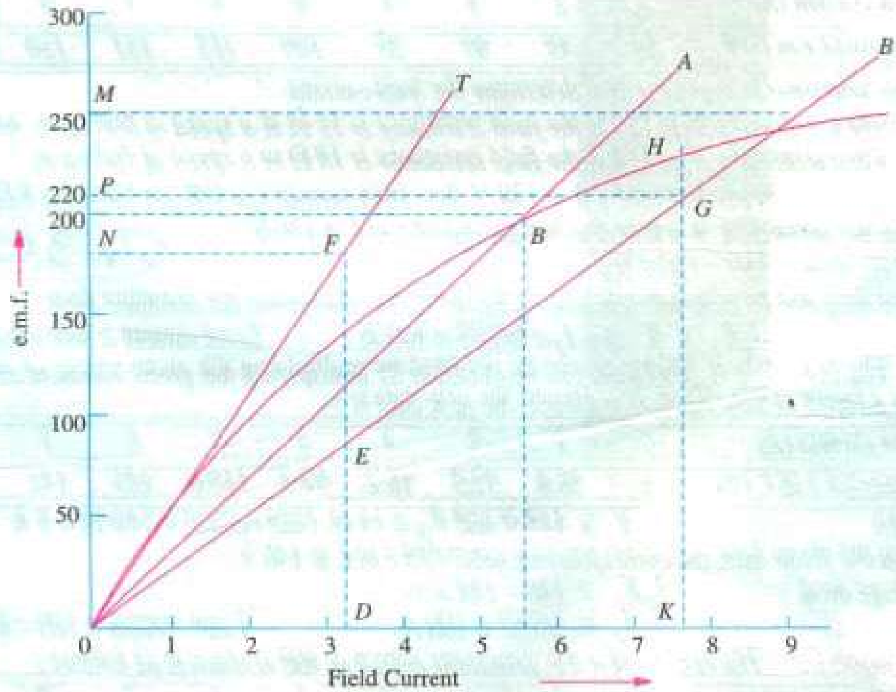


Fig. 28.13

Example 28.8. *The magnetization characteristic for a 4-pole, 110-V, 1000 r.p.m. shunt generator is as follows :*

Field current	0	0.5	1	1.5	2	2.5	3 A
O.C. voltage	5	50	85	102	112	116	120 V

Armature is lap-connected with 144 conductors. Field resistance is 45 ohms. Determine

(*i*) *voltage the machine will build up at no-load.*

(*ii*) *the critical resistance.*

(*iii*) *the speed at which the machine just fails to excite.*

(*iv*) *residual flux per pole.*

(Electrical Machinery- I, Mysore Unit. 1988)

Solution. In Fig. 28.14, *OA* represents the 45-Ω line which is drawn as usual.

(*i*) The voltage to which machine will build up = *OM* = **118 V.**

(*ii*) *OT* is tangent to the initial part of the *O.C.C.* It represents critical resistance. Take point *B* lying on this line. Voltage and exciting current corresponding to this point are 110 V and 1.1 A respectively.

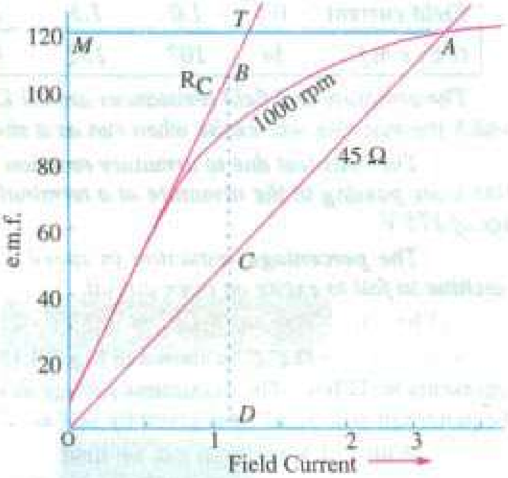


Fig. 28.14

$\therefore \qquad R_c = 110/1.1 = \mathbf{100\ \Omega}$

(*iii*) From any point on *OT*, say point *B*, drop the perpendicular *BD* on *X*-axis.

$$\frac{CD}{BD} = \frac{N_c}{1000} \quad \text{or} \quad \frac{49}{110} = \frac{N_c}{1000}$$

$\therefore \qquad N_c = \mathbf{445\ r.p.m.}$

(*iv*) As given in the table, induced e.m.f. due to residual flux (*i.e.* when there is no exciting current) is 5 V.

$$\therefore \qquad 5 = \frac{\Phi \times 144 \times 1000}{60}\left(\frac{4}{4}\right) \qquad \therefore\ \Phi = \mathbf{2.08\ mWb.}$$

Example 28.9. *A shunt generator gave the following results in the O.C.C. test at a speed of r.p.m.*

Field current (A) :	1	2	3	4	6	8	10
E.M.F. (volt) :	90	185	251	290	324	345	360

The field resistance is adjusted to 50 Ω and the terminal is 300 V on load. Armature resistance is 0.1 Ω and assuming that the flux is reduced by 5% due to armature reaction, find the load supplied by the generator. **(Electromechanic, Allahabad Univ.; 1992)**

Solution. When the terminal voltage is 300 V and R_{sh} = 50 Ω, then field current is

= 300/40 = 6 A

With this shunt current, the induced e.m.f. as seen from the given table (we need not draw the *O.C.C.*) is 324 V.

Due to armature reaction, the flux and hence the induced e.m.f. is reduced to 0.95 of its no-load value.

Hence, induced e.m.f. when generator is no load = 324 × 0.95 = 307.8 V

Armature drop at the given load = 307.8 − 300 = 7.8 V

$$I_a R_a = 7.8, \quad I_a = 7.8/0.1 = 78 \text{ A}$$

Load current = 78 − 6 = 72 A ; Generator output = 72 × 300/1000 = **21.6 kW**

Example 28.10. *A shunt generator gave the following open-circuit characteristic :*

Field current :	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
O.C. e.m.f. :	54	107	152	185	210	230	245 V

The armature and field resistances are 0.1 Ω and 80 Ω respectively. Calculate (a) the voltage to which the machine will excite when run as a shunt generator at the same speed.

(b) The volts lost due to armature reaction when 100 A are passing in the armature at a terminal voltage of 175 V.

(c) The percentage reduction in speed for the machine to fail to excite on open circuit.

(Electrical Machines-I, Bombay Univ. 1988)

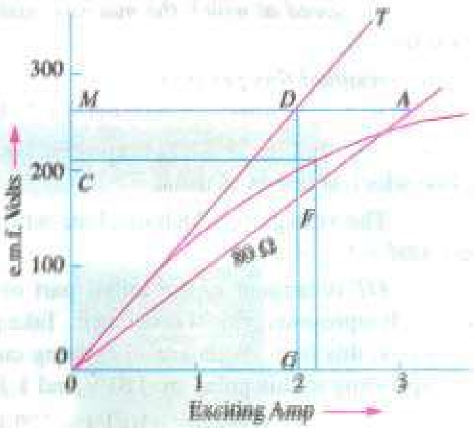


Fig. 28.15

Solution. (a) O.C.C. is shown in Fig. 28.15. *OA* represents 80 Ω line. The maximum voltage to which the generator will build up is given by *OM* = **222 V.**

(b) With 175 V terminal p.d. on load

$$I_{sh} = 175/80 = 2.2 \text{ A}$$

Voltage corresponding to this field current is given by *OC* = 195 V.

Voltage lost due to armature reaction and armature drop = 195 − 175 = 20 V.

Now, armature drop = 0.1 × 100 = 10 V

Let '*x*' be the volts lost due to armature reaction.

Then 10 + *x* = 20 ∴ *x* = **10 V**

(c) Line *OT* is drawn tangential to the curve. *DFG* is perpendicular to the base line.

$$\frac{N_c}{N} = \frac{FG}{DG} = \frac{160}{220} \quad \text{or} \quad \frac{N_c - N}{N} = \frac{-60}{220}$$

$$\text{Percentage reduction in speed} = \frac{-60}{220} \times 100 = \mathbf{-27.3\ \%}$$

Example 28.11. *The O.C.C. of a shunt generator running at 800 r.p.m. is as follows :*

Field current (amp.)	:	1	2	3	4	5	6
Induced e.m.f. (volt).	:	82.5	180	225	252	273	282

If the shunt field resistance is 60 Ω, find

(i) the voltage to which the machine will build up running at the same speed (ii) the value of field regulating resistance if the machine is to build up to 120 V when its field coils are grouped in two parallel circuits and generator is runing at half the speed.

(Electrical Technology, Hyderabad Univ. 1991)

Solution. *O.C.C.* is drawn from the given data and is shown in Fig. 28.16. *OA* represents 60 Ω. The voltage corresponding to point *A* is *OM* = 260 V.

(i) Generator will build up to **260 volts.**

(ii) Lower curve represents the induced e.m.f. for different exciting current values at 400 r.p.m. From point *B* which represents 120 V, draw *BC*. From *C* draw a perpendicular *CD* which gives the exciting current of 3.6 A. It means that current through *each* parallel path of shunt field coils is 3.6 A. Total current which passes through the field regulator resistance is $3.6 \times 2 = 7.2$ A because it is in series with the field coils.

Hence, total shunt field resistance = 120/7.2 = 16.67 Ω

Now, resistance of each shunt parallel path = 60/2 = 30 Ω

Joint resistance of two parallel paths = 30/2 = 15 Ω

∴ Shunt field regulator resistance = 16.67 − 15 = **1.67 Ω**

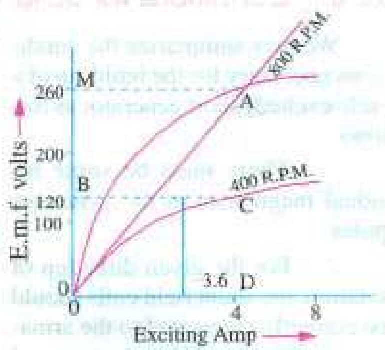


Fig. 28.16

28.7. Voltage Build up of a Shunt Generator

Before loading a shunt generator, it is allowed to build up its voltage. Usually, there is always present some residual magnetism in the poles, hence a small e.m.f. is produced initially. This e.m.f. circulates a small current in the field circuit which increases the pole flux (provided field circuit is properly connected to armature, otherwise this current may wipe off the residual magnetism). When flux is increased, generated e.m.f. is increased which further increases the flux and so on. As shown in Fig. 28.17, *Oa* is the induced e.m.f. due to residual magnetism which appears across the field circuit and causes a field current *Ob* to flow. This current aids residual flux and hence produces, a larger induced e.m.f. *Oc*. In turn, this increased e.m.f. *Oc* causes an even larger current *Od* which creates more flux for a still larger e.m.f. and so on.

Now, the generated e.m.f. in the armature has

(*a*) to supply the ohmic drop $I_f R_{sh}$ in the winding and (*b*) to overcome the opposing self-induced e.m.f. in the field coil *i.e.* $L.\ (d\,I_f/\,dt)$because field coils have appreciable self-inductance.

$$e_g = I_f R_{sh} + L\ .\ d\,I_f/dt$$

If (and so long as), the generated e.m.f. is in excess of the ohmic drop $I_f\, R_{sh}$, energy would continue being stored in the pole fields. For example, as shown in Fig. 28.17, corresponding to field current *OA*, the generated e.m.f. is *AC*. Out of this, *AB* goes to supply ohmic drop $I_f R_{sh}$ and *BC* goes to overcome self-induced e.m.f. in the coil. Corresponding to $I_f = OF$, whole of the generated e.m.f. is used to overcome the ohmic drop. None is left to overcome $L.dI_f/dt$. Hence no energy is stored in the pole fields. Consequently, there is no further increase in pole flux and the generated e.m.f. With the given shunt field resistance represented by line *OP*, the maximum voltage to which the machine will build up is *OE*. If resistance is decreased, it will built up to a somewhat higher voltage. *OR* represents the resistance known as ***critical resistance***. If shunt field resistance is greater than this value, the generator will fail to excite.

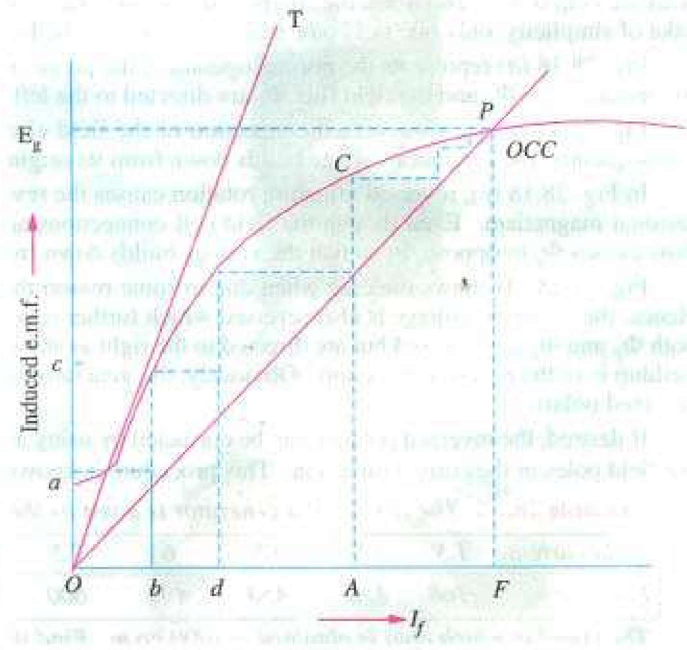


Fig. 28.17

28.8. Conditions for Build-up of a Shunt Generator

We may summarize the conditions necessary for the build-up of a (self-excited) short generator as follows :

1. There must be some residual magnetism in the generator poles.
2. For the given direction of rotation, the shunt field coils should be correctly connected to the armature *i.e.* they should be so connected that the induced current reinforces the e.m.f. produced initially due to residual magnetism.
3. If excited on open circuit, its shunt field resistance should be less than the critical resistance (which can be found from its *O.C.C.*)

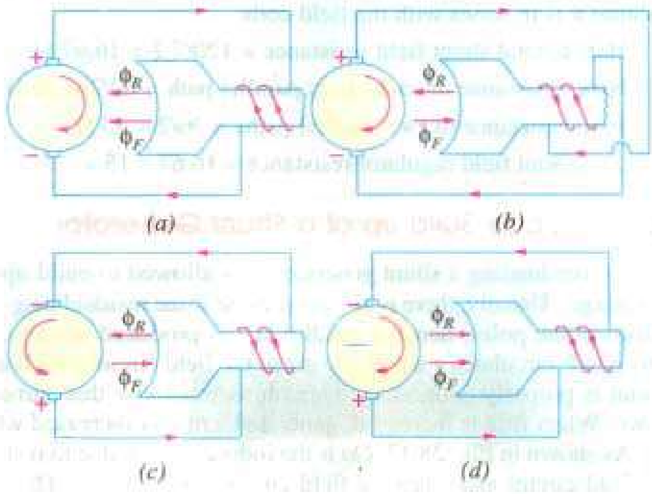


Fig. 28.18

4. If excited on load, then its shunt field resistance should be more than a certain minimum value of resistance which is given by internal characteristic (Art 28.11).

28.9. Other Factors Affecting Voltage Building of a DC Generator

In addition to the factors mentioned above, there are some other factors which affect the voltage building of a self-excited d.c. generator. These factors are *(i)* reversed shunt field connection *(ii)* reversed rotation and *(iii)* reversed residual magnetism. These adverse effects would be explained with the help of Fig. 28.18 and the right-hand rule for finding the direction of the coil flux. For the sake of simplicity, only one field pole has been shown in the Fig. 28.18.

Fig. 28.18 *(a)* represents the normal operation, the prime mover rotation is clockwise and both the residual flux Φ_R and the field flux Φ_F are directed to the left.

Fig. 28.18 *(b)* shows reversed connection of the field circuit which causes Φ_F to oppose Φ_R. Consequently, the generator voltage builds down from its original residual value.

In Fig. 28.18 *(c)*, reversed armature rotation causes the reversal of the voltage produced by the residual magnetism. Even though the field coil connections are correct, the reversed field current flow causes Φ_F to oppose Φ_R so that the voltage builds down from its original residual value.

Fig. 28.18 *(d)* shows the case when due to some reason the residual magnetism gets reversed. Hence, the armature voltage is also reversed which further reverses the field current. Consequently, both Φ_F and Φ_R are reversed but are directed to the right as shown. Under this condition, the voltage buildup is in the reversed direction. Obviously, the generator will operate at rated voltage but with reversed polarity.

If desired, the reversed polarity can be corrected by using an external d.c. source to remagnetise the field poles in the correct direction. This procedure is known as field flashing.

Example 28.12. *The O.C.C. of a generator is given by the following :*

Field current :	*1.5*	*3.5*	*4.5*	*6*	*7.5*	*9*	*10.5*
E.M.F. :	*168*	*330*	*450*	*490*	*600*	*645*	*675*

The speed at which data is obtained is 1000 r.p.m. Find the value of the shunt field resistance that will give a p.d. of 600 V with an armature current of 300 A at the same speed. Due to armature *reaction, the shunt field current is given by I_{sh} (eff.) $= I_{sh} - 0.003\ I_a$. Armature resistance, including brush contact resistance, is 0.1 Ω. What will be the p.d. on open circuit at the same speed ?*

Solution. As shown in Fig. 28.19, the *O.C.C.* has been plotted from the given data.

Voltage drop due to armature resistance = 300 × 0.1 = 30 V.

Reduction of field current due to armature reaction = 0.003 × 300 = 0.9 A.

Any point *A* is taken on the *O.C.C.* A vertical distance *AB* = 30 V is taken and then the horizontal line *BC* = 0.9 A is drawn thus completing triangle *ABC* which is known as drop reaction triangle. Then, point *C* lies on the 300 ampere load saturation curve. This curve can be drawn by finding such similar points like *C′* etc.

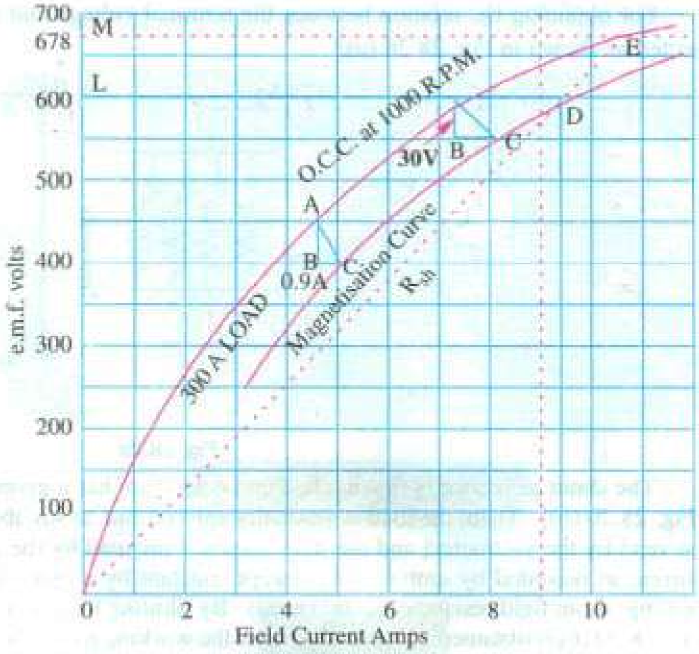


Fig. 28.19

From point *L* representing 600 V, a horizontal line is drawn cutting the load saturation curve at *D*. Join *OD*. Current corresponding to point *D* is 9.4 A. The slope of I_f the line *OD* gives the value of shunt resistance to give 600 V with 300 amperes of armature current.

$\therefore \quad R_{sh} = 600/9.4 =$ **63.8 Ω**

It is seen that e.m.f. on open circuit is **678 V.**

28.10. External Characteristic

After becoming familiar with the no-load characteristic of a shunt generator, we will now proceed to find its external characteristic (*V/I*) when loaded. It is found that if after building up, a shunt generator is loaded, then its terminal voltage *V* drops with increase in load current. Such a drop in voltage is undesirable especially when the generator is supplying current for light and power for which purpose it is desirable that *V* should remain practically constant and independent of the load. This condition of constant voltage is almost impossible to be fulfilled with a shunt generator unless the field current is being automatically adjusted by an automatic regulator. Without such regulation terminal voltage drops considerably as the load on the generator is increased. These are three main reasons for the drop in terminal voltage of a shunt generator when under load.

(*i*) Armature resistance drop :

As the load current increases, more and more voltage is consumed in the ohmic resistance of the armature circuit. Hence, the terminal voltage $V = E - I_a R_a$ is decreased where *E* is the induced e.m.f. in the armature under load condition.

(*ii*) Armature reaction drop

Due to the demagnetising effect of armature reaction, pole flux is weakened and so the induced e.m.f. in the armature is decreased.

(*iii*) The drop in terminal voltage *V* due to (*i*) and (*ii*) results in a decreased field current I_f which further reduces the induced e.m.f.

For obtaining the relation between the terminal voltage and load current, the generator is connected as shown in Fig. 28.20 (*a*).

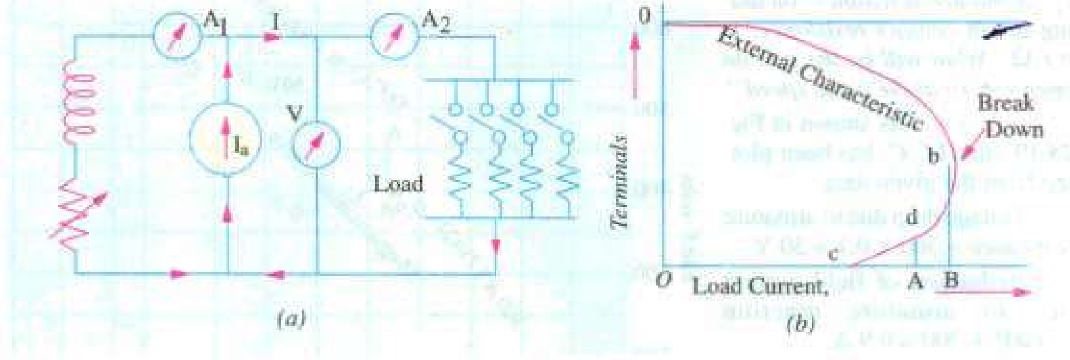


Fig. 28.20

The shunt generator is first excited on no-load so that it gives its full open circuit voltage = *Oa* [Fig. 28.20 (*b*)]. Then, the load is gradually applied and, at suitable intervals, the terminal voltage *V* (as read by the voltmeter) and the load current *I* (as read by the ammeter A_2) are noted. The field current as recorded by ammeter A_1 is kept constant by a rheostat (because during the test, due to heating, shunt field resistance is increased). By plotting these readings, the external characteristic of Fig. 28.20 (*b*) is obtained. The portion *ab* is the working part of this curve. Over this part, if the load resistance is decreased, load current is increased as usual, although this results in a comparatively small additional drop in voltage. These conditions hold good till point *b* is reached. This point is known as breakdown point. It is found that beyond this point (where load is maximum = *OB*) ***any effort to increase load current by further decreasing load resistance results in decreased load current (like OA) due to a very rapid decrease in terminal voltage.***

We will discuss the reason for this unusual behaviour of the generator in more details. Over the earlier portion *ab* [Fig. 28.20 (*b*)] where the load current is comparatively small, when external load resistance is decreased, it results in increased load current as might be expected keeping Ohm's law in mind. It should not, however, be forgotten that due to increase in load current, *V* is also decreased somewhat due to the cause (*iii*) given above. But over the portion *ab*, the effect of decrease in load resistance predominates the effect of decrease in *V* because load current is relatively small.

At point *b*, generator is delivering a very large current *i.e.* current which is many times greater than its normal current. If load resistance is decreased at this point so as to be able to draw a load current greater than *OB*, the current is ***increased*** momentarily. But due to the severe armature reaction for this heavy current and increased $I_a R_a$ drop, the terminal voltage *V* is drastically reduced. The effect of this drastic reduction in *V* results in less load current (= *OA*). In other words, over the portion *bdc* of the curve, the terminal voltage *V* decreases more rapidly than the load resistance. Hence, any, further decrease in load resistance actually causes a***decrease*** in load current (it may seem to contravene Ohm's law but this law is not applicable, here since *V* is not constant). As load resistance is decreased beyond point *b*, the curve turns back till when the generator is actually short-circuited, it cuts the current axis at point *c*. Here, terminal voltage *V* is reduced to zero, though there would be some value of *E* due to residual magnetism (Fig. 28.22).

28.11. Voltage Regulation

By voltage regulation of a generator is meant the change in its terminal voltage with the change in load current when it is running at a constant speed. If the change in voltage between no-load and full load is small, then the generator is said to have good regulation but if the change in voltage is large, then it has poor regulation. The ***voltage regulation of a d.c. generator is the change in voltage when the load is reduced from rated value to zero, expressed as percentage of the rated load***

voltage.

If no-load voltage of a certain generator is 240 V and rated-load voltage is 220 V,

then, $\quad$ regn. $= (240 - 220)/220 = 0.091$ or 9.1 %

28.12. Internal or Total Characteristic

As defined before, internal characteristic gives the relation between E and I_a. Now in a shunt generator

$$I_a = I + I_f \quad \text{and} \quad E = V + I_a R_a$$

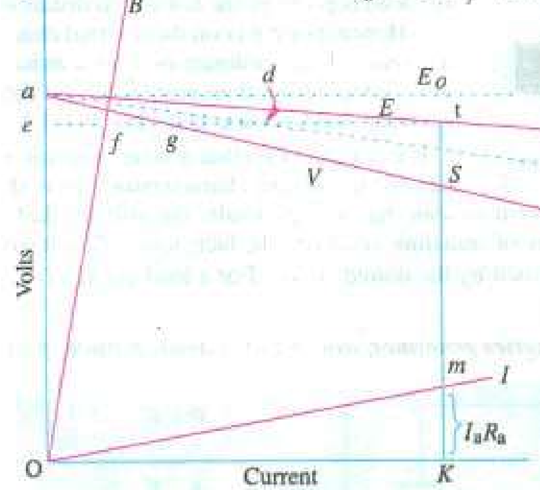


Fig. 28.21

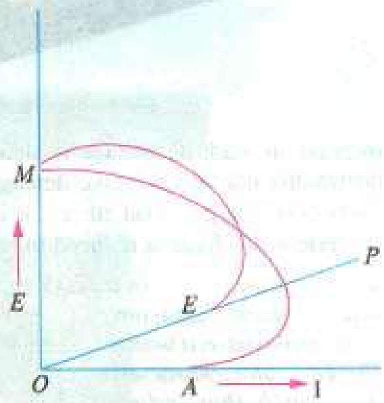


Fig. 28.22

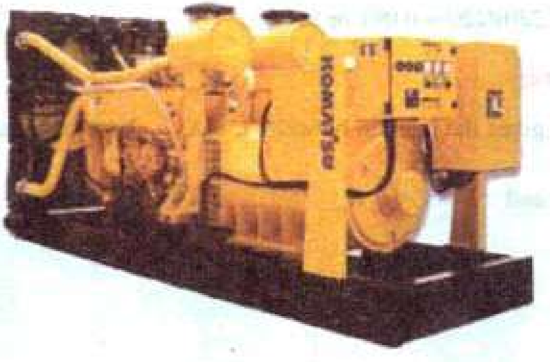
Hence, E/I_a curve can be obtained from V/I curve as shown in Fig. 28.21. In this figure, ab represents the external characteristic as discussed above. The field resistance line OB is drawn as usual. The horizontal distances from OY line to the line OB give the values of field currents for different terminal voltages. If we add these distances horizontally to the external characteristic ab, then we get the curve for the total armature current *i.e.* dotted curve ac. For example, point d on ac is obtained by making $gd = ef$. The armature resistance drop line Or is then plotted as usual. If brush contact resistance is assumed constant, then armature voltage drop is proportional to the armature current. For any armature current $= OK$, armature voltage drop $I_aR_a = mK$. If we add these drops to the ordinates of curve ac, we get the internal characteristic. For example, $St = mK$. The point t lies on the internal characteristic. Other points like t can be found similarly at different armature currents as the total characteristic can be drawn.

It may be noted here, in passing, that product EI_a gives the total power developed within the armature. Some of this power goes to meet I^2R losses in armature and shunt field windings and the rest appears as output.

As explained in Art 28.10 if load resistance is decreased, the armature current increases up to a certain load current value. After that, any decrease in load resistance is not accompanied by increase in load current. Rather, it is decreased and the curve turns back as shown in Fig. 28.22. If the load resistance is too small, then the generator is short-circuited and there is no generated e.m.f. due to heavy demagnetisation of main poles.

Line OP is tangential to the internal characteristic MB and its slope gives the value of the ***minimum*** resistance with which the generator will excite if excited ***on load.***

28.13. Series Generator



Series generator

In this generator, because field windings are in series with the armature [Fig. 28.23 (*a*)], they carry full armature current I_a. As I_a is increased, flux and hence generated e.m.f. is also increased as shown by the curve. Curve *Oa* is the *O.C.C.* The extra exciting current necessary to neutralize the weakening effect of armature reaction at full load is given by the horizontal distance *ab*. Hence, point *b* is on the internal characteristic. If the ordinate *bc* = *gh* = armature voltage drop, then point *c* lies on the external characteristic [Fig. 28.23 (*b*)].

It will be noticed that a series generator has rising voltage characteristic *i.e.* with increase in load, its voltage is also increased. But it is seen that at high loads, the voltage starts decreasing due to excessive demagnetising effects of armature reaction. In fact, terminal voltage starts decreasing as load current is increased as shown by the dotted curve. For a load current *OC′*, the terminal voltage is reduced to zero as shown.

Example 28.13. *In a 220 V supply system, a series generator, working on a linear portion of its magnetisation characteristic is employed as a booster. The generator characteristic is such that induced e.m.f. increases by 1 volt for every increase of 6 amperes of load current through the generator. The total armature resistance of the generator is 0.02 Ω. If supply voltage remains constant, find the voltage supplied to the consumer at a load current of 96 A. Calculate also the power supplied by the booster itself.*

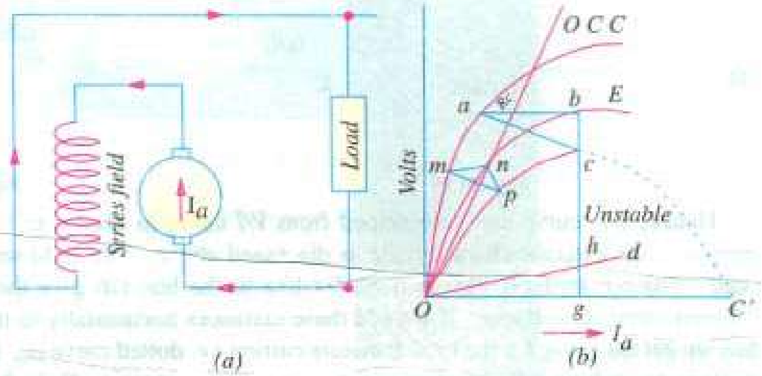


Fig. 28.23

Solution.

Voltage increase for 6 amperes = 1 V ∴ Voltage increase for 96 A = 96/6 = 16 V

Voltage drop in series coils = 96 × 0.02 = 1.9 V

Net Voltage rise due to booster = 16 − 1.9 = 14.1 V

Voltage at consumer end = 220 + 14.1 = **234.1 V**

Power supplied by booster itself = 14.1 × 96 = 1354 W = **1.354 kW**

Example 28.14. *A d.c. series generator, having an external characteristic which is a straight line through zero to 50 V at 200 A is connected as a booster between a station bus bar and a feeder of 0.3 ohm resistance. Calculate the voltage difference between the station bus-bar and the far end of the feeder at a current of (i) 200 A and (ii) 50 A.*

(AIME Sec. B Elect. Machine Summer 1991)

Solution. (*i*) Voltage drop = 200 × 0.3 = 60 V

Booster voltage provided by series generator for 200 A current as given = 50 V.

$\therefore$ Net voltage decrease $= 60 - 50 = 10$ V

(ii) Feeder drop $= 50 \times 0.3 = 15$ V

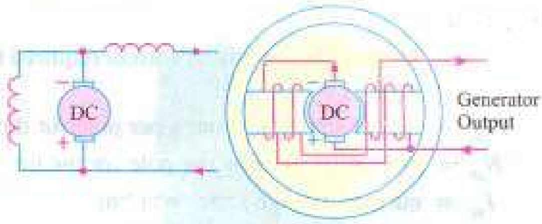
Booster voltage provided by series generator (by proportion) is

$= 50 \times 50/200 = 12.5$ V

$\therefore$ Net decrease in voltage $= 15 - 12.5 = 2.5$ V

28.14. Compound-wound Generator

A shunt generator is unsuitable where constancy of terminal voltage is essential, because its terminal voltage decreases as the load on it increases. This decrease in V is particularly objectionable for lighting circuit where even slight change in the voltage makes an appreciable change in the candle power of the incandescent lamps. A shunt generator may be made to supply substantially constant voltage (or even a rise in voltage as the load increases) by adding to it a few turns joined in series with either the armature or the load (Fig. 28.24). These turns are so connected as to aid to shunt turns when the generator supplies load. As the load current increases, the current through the series windings also increase thereby increasing the flux. Due to the increase in flux, induced e.m.f. is also increased. By adjusting the number of series turns (or series amp-turns), this increase in e.m.f. can be made to balance the combined voltage drop in the generator due to armature reaction and the armature drop. Hence, V remains practically constant which means that field current is also almost unchanged. We have already discussed the three causes which decrease the terminal voltage of a shunt generator (Art 28.10). Out of these three, the first two are neutralized by the series field amp-turns and the third one, therefore, does not occur.



Compound-wound generator

If the series field amp-turns are such as to produce the same voltage at rated load as at no-load, then the generator is ***flat-compounded***. It should be noted, however, that even in the case of a flat-compounded generator, the voltage is not constant from no-load to rated- load. At half the load, the voltage is actually greater than the rated voltage as seen from Fig. 28.24.

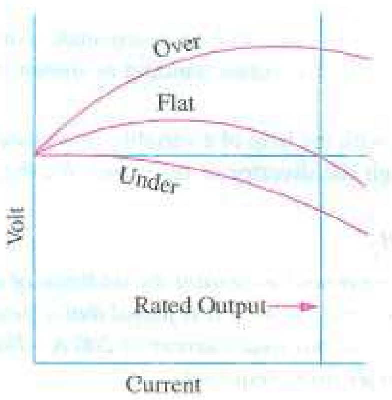


Fig. 28.24

If the series field amp-turns are such that the rated-load voltage is greater than the no-load voltage, then generator is ***over-compounded***. If rated-load voltage is less than the no-load voltage, then the generator is ***under-compounded*** but such generators are seldom used.

For short distances such as in hotels and office buildings, flat-compound generators are used because the loss of voltage over small lengths of the feeder is negligible. But when it is necessary to maintain a constant voltage then an over-compounded generator, which combines the functions of a generator and a booster, is invariably used.

28.15. How to Calculate Required Series Turns ?

Consider a 110-V, 250-ampere generator. Suppose it gives its rated no-load voltage with a field current of 5.8 A. If, now, the series windings are disconnected and the shunt field rheostat is left unchanged then the machine will act as shunt generator, hence its voltage will fall with increase in load current. Further, supply that the field current has to be increased to 6.3 A in order to maintain the rated terminal voltage at full load. If the number of turns of the shunt field winding is 2000, then 2000 × (6.3 − 5.8) = 1000 amp-turns represent the additional excitation that has to be supplied by the series windings. As series turns will be carrying a full load current of 250 A, hence number of series turns = 1000/250 = 4.

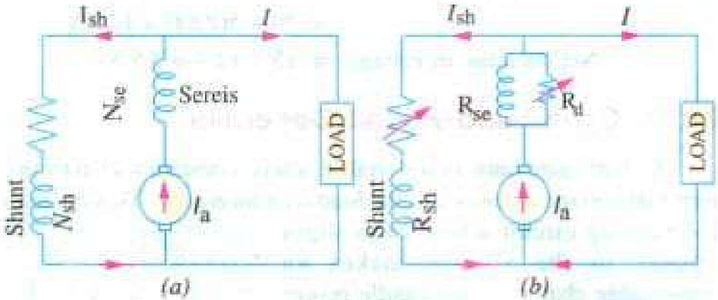


Fig. 28.25

In general, let

ΔI_{sh} = increase in shunt field current required to keep voltage constant from no-load to full-load

N_{sh} = No. of shunt field turns per pole (or the total number of turns)

N_{se} = No. of series turns per pole (or the total number of turns)

I_{se} = current through series winding

= armature current I_a —for long-shunt

= load current I —for short-shunt

It is seen that while running as a simple generator, the increase in shunt field ampere-turns necessary for keeping its voltage constant from no-load to full-load is $N_{sh} \cdot \Delta_{sh}$. This increase in field excitation can be alternatively achieved by adding a few series turns to the shunt generator [Fig. 28.25 (*a*)] thereby converting it into a compound generator.

$$\therefore \qquad N_{sh} \cdot \Delta I_{sh} = N_{se} I_{se}$$

If other things are known, N_{se} may be found from the above equation.

In practice, a few extra series amp-turns are taken in order to allow for the drop in armature. Any surplus amp-turns can be changed with the help of a divertor across the series winding as shown in Fig. 28.25 (*b*).

As said above, the degree of compounding can be adjusted with the help of a variable-resistance, divertor as shown in Fig. 28.25 (*b*). If I_d is the current through the divertor of resistance R_d, then remembering that series windings and divertor are in parallel,

$$\therefore \qquad I_{se} \cdot R_{se} = I_d R_d \quad \text{or} \quad R_d = I_{se} R_{se} / I_d$$

Example 28.15. *A shunt generator is converted into a compound generator by addition of a series field winding. From the test on the machine with shunt excitation only, it is found that a field current of 5 A gives 440 V on no-load and that 6 A gives 440 V at full load current of 200 A. The shunt winding has 1600 turns per pole. Find the number of series turns required.*

(Elect. Machines, A.M.I.E., Sec., B, 1991)

Solution. It would be assumed that shunt generator is converted into a short shunt compound generator. It is given that for keeping the voltage of shunt generator constant at 440 V both at no-load and full-load, shunt field ampere-turns per pole have to be increased from 1600 × 5 = 8000 to

$(1600 \times 6) = 9600$ *i.e.* an increase of $(9600 - 8000) = 1600$ *AT*. The same increase in field *AT* can be brought about by adding a few series turns.

Let n be the number of series turns required per pole. Since they carry 200 A,

$$\therefore \quad n \times 200 = 1600; \; n = \mathbf{8\ turns/pole}$$

Example 28.16. *A long shunt compound generator has a shunt field winding of 1000 turns per pole and series field winding of 4 turns per pole and resistance 0.05 Ω. In order to obtain the rated voltage both at no-load and full-load for operation as shunt generator, it is necessary to increase field current by 0.2 A. The full-load armature current of the compound generator is 80 A. Calculate the divertor resistance connected in parallel with series field to obtain flat compound operation.*

(Elect. Machines A.M.I.E. Sec. B, 1993)

Solution.

Additional *AT* required to maintain rated voltage both at no-load and full-load (Fig. 28.26) $= 1000 \times 0.2 = 200$

No. of series turns/pole = 4

Current required to produce 200 *AT* by the series field $= 200/4 = 50$ A.

Since $I_a = 80$ A, the balance of 30 *A* must pass through the parallel divertor resistance.

$$\therefore \quad 30\,R = 50 \times 0.05, \; R = 0.0833\ \Omega$$

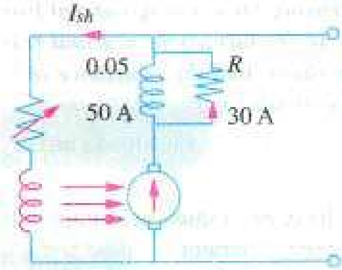


Fig. 28.26

Example 28.17. *A 220-V compound generator is supplying a load of 100 A at 220 V. The resistances of its armature, shunt and series windings are 0.1 Ω, 50 Ω and 0.06 Ω respectively. Find the induced e.m.f. and the armature current when the machine is connected (a) short shunt (b) long shunt (c) how will the series amp-turns be changed in (b) if a divertor of 0.14 Ω is connected in parallel with the series windings ? Neglect armature reaction and brush contact drop.*

Solution. (*a*) Short-shunt (Fig. 28.27).

$$\text{Voltage drop in series} = 100 \times 0.06 = 6\ \text{V}; \; I_{sh} = 220/50 = 4.4\ \text{A}.$$

$$\therefore \quad I_a = 100 + 4.4 = \mathbf{104.4}$$

$$\text{Armature drop} = 104.4 \times 0.1 = 10.4\ \text{V}$$

$$\therefore \quad \text{Induced e.m.f.} = 220 + 6 + 10.5 = \mathbf{236.4\ V}$$

(*b*) Long-shunt (Fig. 28.28)

$$I_{sh} = 220/50 = 4.4\ \text{A} \quad \therefore \quad I_a = 100 + 4.4 = \mathbf{104.4\ A}$$

Voltage drop over armature and series field winding $= 104.4 \times 0.16 = 16.7$ V

$$\therefore \quad \text{Induced e.m.f.} = 200 + 16.7 = \mathbf{216.7\ V}$$

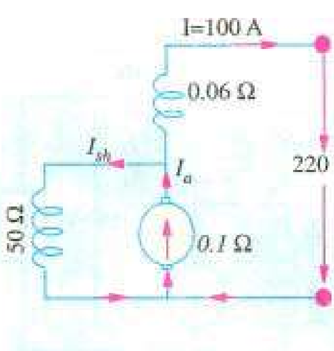


Fig. 28.27

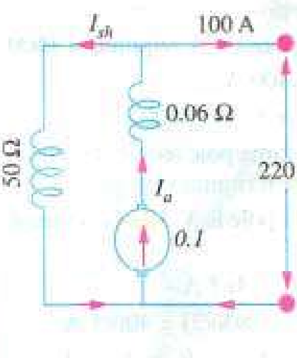


Fig. 28.28

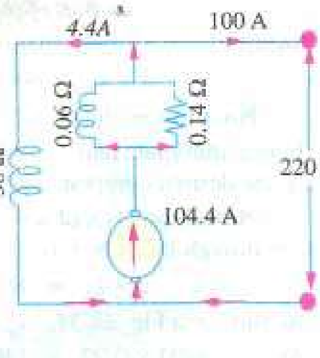


Fig. 28.29

(c) As shown in Fig. 28.29, a divertor of resistance 0.14 Ω is connected in parallel with the series field winding. Let n be the number of series turns.

Number of series amp-turns without divertor = $n \times 104.4 = 104.4\ n$

When divertor is applied, then current through series field is

$$= \frac{104.4 \times 0.14}{(0.14 + 0.06)} = 73.8\ A$$

∴ Series amp-turns = $73.8 \times n$ ∴ Series amp-turns are reduced to $\frac{73.8\ n}{104.4\ n} \times 100 =$ **70 %**

Example 28.18. *A 250-kW, 240-V generator is to be compounded such that its voltage rises from 220 volts at no-load to 240 V at full load. When series field is cut out and shunt field is excited from an external source, then from the load test it is found that this rise in voltage can be obtained by increasing the exciting current from 7 A at no-load to 12 A at full-load. Given shunt turns/pole = 650, series turns/pole = 4 and resistance of series winding, 0.006 Ω. If the machine is connected long-shunt, find the resistance of the series amp-turns at no-load and drop in series winding resistance at full-load.*

Solution. Full-load current $= 250 \times 10^3/240$

$= 1042$ A

Increase in shunt field ampere-turns to over-compound the shunt generator

$= 650\,(12 - 7) = 3{,}250$. As seen from Fig. 28.30

$4 \times I_{se} = 3250$

$I_{se} = 3250/4 = 812.5$ A

∴ $I_d = 1042 - 812.5 = 229.5$ A

It is so because no-load shunt current being negligible, $I_a = I = 1042$ A.

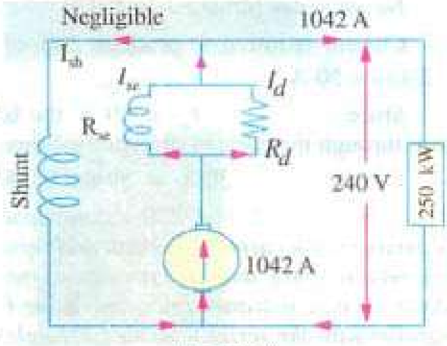


Fig. 28.30

Since series winding and divertor are in parallel, $I_d R_d = I_{se} R_{se}$ or $229.5\ R_d = 812.5 \times 0.006$

∴ $R_d =$ **0.0212 Ω**

Example 28.19. *A 60-kW d.c. shunt generator has 1600 turns/pole in its shunt winding. A shunt field current of 1.25 A is required to generate 125 V at no-load and 1.75 A to generate 150 V at full load. Calculate*

(i) the minimum number of series turns/pole needed to produce the required no-load and full-load voltages as a short-shunt compound generator.

(ii) if the generator is equipped with 3 series turns/pole having a resistance of 0.02 Ω, calculate divertor resistance required to produce the desired compounding.

(iii) voltage regulation of the compound generator.

Solution. (i) Extra excitation ampere-turns required = 1600 (1.75 − 1.25) = 800

$I_{se} = I = 60{,}000/150 = 400$ A

∴ No. of series turns/pole required = 800/400 = **2**

Hence, minimum number of series turns/pole required for producing the desired compound generator terminal voltage is **2**.

(ii) Now, actual No. of series turns/pole is 3. Hence, current passing through it can be found from

$3 \times I_{se} = 800;\ I_{se} = 800/3$ A

As shown in Fig. 28.31, $I_d = 400 - (800/3) = 400/3$ A

Also $(800/3) \times 0.02 = (400/3) \times R_d$; $R_d =$ **0.04 Ω**

(iii) % regn. $= (125 - 150) \times 100/150 =$ **− 16.7%**

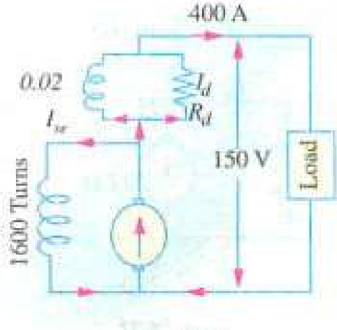


Fig. 28.31

Example 28.20. *A D.C. generator having an external characteristics which is a straight line through zero to 50 V at 200 Amp, is connected as a booster between a station bus-bar and a feeder of 0.3 ohm resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (i) 160 A, (ii) 50 A.* **(Manomaniam Surdaranar Univ. Nov. 1998)**

Solution. Due to the feeder resistance of 0.3 ohm there is a voltage drop of $I \times 0.3$ volts in the direction of current, where I refers to the current flowing. Due to booster, there is a rise in voltage given by $I \times (50/200)$ according to the V-I characteristic given for the series generator. As a sum total of these two, the net feeder drop will be $I \times (0.3 - 0.25)$ or I volts, and that represents the voltage between far end of the feeder and the bus-bar.

(*i*) **At 160 A,** the net voltage drop in feeder = 160×0.05 or $48 - 40 = 9$ volts.

Since 48 volts drop is partially compensated by 40 volts boosted up.

(*ii*) **At 50 A,** the required answer = (15 volts drop 12.5 volts boosted up) = 2.5 volts drop.

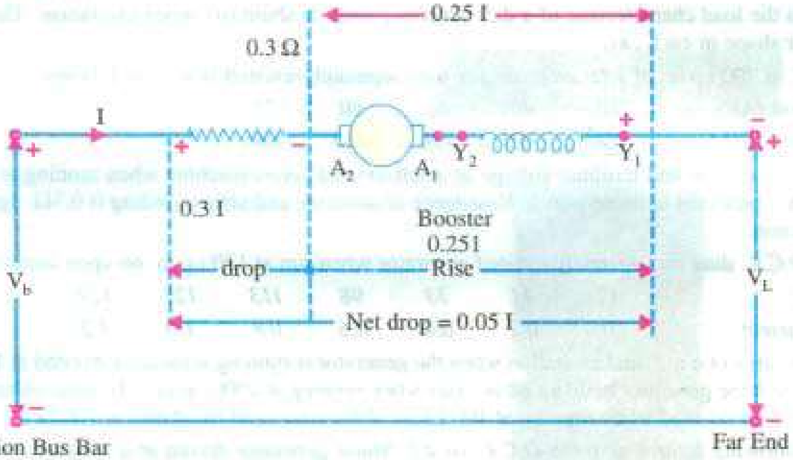


Fig. 28.32. Feeder and Booster

28.16. Uses of D.C. Generators

1. Shunt generators with field regulators are used for ordinary lighting and power supply purposes. They are also used for charging batteries because their terminal voltages are almost constant or can be kept constant.

2. Series generators are not used for power supply because of their rising characteristics. However, their rising characteristic makes them suitable for being used as boosters (Ex. 28.15) in certain types of distribution systems particularly in railway service.

Compound generators are used in electric railways

3. Compound generators

The cumulatively-compound generator is the most widely used d.c. generator because its external characteristic can be adjusted for compensating the voltage drop in the line resistance. Hence, such generators are used for motor driving which require d.c. supply at constant voltage, for lamp loads and for heavy power service such as electric railways.

The differential-compound generator has an external characteristic similar to that of a shunt generator but with large demagnetization armature reaction. Hence, it is widely used in arc welding where larger voltage drop is desirable with increase in current.

Tutorial Problem in 28.1

1. The *OC* curve of a d.c. shunt generator for a speed of 1000 r.p.m. is given by the following table.

Field current :	2.0	3.0	4.0	5.0	6.0	7.0
E.M.F. volts :	102	150	188	215	232	245

The shunt has a resistance of 37 Ω. Find the speed at which excitation may be expected to build up.

The aramture resistance of 0.04 Ω. Neglecting the effects of brush drop and armature reaction, estimate the p.d. when the speed is 1000 r.p.m. and the armature delivers a current of 100 A. **(725 r.p.m.; 231 V)**

2. A d.c. shunt generator running at 850 r.p.m. gave the followig *O.C.C.* data :

Field current (A) :	0	0.5	1	2	3	4	5
E.M.F. (V) :	10	60	120	199	232	248	258

If the resistance of the shunt field is 50 Ω, determine the additional resistance required in the shunt field circuit to give 240 V at a speed of 1000 r.p.m. **[64.3 Ω)**

3. Sketch the load characteristic of a d.c. generator with (*i*) shunt (*ii*) series excitation. Give reasons for the particular shape in each case.

The *O.C.C.* at 700 r.p.m. of a series generator with separately-excited field is as follows :

Field current (A) :	20	40	50	60	75
Armature e.m.f. (V) :	190	360	410	450	480

Determine the current and terminal voltage as a self-excited series machine when running at 600 r.p.m. with a load of 6 Ω connected to the terminal. Resistance of armature and series winding is 0.3Ω. Ignore effect of armature reaction. **[369 V; 61.5 A]**

4. The *O.C.C.* data for separately-excited generator when run at 130 r.p.m. on open circuit is

E.M.F. (V) :	12	44	73	98	113	122	127
Exciting current :	0	0.2	0.4	0.6	0.8	1.0	1.2

Deduce the curve of e.m.f. and excitation when the generator is running separately-excited at 1000 r.p.m. To what voltage will the generator build up on no-load when running at 1000 r.p.m.? To what voltage will the generator build up on no-load when running at 100 r.p.m. if the total field resistance is 100 Ω ? **[91 V]**

5. The following figures give the *O.C.C.* of d.c. Shunt generator driven at a constant speed of 700 r.p.m.

Terminal voltage (V):	10	20	40	80	120	160	200	240	260
Field Current (A) :	0	0.1	0.24	0.5	0.77	1.2	1.92	3.43	5.2

Determine the critical resistance at (*a*) 700 r.p.m. (*b*) 850 r.p.m. If resistance of field coils is 50 Ω, find the range of the field rheostat required to vary the voltage between the limits of 180 V and 250 V on open circuit at a speed of 700 r.p.m. **[160 Ω ; 194 Ω; 70 Ω to 10 Ω]**

6. The *O.C.C.* of a shunt generator when separately-excited and running at 1000 r.p.m. is given by :

O.C.C. volt :	56	112	150	180	200	216	230
Field amp. :	0.5	1.0	1.5	2.0	2.5	3.0	3.5

If the generator is shunt-connected and runs at 1100 r.p.m. with a total field resistance of 80 Ω, determine

(*a*) no-load e.m.f.

(*b*) the output when the terminal voltage is 200 V if the armature resistance is 0.1 Ω.

(*c*) the terminal voltage of the generator when giving the maximum output current.

Neglect the effect of armature reaction and of brush contact drop.

[236 V; 200 V; 460 V; 150 V (approx.)]

7. A long-shunt compound d.c. generator with armature, series field and shunt field resistance of 0.5, 0.4 and 250 Ω respectively gave the following readings when run at constant speed :

Load current (A) :	0	10	20	30	40
Terminal p.d. (V) :	480	478	475	471	467

Plot the curve of internal generated e.m.f. against load current. Explain fully the steps by which this curve is obtained and tabulate the values from which it is plotted. **[For 40 A; E = 504.7 V (approx.)]**

8. A shunt generator has the following open-circuit characteristic at 800 r.p.m.

Field amperes	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5
E.M.F. Volt	:	54	107	152	185	210	230	245

Armature and shunt field resistances are respectively 0.1 Ω and 80 Ω. The terminal p.d. falls to 175 V when the armature current is 100 A. Find the O.C. volts and the volts lost due to (*i*) reduction in the field current (*ii*) armature resistance (*iii*) armature reaction. **[220 V (*i*) 27 V (*ii*) 120 V (*iii*) 8 V]**

9. The open-circuit characteristic of a shunt generator when driven at normal speed is as follows :

Field current	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
O.C. volts	:	54	107	152	185	210	230	240 V

The resistance of armature circuit is 0.1 Ω. Due to armature reaction the effective field current is given by the relation I_{sh} (eff.) = $I_{sh} - 0.003\, I_a$. Find the shunt field circuit resistance that will give a terminal voltage of 220 V with normal speed (*a*) on open circuit (*b*) at a load current of 100 A. Also find (*c*) number of series turns for level compounding at 220 V with 100 A armature current ; take number of shunt turns per pole as 1200 and (*d*) No. of series turns for over-compounding giving a terminal voltage of 220 V at no-load and 230 V with 100 A armature current. **[(*a*) 80 Ω (*b*) 66 Ω (*c*) 6.8 Ω (*d*) 9.1 turns]**

10. Find how many series turns per pole are needed on a 500-kW compound generator required to give 450 V on no-load and 500 V on full-load, the requisite number of ampere-turns per pole being 9,000 and 6,500 respectively. The shunt winding is designed to give 450 V at no-load when its temperature is 20°C. The final temperature is 60°C. Take $\alpha_0 = 1/234.5$ per °C.

[2.76] (Electrical Technology, Allahabad Univ. 1977)

OBJECTIVE TESTS – 28

1. The external characteristic of a shunt generator can be obtained directly from its —— characteristic.
   (*a*) internal (*b*) open-circuit
   (*c*) load-saturation (*d*) performance
2. Load saturation characteristic of a d.c. generator gives relation between
   (*a*) V and I_a (*b*) E and I_a
   (*c*) E_0 and I_f (*d*) V and I_f
3. The slight curvature at the lower end of the *O.C.C.* of a self-excited d.c. generator is due to
   (*a*) residual pole flux
   (*b*) high armature speed
   (*c*) magnetic inertia
   (*d*) high field circuit resistance.
4. For the voltage built-up of a self-excited d.c. generator, which of the following is not an essential condition ?
   (*a*) There must be some residual flux
   (*b*) Field winding mmf must aid the residual flux
   (*c*) Total field circuit resistance must be less than the critical value
   (*d*) Armature speed must be very high.
5. The voltage build-up process of a d.c. generator is
   (*a*) difficult (*b*) delayed
   (*c*) cumulative (*d*) infinite
6. Which of the following d.c. generator cannot build up on open-circuit ?
   (*a*) shunt (*b*) series
   (*c*) short shunt (*d*) long shunt
7. If a self-excited d.c. generator after being installed, fails to build up on its first trial run, the first thing to do is to
   (*a*) increase the field resistance
   (*b*) check armature insulation
   (*c*) reverse field connections
   (*d*) increase the speed of prime mover.
8. If residual magnetism of a shunt generator is destroyed accidentally, it may be restored by connecting its shunt field
   (*a*) to earth
   (*b*) to an a.c. source
   (*b*) in reverse
   (*d*) to a d.c. source.
9. The three factors which cause decrease in the terminal voltage of a shunt generator are
   (*a*) armature reactances
   (*b*) armature resistance
   (*c*) armature leakages
   (*d*) armature reaction
   (*e*) reduction in field current
10. If field resistance of a d.c. shunt generator is increased beyond its critical value, the generator
    (*a*) output voltage will exceed its name-plate rating
    (*b*) will not build up

(c) may burn out if loaded to its name-plate rating
(d) power output may exceed its name-plate rating

11. An ideal d.c. generator is one that has .... voltage regulation.
(a) low (b) zero
(c) positive (d) negative

12. The .... generator has poorest voltage regulation.
(a) series (b) shunt
(c) compound (d) high

13. The voltage regulation of an overcompound d.c. generator is always ....
(a) positive
(b) negative
(c) zero
(d) high

14. Most commercial compound d.c. generator are normally supplied by the manufacturers as over compound machines because
(a) they are ideally suited for transmission of d.c. energy to remotely-located loads
(b) degree of compounding can be adjusted by using a divertor across series field
(c) they are more cost effective than shunt generators
(d) they have zero percent regulation.

ANSWERS

1. (b) 2. (d) 3. (c) 4. (d) 5. (c) 6. (b) 7. (c) 8. (d) 9. (b,d,e) 10. (b) 11. (b) 12. (a) 13. (b) 14. (b).

QUESTIONS AND ANSWERS ON D.C. GENERATORS

Q. 1. How may the number of parallel paths in an armature be increased ?
Ans. By increasing the number of magnetic poles.

Q. 2. How are brushes connected in a d.c. generator ?
Ans. Usually, all positive brushes are connected together and all the negative brushes together (Fig. 28.33).

Q. 3. What is meant by armature reaction ?
Ans. It is the effect of armature magnetic field on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :
(i) It demagnetises or weakens the main flux and
(ii) It cross-magnetises or distorts it.

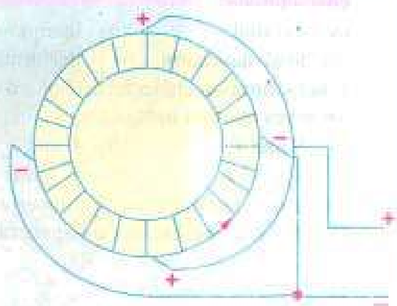


Fig. 28.33

Q. 4. What is the effect of this distortion on the operation of the machine ?
Ans. It acts as a magnetic drag on the armature which consequently requires more power to turn it.

Q. 5. How can field distortion be remedied ?
Ans. By using compensating windings which are embedded in the slots in the pole-shoe and are connected in series with the armature.

Q. 6. What is meant by normal neutral plane ?
Ans. It is a plane which passes through the axis of the armature perpendicular to the magnetic field of the generator when there is no flow of current through the armature.

Q. 7. What is the importance of this plane in the working of the machine ?
Ans. It gives the position where brushes would be placed to prevent sparking during the operation of the generator where the main pole field not distorted by armature field and were there no self-induction in the coils.

Q. 8. How do you differentiate between normal neutral plane ?
Ans. The *NNP* is the position of zero induction and hence minimum sparking assuming no field distortion *i.e.* on no-load. It is perpendicular to magnetic axis. *NA* is the position of zero induction and hence minimum sparking with distorted field *i.e.* ***when generator is on load.***

Q. 9. How do you define 'commutating plane' ?
Ans. It is the plane which passes through the axis of the armature and through centre of contact of the brushes as shown in Fig. 28.34.

Q. 10. What is the angle of lead ?

Ans. It is the angle between the *NNP* and the commutating plane.

Q. 11. What affects this angle ?

Ans. For sparkless commutation, the angle of lead varies directly with load. Its value can be kept small by making main pole field considerably more powerful than the armature field.

Q. 12. What is the best way of minimizing eddy currents in an armature ?

Ans. Lamination.

Q. 13. How should the armature be laminated for the purpose ?

Ans. It should be laminated at right angle to its axis.

Q. 14. How does field distortion affect communication ?

Ans. The neutral plane no longer coincides with the normal neutral plane but is advanced by a certain angle in the direction of rotation of the armature.

Q. 15. Should the brushes of a loaded generator be placed in the neutral plane ?

Ans. No.

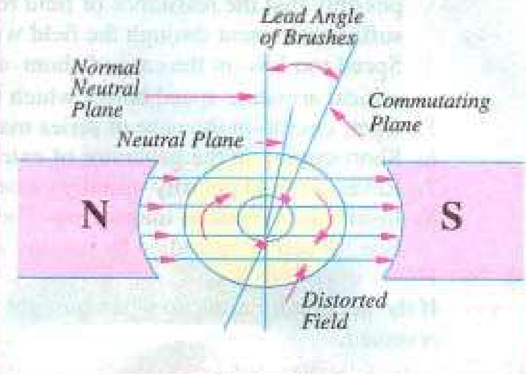


Fig. 28.34

Q. 16. Why not ?

Ans. The brushes must be advanced by a certain angle (called brush lead) beyond the neutral plane to prevent sparking.

Q. 17. What causes sparking at the brushes ?

Ans. It is due to the self-induction of the coil undergoing commutation.

Q. 18. What is the standard direction of rotation of the d.c. generators ?

Ans. Clockwise when viewed from the end opposite to the driven end.

Q. 19. What is meant by build-up of a generator ?

Ans. It means the gradual increase in the generator voltage to its maximum value after the generator is started from rest.

Q. 20. How should a generator be started ?

Ans. It is usually brought up to speed with the help of the driving engine called prime-mover.

Q. 21. How should a shunt or compound generator be started ?

Ans. Such machines excite best when all switches controlling the external circuit are open.

Q. 22. How about a series generator ?

Ans. In this case, the external circuit must be closed otherwise the generator will not build-up.

Q. 23. What is the procedure for shunting down a generator ?

Ans. First, the load should be gradually reduced, if possible, by easing down the driving engine, then when the generator is supplying little or no current, the main switch should be opened. When the voltmeter reads almost zero, then brushes should be raised from the commutator.

Q. 24. What are the indications and causes of an overloaded generator ?

Ans. A generator is said to be overloaded if a greater output is taken from it that it can safely carry. Overloading is indicated by *(i)* excessive sparking at brushes and *(ii)* overheating of the armature and other parts of the generator. Most likely causes of overloading are :

1. Excessive voltage-as indicated by the voltmeter or the increased brilliancy of the pilot lamp. This could be due to over-excitation of field magnets or too high speed of the engine.
2. Excessive current–which could be due to bad feeding of the load.
3. Reversal of polarity–this happens occasionally when the series or compound-wound generators are running in parallel. Polarity reversal occurs during stopping by the current from the machines at work.
4. Short-circuit or ground in the generator itself or in the external circuit.

Q. 25. Mention and explain the various causes for the failure of the generator to build up.

Ans. Principal causes due to which a generator may fail to excite are :

1. Brushes not properly adjusted–if brushes are not in their proper positions, then whole of the armature voltage will not be utilized and so would be insufficient to excite the machine.
2. Defective contacts-unclean contacts may interpose large resistance in the path of the exciting current and reduce it to such a small value that it fails to excite the machine.
3. Incorrect adjustment of regulators–in the case of shunt and compound generators, it is possibly that the resistance of field regulator may be too high to permit the passage of sufficient current through the field windings.
4. Speed too low–in the case of shunt–and compound-wound generators, there is certain critical armature speed below which they will not excite.
5. Open-circuit–in the case of series machines.
6. Short-circuit–in the generator of external circuit.
7. Reversed field polarity–usually caused by the reversed connections of the field coils.
8. Insufficient residual magnetism–The trouble normally occurs when the generator is new. It can be remedied by passing a strong direct current through the field coils.

Q. 26. How do we conclude that connections between field coils and armature are correct ?

Ans. If the generator builds up when brought to full speed. If it does not, then connections are reversed.

Q. 27. When a generator loses its residual magnetism either due to lighting or short circuit, how can it be made to build up ?

Ans. By temporarily magnetisng the main poles with the help of current from an external battery.

Q. 28. Can a generator be reversed by reversing the connections between the armature and field coils ?

Ans. No, because if these connections are reversed, the generator will not build up at all.

Q. 29. Will a generator build up if it becomes reversed ?

Ans. Yes.

Q. 30. Then, what is the objection to a reversed generator ?

Ans. Since the current of such a reversed generator is also reversed, serious trouble can occur if attempt is made to connect it in parallel with other machines which are not reversed.

Q. 31. What are the two kinds of sparking produced in a generator ?

Ans. One kind of sparking is due to bad adjustment of brushes and the other due to bad condition of the commutator. The sparking of the first are bluish whereas those of the other are reddish in colour.

Q. 32. What is the probable reason if sparking does not disappear in any position when brushes are rocked around the commutator ?

Ans. (*i*) The brushes may not be separated at correct distance.

(*ii*) The neutral plane may not be situated in the true theoretical position on the commutator due to faulty winding.

Q. 33. What is the permissible rise of temperature in a well-designed generator ?

Ans. 27°C above the surrounding air.

Q. 34. What are the causes of hot bearings ?

Ans. (*i*) lack of oil (*ii*) belt too tight (*iii*) armature not centered with respect of pole pieces (*iv*) bearing too tight or not in line.

Q. 35. What causes heating of armature ?

Ans.
1. Eddy currents.
2. Moisture which almost short-circuits the armature.
3. Unequal strength of magnetic poles.
4. Operation above rated voltage and below normal speed.

Q. 36. What is the commutator pitch of a 4-pole d.c. armature having 49 commutator bars?

Ans. $Y_c = (49 \pm 1)/2 = 24$ or 25.

Q. 37. Will it make any difference if lower figure of 24 is selected in preference to other.

Ans. Yes. Direction of armature rotation would be reversed.

C H A P T E R 29

D.C. MOTOR

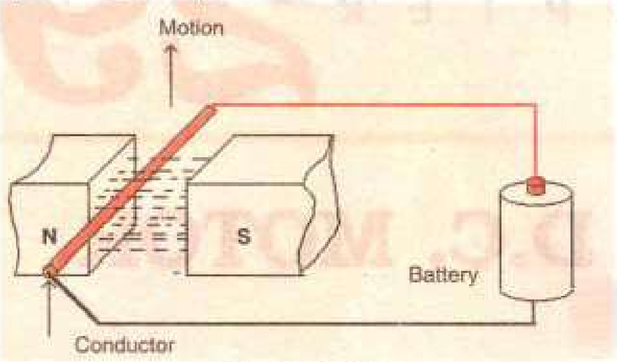
Learning Objectives

- Motor Principle
- Comparison of Generator and Motor Action
- Significance of the Back e.m.f.—Voltage Equation of a Motor
- Conditions for Maximum Power
- Torque
- Armature Torque of a Motor
- Shaft Torque
- Speed of a D.C. Motor
- Speed RegulationTorque and Speed of a D.C. Motor
- Motor Characteristics
- Characteristics of Series Motors
- Characteristics of Shunt Motors
- Compound Motors
- Performance Curves
- Comparison of Shunt and Series Motors
- Losses and Efficiency
- Power Stages

Design for optimum performance and durability in demanding variable speed motor applications. D.C. motors have earned a reputation for dependability in severe operating conditions

29.1. Motor Principle

An Electric motor is a machine which converts electric energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by $F = BIl$ Newton.



Principle of Motor

Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 29.1 a part of multipolar d.c. motor is shown. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under *N*-pole are assumed to carry current downwards (crosses) and those under *S*-poles, to carry current upwards (dots). By applying Fleming's Left-hand Rule, the direction of the force on each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor can be found. It will be seen that each conductor experiences a force *F* which tends to rotate the armature in anticlockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

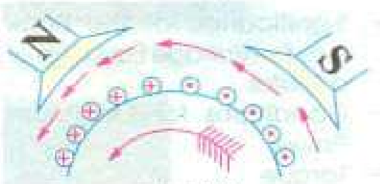


Fig. 29.1

It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

29.2. Comparison of Generator and Motor Action

As said above, the same d.c. machine can be used, at least theoretically, interchangeably as a generator or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation.

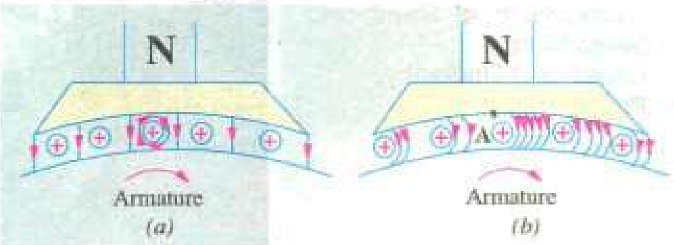


Fig. 29.2

Let us first consider its operation as a generator and see how exactly and through which agency, mechanical power is converted into electric power.

In Fig. 29.2 part of a generator whose armature is being driven clockwise by its prime mover is shown.

Fig. 29.2 (*a*) represents the fields set up independently by the main poles and the armature conductors like *A* in the figure. The resultant field or magnetic lines on flux are shown in Fig. 29.2 (*b*).

It is seen that there is a crowding of lines of flux on the right-hand side of A. These magnetic lines of flux may be likened to the rubber bands under tension. Hence, the bent lines of flux up a mechanical force on A much in the same way as the bent elastic rubber band of a catapult produces a mechanical force on the stone piece. It will be seen that this force is in a direction opposite to that of armature rotation. Hence, it is known as backward force or magnetic drag on the conductors. It is against this drag action on all armature conductor that the prime mover has to work. The work done in overcoming this opposition is converted into electric energy. Therefore, it should be clearly understood that it is only through the instrumentality of this magnetic drag that energy conversion is possible in a d.c. generator*.

Next, suppose that the above d.c. machine is uncoupled from its prime mover and that current is sent through the armature conductors under a N-pole in the downward direction as shown in Fig. 29.3 (*a*). The conductors will again experience a force in the anticlockwise direction (Fleming's Left hand Rule). Hence, the machine will start rotating anticlockwise, thereby developing a torque which can produce mechanical rotation. The machine is then said to be motoring.

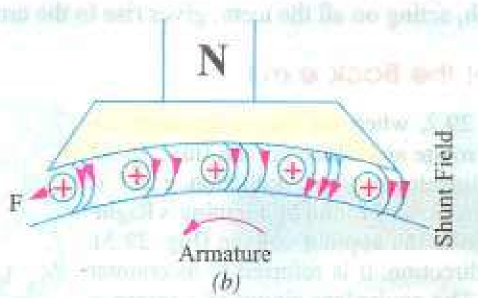


Fig. 29.3 (*a*)

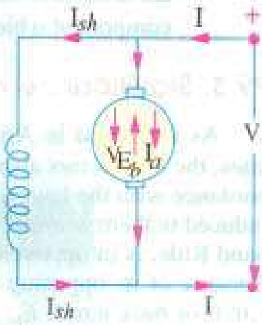


Fig. 29.3 (*b*)

As said above, energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In the case of a generator, it was the magnetic drag which provided the necessary opposition. But what is the equivalent of that drag in the case of a motor ? Well, it is the back e.m.f. It is explained in this manner :

As soon as the armature starts rotating, dynamically (or motionally) induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f. as found by Fleming's Right-hand Rule, is outwards *i.e.*, in direct opposition to the applied voltage (Fig. 29.3 (*b*)). This is why it is known as back e.m.f. E_b or counter e.m.f. Its value is the same as for the motionally induced e.m.f. in the generator *i.e.* $E_b = (\Phi ZN) \times (P/A)$ volts. The applied voltage V has to be force current through the armature conductors against this back e.m.f. E_b. The electric work done in overcoming this opposition is converted into mechanical energy developed in the armature. Therefore, it is obvious that but for the production of this opposing e.m.f. energy conversion would not have been possible.

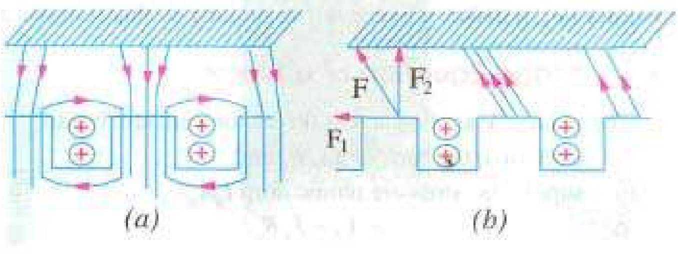


Fig. 29.4

Now, before leaving this topic, let it be pointed out that in an actual motor with slotted armature, the torque is not due to mechanical force on the conductors themselves, but due to tangential pull on the armature teeth as shown in Fig. 29.4.

It is seen from Fig. 29.4 (*a*) that the main flux is concentrated in the form of tufts at the armature teeth while the armature flux is shown by the dotted lines embracing the armature slots. The effect of

* In fact, it seems to be one of the fundamental laws of Nature that no energy conversion from one form to another is possible until there is some one to oppose the conversion. But for the presence of this opposition, there would simply be no energy conversion. In generators, opposition is provided by magnetic drag whereas in motors, back e.m.f. does this job. Moreover, it is only that part of the input energy which is used for overcoming this opposition that is converted into the other form.

armature flux on the main flux, as shown in Fig. 29.4 (*b*), is two-fold :

(*i*) It increases the flux on the left-hand side of the teeth and decreases it on the right-hand side, thus making the distribution of flux density across the tooth section unequal.

(*ii*) It inclines the direction of lines of force in the air-gap so that they are not radial but are disposed in a manner shown in Fig. 29.4 (*b*). The pull exerted by the poles on the teeth can now be resolved into two components. One is the tangential component F_1 and the other vertical component F_2. The vertical component F_2, when considered for all the teeth round the armature, adds up to zero. But the component F_1 is not cancelled and it is this tangential component which, acting on all the teeth, gives rise to the armature torque.

29.3. Significance of the Back e.m.f.

As explained in Art 29.2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's Right-hand Rule, is in opposition to the applied voltage (Fig. 29.5). Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f. E_b. The equivalent circuit of a motor is shown in Fig. 29.6. The rotating armature generating the back e.m.f. E_b is like a battery of e.m.f. E_b put across a supply mains of V volts. Obviously, V has to drive I_a against the opposition of E_b. The power required to overcome this opposition is E_bI_a.

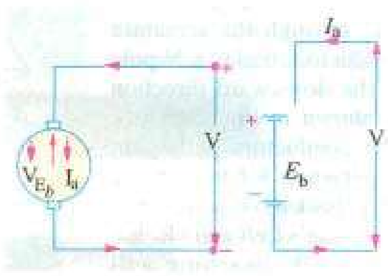


Fig. 29.5

In the case of a cell, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

$$\text{It will be seen that } I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{V - V_b}{R_a}$$

where R_a is the resistance of the armature circuit. As pointed out above,

$E_b = \Phi ZN \times (P/A)$ volt where N is in r.p.s.

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, E_b is large, hence armature current I_a, seen from the above equation, is small. If the speed is less, then E_b is less, hence more current flows which develops motor torque (Art 29.7). So, we find that E_b acts like a governor *i.e.*, it makes a motor self-regulating so that it draws as much current as is just necessary.

29.4. Voltage Equation of a Motor

The voltage V applied across the motor armature has to

(*i*) overcome the back e.m.f. E_b and

(*ii*) supply the armature ohmic drop I_aR_a.

$$\therefore \quad V = E_b + I_a R_a$$

This is known as voltage equation of a motor.

Now, multiplying both sides by I_a, we get

$$V I_a = E_bI_a + I_a^2 R_a$$

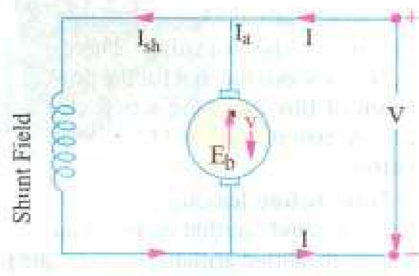


Fig.29.6

As shown in Fig. 29.6,

$V I_a$ = Eectrical input to the armature

E_bI_a = Electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$ = Cu loss in the armature

Hence, out of the armature input, some is wasted in I^2R loss and the rest is converted into mechanical power *within* the armature.

It may also be noted that motor efficiency is given by the ratio of power developed by the armature to its input *i.e.*, $E_b I_a / V I_a = E_b/V$. Obviously, higher the value of E_b as compared to V, higher the motor efficiency.

29.5. Condition for Maximum Power

The gross mechanical power developed by a motor is $P_m = V I_a - I_a^2 R_a$.

Differentiating both sides with respect to I_a and equating the result to zero, we get

$$d P_m / d I_a = V - 2 I_a R_a = 0 \quad \therefore \quad I_a R_a = V/2$$

As $\quad V = E_b + I_a R_a$ and $I_a R_a = V/2 \quad \therefore \quad E_b = V/2$

Thus gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

Example 29.1. *A 220-V d.c. machine has an armature resistance of 0.5 Ω. If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as (i) generator (ii) motor.*

(Electrical Technology-I, Bombay Univ. 1987)

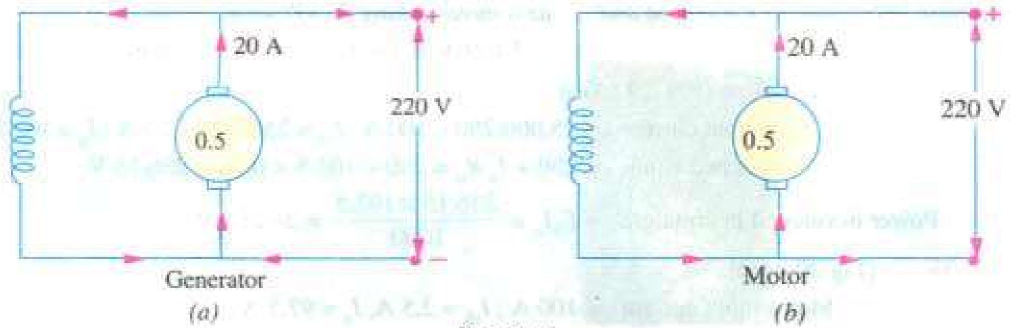


Fig. 29.7

Solution. As shown in Fig. 29.7, the d.c. machine is assumed to be shunt-connected. In each case, shunt current is considered negligible because its value is not given.

(*a*) As Generator [Fig. 29.7(*a*)] $\quad E_g = V + I_a R_a = 220 + 0.5 \times 20 = $ **230 V**

(*b*) As Motor [Fig 29.7 (*b*)] $\quad E_b = V - I_a R_a = 220 - 0.5 \times 20 = $ **210 V**

Example 29.2. *A separately excited D.C. generator has armature circuit resistance of 0.1 ohm and the total brush-drop is 2 V. When running at 1000 r.p.m., it delivers a current of 100 A at 250 V to a load of constant resistance. If the generator speed drop to 700 r.p.m., with field-current unaltered, find the current delivered to load.* **(AMIE, Electrical Machines, 2001)**

Solution. $R_L = 250/100 = 2.5$ ohms.

$E_{g1} = 250 + (100 \times 0.1) + 2 = 262$ V.

At 700 r.p.m., $E_{g2} = 262 \times 700/1000 = 183.4$ V

If I_a is the new current, $E_{g2} - 2 - (I_a \times 0.1) = 2.5\, I_a$

This gives $I_a = 96.77$ amp.

Extension to the Question : *With what load resistance will the current be 100 amp, at 700 r.p.m.?*

Solution. $E_{g2} - 2 - (I_a \times 0.1) = R_L \times I_a$

For $I_a = 100$ amp, and $E_{g2} = 183.4$ V, $R_L = 1.714$ ohms.

Example 29.3. *A 440-V, shunt motor has armature resistance of 0.8 Ω and field resistance of 200 Ω. Determine the back e.m.f. when giving an output of 7.46 kW at 85 percent efficiency.*

Solution. Motor input power $= 7.46 \times 10^3 / 0.85$ W

$$\text{Motor input current} = 7460/0.85 \times 440 = 19.95 \text{ A} ; I_{sh} = 440/200 = 2.2 \text{ A}$$

$$I_a = 19.95 - 2.2 = 17.75 \text{ A} ; \text{Now, } E_b = V - I_a R_a$$

$$\therefore \quad E_b = 440 - (17.75 \times 0.8) = \mathbf{425.8\ V}$$

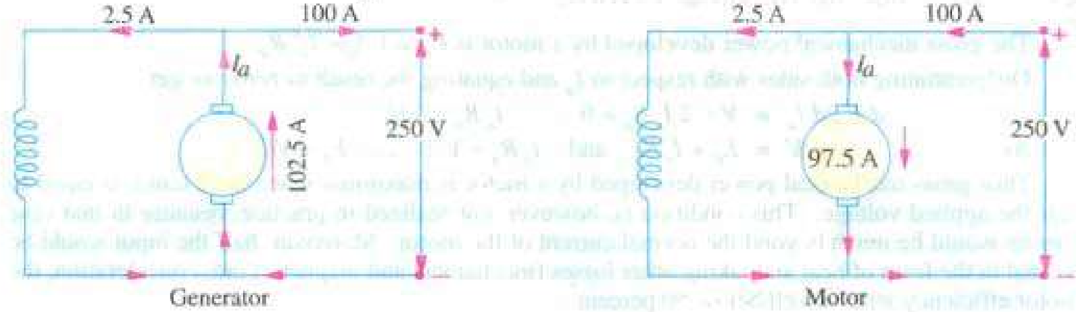


Fig. 29.8 (*a*) Fig. 29.8 (*b*)

Example 29.4. *A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working* **(i)** *as a generator delivering 25 kW output and* **(ii)** *as a motor taking 25 kW input.*

(Electrical Technology, Punjab Univ., June 1991)

Solution. As Generator [Fig. 29.8 (*a*)]

$$\text{Output current} = 25{,}000/250 = 100 \text{ A} ; I_{sh} = 250/100 = 2.5 \text{ A} ; I_a = 102.5 \text{ A}$$

$$\text{Generated e.m.f.} = 250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15 \text{ V}$$

$$\text{Power developed in armature} = E_b I_a = \frac{256.15 \times 102.5}{1000} = \mathbf{26.25\ kW}$$

As Motor [Fig 29.8 (*b*)]

$$\text{Motor input current} = 100 \text{ A} ; I_{sh} = 2.5 \text{ A}, I_a = 97.5 \text{ A}$$

$$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15 \text{ V}$$

$$\text{Power developed in armature} = E_b I_a = 244.15 \times 97.5/1000 = \mathbf{23.8\ kW}$$

Example 29.5. *A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has r_a = 2 and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine.* **[Sambalpur University, 1998]**

Solution. Current distributions during two actions are indicated in Fig. 29.9 (*a*) and (*b*). As a generator, I_a = 13 amp

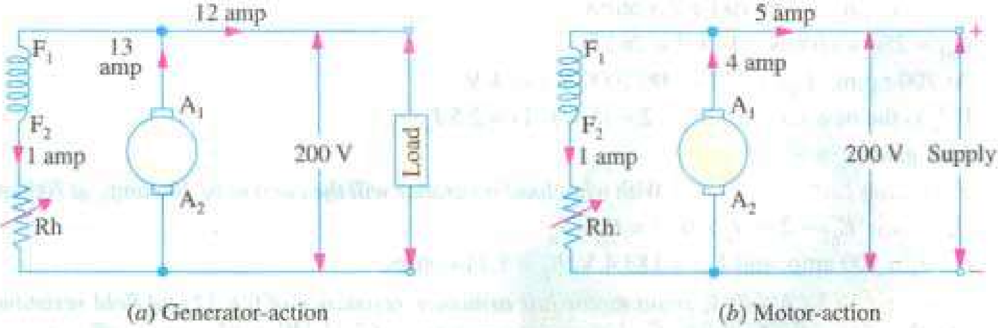


Fig. 29.9

$$E_g = 200 + 13 \times 2 = 226 \text{ V}$$

$$\phi \frac{ZN}{60} \times \frac{P}{a} = 226$$

For a Lap-wound armature,

$$P = a$$

$$\therefore \quad \phi = \frac{226 \times 60}{1000 \times 32} = 0.42375 \text{ wb}$$

As a motor,

$$I_a = 4 \text{ amp}$$

$$E_b = 200 - 4 \times 2 = 192 \text{ V}$$

$$= \phi ZN/60$$

$$\text{Giving } N = \frac{60 \times 192}{0.42375 \times 32}$$

$$= 850 \text{ r.p.m.}$$

Tutorial Problems 29.1

1. What do you understand by the term 'back e.m.f.' ? A d.c. motor connected to a 460-V supply has an armature resistance of 0.15 Ω. Calculate

(a) The value of back e.m.f. when the armature current is 120 A.

(b) The value of armature current when the back e.m.f. is 447.4 V. [(a) 442 V (b) 84 A]

2. A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of 0.25 Ω, calculate the value of the back e.m.f. at this load. [430 V]

3. A 4-pole d.c. motor takes an armature current of 150 A at 440 V. If its armature circuit has a resistance of 0.15 Ω, what will be the value of back e.m.f. at this load ? [417.5 V]

29.6. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius r metre acted upon by a circumferential force of F Newton which causes it to rotate at N r.p.m. (Fig. 29.10).

Then torque $T = F \times r$ Newton-metre (N - m)

Work done by this force in one revolution

$= \text{Force} \times \text{distance} = F \times 2\pi r$ Joule

Power developed $= F \times 2\pi r \times N$ Joule/second or Watt

$= (F \times r) \times 2\pi N$ Watt

Now $2\pi N$ = Angular velocity ω in radian/second and $F \times r$ = Torque T

$\therefore$ Power developed $= T \times \omega$ watt or $P = T\omega$ Watt

Moreover, if N is in r.p.m., then

$$\omega = 2\pi N/60 \text{ rad/s}$$

$$\therefore \quad P = \frac{2\pi N}{60} \times T \quad \text{or} \quad P = \frac{2\pi}{60} \cdot NT = \frac{NT}{9.55}$$

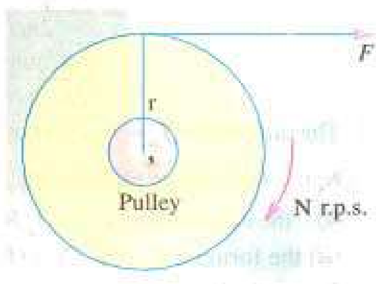


Fig. 29.10

29.7. Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N r.p.s. If T_a is in N/M, then power developed $= T_a \times 2\pi N$ watt ...(i)

We also know that electrical power converted into mechanical power in the armature (Art 29.4)

$$= E_b I_a \text{ watt} \quad ...(ii)$$

Equating (i) and (ii), we get $T_a \times 2\pi N = E_b I_a$ ...(iii)

Since $E_b = \Phi ZN \times (P/A)$ volt, we have

$$T_a \times 2\pi N = \Phi\, ZN\left(\frac{P}{A}\right) . I_a \text{ or } T_a = \frac{1}{2\pi} . \Phi Z I_0 \left(\frac{P}{A}\right) N\text{-}m$$

$$= 0.159 \text{ N newton metre}$$

$$\therefore \quad T_a = 0.159\, \Phi Z I_a \times (P/A)\ N\text{-}m$$

Note. From the above equation for the torque, we find that $T_a \propto \Phi I_a$.

(a) In the case of a series motor, Φ is directly proportional to I_a (before saturation) because field windings carry full armature current $\therefore \ T_a \propto I_a^2$

(b) For shunt motors, Φ is practically constant, hence $T_a \propto I_a$.

As seen from (iii) above

$$T_a = \frac{E_b\, I_a}{2\pi\, N} \text{ N - m - N in r.p.s.}$$

If N is in r.p.m., then

$$T_a = \frac{E_b\, I_a}{2\pi\, N/60} = 60\,\frac{E_b\, I_a}{2\pi N} = \frac{60}{2\pi}\,\frac{E_b\, I_a}{N} = 9.55\,\frac{E_b I_a}{N} \text{ N-m}$$

29.8. Shaft Torque (T_{sh})

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft. The motor output is given by

Output = $T_{sh} \times 2\pi N$ Watt provided T_{sh} is in N-m and N in r.p.s.

$$\therefore \quad T_{sh} = \frac{\text{Output in watts}}{2\pi N} \text{ N- m} - N \text{ in r.p.s}$$

$$= \frac{\text{Output in watts}}{2\pi N/60} \text{ N-m} - N \text{ in r.p.m.}$$

$$= \frac{60}{2\pi}\,\frac{\text{output}}{N} = 9.55\,\frac{\text{Output}}{N} \text{ N-m.}$$

The difference $(T_a - T_{sh})$ is known as lost torque and is due to iron and friction losses of the motor.

Note. The value of back e.m.f. E_b can be found from

(i) the equation, $E_b = V - I_a R_a$

(ii) the formula $E_b = \Phi\, ZN \times (P/A)$ volt

Example 29.6. *A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2 Ω. The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate (i), the speed and (ii) the gross torque developed by the armature.* **(Elect. Machines, A.M.I.E. Sec B, 1989)**

Solution. $E_b = 480 - 110 \times 0.2 = 458$ V, $\Phi = 0.05$ W, Z = 864

$$\text{Now,} \quad E_b = \frac{\Phi\, ZN}{60}\left(\frac{P}{A}\right) \text{ or } 458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right)$$

$$\therefore \quad N = 636 \text{ r.p.m.}$$

$$T_a = 0.159 \times 0.05 \times 864 \times 110\ (6/6) = 756.3 \text{ N-m}$$

Example 29.7. *A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.*

(Elect. Engg.-II, Pune Univ. 1991)

Solution. $E_b = \Phi ZN\ (P/A)$

Now, $E_b = V - I_a R_a = 50 - 40 \times 0.75 = 220$ V

$\therefore \quad 220 = 25 \times 10^{-3} \times 782 \times \text{N} \times 0.75 = 220$ V

$\therefore \quad 220 = 0.159\ \Phi\ ZI_a\ (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) =$ **249 N-m**

Example 29.8. *A d.c. shunt machine develops an a.c. e.m.f. of 250 V at 1500 r.p.m. Find its torque and mechanical power developed for an armature current of 50 A. State the simplifying assumptions.*

(Basic Elect. Machine Nagpur Univ., 1993)

Solution. A given d.c. machine develops the same e.m.f. in its armature conductors whether running as a generator or as a motor. Only difference is that this armature e.m.f. is known as back e.m.f. when the machine is running as a motor.

Mechanical power developed in the arm $= E_b I_a = 250 \times 50 = 12{,}500$ W

$T_a = 9.55\ E_b\ I_a/N = 9.55 \times 250 \times 50/1500 =$ **79.6 N-m.**

Example 29.9. *Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.*

(Elect. Machine AMIE Sec. B Winter 1991)

Solution. Developed torque or gross torque is the same thing as armature torque.

$\therefore \quad T_a = 0.159\ \Phi\ ZA\ (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45\ (4/2) = 286.2$ N-m

$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193$ V

Now, $E_b = \Phi\ ZN\ (P/A)$ or $193 = 25 \times 10^{-3} \times 800 \times N\ \pi \times (4/2)$

$\therefore \quad N = 4.825$ r.p.s.

Also, $2\pi N T_{sh} =$ output or $2\pi \times 4.825\ T_{sh} = 8200$ $\quad \therefore T_{sh} = 270.5$ N-m

Example 29.10. *A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that $R_a = 0.2\ \Omega$.*

(Electrical Technology-II, Gwalior Univ. 1985)

Solution. As seen from Art 27.7, $T_a \propto \Phi I_a$. Since Φ is constant, $T_a \propto I_a$,

$\therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2} \quad \therefore \quad T_{a2}/T_{a1} = I_{a2}/I_{a1}$

$\therefore \quad 2 = I_{a2}/50$ or $I_{a2} = 100$ A

Now, $N_2/N_1 = E_{b2}/E_{b1}$ — since Φ remains constant.

$F_{b1} = 220 - (50 \times 0.2) = 210$ V $\qquad E_{b2} = 220 - (100 \times 0.2) = 200$ V

$\therefore \quad N_2/500 = 200/210 \qquad \therefore N_2 =$ **476 r.p.m.**

Example 29.11. *A 500-V, 37.3 kW, 1000 r.p.m. d.c. shunt motor has on full-load an efficiency of 90 percent. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine (i) full-load line current (ii) full load shaft torque in N-m and (iii) total resistance in motor starter to limit the starting current to 1.5 times the full-load current.*

(Elect. Engg. I; M.S. Univ. Baroda 1987)

Solution. (*i*) Motor input = 37,300/0.9 = 41,444 W

F.L. line current = 41,444/500 = **82.9 A**

(ii) $$T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{37,300}{1000} = 356 \text{ N-m}$$

(iii) Starting line current $= 1.5 \times 82.9 = 124.3$ A

Arm. current at starting $= 124.3 - 1.8 = 122.5$ A

If R is the starter resistance (which is in series with armature), then

$122.5\,(R + 0.24) + 2 = 500 \quad \therefore \quad R = \mathbf{3.825\ \Omega}$

Example 29.12. *A 4-pole, 220-V shunt motor has 540 lap-wound conductor. It takes 32 A from the supply mains and develops output power of 5.595 kW. The field winding takes 1 A. The armature resistance is 0.09 Ω and the flux per pole is 30 mWb. Calculate (i) the speed and (ii) the torque developed in newton-metre.* **(Electrical Technology, Nagpur Univ. 1992)**

Solution. $I_a = 32 - 1 = 31$ A ; $E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2\ V$

Now, $$E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \quad \therefore \quad 217.2 = \frac{30 \times 10^{-3} \times 540 \times N}{60}\left(\frac{4}{4}\right)$$

(i) $\therefore$ $N = \mathbf{804.4\ r.p.m.}$

(ii) $$T_{sh} = 9.55 \times \frac{\text{output in watts}}{N} = 9.55 \times \frac{5,595}{804.4} = \mathbf{66.5\ N\text{-}m}$$

Example 29.13 (a). *Find the load and full-load speeds for a four-pole, 220-V, and 20-kW, shunt motor having the following data :*

Field–current = 5 amp, armature resistance = 0.04 ohm,

Flux per pole = 0.04 Wb, number of armature-conductors = 160, Two-circuit wave-connection, full load current = 95 amp, No load current =9 A. Neglect armature reaction.

(Bharathithasan Univ. April 1997)

Solution. The machine draws a supply current of 9 amp at no load. Out of this, 5 amps are required for the field circuit, hence the armature carries a no-load current of 4 amp.

At load, armature-current is 90 amp. The armature-resistance-drop increases and the back e.m.f. decreases, resulting into decrease in speed under load compared to that at No-Load.

At No Load : $E_{ao} = 220 - 4 \times 0.04 = 219.84$ volts

Substituting this,

$0.04 \times 160 \times (N/60) \times (4/2) = 219.84$

No-Load speed, $N_0 = 1030.5$ r.p.m.

At Full Load : Armature current $= 90\ A$, $E_a = 200 - 90 \times 0.04 = 216.4$ V

$N = (216.4/219.84) \times 1030.5 = 1014.4$ r.p.m.

Example 29.13 (b). *Armature of a 6-pole, 6-circuit D.C. shunt motor takes 400 A at a speed of 350 r.p.m. The flux per pole is 80 milli-webers, the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron-loss. Calculate the brake-horse-power.*

(Manonmaniam Sundaranar Univ. Nov. 1998)

Solution. Number of armature turns = 600

Therefore, Z = Number of armature conductors = 1200

If electromagnetic torque developed is T Nw – m,

Armature power $= T\,\omega = T \times 2\,\pi\ 350/60$

$= 36.67\ T$ watts

To calculate armature power in terms of Electrical parameters, E must be known.

$$E = \phi\, Z\,(N/60)\,(P/A)$$

$$= 80 \times 10^{-3} \times 1200 \times (350/60) \times (6/6)$$
$$= 560 \text{ volts}$$

With the armature current of 400 *A*, Armature power = 560 × 400 watts

Equating the two,

$T = 560 \times 400/36.67 = 6108.5$ Nw – m. Since 3 % of this torque is required for overcoming different loss-terms,

$$\text{Net torque} = 0.97 \times 6180.5 = 5925 \text{ Nw - m}$$

For Brake-Horse-Power, net output in kW should be computed first. Then "kW" is to be converted to "BHP", with 1 HP = 0.746 kW.

Net output in kW = $5925 \times 36.67 \times 10^{-3} = 217.27$ kW

Converting this to BHP, the output = 291.25 HP

Example 29.13 (*c*). *Determine the torque established by the armature of a four-pole D.C. motor having 774 conductors, two paths in parallel, 24 milli-webers of pole-flux and the armature current is 50 Amps.* **(Bharathiar Univ. April 1998)**

Solution. Expression for torque in terms of the parameters concerned in this problem is as follows :

$$T = 0.159\, \phi\, Z\, I_a\, p/a \text{ Nw - m}$$

Two paths in parallel for a 4-pole case means a wave winding.

$$T = 0.159 \times (24 \times 10^{-3}) \times 774 \times 50 \times 4/2$$
$$= 295.36 \text{ Nw-m}$$

Example 29.13 (*d*). *A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms, determine the efficiency of the machine running as a generator delivering a load current of 40 Amps.*

(Bharathiar Univ. April 1998)

Solution. (*i*) *No Load, running as a motor* :

$$\text{Input Power} = 500 \times 5 = 2500 \text{ watts}$$
$$\text{Field copper-loss} = 500 \times 2.5 = 1250 \text{ watts}$$

Neglecting armature copper-loss at no load (since it comes out to be $2.5^2 \times 0.15 = 1$ watt), the balance of 1250 watts of power goes towards no load losses of the machine running at rated speed. These losses are mainly the no load mechanical losses and the core-loss.

(*ii*) *As a Generator, delivering 40 A to load* :

$$\text{Output delivered} = 500 \times 40 \times 10^{-3} = 20 \text{ kW}$$

Losses : (*a*) Field copper-loss = 1250 watts

(*b*) Armature copper-loss = $42.5^2 \times 0.15 = 271$ watts

(*c*) No load losses = 1250 watts

Total losses = 2.771 kW

Generator Efficiency = (20/22.771) × 100 % = 87.83 %

Extension to the Question : *At what speed should the Generator be run, if the shunt-field is not changed, in the above case ? Assume that the motor was running at 600 r.p.m. Neglect armature reaction.*

Solution. As a motor on no-load,

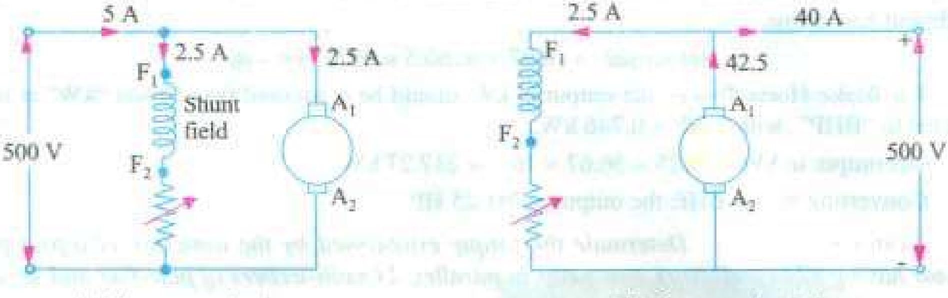
$$E_{b0} = 500 - I_a r_a = 500 - 0.15 \times 2.5 = 499.625 \text{ V}$$

As a Generator with an armature current of 42.5 A,

$$E_{b0} = 500 + 42.5 \times 0.15 = 506.375 \text{ V}$$

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp. With armature reaction is ignored, the flux/pole remains same. The e.m.f. then becomes proportional to the speed. If the generator must be driven at N r.p.m.

$$N = (506.375/449.625) \times 600 = 608.1 \text{ r.p.m.}$$



(a) Motor at no load (b) Generator loaded

Fig. 29.11

Note. Alternative to this slight increase in the speed is to increase the field current with the help of decreasing the rheostatic resistance in the field-circuit.

Example 29.13 (e). *A d.c. series motor takes 40 A at 220 V and runs at 800 r.p.m. If the armature and field resistance are 0.2 Ω and 0.1 Ω respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor ?*

Solution. Armature torque is given by $T_a = 9.55 \dfrac{E_b I_a}{N}$ N-m

Now $\qquad E_b = V - I_a (R_a + R_{se}) = 220 - 40 (0.2 + 0.1) = 208$ V

$\therefore \qquad T_a = 9.55 \times 208 \times 40/800 =$ **99.3 N-m**

Cu loss in armature and series-field resistance $= 40^2 \times 0.3 = 480$ W

Iron and friction losses = 500 W ; Total losses = 480 + 500 = 980 W

Motor power input = 220 × 40 = 8,800 W

Motor output = 8,800 − 980 = 7,820 W = **7.82 kW**

Example 29.14. *A cutting tool exerts a tangential force of 400 N on a steel bar of diameter 10 cm which is being turned in a simple lathe. The lathe is driven by a chain at 840 r.p.m. from a 220 V d.c. Motor which runs at 1800 r.p.m. Calculate the current taken by the motor if its efficiency is 80 %. What size is the motor pulley if the lathe pulley has a diameter of 24 cm ?*

(Elect. Technology-II, Gwalior Univ. 1985)

Solution. Torque $\qquad T_{sh}$ = Tangential force × radius = 400 × 0.05 = 20 N-m

Output power $= T_{sh} \times 2\pi N$ watt $= 20 \times 2\pi \times (840/60)$ watt = 1,760 W

Motor η = 0.8 ∴ Motor input = 1,760/0.8 = 2,200 W

Current drawn by motor = 2200/220 = **10 A**

Let N_1 and D_1 be the speed and diameter of the driver pulley respectively and N_2 and D_2 the respective speed and diameter of the lathe pulley.

Then $\qquad N_1 \times D_1 = N_2 \times D_2$ or $1,800 \times D_1 = 840 \times 0.24$

$\therefore \qquad D_1 = 840 \times 0.24/1,800 = 0.112$ m = **11.2 cm**

Example 29.15. *The armature winding of a 200-V, 4-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3 Ω; the armature resistance 0.5 Ω and the iron and friction losses total 900 W. The pulley diameter is 0.41 m. Find the pull in newton at the rim of the pulley.*

(Elect. Engg. AMIETE Sec. A. 1991)

Solution. $E_b = V - I_a R_a = 200 - 45\,(0.5 + 0.3) = 164$ V

Now $E_b = \dfrac{\Phi ZN}{60} \cdot \left(\dfrac{P}{A}\right)$ volt

$\therefore$ $164 = \dfrac{18 \times 10^{-3} \times 280 \times 4 \times N}{60} \times \dfrac{4}{4}$ $\therefore$ $N = 488$ r.p.m.

Total input $= 200 \times 45 = 9{,}000$ W ; Cu loss $= I_a^2 R_a = 45^2 \times 0.8 = 1{,}620$ W

Iron + Friction losses $= 800$ W ; Total losses $= 1{,}620 + 800 = 2{,}420$ W

Output $= 9{,}000 - 2{,}420 = 6{,}580$ W

$\therefore$ $T_{sh} = 9 \times 55 \times \dfrac{6580}{488} = 128$ N-m

Let F be the pull in newtons at the rim of the pulley.

Then $F \times 0.205 = 128.8$ $\therefore$ $F = 128.8/0.205\ N =$ **634 N**

Example 29.16. *A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find* (a) *total torque* (b) *useful torque* (c) *useful flux / pole* (d) *rotational losses and* (e) *efficiency.*

Solution. $E_b = V - I_a R_a -$ brush drop $= 240 - (50 \times 0.1) - 2 = 233$ V

Also $I_a = 50$ A

(*a*) Armature torque $T_a = 9.55\,\dfrac{E_b I_a}{N}$ N-m $= 9.55 \times \dfrac{233 \times 50}{1000} =$ **111 N-m**

(*b*) $T_{sh} = 9.55\,\dfrac{\text{output}}{N} = 9.55 \times \dfrac{11{,}190}{1000} =$ **106.9 N-m**

(*c*) $E_b = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

$\therefore$ $233 = \dfrac{\Phi \times 540 \times 1000}{60} \times \left(\dfrac{4}{2}\right)$ $\therefore$ **$\Phi = 12.9$ mWb**

(*d*) Armature input $= V I_a = 240 \times 50 =$ **12,000 W**

Armature Cu loss $= I_a^2 R_a = 50^2 \times 0.1 = 250$ W ; Brush contact loss $= 50 \times 2 = 100$ W

$\therefore$ Power developed $= 12{,}000 - 350 = 11{,}650$ W ; Output $= 11.19$ kW $= 11{,}190$ W

$\therefore$ Rotational losses $= 11{,}650 - 11{,}190 =$ **460 W**

(*e*) Total motor input $= VI = 240 \times 51 = 12{,}340$ W ; Motor output $= 11{,}190$ W

$\therefore$ Efficiency $= \dfrac{11{,}190}{12{,}240} \times 100 =$ **91.4 %**

Example 29.17. *A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux is proportional to the field current.*

(Elect. Engg.-II, Kerala Univ. 1988)

Solution. Since $\Phi \propto I_a$, hence $T \propto I_a^2$

$\therefore$ $T_1 \propto 40^2$ and $T_2 \propto 30^2$ $\therefore$ $\dfrac{T_2}{T_1} = \dfrac{9}{16}$

$\therefore$ Percentage change in torque is

$$= \frac{T_1 - T_2}{T_1} \times 100 = \frac{7}{16} \times 100 = \mathbf{43.75\ \%}$$

Now $E_{b1} = 460 - (40 \times 0.8) = 428$ V ; $E_{b2} = 460 - (30 \times 0.8) = 436$ V

$\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{I_{a1}}{I_{a2}}$ $\therefore$ $\dfrac{N_2}{500} = \dfrac{436}{428} \times \dfrac{40}{30}$ $\therefore$ $N_2 =$ **679 r.p.m.**

Example 29.18. *A 460-V, 55.95 kW, 750 r.p.m. shunt motor drives a load having a moment of inertia of 252.8 kg-m^2. Find approximate time to attain full speed when starting from rest against full-load torque if starting current varies between 1.4 and 1.8 times full-load current.*

Solution. Let us suppose that the starting current has a steady value of (1.4 + 1.8)/2 = 1.6 times full-load value.

$$\text{Full-load output} = 55.95 \text{ kW} = 55{,}950 \text{ W} ; \text{Speed} = 750 \text{ r.p.m.} = 12.5 \text{ r.p.s.}$$

$$\text{F.L. shaft torque } T = \text{power}/\omega = \text{power}/2\pi N = 55{,}950\ \pi \times (750/60) = 712.4 \text{ N-m}$$

During starting period, average available torque

$$= 1.6\,T - T = 0.6\,T = 0.6 \times 712.4 = 427.34 \text{ N-m}$$

This torque acts on the moment of inertial $I = 252.8$ km-m^2.

$$\therefore \quad 427.4 = 252.8 \times \frac{d\omega}{dt} = 252.8 \times \frac{2\pi \times 12.5}{dt}, \quad \therefore \quad dt = \mathbf{46.4\ s}$$

Example 29.19. *A 14.92 kW, 400 V, 400 -r.p.m. d.c. shunt motor draws a current of 40 A when running at full-load. The moment of inertia of the rotating system is 7.5 kg-m^2. If the starting current is 1.2 times full-load current, calculate*

(*a*) *full-load torque*

(*b*) *the time required for the motor to attain the rated speed against full-load.*

(Electrical Technology, Gujarat Univ. 1988)

Solution. (*a*) F.L. output 14.92 kW = 14,920 W ; Speed = 400 r.p.m. = 20/3 r.p.s

Now, $T\omega$ = output $\therefore$ $T = 14{,}920/2\pi \times (20/3) =$ **356 N-m**

(*b*) During the starting period, the torque available for accelerating the motor armature is

$$= 1.2\,T - T = 0.2\,T = 0.2 \times 356 = 71.2 \text{ N-m}$$

$$\text{Now, torque} = I\,\frac{d\omega}{dt} \quad \therefore \quad 71.2 = 7.5 \times \frac{2\pi \times (20/3)}{dt} \quad \therefore \quad dt = \mathbf{4.41\ second}$$

29.9. Speed of a D.C. Motor

From the voltage equation of a motor (Art. 27.4), we get

$$E_b = V - I_a R_a \quad \text{or} \quad \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a$$

$$\therefore \quad N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{60A}{ZP}\right) \text{ r.p.m.}$$

$$\text{Now} \quad V - I_a R_a = E_b \quad \therefore \quad N = \frac{E_b}{\Phi} \times \left(\frac{60A}{ZP}\right) \text{ r.p.m. or } N = K\,\frac{E_b}{\Phi}$$

It shows that speed is directly proportional to back e.m.f. E_b and inversely to the flux Φ on $N \propto E_b/\Phi$.

For Series Motor

Let
N_1 = Speed in the 1st case ; I_{a1} = armature current in the 1st case
Φ_1 = flux/pole in the first case
N_2, I_{a2}, Φ_2 = corresponding quantities in the 2nd case.

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\Phi_1} \text{ where } E_{b1} = V - I_{a1} R_a ; \; N_2 \propto \frac{E_{b2}}{\Phi_2} \text{ where } E_{b2} = V - I_{a2} R_a$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Prior to saturation of magnetic poles ; $\Phi \propto I_a$ $\quad \therefore \quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{I_{a1}}{I_{a2}}$

For Shunt Motor

In this case the same equation applies,

i.e., $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$ If $\Phi_2 = \Phi_1$, then $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$.

29.10. Speed Regulation

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed-controlling devices.

The speed regulation is defined *as the change in speed when the load on the motor is reduced from rated value to zero, expressed as percent of the rated load speed*.

$$\therefore \quad \% \text{ speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100 = \frac{dN}{N} \times 100$$

29.11. Torque and Speed of a D.C. Motor

It will be proved that though torque of a motor is admittedly a function of flux and armature current, *yet it is independent of speed*. In fact, it is the speed which depends on torque and not *vice-versa*. It has been proved earlier that

$$N = K\frac{V - I_a R_a}{\phi} = \frac{KE_b}{\Phi} \qquad \text{...Art. 27.9}$$

Also, $$T_a \propto \Phi I_a \qquad \text{...Art. 27.7}$$

It is seen from above that increase in flux would decrease the speed but increase the armature torque. It cannot be so because torque always tends to produce rotation. If torque increases, motor speed must *increase* rather than *decrease*. The apparent inconsistency between the above two equations can be reconciled in the following way :

Suppose that the flux of a motor is decreased by decreasing the field current. Then, following sequence of events take place :

1. Back e.m.f. E_b (= $N\Phi/K$) drops instantly (the speed remains constant because of inertia of the heavy armature).
2. Due to decrease in E_b, I_a is increased because $I_a = (V - E_b)/R_a$. Moreover, a small reduction in flux produces a proportionately large increase in armature current.
3. Hence, the equation $T_a \propto \Phi I_a$, a small decrease in ϕ is more than counterbalanced by a *large* increase in I_a with the result that there is a net *increase* in T_a.
4. This increase in T_a produces an increase in motor speed.

It is seen from above that with the applied voltage V held constant, motor speed varies inversely as the flux. However, it is possible to increase flux and, at the same time, increase the speed provided I_a is held constant as is actually done in a d.c. servomotor.

Example 29.20. *A 4-pole series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.*

(Elect. Engg. AMIETE Sec. A Part II June 1991)

Solution. $$T_a = 0.159\,\phi\, Z I_a\,(P/A) \text{ N-m}$$

$$\therefore \quad 209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a\,(4/2);\; I_a = 20.1 \text{ A}$$

$$E_a = V - I_a R_a = 500 - 20.1 \times 3 = 439.7 \text{ V}$$

Now, speed may be found either by using the relation for E_b or T_a as given in Art.

$$E_b = \Phi ZN \times (P/A) \text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2$$

$$\therefore \quad N = 6.73 \text{ r.p.s. or } \mathbf{382.2 \text{ r.p.m.}}$$

Example 29.21. *A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2 Ω and 250 Ω. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.* **(Elect. Engg. A.M.Ae. S.I. June 1991)**

Solution. Formula used : $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}} \times \dfrac{\Phi_0}{\Phi}$; Since $\Phi_0 = \Phi$ (given); $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}}$

$$I_{sh} = 250/250 = 1 \text{ A}$$

$$E_{b0} = V - I_{a0} R_a = 250 - (7 \times 0.2) = 248.6 \text{ V}; E_b = V - I_a R_a = 250 - (49 \times 0.2) = 240.2 \text{ V}$$

$$\therefore \quad \frac{N}{1000} = \frac{240.2}{248.6}; \; N = \mathbf{9666.1 \text{ r.p.m.}}$$

Example 29.22. *A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15 Ω and its field resistance 0.1 Ω. Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A.* **(Electrical Machinery - I, Banglore Univ. 1986)**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \; \Phi_2 = 0.45\,\Phi_1 \text{ or } \frac{\Phi_1}{\Phi_2} = \frac{1}{0.45}$$

$$E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ V}; E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ V}$$

$$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{1}{0.45}; \; N_2 = \mathbf{1940 \text{ r.p.m.}}$$

Example 29.23. *A 230-V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 115 Ω. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.* **(Elect. Technology, Kerala Univ. 1988)**

Solution.

$$N_1 = 1200 \text{ r.p.m.}, E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V}$$

$$N_2 = 1120 \text{ r.p.m.}, E_{b2} = 230 - 0.5\, I_{a2}$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \therefore \quad \frac{1120}{1200} = \frac{230 - 0.5 I_{a2}}{228.75}; \; I_{a2} = \mathbf{33 \text{ A}}$$

Line current drawn by motor $= I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35\ A}$

Power input at rated load $= 230 \times 35 = \mathbf{8{,}050\ W}$

Example 29.24. *A belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V bus-bars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed ? Given armature resistance = 0.025 Ω, field resistance = 60 Ω and contact drop under each brush = 1 V, Ignore armature reaction.* **(Elect. Machines (E-3) AMIE Sec.C Winter 1991)**

Solution. As Generator [Fig. 29.12 (*a*)]

Load current, $I = 100{,}000/220 = 454.55\ A$; $I_{sh} = 220/60 = 3.67$ A

$$I_a = I + I_{sh} = 458.2 \text{ A}; \; I_a R_a = 458.2 \times 0.025 = 11.45$$

$$E_b = 220 + 11.45 + 2 \times 1 = 233.45\ V; \; N_1 = 300 \text{ r.p.m.}$$

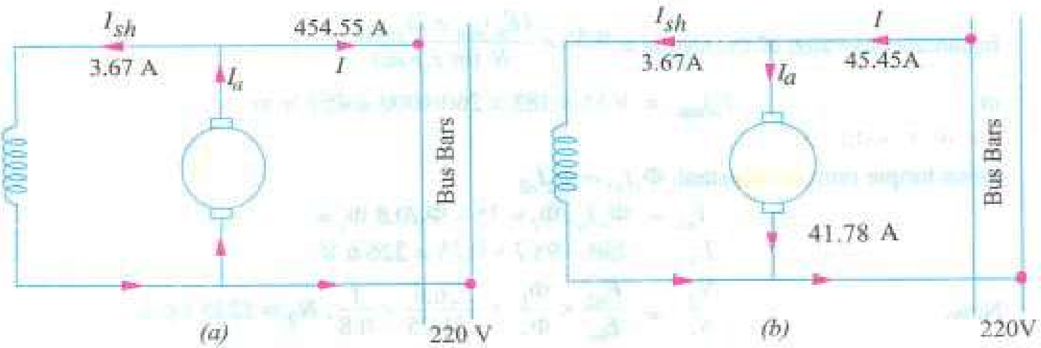


Fig. 29.12

As Motor [Fig. 29.12 (*b*)]

Input line current $= 100{,}000/220 = 45.45$ A; $I_{sh} = 220/60 = 3.67$ A

$I_a = 45.45 - 3.67 = 41.78$ A; $I_a R_a = 41.78 \times 0.025 = 1.04$ V; $E_{b2} = 220 - 1.04 - 2 \times 1 = 216.96$ V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \text{ since } \Phi_1 = \Phi_2 \text{ because } I_{sh} \text{ is constant}$$

$$\therefore \quad \frac{N_2}{300} = \frac{216.96}{233.45}; \; N_2 = 279 \text{ r.p.m.}$$

Example 29.25. *A d.c. shunt machine generates 250-V on open circuit at 1000 r.p.m. Effective armature resistance is 0.5 Ω, field resistance is 250 Ω, input to machine running as a motor on no-load is 4 A at 250 V. Calculate speed of machine as a motor taking 40 A at 250 V. Armature reaction weakens field by 4%.* **(Electrical Machines-I, Gujarat Univ. 1987)**

Solution. Consider the case when the machine runs as a motor on no-load.

Now, $I_{sh} = 250/250 = 1$ A; Hence, $I_{a0} = 4 - 1 = 3$A; $E_{b0} = 250 - 0.5 \times 3 = 248.5$ V

It is given that when armature runs at 1000 r.p.m., it generates 250 V. When it generates 248.5 V, it must be running at a speed = 1000 × 248.5/250 = 994 r.p.m.

Hence, $\quad N_0 = 994$ r.p.m.

When Loaded

$I_a = 40 - 1 = 39$ *A*; $E_b = 250 - 39 \times 0.5 = 230.5$ *V* Also, $\Phi_0/\Phi = 1/0.96$

$$\frac{N}{E} = \frac{E_b}{E_{b0}} \quad \therefore \quad \frac{N}{994} = \frac{230.5}{248.5} \times \frac{1}{0.96} \qquad N = \mathbf{960 \text{ r.p.m.}}$$

Example 29.26. *A 250-V shunt motor giving 14.92 kW at 1000 r.p.m. takes an armature current of 75 A. The armature resistance is 0.25 ohm and the load torque remains constant. If the flux is reduced by 20 percent of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of the armature current and speed.*

(Elect. Engg. AMIETE (New Scheme) 1990)

Solution. $E_{b1} = 250 - 75 \times 0.25 = 231.25$ V, as in Fig. 29.13. When flux is reduced by 20%, the back e.m.f. is also reduced instantly by 20% because speed remains constant due to inertia of the heavy armature (Art. 29.11).

$\therefore$ Instantaneous value of back e.m.f. $(E_b)_{inst} = 231.25 \times 0.8 = 185$ V

$$(I_a)_{inst} = [V - (E_b)_{inst}]/R_a = (250 - 185)/0.25 = 260 \text{ A}$$

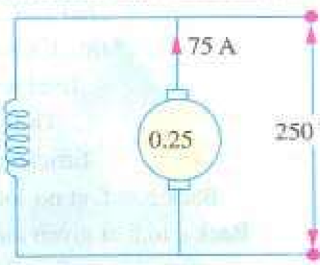


Fig. 29.13

Instantaneous value of the torque $= 9.55 \times \dfrac{(E_b)_{inst} \times (I_a)_{inst}}{N \text{ (in r.p.m.)}}$

or $(T_a)_{inst} = 9.55 \times 185 \times 260/1000 = 459$ N-m

Steady Conditions

Since torque remains constant, $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$I_{a2} = \Phi_1 I_{a1}/\Phi_2 = 75 \times \Phi_1/0.8\ \Phi_1 = \mathbf{93.7\ A}$$

$$\therefore \quad E_{b2} = 250 - 93.7 \times 0.25 = 226.6 \text{ V}$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{226.6}{231.25} \times \frac{1}{0.8};\ N_2 = 1225 \text{ r.p.m.}$$

Example 29.27. *A 220-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100 Ω. The resistance of armature at standstill gives a drop of 6 volts across armature terminals when 10 A were passed through it. Calculate (a) speed on load (b) torque in N-m and (c) efficiency. The normal input of the motor is 8 kW.*

(Electrotechnics-II; M.S. Univ. Baroda 1988)

Solution. (*a*) $I_{sh} = 200/100 = 2$ A

F.L. Power input $= 8{,}000$ W; F.L. line current $= 8{,}000/200 = 40$ A

$$I_a = 40 - 2 = 38 \text{ A}; \quad R_a = 6/10 = 0.6\ \Omega$$

$$E_{b0} = 200 - 2 \times 0.6 = 198.8 \text{ V}; \ E_b = 200 - 38 \times 0.6 = 177.2 \text{ V}$$

Now, $$\frac{N}{N_0} = \frac{E_b}{E_{b0}} \text{ or } \frac{N}{700} = \frac{177.2}{198.8};\ N = \mathbf{623.9\ r.p.m.}$$

(*b*) $T_a = 9.55\, E_b I_a/N = 9.55 \times 177.2 \times 38/623.9 = \mathbf{103\ N\text{-}m}$

(*c*) N.L. power input $= 200 \times 4 = 800$ W; Arm. Cu loss $= I_a^2 R_a = 2^2 \times 0.6 = 2.4$ W

Constant losses $= 800 - 2.4 = 797.6$ W; F.L. arm. Cu loss $= 38^2 \times 0.6 = 866.4$ W

Total F.L. losses $= 797.6 + 866.4 = 1664$ W; F.L. output $= 8{,}000 - 1664 = 6336$ W

F.L. Motor efficiency $= 6336/8{,}000 = 0.792$ or **79.2 %**

Example 29.28. *The input to 230-V, d.c. shunt motor is 11kW. Calculate (a) the torque developed (b) the efficiency (c) the speed at this load. The particulars of the motor are as follows :*

No-load current = 5 A; No-load speed = 1150 r.p.m.

Arm. resistance = 0.5 Ω; shunt field resistance = 110 Ω.

(Elect. Technology ; Bombay University 1988)

Solution. No-load input $= 220 \times 5 = 1{,}100$ W; $I_{sh} = 220/110 = 2$A; $I_{ao} = 5 - 2 = 3$A

No-load armature Cu loss $= 3^2 \times 0.5 = 4.5$ W

$\therefore$ Constant losses $= 1{,}100 - 4.5 = 1{,}095.5$ W

When input is 11 kW.

Input current $= 11{,}000/220 = 50$A ; Armature current $= 50 - 2 =$ **48 A**

Arm. Cu loss $= 48^2 \times 0.5 = 1{,}152$ W ;

Total loss = Arm. Cu loss + Constant losses $= 1152 + 1095.5 = 2248$ W

Output $= 11{,}000 - 2{,}248 = 8{,}752$ W

(*b*) Efficiency $= 8{,}752 \times 100/11{,}000 =$ **79.6%**

(*c*) Back e.m.f. at no-load $= 220 - (3 \times 0.5) = 218.5$ V

Back e.m.f. at given load $= 220 - (48 \times 0.5) = 196$ V

$\therefore$ Speed $N = 1{,}150 \times 196/218.5 =$ **1,031 r.p.m.**

(a) $$T_a = 9.55 \times \frac{196 \times 48}{1031} = \mathbf{87.1\ N\text{-}m}$$

Example 29.29. *The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.*

(Elect. Machines, A.M.I.E. Sec. B, 1993)

Solution. $$E_{b1} = 250 - 80\,(0.1 + 0.05) - 3 = 235\text{ V}$$
$$E_{b2} = 250 - 100\,(0.1 + 0.05) - 3 = 232\text{ V}$$

Since $\Phi \propto I_a$, hence, $\Phi_1 \propto 80$, $\Phi_2 \propto 100$, $\Phi_1/\Phi_2 = 80/100$

Now $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100}\ ; \quad N_2 = 474\text{ r.p.m.}$$

Example 29.30. *A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.*

(Elect. Machines ; A.M.I.E. Sec. B, 1991)

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \qquad (\because\ \Phi \propto I_a)$$

Since field is unsaturated, $T_a \propto \Phi I_a \propto I_a^2$. $\qquad (\because\ T_1 \propto I_{a1}^2$ and $T_2 \propto I_{a2}^2)$

or $$T_2/T_1 = (I_{a2}/I_{a1})^2 \text{ or } 1/2 = (I_{a2}/I_{a1})^2\ ; \ I_{a1} = I_{a1}/\sqrt{2} = 70.7\text{ A}$$
$$E_{b1} = 220 - 100 \times 0.1 = 210\text{ V}\ ; \ E_{b2} = 220 - 0.1 \times 70.7 = 212.9\text{ V}$$

$\therefore$ $$\frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7}\ ; \quad N_2 = \mathbf{1147\ r.p.m.}$$

Example 29.31. *A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.*

$$\Phi = (1.7 \times 10^{-2} \times I^{0.5})\text{ weber}$$

where I is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses. **(Elect. Machines, Nagpur Univ. 1993)**

Solution. Let us first find R_a.

Now $$N = \frac{E_b}{Z\Phi}\left(\frac{60\ A}{P}\right)\text{r.p.m.}$$

$\therefore$ $$600 = \frac{E_b}{1.7 \times 10^{-2} \times 25^{0.5}} \times \frac{60 \times 4}{500 \times 4}$$

$\therefore$ $$E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425\text{ V}$$
$$I_a R_a = 450 - 425 = 25\text{ V}\ ; \ R_a = 25/25 = 1.0\ \Omega$$

Now in the Ist Case

$$T_1 \propto \Phi_1 I_1 \quad \therefore \quad T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25$$

Similarly $$T_2 \propto 1.7 \times 10^{-2} \times \sqrt{1 \times I}\ ; \quad \text{Now} \quad T_1 = 2T_2$$

$\therefore$ $$1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I^{3/2} \times 2 \quad \therefore \quad I = (125/2)^{2/3} = 15.75\text{ A}$$
$$E_{b1} = 425\text{ V}\ ; \quad E_{b2} = 225 - (15.75 \times 1) = 209.3\text{ V}$$

Using the relation $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$; we have

$$\frac{N_2}{600} = \frac{209.3}{425} \times \frac{1.7 \times 10^{-2} \times 5}{1.7 \times 10^{-2} \times \sqrt{15.75}}; \quad N_2 = \mathbf{372\ r.p.m.}$$

Tutorial Problems 29.2

1. Calculate the torque in newton-metre developed by a 440-V d.c. motor having an armature resistance of 0.25 Ω and running at 750 r.p.m. when taking a current of 60 A. **[325 N-m]**

2. A 4-pole, lap-connected d.c. motor has 576 conductors and draws an armature current of 10 A. If the flux per pole is 0.02 Wb, calculate the armature torque developed. **[18.3 N-m]**

3. (*a*) A d.c. shunt machine has armature and field resistances of 0.025 Ω and 80 Ω respectively. When connected to constant 400-V bus-bars and driven as a generator at 450 r.p.m., it delivers 120 kW. Calculate its speed when running as a motor and absorbing 120 kW from the same bus-bars.

(*b*) Deduce the direction of rotation of this machine when it is working as a motor assuming a clockwise rotation as a generator. **[(*a*) 435 r.p.m. (*b*) Clockwise]**

4. The armature current of a series motor is 60 A when on full-load. If the load is adjusted to that this current decreases to 40-A, find the new torque expressed as a percentage of the full-load torque. The flux for a current of 40 A is 70% of that when current is 60 A. **[46%]**

5. A 4-pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap-wound with 720 conductors. The shunt field resistance is 240 Ω and the armature resistance is 0.2 Ω. Brush contact drop is 1V per brush. Determine the speed of the machine when running (*a*) as a motor taking 60 A and (*b*) as a generator supplying 120 A. The terminal voltage in each case is 480 V. **[972 r.p.m. ; 1055 r.p.m.]**

6. A 25-kW shunt generator is delivering full output to 400-V bus-bars and is driven at 950 r.p.m. by belt drive. The belt breaks suddenly but the machine continues to run as a motor taking 25 kW from the bus-bars. At what speed does it run ? Take armature resistance including brush contact resistance as 0.5 Ω and field resistance as 160 Ω. **[812.7 r.p.m.]** *(Elect. Technology, Andhra Univ. Apr. 1977)*

7. A 4-pole, d.c. shunt motor has a wave-wound armature with 65 slots each containing 6 conductors. The *flux* per pole is 20 mWb and the armature has a resistance of 0.15 Ω. Calculate the motor speed when the machine is operating from a 250-V supply and taking a current of 60 A. **[927 r.p.m.]**

8. A 500-V, d.c. shunt motor has armature and field resistances of 0.5 Ω and 200 Ω respectively. When loaded and taking a total input of 25 kW, it runs at 400 r.p.m. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. **[442 r.p.m.]**

9. A d.c. shunt motor runs at 900 r.p.m. from a 400 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230 V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 400 V. **[595 r.p.m.]**

10. A shunt machine connected to 250-A mains has an armature resistance of 0.12 Ω and field resistance of 100 Ω. Find the ratio of the speed of the machine as a generator to the speed as a motor, if line current is 80 A in both cases. **[1.08]** *(Electrical Engineering-II, Bombay Univ. April. 1977, Madras Univ. Nov. 1978)*

11. A 20-kW d.c. shunt generator delivering rated output at 1000 r.p.m. has a terminal voltage of 500 V. The armature resistance is 0.1 Ω, voltage drop per brush is 1 volt and the field resistance is 500 Ω.

Calculate the speed at which the machine will run as a motor taking an input of 20 kW from a 500 V d.c. supply. **[976.1 r.p.m.]** *(Elect. Engg-I Bombay Univ. 1975)*

12. A 4-pole, 250-V, d.c. shunt motor has a lap-connected armature with 960 conductors. The flux per pole is 2×10^{-2} Wb. Calculate the torque developed by the armature and the useful torque in newton-metre when the current taken by the motor is 30A. The armature resistance is 0.12 ohm and the field resistance is 125 Ω. The rotational losses amount to 825 W.

[85.5 N-m ; 75.3 N-m] *(Electric Machinery-I, Madras Univ. Nov. 1979)* (*a*) $$T_a = 9.55 \times \frac{196 \times 48}{1031} = \mathbf{87.1\ N\text{-}m}$$

Example 29.29. *The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.*

(Elect. Machines, A.M.I.E. Sec. B, 1993)

Solution. $$E_{b1} = 250 - 80\,(0.1 + 0.05) - 3 = 235\text{ V.}$$

$$E_{b2} = 250 - 100\,(0.1 + 0.05) - 3 = 232\text{ V}$$

Since $\Phi \propto I_a$, hence, $\Phi_1 \propto 80$, $\Phi_2 \propto 100$, $\Phi_1/\Phi_2 = 80/100$

Now $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100}\ ; \quad N_2 = 474\text{ r.p.m.}$$

Example 29.30. *A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.*

(Elect. Machines ; A.M.I.E. Sec. B, 1991)

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \qquad (\because\ \Phi \propto I_a)$$

Since field is unsaturated, $T_a \propto \Phi I_a \propto I_a^2$. $\qquad (\because\ T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2)$

or $$T_2/T_1 = (I_{a2}/I_{a1})^2 \text{ or } 1/2 = (I_{a2}/I_{a1})^2\ ;\ I_{a1} = I_{a1}/\sqrt{2} = 70.7\text{ A}$$

$$E_{b1} = 220 - 100 \times 0.1 = 210\text{ V}\ ;\ E_{b2} = 220 - 0.1 \times 70.7 = 212.9\text{ V}$$

$\therefore$ $$\frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7}\ ;\quad N_2 = \mathbf{1147\ r.p.m.}$$

Example 29.31. *A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.*

$$\Phi = (1.7 \times 10^{-2} \times I^{0.5})\text{ weber}$$

where I is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses. **(Elect. Machines, Nagpur Univ. 1993)**

Solution. Let us first find R_a.

Now $$N = \frac{E_b}{Z\Phi}\left(\frac{60\ A}{P}\right)\text{r.p.m.}$$

$\therefore$ $$600 = \frac{E_b}{1.7 \times 10^{-2} \times 25^{0.5}} \times \frac{60 \times 4}{500 \times 4}$$

$\therefore$ $$E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425\text{ V}$$

$$I_a R_a = 450 - 425 = 25\text{ V}\ ;\ R_a = 25/25 = 1.0\ \Omega$$

Now in the Ist Case

$$T_1 \propto \Phi_1 I_1 \quad \therefore \quad T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25$$

Similarly $$T_2 \propto 1.7 \times 10^{-2} \times \sqrt{1 \times I}\ ; \quad \text{Now} \quad T_1 = 2T_2$$

$\therefore$ $$1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I^{3/2} \times 2 \quad \therefore \quad I = (125/2)^{2/3} = 15.75\text{ A}$$

$$E_{b1} = 425\text{ V}\ ; \quad E_{b2} = 225 - (15.75 \times 1) = 209.3\text{ V}$$

Using the relation $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$; we have

29.12. Motor Characteristics

The characteristic curves of a motor are those curves which show relationships between the following quantities.

1. **Torque and armature current** *i.e.* T_a/I_a characteristic. It is known as ***electrical characteristic***.
2. **Speed and armature current** *i.e.* N/I_a characteristic.
3. **Speed and torque** *i.e.* N/T_a characteristic. It is also known as ***mechanical characteristic***. It can be found from (1) and (2) above.

While discussing motor characteristics, the following two relations should always be kept in mind :

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

29.13. Characteristics of Series Motors

1. T_a/I_a Characteristic. We have seen that $T_a \propto \Phi I_a$. In this case, as field windings also carry the armature current, $\Phi \propto I_a$ up to the point of magnetic saturation. Hence, before saturation,

$$T_a \propto \Phi I_a \quad \text{and} \quad \therefore \quad T_a \propto I_a^2$$

At light loads, I_a and hence Φ is small. But as I_a increases, T_a increases as the square of the current. Hence, T_a/I_a curve is a parabola as shown in Fig. 29.14. After saturation, Φ is almost independent of I_a hence $T_a \propto I_a$ only. So the characteristic becomes a straight line. The shaft torque T_{sh} is less than armature torque due to stray losses. It is shown dotted in the figure. So we conclude that (prior to magnetic saturation) on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in cases where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc., series motors are used.

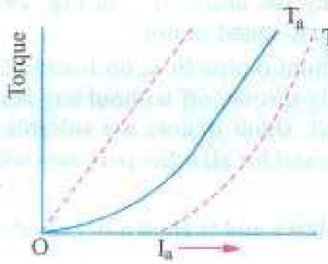


Fig. 29.14

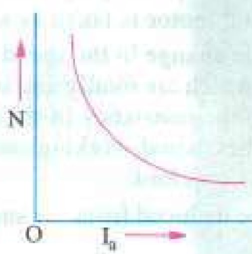


Fig. 29.15

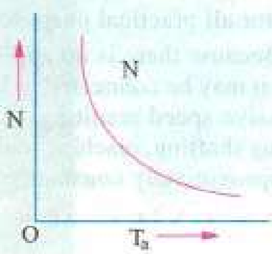


Fig. 29.16

2. N/I_a Characteristics. Variations of speed can be deduced from the formula :

$$N \propto \frac{E_b}{\Phi}$$

Change in E_b, for various load currents is small and hence may be neglected for the time being. With increased I_a, Φ also increases. Hence, speed varies inversely as armature current as shown in Fig. 29.15.

When load is heavy, I_a is large. Hence, speed is low (this decreases E_b and allows more armature current to flow). But when load current and hence I_a falls to a small value, speed becomes dangerously high. Hence, a series motor should never be started without some mechanical (not belt-driven) load on it otherwise it may develop excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

3. N/T_a or mechanical characteristic. It is found from above that when speed is high, torque is low and ***vice-versa***. The relation between the two is as shown in Fig. 29.16.

29.14. Characteristics of Shunt Motors

1. T_a/I_a Characteristic

Assuming Φ to be practically constant (though at heavy loads, ϕ decreases somewhat due to increased armature reaction) we find that $T_a \propto I_a$.

Hence, the electrical characteristic as shown in Fig. 29.17. is practically a straight line through the origin. Shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on (heavy) load.

2. N/I_a Characteristic

If Φ is assumed constant, then $N \propto E_b$. As E_b is also practically constant, speed is, for most purposes, constant (Fig. 29.18).

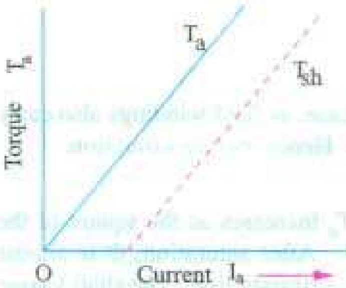


Fig. 29.17

Fig. 29.18

Fig. 29.19

But strictly speaking, both E_b and Φ decrease with increasing load. However, E_b decreases slightly more than ϕ so that on the whole, there is some decrease in speed. The drop varies from 5 to 15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly drooping as shown by the dotted line in Fig. 29.18. But, for all practical purposes, shunt motor is taken as a constant-speed motor.

Because there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

3. N/T_a Characteristic can be deduced from (1) and (2) above and is shown in Fig. 29.19.

29.15. Compound Motors

These motors have both series and shunt windings. If series excitation helps the shunt excitation *i.e.* series flux is in the *same* direction (Fig. 29.20); then the motor is said to be cummulatively compounded. If on the other hand, series field opposes the shunt field, then the motor is said to be differentially compounded.

Compound Motors

The characteristics of such motors lie in between those of shunt and series motors as shown in Fig. 29.21.

(a) Cumulative-compound Motors

Such machines are used where series characteristics are required and where, in addition, the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to take sudden cuts quite often. Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy loads. In conjunction with fly-wheel (functioning as load equalizer), it is employed where there are sudden temporary loads as in rolling mills. The fly-wheel supplies its stored kinetic energy when motor slows down due to sudden heavy load. And when due to the removal of load motor speeds up, it gathers up its kinetic energy.

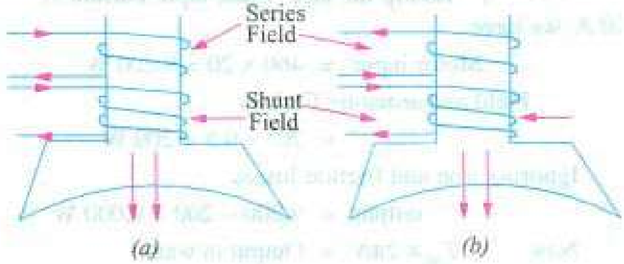


Fig. 29.20

Compound-wound motors have greatest application with loads that require high starting torques or pulsating loads (because such motors smooth out the energy demand required of a pulsating load). They are used to drive electric shovels, metal-stamping machines, reciprocating pumps, hoists and compressors etc.

(b) Differential-compound Motors

Since series field opposes the shunt field, the flux is decreased as load is applied to the motor. This results in the motor speed remaining almost constant or even increasing with increase in load (because, $N \propto E_b/(\Phi)$. Due to this reason, there is a decrease in the rate at which the motor torque increases with load. Such motors are not in common use. But because they can be designed to give an accurately constant speed under all conditions, they find limited application for experimental and research work.

One of the biggest drawback of such a motor is that due to weakening of flux with increases in load, there is a tendency towards speed instability and motor running away unless designed properly.

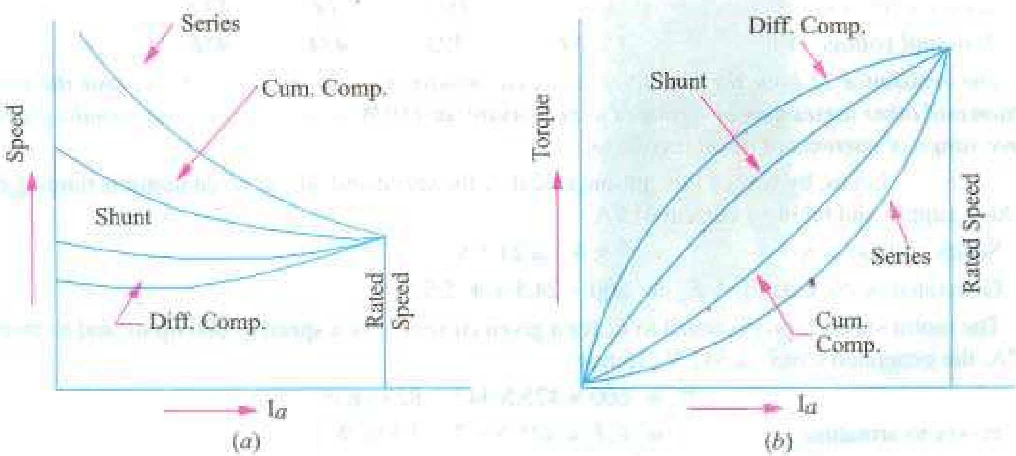


Fig. 29.21

Example 29.32. *The following results were obtained from a static torque test on a series motor :*

Current (A) :	20	30	40	50
Torque (N - m) :	128.8	230.5	349.8	46.2

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 Ω. Ignore iron and friction losses.

Solution. Taking the case when input current is 20 A, we have

Motor input $= 460 \times 20 = 9{,}200$ W

Field and armature Cu loss

$= 20^2 \times 0.5 = 200$ W

Ignoring iron and friction losses,

output $= 9{,}200 - 200 = 9{,}000$ W

Now, $T_{sh} \times 2\pi N$ = Output in watts.

$\therefore \quad 128.8 \times 2\pi \times N = 9{,}000$

$\therefore \quad N = 9{,}000/2\pi \times 128.8$

$= 11.12$ r.p.s. $= 667$ r.p.m.

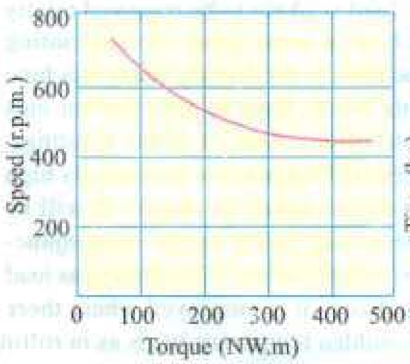


Fig. 29.22

Similar calculations for other values of current are tabulated below :

Current (A)	20	30	40	50
Input (W)	9,200	13,800	18,400	23,000
I^2R loss (W)	200	450	800	1,250
Output (W)	9,200	13,350	17,600	21,850
Speed (r.p.m.)	667	551	480	445
Torque (N-m)	128.8	230.5	349.8	469.2

From these values, the speed/torque curve can be drawn as shown in Fig. 29.22.

Example 29.33. *A fan which requires 8 h.p. (5.968 kW) at 700 r.p.m. is coupled directly to a d.c. series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the cube of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as follows :*

Load current (A)	:	7	10.5	14	27.5
Terminal voltage (V)	:	347	393	434	458

The resistance of both the armature and field windings of the motor is 3.5 Ω and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage. **(I.E.E. London)**

Solution. Let us, by way of illustration, calculate the speed and output when motor is running off a 500-V supply and taking a current of 7A.

Series voltage drop $= 7 \times 3.5 = 24.5$ V

Generated or back e.m.f. $E_b = 500 - 24.5 = 475.5$ V

The motor speed is proportional to E_b for a given current. For a speed of 600 r.p.m. and a current of 7A, the generated e.m.f is 347 V. Hence,

$$N = 600 \times 475.5/347 = 823 \text{ r.p.m.}$$

Power to armature $= E_b I_a = 475.5 \times 7 = 3{,}329$ W

Output = Armature power − 450 = 3,329 − 450 = 2,879 W = **2.879 kW**

Power required by the fan at 823 r.p.m. is $= 5.968 \times 823^2/700^2 = 9.498$ kW

These calculations are repeated for the other values of current in the following table.

Input current (A)	7	10.5	14	27.5
Series drop (V)	24.5	36.7	49	96.4
Back e.m.f. (V)	475.5	463.3	451	403.6

E.M.F. at 600 r.p.m. (V)	347	393	434	458
Speed N (r.p.m.)	823	707	623	528
Armature power (W)	3329	4870	6310	11,100
Motor output (kW)	2.879	4.420	5.860	10.65
Power required by fan (kW)	9.698	6.146	4.222	2.566

In Fig. 29.23 (*i*) the motor output in kW and (*ii*) power required by fan in kW against input currentis plotted. Since motor output equals the input to fan, hence the intersection point of these curves gives the value of motor input current under the given conditions.

Input current corresponding to intersection point = 12 A

∴ Motor input = 500 × 12 = **6,000 W**

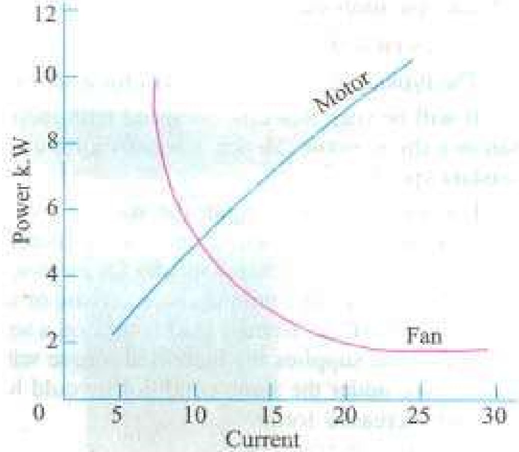


Fig. 29.23

29.16. Performance Curves

(*a*) Shunt Motor

In Fig. 29.24 the four essential characteristics of a shunt motor are shown *i.e.* torque, current speed and efficiency, each plotted as a function of motor output power. These are known as the ***performance curves*** of a motor.

It is seen that shunt motor has a definite no-load speed. Hence, it does not 'run away' when load is suddenly thrown off provided the field circuit remains closed. The drop in speed from no-load to full-load is small, hence this motor is usually referred to as ***constant speed*** motor. The speed for any load within the operating range of the motor can be readily obtained by varying the field current by means of a field rheostat.

The efficiency curve is usually of the same shape for all electric motors and generators. The shape of efficiency curve and the point of maximum efficiency can be varied considerably by the designer, though it is advantageous to have an efficiency curve which is farily flat, so that there is little change in efficiency between load and 25% overload and to have the maximum efficiency as near to the full load as possible.

It will be seen from the curves, that a certain value of current is required even when output is zero. The motor input under no-load conditions goes to meet the various losses occuring within the machine.

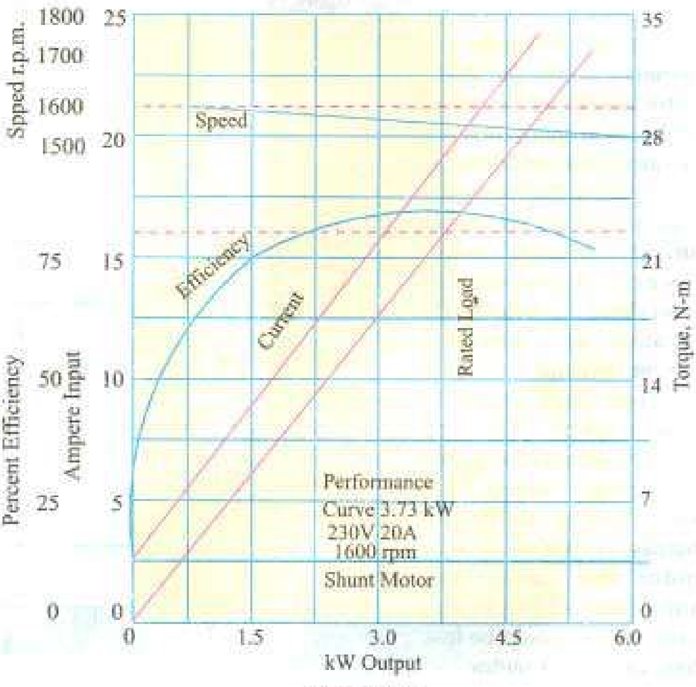


Fig. 29.24

As compared to other motors, a shunt motor is said to have a lower starting torque. But this should not be taken of mean that a shunt motor is incapable of starting a heavy load. Actually, it means that series and compound motors are capable of starting heavy loads with less excess of current inputs over normal values than the shunt motors and that consequently the depreciation on the motor will be relatively less. For example, if twice full load torque is required at start, then shunt motor draws twice the full-load current ($T_a \propto I_a$ or $I_a \propto \sqrt{T_a}$) whereas series motor draws only approximately one and a half times the full load current ($T_a \propto I_a^2$ or $I_a \propto \sqrt{T_a}$).

The shunt motor is widely used with loads that require essentially constant speed but where high starting torques are not needed. Such loads include centrifugal pumps, fans, winding reels conveyors and machine tools etc.

(b) Series Motor

The typical performance curves for a series motor are shown in Fig. 29.25.

It will be seen that drop in speed with increased load is much more prominent in series motor than in a shunt motor. Hence, a series motor is not suitable for applications requiring a substantially constant speed.

For a given current input, the starting torque developed by a series motor is greater than that developed by a shunt motor. Hence, series motors are used where huge starting torques are necessary *i.e.* for street cars, cranes, hoists and for electric-railway operation. In addition to the huge starting torque, there is another unique characteristic of series motors which makes them especially desirable for traction work *i.e.* when a load comes on a series motor, it responds by decreasing its speed (and hence, E_b) and supplies the increased torque with a small increase in current. On the other hand a shunt motor under the same conditions would hold its speed nearly constant and would supply the required increased torque with a large increase of input current. Suppose that instead of a series motor, a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed for the car at approximately the same value it had on the level ground, but the motor tends to take an excessive current. A series motor, however, automatically slows down on such a grade because of increased current demand, and so it develops more torque at reduced speed. The drop in speed permits the motor to develop a large torque with but a moderate increase of power. Hence, under the same load conditions, rating of the series motor would be less than for a shunt motor.

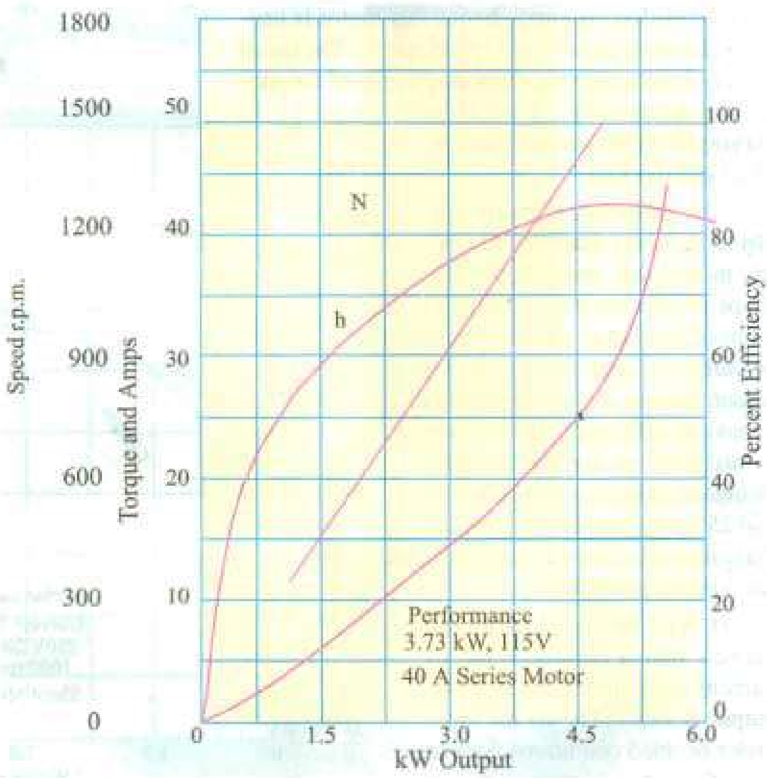


Fig. 29.25

29.17. Comparison of Shunt and Series Motors

(a) Shunt Motors

The different characteristics have been discussed in Art. 29.14. It is clear that

(a) speed of a shunt motor is sufficiently constant.

(b) for the same current input, its starting torque is not a high as that of series motor. Hence, it is used.

(i) When the speed has to be maintained approximately constant from N.L. to F.L. *i.e.* for driving a line of shafting etc.

(ii) When it is required to drive the load at various speeds, any one speed being kept constant for a relatively long period *i.e.* for individual driving of such machines as lathes. The shunt regulator enables the required speed control to be obtained easily and economically.

Shunt Motors

Summary of Applications

Type of motor	*Characteristics*	*Applications*
Shunt	Approximately constant speed Adjustable speed Medium starting torque (Up to 1.5 F.L. torque)	For driving constant speed line shafting Lathes Centrifugal pumps Machine tools Blowers and fans Reciprocating pumps
Series	Variable speed Adjustable variying speed High Starting torque	For traction work *i.e.* Electric locomotives Rapid transit systems Trolley, cars etc. Cranes and hoists Conveyors
Comulative Compound	Variable speed Adjustable varying speed High starting torque	For intermittent high torque loads For shears and punches Elevators Conveyors Heavy planers Heavy planers Rolling mills; Ice machines; Printing presses; Air compressors

(b) Series Motors

The operating characteristics have been discussed in Art 29.13. These motors

1. have a relatively huge starting torques.
2. have good accelerating torque.
3. have low speed at high loads and dangerously high speed at low loads.

Hence, such motors are used

1. when a large starting torque is required *i.e.* for driving hoists, cranes, trams etc.

2. when the motor can be directly coupled to a load such as a fan whose torque increases with speed.

3. if constancy of speed is not essential, then, in fact, the decrease of speed with increase of load has the advantage that the power absorbed by the motor does not increase as rapidly as the torque. For instance, when torque is doubled, the power approximately increases by about 50 to 60% only. ($\therefore I_a \propto \sqrt{T_a}$).

4. a series motor should not be used where there is a possibility of the load decreasing to a very small value. Thus, it should not be used for driving centrifugal pumps or for a belt-drive of any kind.

Series Motors

29.18. Losses and Efficiency

The losses taking place in the motor are the same as in generators. These are (i) Copper losses (ii) Magnetic losses and (iii) Mechanical losses.

The condition for maximum *power* developed by the motor is

$$I_a R_a = V/2 = E_b$$

The condition for maximum *efficiency* is that armature Cu losses are equal to constant losses. (Art. 26.39).

29.19. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Fig. 29.26.

Overall or commercial efficiency $\eta_c = \frac{C}{A}$, Electrical efficiency $\eta_e = \frac{B}{A}$, Mechanical efficiency $\eta_m = \frac{C}{B}$.

The efficiency curve for a motor is similar in shape to that for a generator (Art. 24.35).

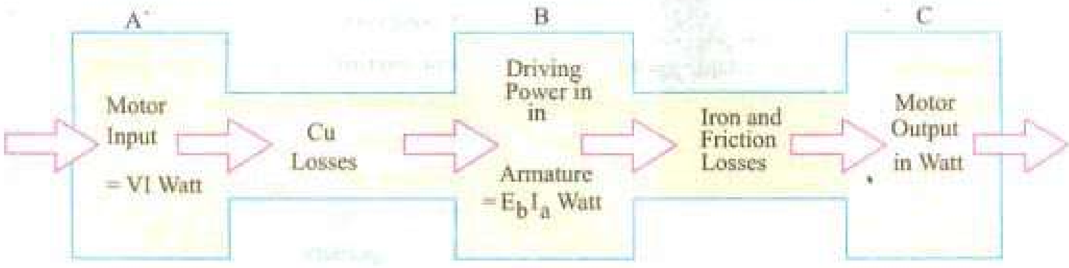


Fig. 29.26

It is seen that $A - B$ = copper losses and $B - C$ = iron and friction losses.

Example 29.34. *One of the two similar 500-V shunt machines A and B running light takes 3 A. When A is mechanically coupled to B, the input to A is 3.5 A with B unexcited and 4.5 A when B is separately-excited to generate 500 V. Calculate the friction and windage loss and core loss of each machine.* **(Electric Machinery-I, Madras Univ. 1985)**

Solution. When running light, machine input is used to meet the following losses (i) armature Cu loss (ii) shunt Cu loss (iii) iron loss and (iv) mechanical losses *i.e.* friction and windage losses. Obviously, these no-load losses for each machine equal $500 \times 3 = 1500$ W.

(*a*) **With B unexcited**

In this case, only mechanical losses take place in B, there being neither Cu loss nor iron-loss because B is unexcited. Since machine A draws 0.5 A more current.

Friction and windage loss of $B = 500 \times 0.5 =$ **250 W**

(*b*) **With B excited**

In this case, both iron losses as well as mechanical losses take place in machine B. Now, machine A draws, $4.5 - 3 = 1.5$ A more current.

Iron and mechanical losses of $B = 1.5 \times 500 = 750$ W

Iron losses of $B = 750 - 250 =$ **500 W**

Example 29.35. *A 220 V shunt motor has an armature resistance of 0.2 ohm and field resistance of 110 ohm. The motor draws 5 A at 1500 r.p.m. at no load. Calculate the speed and shaft torque if the motor draws 52 A at rated voltage.* **(Elect. Machines Nagpur Univ. 1993)**

Solution.

$$I_{sh} = 220/110 = 2\text{ A};\ I_{a1} = 5 - 2 = 3\text{ A};\ I_{a2} = 52 - 2 = 50\text{ A}$$

$$E_{b1} = 220 - 3 \times 0.2 = 219.4\text{ V};\ E_{b2} = 220 - 50 \times 0.2 = 210\text{ V}$$

$$\frac{N_2}{1500} = \frac{210}{219.4};\ N_2 = 1436\text{ r.p.m.} \quad (\because \Phi_1 = \Phi_2)$$

For finding the shaft torque, we will find the motor output when it draws a current of 52 A. First we will use the no-load data for finding the constant losses of the motor.

No load motor input $= 220 \times 5 = 1100$ W; Arm. Cu loss $= 3^2 \times 0.2 = 2$ W

$\therefore$ Constant or standing losses of the motor $= 1100 - 2 = 1098$

When loaded, arm. Cu loss $= 50^2 \times 0.2 = 500$ W

Hence, total motor losses $= 1098 + 500 = 1598$ W

Motor input on load $= 220 \times 52 = 11{,}440$ W; output $= 11{,}440 - 1598 = 9842$ W

$\therefore$ $T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 9842/1436 =$ **65.5 N-m**

Example 29.36. *250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 amperes. Armature and shunt field resistances are 0.2 and 250 ohms respectively. Calculate the speed when loaded taking a current of 50 A. The armature reaction weakens the field by 3%.*

(Elect. Engg.-I Nagpur Univ. 1993)

Solution.

$$I_{sh} = 250/250 = 1\text{ A};\ I_{a1} = 5 - 1 = 4\text{ A};\ I_{a2} = 50 - 1 = 49\text{ A}$$

$$E_{b1} = 250 - 4 \times 0.2 = 249.2\text{ V};\ E_{b2} = 250 - 49 \times 0.2 = 240.2\text{ V}$$

$$\frac{N_2}{1000} = \frac{240.2}{249.2} \times \frac{\Phi_1}{0.97\,\Phi_1};\ N_2 = 944\text{ r.p.m.}$$

Example 29.37. *A 500 V d.c. shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuit are 0.22 ohm and 250 ohm respectively. Find* (*a*) *the efficiency when loaded and taking a current of 100 A* (*b*) *the percentage change of speed. State precisely the assumptions made.* **(Elect. Engg-I, M.S. Univ. Baroda 1987)**

Solution. No-Load condition

$$I_{sh} = 500/250 = 2\text{ A};\ I_{a0} = 5 - 2 = 3\text{ A};\ E_{b0} = 500 - (3 \times 0.22) = 499.34\text{ V}$$

Arm. Cu loss $= 3^2 \times 0.22 = 2$ W; Motor input $= 500 \times 5 = 2500$ W

Constant losses $= 2500 - 2 = 2498$ W

It is assumed that these losses remain constant under all load conditions.

Load condition

(*a*) Motor current $= 100$ A; $I_a = 100 - 2 = 98$ A; $E_b = 500 - (98 \times 0.22) = 478.44$ V

$$\text{Arm. Cu loss} = 98^2 \times 0.22 = 2110 \text{ W, Total losses} = 2110 + 2498 = 4608 \text{ W}$$

$$\text{Motor input} = 500 \times 100 = 50{,}000 \text{ W, Motor output} = 50{,}000 - 4{,}608 = 45{,}392 \text{ W}$$

$$\text{Motor } \eta = 45{,}392/50{,}000 = 0.908 \text{ or } \mathbf{90.8\%}$$

$$\frac{N}{N_0} = \frac{E_b}{E_{b0}} = \frac{478.44}{499.34} \text{ or } \frac{N - N_0}{N_0} = \frac{-20.9}{499.34} = -0.0418 \text{ or } \mathbf{-4.18\%}$$

Example 29.38. *A 250 V d.c. shunt motor runs at 1000 r.p.m. while taking a current of 25 A. Calculate the speed when the load current is 50 A if armature reaction weakens the field by 3%. Determine torques in both cases.*

$$R_a = 0.2 \text{ ohm}; R_f = 250 \text{ ohms}$$

Voltage drop per brush is 1 V. **(Elect. Machines Nagpur Univ. 1993)**

Solution.

$$I_{sh} = 250/250 = 1 \text{ A}; I_{a1} = 25 - 1 = 24 \text{ A}$$

$$E_{b1} = 250 - \text{arm. drop} - \text{brush drop}$$

$$= 250 - 24 \times 0.2 - 2 = 243.2 \text{ V}$$

$$I_{a2} = 50 - 1 = 49 \text{ A}; E_{b2} = 250 - 49 \times 0.2 - 2 = 238.2 \text{ V}$$

$$\frac{N_2}{1000} = \frac{238.2}{243.2} \times \frac{\Phi_1}{0.97\,\Phi_1}; N_2 = 1010 \text{ r.p.m.}$$

$$T_{a1} = 9.55\, E_{b1}\, I_{a1}/N_1 = 9.55 \times 243.2 \times 24/1000 = 55.7 \text{ N-m}$$

$$T_{a2} = 9.55 \times 238.2 \times 49/1010 = 110.4 \text{ r.p.m.}$$

Example 29.39. *A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5 Ω and field resistance of 250 Ω. When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.* **(Electrical Technology, Aligarh Muslim Univ. 1989)**

Solution.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back e.m.f. of 250 V produced in its armature. Hence $N_1 = 1000$ r.p.m. and $E_{b1} = 250$ V.

When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is

$$E_{b2} = 250 - (40 - 1) \times 0.5 = 230.5 \text{ V; Also } \Phi_2 = 0.96\,\Phi_1, N_2 = ?$$

Using the above equation we have

$$\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96\,\Phi_1}; N_2 = \mathbf{960 \text{ r.p.m.}}$$

Efficiency

No-load input represents motor losses which consists of

(*a*) armature Cu loss $= I_a^2 R_a$ which is variable.

(*b*) constant losses W_c which consists of (*i*) shunt Cu loss (*ii*) magnetic losses and (*iii*) mechanical losses.

No-load input or total losses = 250 × 4 = 1000 W

Arm. Cu loss $= I_a^2 R_a = 3^2 \times 0.5 = 4.5$ W, $\therefore W_c = 1000 - 4.5 = 995.5$ W

When motor draws a line current of 40 A, its armature current is (40 – 1) = 39 A

$$\text{Arm. Cu loss} = 39^2 \times 0.5 = 760.5 \text{ W; Total losses} = 760.5 + 955.5 = 1756 \text{ W}$$

$$\text{Input} = 250 \times 40 = 10{,}000 \text{ W; output} = 10{,}000 - 1756 = 8{,}244 \text{ W}$$

$$\therefore \quad \eta = 8{,}244 \times 100/10{,}000 = \mathbf{82.44\%}$$

Example 29.40. *The armature winding of a 4-pole, 250 V d.c. shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125 Ω respectively. If the rotational losses amount to be 810 W find,*

(i) gross torque (ii) useful torque and (iii) efficiency. **(Elect. Machines Nagpur Univ. 1993)**

Solution. $I_{sh} = 250/125 = 2$ A; $I_a = 25 - 2 = 23$ A; $E_b = 250 - (23 \times 0.1) = 247.7$ V

Now, $E_b = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right)$ $\quad\therefore 247.7 = \dfrac{20 \times 10^{-3} \times 960 \times N}{60}\left(\dfrac{4}{4}\right)$; $N = 773$ r.p.m.

(i) Gross torque or armature torque $T_a = 9.55\dfrac{E_b I_a}{N} = 9.55 \times \dfrac{247.7 \times 23}{773} = $ **70.4 N - m**

(ii) Arm Cu loss $= 23^2 \times 0.1 = 53$ W; Shunt Cu loss $= 250 \times 2 = 500$ W

Rotational losses = 810 W; Total motor losses = 810 + 500 + 53 = 1363 W

Motor input = 250 × 25 = 6250 W; Motor output = 6250 − 1363 = 4887 W

$T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 4887/773 = 60.4$ N-m

(iii) Efficiency = 4887/6250 = 0.782 = 78.2%

Example 29.41. *A 20-hp (14.92 kW); 230-V, 1150-r.p.m. 4-pole, d.c. shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature circuit resistance of 0.2 Ω. When it delivers rated power at rated speed, it draws a line current of 74.8 A and a field current of 3 A. Calculate (i) the flux per pole (ii) the torque developed (iii) the rotational losses (iv) total losses expressed as a percentage of power.* **(Electrical Machinery-I, Banglore Univ. 1987)**

Solution. $I_a = 74.8 - 3 = 71.8$ A; $E_b = 230 - 71.8 \times 0.2 = 215.64$ V

(i) Now, $E_b = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right)$; $215.64 = \dfrac{\Phi \times 620 \times 1150}{60}\left(\dfrac{4}{2}\right)$; $\Phi = $ **9 mWb**

(ii) Armature Torque, $T_a = 9.55 \times 215.64 \times 71.8/1150 = $ **128.8 N-m**

(iii) Driving power in armature $= E_b I_a = 215.64 \times 71.8 = 15{,}483$ W

Output = 14,920 W; Rotational losses = 15,483 − 14,920 = **563 W**

(iv) Motor input $= VI = 230 \times 74.8 = 17{,}204$ W; Total loss = 17,204 − 14,920 = 2,284 W

Losses expressed as percentage of power input = 2284/17,204 = 0.133 or **13.3%**

Example 29.42. *A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5 Ω and 250 Ω respectively. At what output power will the efficiency be maximum ? Is it possible to obtain this output from the machine ?* **(Electrotechnics-II, M.S. Univ. Baroda 1985)**

Solution. When loaded lightly

Total motor input (or total no-load losses) = 250 × 5 = 1,250 W

$I_{sh} = 250/250 = 1A \quad \therefore I_a = 5 - 1 = 4$ A

Field Cu loss = 250 × 1 = 250 W; Armature Cu loss $= 4^2 \times 0.5 = 8$ W

∴ Iron losses and friction losses = 1250 − 250 − 8 = 992 W

These losses would be assumed constant.

Let I_a be the full-load armature current, then armature input is $= (250 \times I_a)$ W

F.L. output = 7.46 × 1000 = 7,460 W

The losses in the armature are :

(i) Iron and friction losses = 992 W

(ii) Armature Cu loss $= I_a^2 \times 0.05$ W $\qquad \therefore \quad 250\, I_a = 7{,}460 + 992 + I_a^2 \times 0.5$

or $\quad 0.5\,I_a^2 - 250\,I_a + 8{,}452 = 0 \qquad \therefore \quad I_a = 36.5\text{ A}$

$\therefore$ F.L. input current $= 36.5 + 1 = 37.5$ A ; Motor input $= 250 \times 37.5$ W

F.L. output $= 7{,}460$ W

$\therefore$ F.L. efficiency $= 7460 \times 100/250 \times 37.5 = 79.6\%$

Now, efficiency is maximum when armature Cu loss equals constant loss.

i.e. $\quad I_a^2 R_a = I_a^2 \times 0.5 = (1{,}250 - 8) = 1{,}242$ W or $I_a = 49.84$ A

$\therefore$ Armature input $= 250 \times 49.84 = 12{,}460$ W

Armature Cu loss $= 49.84^2 \times 0.5 = 1242$ W; Iron and friction losses $= 992$ W

$\therefore$ Armature output $= 12{,}460 - (1{,}242 + 992) = 10{,}226$ W

$\therefore$ Output power $= 10{,}226$ W $=$ **10.226 kW**

As the input current for maximum efficiency is beyond the full-load motor current, it is never realised in practice.

Example 29.43. *A d.c. series motor drives a load, the torque of which varies as the square of the speed. Assuming the magnetic circuit to be remain unsaturated and the motor resistance to be negligible, estimate the percentage reduction in the motor terminal voltage which will reduce the motor speed to half the value it has on full voltage. What is then the percentage fall in the motor current and efficiency ? Stray losses of the motor may be ignored.*

(Electrical Engineering-III, Pune Univ. 1987)

Solution. $T_a \propto \Phi\, I_a \propto I_a^2$. Also, $T_a \propto N^2$. Hence $N^2 \propto I_a^2$ or $N \propto I_a$

$\therefore \quad N_1 \propto I_{a1}$ and $N_2 \propto I_{a2}$ or $N_2/N_1 = I_{a2}/I_{a1}$

Since, $\quad N_2/N_1 = 1/2 \quad \therefore \quad I_{a2}/I_{a1} = 1/2$ or $I_{a2} = I_{a1}/2$

Let V_1 and V_2 be the voltages across the motor in the two cases. Since motor resistance is negligible, $E_{b1} = V_1$ and $E_{b2} = V_2$. Also $\Phi_1 \propto I_{a1}$ and $\Phi_2 \propto I_{a2}$ or $\Phi_1/\Phi_2 = I_{a1}/I_{a2} = I_{a1} \times 2/I_{a1} = 2$

Now,
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{1}{2} = \frac{V_2}{V_1} \times 2 \quad \text{or} \quad \frac{V_2}{V_1} = \frac{1}{4}$$

$$\therefore \quad \frac{V_1 - V_2}{V_1} = \frac{4-1}{4} = 0.75$$

$\therefore$ Percentage reduction in voltage $= \dfrac{V_1 - V_2}{V_1} \times 100 = 0.75 \times 100 =$ **75%**

Percentage change in motor current $= \dfrac{I_{a1} - I_{a2}}{I_{a1}} \times 100 = \dfrac{I_{a1} - I_{a1}/2}{I_{a1}} \times 100 =$ **50%**

Example 29.44. *A 6-pole, 500-V wave-connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. The armature and field resistance are 0.5 Ω and 250 Ω respectively. What will be the speed and torque developed by the motor when it draws 20 A from the supply mains ? Neglect armature reaction. If magnetic and mechanical losses amount to 900 W, find (i) useful torque (ii) output in kW and (iii) efficiency at this load.*

Solution. (*i*) $\quad I_{sh} = 500/250 = 2$ A $\quad \therefore \quad I_a = 20 - 2 = 18$ A

$\therefore \quad E_b = 500 - (18 \times 0.5) = 491$ V; Now, $E_b = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

$\therefore \quad 491 = \dfrac{20 \times 10^{-3} \times 1200 \times N}{60} \times \left(\dfrac{6}{2}\right)$; $N =$ **410 r.p.m. (approx.)**

Now $\quad T_a = 9.55\,\dfrac{E_b I_a}{N} = 9.55\,\dfrac{491 \times 18}{410} =$ **206 N - m**

$$\text{Armature Cu loss} = 18^2 \times 0.5 = 162 \text{ W; Field Cu loss} = 500 \times 2 = 1000 \text{ W}$$

$$\text{Iron and friction loss} = 900 \text{ W ; Total loss} = 162 + 1000 + 900 = 2{,}062 \text{ W}$$

$$\text{Motor input} = 500 \times 20 = 10{,}000 \text{ W}$$

(*i*) $T_{sh} = 9.55 \times \dfrac{7938}{410} = \mathbf{184.8 \text{ N - m}}$

(*ii*) Output = 10,000 − 2062 = **7,938 kW**

(*iii*) $\% \, \eta = \dfrac{\text{Output}}{\text{Input}} \times 100 = \dfrac{7.938 \times 100}{10{,}000} = 0.794 = \mathbf{79.4\%}$

Example 29.45. *A 50-h.p. (37.3 kW), 460-V d.c. shunt motor running light takes a current of 4 A and runs at a speed of 660 r.p.m. The resistance of the armature circuit (including brushes) is 0.3 Ω and that of the shunt field circuit 270 Ω.*

Determine when the motor is running at full load

(i) the current input (ii) the speed. Determine the armature current at which efficiency is maximum. Ignore the effect of armature reaction. **(Elect. Technology Punjab, Univ. 1991)**

Solution. $I_{sh} = 460/270 = 1.7$ A; Field Cu loss = 460 × 1.7 = 783 W

When running light

$I_a = 4 - 1.7 = 2.3$ A; Armature Cu loss = $2.3^2 \times 0.3 = 1.5$ W (negligible)

No-load armature input = 460 × 2.3 = 1,058 W

As armature Cu loss is negligible, hence 1,058 W represents iron, friction and windage losses which will be assumed to be constant.

Let full-load armature input current be I_a. Then

Armature input $= 460 \, I_a$ W; Armature Cu loss $= I_a^2 \times 0.3$ W

Output = 37.3 kW = 37,300 W

$\therefore \quad 460 \, I_a = 37{,}300 + 1{,}058 + 0.3 \, I_a^2$ or $0.3 \, I_a^2 - 460 \, I_a + 38{,}358 = 0$

$\therefore \quad I_a = 88.5$ A

(*i*) Current input $= 88.5 + 1.7 = \mathbf{90.2 \text{ A}}$

(*ii*) $E_{b1} = 460 - (2.3 \times 0.3) = 459.3\text{V}; \; E_{b2} = 460 - (88.5 \times 0.3) = 433.5$ V

$\therefore \quad N_2 = 660 \times 433.5/459.3 = \mathbf{624 \text{ r.p.m.}}$

For maximum efficiency, $I_a^2 R_a$ = constant losses (Art. 24.37)

$\therefore \quad I_a^2 \times 0.3 = 1058 + 783 = 1{,}841 \; \therefore \; I_a = (1841/0.3)^{1/2} = \mathbf{78.33 \text{ A}}$

Tutorial Problems 29.3

1. A 4-pole 250-V, d.c. series motor has a wave-wound armature with 496 conductors. Calculate
   (*a*) the gross torque (*b*) the speed
   (*b*) the output torque and (*d*) the efficiency, if the motor current is 50 A
   The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19 Ω, field resistance = 0.14 Ω.
   [(*a*) 173.5 N-m (*b*) 642 r.p.m. (*c*) 161.4 N-m (*d*) 86.9%]
2. On no-load, a shunt motor takes 5 A at 250 V, the resistances of the field and armature circuits are 250 Ω and 0.1 Ω respectively. Calculate the output power and efficiency of the motor when the total supply current is 81 A at the same supply voltage. State any assumptions made.
   [18.5 kW; 91%. It is assumed that windage, friction and eddy current losses are independent of the current and speed]

3. A 230 V series motor is taking 50 A. Resistance of armature and series field windings is 0.2 Ω and 0.1 Ω respectively. Calculate :

(a) brush voltage (b) back e.m.f.

(c) power wasted in armature (d) mechanical power developed

[(a) 215 V (b) 205 V (c) 500 W (d) 13.74 h.p.] (10.25 kW)

4. Calculate the shaft power of a series motor having the following data; overall efficiency 83.5%, speed 550 r.p.m. when taking 65 A; motor resistance 0.2 Ω, flux per pole 25 mWb, armature winding lap with 1200 conductor.
(15.66 kW)

5. A shunt motor running on no-load takes 5 A at 200 V. The resistance of the field circuit is 150 Ω and of the armature 0.1 Ω. Determine the output and efficiency of motor when the input current is 120 A at 200 V. State any conditions assumed. (89.8%)

6. A d.c. shunt motor with interpoles has the following particulars :

Output power ; 8,952 kW, 440-V, armature resistance 1.1 Ω, brush contact drop 2 V, interpole winding resistance 0.4 Ω shunt resistance 650 Ω, resistance in the shunt regulator 50 Ω. Iron and friction losses on full-load 450 W. Calculate the efficiency when taking the full rated current of 24 A. (85%)

7. A d.c. series motor on full-load takes 50 A from 230 V d.c. mains. The total resistance of the motor is 0.22 Ω. If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 A. (10.275 kW)

8. A 2-pole d.c shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500 Ω and the armature has a resistance of 0.6 Ω. Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm. Neglect temperature rise.

[*Rajiv Gandhi Tech. Univ. Bhopal,2000*] (82.63%)

OBJECTIVE TESTS – 29

1. In a d.c. motor, undirectional torque is produced with the help of
   (a) brushes (b) commutator
   (c) end-plates (d) both (a) and (b)
2. The counter e.m.f. of a d.c. motor
   (a) often exceeds the supply voltage
   (b) aids the applied voltage
   (c) helps in energy conversion
   (d) regulates its armature voltage
3. The normal value of the armature resistance of a d.c. motor is
   (a) 0.005 (b) 0.5
   (c) 10 (d) 100
   (*Grad. I.E.T.E. June 1987*)
4. The E_b/V ratio of a d.c. motor is an indication of its
   (a) efficiency (b) speed regulation
   (c) starting torque (d) Running Torque
   (*Grad. I.E.T.E. June 1987*)
5. The mechanical power developed by the armature of a d.c. motor is equal to
   (a) armature current multiplied by back e.m.f.
   (b) power input minus losses
   (c) power output multiplied by efficiency
   (d) power output plus iron losses
6. The induced e.m.f. in the armature conductors of a d.c. motor is
   (a) sinusoidal (b) trapezoidal
   (c) rectangular (d) alternating
7. A d.c. motor can be looked upon as d.c. generator with the power flow
   (a) reduced (b) reversed
   (c) increased (d) modified
8. In a d.c. motor, the mechanical output power actually comes from
   (a) field system
   (b) air-gap flux
   (c) back e.m.f.
   (d) electrical input power
9. The maximum torque of d.c. motors is limited by
   (a) commutation (b) heating

(c) speed (d) armature current

10. Which of the following quantity maintains the same direction whether a d.c. machine runs as a generator or as a motor ?

(a) induced e.m.f. (b) armature current

(c) field current (d) supply current

11. Under constant load conditions, the speed of a d.c. motor is affected by

(a) field flux (b) armature current

(c) back e.m.f. (d) both (b) and (c)

12. It is possible to increase the field flux and, at the same time, increase the speed of a d.c. motor provided its .... is held constant.

(a) applied voltage

(b) torque

(c) Armature circuit resistance

(d) armature current

13. The current drawn by a 120 - V d.c. motor of armature resistance 0.5 Ω and back e.m.f. 110 V is .... ampere.

(a) 20 (b) 240

(c) 220 (d) 5

14. The shaft torque of a d.c. motor is less than its armature torque because of .... losses.

(a) copper (b) mechanical

(c) iron (d) rotational

15. A d.c. motor develops a torque of 200 N-m at 25 rps. At 20 rps it will develop a torque of .... N-m.

(a) 200 (b) 160

(c) 250 (d) 128

16. Neglecting saturation, if current taken by a series motor is increased from 10 A to 12 A, the percentage increase in its torque is .... percent

(a) 20 (b) 44

(c) 30.5 (d) 16.6

17. If load on a d.c. shunt motor is increased, its speed is decreased due primarily to

(a) increase in its flux

(b) decrease in back e.m.f.

(c) increase in armature current

(d) increase in brush drop

18. If the load current and flux of a d.c. motor are held constant and voltage applied across its armature is increased by 10 per cent, its speed will

(a) decrease by about 10 per cent

(b) remain unchanged

(c) increase by about 10 per cent

(d) increase by 20 per cent.

19. If the pole flux of a d.c. motor approaches zero, its speed will

(a) approach zero

(b) approach infinity

(c) no change due to corresponding change in back e.m.f.

(d) approach a stable value somewhere between zero and infinity.

20. If the field circuit of a loaded shunt motor is suddenly opened

(a) it would race to almost infinite speed

(b) it would draw abnormally high armature current

(c) circuit breaker or fuse will open the circuit before too much damage is done to the motor

(d) torque developed by the motor would be reduced to zero.

21. Which of the following d.c. motor would be suitable for drives requiring high starting torque but only fairly constant speed such as crushers ?

(a) shunt (b) series

(c) compound (d) permanent magnet

22. A d.c. shunt motor is found suitable to drive fans because they require

(a) small torque at start up

(b) large torque at high speeds

(c) practically constant voltage

(d) both (a) and (b)

23. Which of the following load would be best driven by a d.c. compound motor ?

(a) reciprocating pump

(b) centrifugal pump

(c) electric locomotive

(d) fan

24. As the load is increased, the speed of a d.c. shunt motor

(a) increases proportionately

(b) remains constant

(c) increases slightly

(d) reduces slightly

25. Between no-load and full-load, ... motor develops the *least* torque

(a) series

(b) shunt

(c) cumulative compound

(d) differential compound

26. The T_a/I_a graph of a d.c. series motor is a
(a) parabola from no-load to overload
(b) straight line throughout
(c) parabola throughout
(d) parabola upto full-load and a straight line at overloads.

27. As compared to shunt and compound motors, series motor has the highest torque because of its comparatively .... at the start.
(a) lower armature resistance
(b) stronger series field
(c) fewer series turns
(d) larger armature current

28. Unlike a shunt motor, it is difficult for a series motor to stall under heavy loading because
(a) it develops high overload torque
(b) its flux remains constant
(c) it slows down considerably
(d) its back e.m.f. is reduced to almost zero.

29. When load is removed, ... motor will run at the *highest* speed.
(a) shunt
(b) cumulative-compound
(c) differential compound
(d) series

30. A series motor is best suited for driving
(a) lathes
(b) cranes and hoists
(c) shears and punches
(d) machine tools

31. A 220 V shunt motor develops a torque of 54 N-m at armature current of 10 A. The torque produced when the armature current is 20 A, is
(a) 54 N-m (b) 81 N-m
(c) 108 N-m (d) None of the above
(Elect. Machines, A.M.I.E. Sec. B, 1993)

32. The d.c. series motor should never be switched on at no load because
(a) the field current is zero
(b) The machine does not pick up
(c) The speed becomes dangerously high
(d) It will take too long to accelerate.
(Grad. I.E.T.E. June 1988)

33. A shunt d.c. motor works on a.c. mains
(a) unsatisfactorily (b) satisfactorily
(c) not at all (d) none of the above
(Elect. Machines, A.M.I.E. Sec. B, 1993)

34. A 200 V, 10 A motor could be rewound for 100 V, 20 A by using .... as many turns per coil of wire, having .... the cross-sectional area.
(a) twice, half
(b) thrice, one third
(c) half, twice
(d) four times, one-fourth

Answers

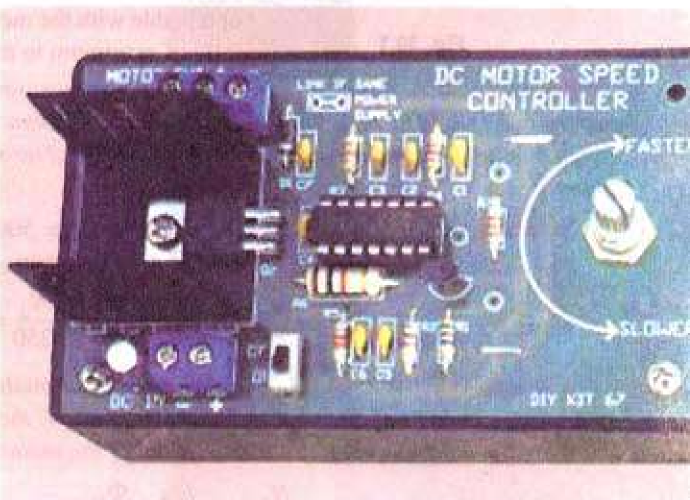
1. (d) 2. (c) 3. (b) 4. (a) 5. (a) 6. (a) 7. (b) 8. (d) 9. (a) 10. (a) 11. (a) 12. (d) 13. (a) 14. (d) 15. (a) 16. (b) 17. (b) 18. (c) 19. (b) 20. (c) 21. (c) 22. (d) 23. (a) 24. (d) 25. (a) 26. (d) 27. (b) 28. (a) 29. (d) 30. (b) 31. (c) 32. (c) 33. (a) 34. (c).

CHAPTER 30

SPEED CONTROL OF D.C. MOTORS

Learning Objectives

- Factors Controlling Motor Speed
- Speed Control of Shunt Motors
- Speed Control of Series Motors
- Merits and Demerits of Rheostatic Control Method
- Series-Parallel Control
- Electric Braking
- Electric Braking of Shunt Motor
- Electric Braking of Series Motors
- Electronic Speed control Method for D.C. Motors
- Uncontrolled Rectifiers
- Controlled Rectifiers
- Thyristor Choppers
- Thyristor Inverters
- Thyristor Speed Control of Separately-excited D.C. Motor
- Thyristor Speed Control of D.C. Series Motor
- Full-wave Speed Control of a Shunt Motor
- Thyristor Control of a Shunt Motor
- Thyristor Speed Control of a Series D.C. Motor
- Necessity of a Starter
- Shunt Motor Starter
- Three-point Starter
- Four-point Starter
- Starting and Speed Control of Series Motors
- Grading of Starting Resistance
- Shunt Motors
- Series Motor Starters
- Thyristor Controller Starters



DC motor speed controller control the speed of any common dc motor rated upto 100 V. It operates on 5V to 15 V.

30.1. Factors Controlling Motor Speed

It has been shown earlier that the speed of a motor is given by the relation

$$N = \frac{V - I_a R_a}{Z\Phi} \cdot \left(\frac{A}{P}\right) = K\frac{V - I_a R_a}{\Phi} \text{ r.p.s.}$$

where R_a = armature circuit resistance.

It is obvious that the speed can be controlled by varying (*i*) flux/pole, Φ (Flux Control) (*ii*) resistance R_a of armature circuit (Rheostatic Control) and (*iii*) applied voltage V (Voltage Control). These methods as applied to shunt, compound and series motors will be discussed below.

30.2. Speed Control of Shunt motors

(*i*) Variation of Flux or Flux Control Method

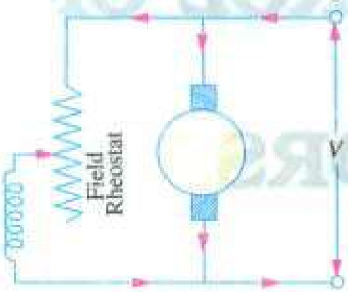


Fig. 30.1

It is seen from above that $N \propto 1/\Phi$. By decreasing the flux, the speed can be increased and *vice versa*. Hence, the name *flux* or *field control* method. The flux of a d.c. motor can be changed by changing I_{sh} with help of a shunt field rheostat (Fig. 30.1). Since I_{sh} is relatively small, shunt field rheostat has to carry only a small current, which means I^2R loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpolar machine, the speed can be increased by this method in the ratio 2 : 1. Any further weakening of flux Φ adversely affects the communication and hence puts a limit to the maximum speed obtainable with the method. In machines fitted with interpoles, a ratio of maximum to minimum speed of 6 : 1 is fairly common.

Example 30.1. *A 500 V shunt motor runs at its normal speed of 250 r.p.m. when the armature current is 200 A. The resistance of armature is 0.12 ohm. Calculate the speed when a resistance is inserted in the field reducing the shunt field to 80% of normal value and the armature current is 100 ampere.* **(Elect. Engg. A.M.A.E. S.I. June 1992)**

Solution. $E_{b1} = 500 - 200 \times 0.12 = 476$ V; $E_{b2} = 500 - 100 \times 0.12 = 488$ V

$$\Phi_2 = 0.8\,\Phi_1 \;;\; N_1 = 250 \text{ rpm} \;;\; N_2 = ?$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{250} = \frac{488}{476} \times \frac{\Phi_1}{0.8\Phi_1},\; N_2 = \textbf{320 r.p.m.}$$

Example 30.2. *A 250 volt d.c. shunt motor has armature resistance of 0.25 ohm, on load it takes an armature current of 50 A and runs at 750 r.p.m. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor.* **(Elect. Eng-II, Pune Univ. 1987)**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, $T_a \propto \Phi I_a$. Hence $T_{a1} \propto \Phi_1 I_{a1}$ and $T_{a2} \propto \Phi_2 I_{a2}$.

Since $T_{a1} = T_{a2} \quad \therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

Now, $\Phi_2 = 0.9\,\Phi_1 \quad \therefore \quad 50\,\Phi_1 = 0.9\,\Phi, I_{a2} = 55.6$ A

$\therefore \quad E_{b1} = 250 - (50 \times 0.25) = 237.5$ V ; $E_{b2} = 250 - (55.6 \times 0.25) = 231.1$ V

$$\therefore \quad \frac{N_2}{750} = \frac{231.1}{237.5} \times \frac{\Phi_1}{0.9\,\Phi_1} \;;\; N_2 = \textbf{811 r.p.m.}$$

Example 30.3. *Describe briefly the method of speed control available for dc motors.*

A 230 V d.c. shunt motor runs at 800 r.p.m. and takes armature current of 50 A. Find resistance *to be added to the field circuit to increase speed to 1000 r.p.m. at an armature current of 80 A. Assume flux proportional to field current. Armature resistance = 0.15 Ω and field winding resistance = 250 Ω.*

(Elect. Technology, Hyderabad Univ. 1991)

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ since flux ∝ field current

$E_{b1} = 230 - (50 \times 0.15) = 222.5$ V ; $E_{b2} = 230 - (80 \times 0.15) = 218$V

Let R_t = total shunt resistance = $(250 + R)$ where R is the additional resistance

$I_{sh1} = 230/250 = 0.92$ A, $I_{sh2} = 230/R_t$; $N_1 = 800$ r.p.m. ; $N = 1000$ r.p.m.

$$\therefore \quad \frac{1000}{800} = \frac{218}{222.5} \times \frac{0.92}{230/R_t} ; R_t = 319\ \Omega \quad \therefore \quad R = 319 - 250 = \mathbf{69\ \Omega.}$$

$$I_{sh2} = \frac{230}{319} = 0.721$$

$$\text{Ratio of torque in two cases} = \frac{T_2}{T_1} = \frac{I_{sh2}\ I_{a2}}{I_{sh1}\ I_{a1}} = \frac{0.721 \times 80}{0.92 \times 50} = \mathbf{1.254}$$

Example 30.4. *A 250 V, d.c. shunt motor has shunt field resistance of 250 Ω and an armature resistance of 0.25 Ω. For a given load torque and no additional resistance included in the shunt field circuit, the motor runs at 1500 r.p.m. drawing an armature current of 20 A. If a resistance of 250 Ω is inserted in series with the field, the load torque remaining the same, find out the new speed and armature current. Assume the magnetisation curve to be linear.*

(Electrical Engineering-I, Bombay Univ. 1987)

Solution. In this case, the motor speed is changed by changing the flux.

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Since it is given that magnetisation curve is linear, it means that flux is directly proportional to shunt current. Hence $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ where $E_{b2} = V - I_{a2} R_a$ and $E_{b1} = V - I_{a1} R_a$.

Since load torque remains the same $\therefore$ $T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2}$ or $\Phi_1 I_{a1} = \Phi_2 Ia_2$

$$\therefore \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$$

Now, $I_{sh1} = 250/250 = 1$ A; $I_{sh2} = 250/(250 + 250) = 1/2$ A

$$\therefore \quad I_{a2} = 20 \times \frac{1}{1/2} = \mathbf{40\ A} \quad \therefore \quad E_{b2} = 250 - (40 \times 0.25) = 240 \text{ V and}$$

$$E_{b1} = 250 - (20 \times 0.25) = 245 \text{ V} \quad \therefore \quad \frac{N_2}{1500} = \frac{240}{245} \times \frac{1}{1/2}$$

$$\therefore \quad N_2 = \mathbf{2,930\ r.p.m.}$$

Example 30.5. *A 250 V, d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω. When driving a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance should be inserted in the shunt field circuit ? Assume that the magnetic circuit is unsaturated.*

(Elect. Engg. AMIETE, June 1992)

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Since the magnetic circuit is unsaturated, it means that flux is directly proportional to the shunt current.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}} \quad \text{where } E_{b2} = V - I_{a2} R_a \text{ and } E_{b1} = V - I_{a1} R_a$$

Since motor is driving at load of constant torque,

$$T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \therefore \quad \Phi_2 Ia_2 = \Phi_1 I_{a1} \quad \text{or} \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$$

Now, $I_{sh1} = 250/250 = 1 \text{ A} ; I_{sh2} = 250/R_t$

where R_t is the total resistance of the shunt field circuit.

$$\therefore \quad I_{a2} = 20 \times \frac{1}{250/R_t} = \frac{2R_t}{25} \; ; E_{b1} = 250 - (20 \times 0.5) = 240 \text{ V}$$

$$E_{b2} = 250 - \left(\frac{2R_t}{25} \times 0.5\right) = 250 - (R_t/25) \quad \therefore \frac{800}{600} = \frac{250 - (R_t/25)}{240} \times \frac{1}{250/R_t}$$

$$0.04 R_t^2 - 250 R_t + 80{,}000 = 0$$

or
$$R_t = \frac{250 \pm \sqrt{62{,}500 - 12{,}800}}{0.08} = \frac{27}{0.08} = 337.5 \,\Omega$$

Additional resistance required in the shunt field circuit = 337.5 − 250 = **87.5 Ω**.

Example 30.6. *A 220 V shunt motor has an armature resistance of 0.5 Ω and takes a current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ?* **(Elect. Machines, AMIE, Sec B, 1991)**

Solution. Formula used is $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$. Since torque remains constant,

Hence
$$\Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \therefore \quad I_{a2} = I_{a1} \cdot \frac{\Phi_1}{\Phi_2} = 40x \text{ where } x = \frac{\Phi_1}{\Phi_2}$$

$$E_{b1} = 220 - (40 \times 0.5) = 200 \text{ V} ; E_{b2} = 220 - (40x \times 0.5) = (220 - 20x) \text{ V}$$

$$\frac{N_2}{N_1} = \frac{3}{2} \quad \text{... given} \quad \therefore \quad \frac{3}{2} = \frac{(220 - 20x)}{200} \times x \quad \therefore \quad x^2 - 11x + 15 = 0$$

or
$$x = \frac{11 \pm \sqrt{121 - 60}}{2} = \frac{11 \pm 7.81}{2} = 9.4^* \quad \text{or} \quad 1.6$$

$$\therefore \quad \frac{\Phi_1}{\Phi_2} = 1.6 \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{1}{1.6}$$

$$\therefore \quad \frac{\Phi_1 - \Phi_2}{\Phi_1} = \frac{1.6 - 1}{1.6} = \frac{3}{8} \quad \therefore \quad \text{percentage change in flux} = \frac{3}{8} \times 100 = \mathbf{37.5\%}$$

Example 30.7. *A 220-V, 10-kW, 2500 r.p.m. shunt motor draws 41 A when operating at rated conditions. The resistances of the armature, compensating winding, interpole winding and shunt field winding are respectively 0.2 Ω, 0.05 Ω, 0.1 Ω and 110 Ω. Calculate the steady-state values of armature current and motor speed if pole flux is reduced by 25%, a 1 Ω resistance is placed in series with the armature and the load torque is reduced by 50%.*

Solution. $I_{sh} = 220/110 = 2 \text{ A}; I_{a1} = 41 - 2 = 39 \text{ A}$ (Fig. 30.2)

$$T_1 \propto \Phi_1 I_{a1} \text{ and } T_2 \propto \Phi_2 I_{a2}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_{a1}}{I_{a2}}$$

* This figure is rejected as it does not give the necessay increase in speed.

or $$\frac{1}{2} = \frac{3}{4} \times \frac{I_{a2}}{39} \quad \therefore \quad I_{a2} = 26 \text{ A}$$

$$E_{b1} = 220 - 39\,(0.2 + 0.1 + 0.05) = 206.35 \text{ V}$$

$$E_{b2} = 220 - 26\,(1 + 0.35) = 184.9 \text{ V}$$

Now, $$\frac{N_2}{2500} = \frac{184.9}{206.35} \times \frac{4}{3}\,;\; N_2 = \textbf{2987 r.p.m.}$$

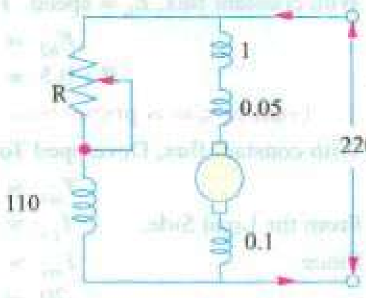


Fig. 30.2

Example 30.8. *A 220 V, 15 kW, 850 r.p.m. shunt motor draws 72.2 A when operating at rated condition. The resistances of the armature and shunt field are 0.25 Ω and 100 Ω respectively. Determine the percentage reduction in field flux in order to obtain a speed of 1650 r.p.m. when armature current drawn is 40 A.*

Solution. $$I_{sh} = 220/100 = 2.2 \text{ A};\; I_{a1} = 72.2 - 2.2 = 70 \text{ A}$$

$$E_{b1} = 220 - 70 \times 0.25 = 202.5 \text{ V},\; E_{b2} = 220 - 40 \times 0.25 = 210 \text{ V.}$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{1650}{850} = \frac{210}{202.5} \times \frac{\Phi_1}{\Phi_2}$$

$$\therefore \quad \Phi_2 = 0.534\,\Phi_1\,;$$

$$\therefore \quad \text{Reduction in field flux} = \frac{\Phi_1 - 0.534\,\Phi_1}{\Phi_1} \times 100 = \mathbf{46.6\%}.$$

Example 30.9. *A 220 V shunt motor has an armature resistance of 0.5 ohm and takes an armature current of 40 A on a certain load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ? Neglect saturation and armature reaction.*

(Elect. Machines, AMIE, Sec B, 1991)

Solution. $$T_1 \propto \Phi_1 \times \Phi_1, \rightarrow \Phi_1 I_{a1}, \text{ and } T_2 \propto \Phi_2 I_{a2} \text{ Since, } T_1 = T_2$$

$$\therefore \quad \Phi_2 I_{a2} = \Phi_1 I_{a1} \quad \text{or} \quad \Phi_2/\Phi_1 = I_{a1}/I_{a2} = 40/I_{a2}$$

$$E_{b1} = 220 - 40 \times 0.5 = 200 \text{ V}\,;\; E_{b2} = 220 - 0.5\,I_{a2}$$

Now, $$\frac{E_{b2}}{E_{b1}} = \frac{N_2\Phi_2}{N_1\Phi_1} \quad \text{or} \quad \frac{220 - 0.5\,I_{a2}}{200} = \frac{1.5\,N_1}{N_1} \times \frac{40}{I_{a2}}$$

$$\therefore \quad I_{a2} - 40\,I_{a2} + 24{,}000 = 0 \quad \text{or} \quad I_{a2} = 63.8 \text{ A}$$

$$\therefore \quad \frac{\Phi_2}{\Phi_1} = \frac{40}{63.8} = 0.627 \quad \text{or} \quad \Phi_2 = 0.627\,\Phi_1 = \mathbf{62.7\%\ of\ \Phi_1}$$

Example 30.10. *A d.c. shunt motor takes an armature current of 20 A from a 220 V supply. Armature circuit resistance is 0.5 ohm. For reducing the speed by 50%, calculate the resistance required in the series, with the armature, if*

(*a*) *the load torque is constant*

(*b*) *the load torque is proportional to the square of the speed.* **(Sambalpur Univ., 1998)**

Solution. $$E_{b1} = V - I_a r_a = 220 - 20 \times 0.5 = 210 \text{ V}$$

$$210 \propto N_1$$

(*a*) Constant Load torque

In a shunt motor, flux remains constant unless there is a change in terminal voltage or there is a change in the field-circuit resistance.

If torque is constant, armature-current then must remain constant. $I_a = 20$ amp

With an external armature-circuit resistor of *R* ohms, $20 \times (R + 0.5) = 220 - E_{b2}$

The speed required now is $0.5\,N_1$.

With constant flux, $E_b \propto$ speed. Hence, $210 \propto N_1$

$$E_{b2} \propto 0.5\,N_1,\ E_{b2} = 105$$

$$R + 0.5 = (220 - 105)/20 = 5.75, \text{ giving } R = \mathbf{5.25\ ohms}$$

(b) Load torque is proportional to the square of speed.

With constant flux, Developed Torque at N_1 r.p.m. $\propto I_{a1}$

$$T_{m1} \propto 20$$

From the Load Side, $T_{L1} \propto N_1^2$

Since $T_{m1} = T_{L1}$

$$20 \propto N_1^2 \quad \text{...(a)}$$

At 50% speed, Load Torque, $T_{L2} \propto (0.5\,N_1)^2$

For motor torque, $T_{m2} \propto I_{a2}$

Since $T_{L2} = T_{m2},\ I_{a2} \propto (0.5\,N_1)^2 \quad \text{...(b)}$

From eqn. (a) and (b) above

$$\frac{I_{a2}}{I_{a1}} = 0.25,\ I_{a2} = 5 \text{ amp}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{220 - I_{a2}\,(R + 0.5)}{210} = 0.5\,N_1$$

$$220 - I_{a2}\,(R + 0.5) = 0.5 \times 210$$

$$R + 0.5 = \frac{220 - 105}{5} = 23,\ R = \mathbf{22.5\ ohms}$$

Check : With the concept of armature power output : (applied here for part (b) only as an illustration).

$$\text{Armature power–output} = E_b \times I_a = T \times \omega$$

When $T \propto (\text{speed})^2$, $\quad E_b\,I_a = k_1\,T_\omega = K_2\,\omega^3$

At N_1 r.p.m. $\quad 210 \times 20 = K_3\,N_1^3 \quad \text{...(c)}$

With constant flux, at half speed

$$E_{b2} = 105$$

$$105 \times I_{a2} = K_3\,(0.5\,N_1)^3 \quad \text{...(d)}$$

From eqs. (c) and (d),

$$\frac{105 \times I_{a2}}{210 \times 20} = \frac{0.125 \times N_1^3}{N_1^3}, \text{ giving } I_{a2} = 5 \text{ amp}$$

This gives $\quad R = \mathbf{22.5\ ohms.}$

Example 30.11. *A 250 V shunt motor runs at 1000 r.p.m. at no-load and takes 8 A. The total armature and shunt field resistances are respectively 0.2 ohm, 250 ohm. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.*

(Nagpur Univ. Summer 2000)

Solution. The current distribution is shown in Fig. 30.3.

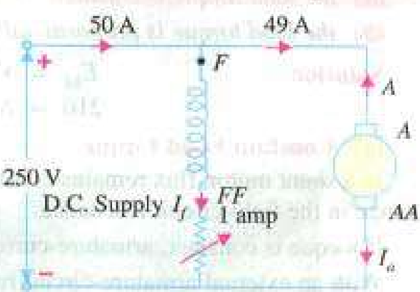


Fig. 30.3

At no load, $\quad I_L = 8 \text{ amp},\ I_f = 1 \text{ amp},$

Hence, $\quad I_a = 7 \text{ amp}$

$$E_{b0} = 250 - 7 \times 0.2 = 248.6 \text{ volts},$$

$$= K\phi \times 1000$$

$$\therefore \quad K\phi = 0.2486$$

At load, $I_a = 49$ amp

$$E_{b1} = 250 - 49 \times 0.2 = 240.2$$

$$N_1 = \frac{240.2}{0.2486} = 966.2 \text{ r.p.m.}$$

Notes : (*i*) The assumption of constant flux has simplified the issue. Generally, armature reaction tends to weaken the flux and then the speed tends to increase slightly.

(*ii*) The no load armature current of 7 amp is required to overcome the mechanical losses of motor as well as driven load, at about 1000 r.p.m.

Example 30.12. *A 240 V d.c. shunt motor has an armature-resistance of 0.25 ohm, and runs at 1000 r.p.m., taking an armature current 40 A. It is desired to reduce the speed to 800 r.p.m.*

(*i*) *If the armature current remains the same, find the additional resistance to be connected in series with the armature-circuit.*

(*ii*) *If, with the above additional resistance in the circuit, armature current decreases to 20 A, find the speed of the motor.* **(Bhartiar Univ., November 1997)**

Solution. $E_b = 240 - 0.25 \times 40 = 230 \text{ V}, 230 \propto 1000$ ...(*a*)

(*i*) For 800 r.p.m., $E_{b2} \propto 800$ ...(*b*)

From (*a*) and (*b*), $E_{b2} = \frac{800}{1000} \times 230 = 184$

$$240 - (R + 0.25) \times 40 = 184, \; R = \mathbf{1.15 \text{ ohm}}$$

(*ii*) $E_{b3} = 240 - 20\,(1.40) = 212$

$$E_{b3} = \frac{N_3}{1000} \times 230 = 212, \; N_3 = \frac{212}{230} \times 1000 = \mathbf{922 \text{ r.p.m.}}$$

Example 30.13. *A 7.48 kW, 220 V, 990 r.p.m. shunt motor has a full load efficiency of 88%, the armature resistance is 0.08 ohm and shunt field current is 2 A. If the speed of this motor is reduced to 450 r.p.m. by inserting a resistance in the armature circuit, find the motor output, the armature current, external resistance to be inserted in the armature circuit and overall efficiency. Assume the load torque to remain constant.* **(Nagpur Univ., November 1998)**

Solution. With an output of 7.48 kW and an efficiency of 88%, the input power is 8.50 kW. Losses are 1.02 kW.

Input Current = 85000/220 = 38.64 A

Armature-Current = 38.64 − 2.00 = **36.64 A**

Power Loss in the shunt field circuit = $220 \times 2 = 440$ W

Copper-Loss in armature-circuit = $36.64^2 \times 0.08 = 107.4$ W

No-Load-Loss at 900 r.p.m. = 1020 − 107.4 − 440 = 473 W

At 900 r.p.m. Back-emf = $E_{b1} = 200 - (36.64 \times 0.08) \risingdotseq 217.3$ V

Motor will run at 450 r.p.m. with flux per pole kept constant, provided the back-emf = E_{b2}

$= (459/900) \times 217.1\text{V} = 108.5$ V

There are two simplifying assumptions in this case, which must be stated before further calculations :

1. Load-torque is constant,
2. No Load Losses are constant.

(These statements can be different which leads to variations in the next steps of calculations.)

For constant load-torque, the condition of constant flux per pole results into constant armature current, which is 36.64 A.

With an armature current of 36.64 A, let the external resistance required for this purpose be R.

$$36.64\,R = 217.1 - 108.5 = 108.6 \text{ V}, \; R = 2.964 \text{ ohms}$$

Total Losses

Total i^2r - loss in armature = $36.64^2 \times (2.964 + 0.08) = 4086$ W

Field-copper-loss + No load loss = 440 + 473 = 913

$$\text{Total Loss} = 4999 \text{ W}$$

$$\text{Total Output} = 8500 - 4999 = 3501 \text{ W}$$

Hence, $\quad$ Efficiency = (3501/8500) × 100 = **41.2%**

(**Note :** Because of missing data and clarification while making the statements in the question, there can be variations in the assumption and hence in the final solutions.)

Example 30.14. *A d.c. shunt motor supplied at 230 V runs at 990 r.p.m. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 r.p.m. assuming that armature current is 25 amp.* **(Nagpur Univ., November 1997)**

Solution. It is assumed that armature resistance is to

(*a*) At 900 r.p.m. : $\quad E_{b1} = 230 = K \times 900$

(*b*) At 500 r.p.m. : $\quad E_{b2} = K \times 500$

Therefore, $\quad E_{b2} = E_{b1} \times 500/900 = 127.8$ volts

The difference between E_{b1} and E_{b2} must be the drop in the external resistance to be added to the armature circuit for the purpose of reducing the speed to 500 r.p.m.

$$E_{b1} - E_{b2} = 25 \times R$$

$$R = (230 - 127.8)/25 = \mathbf{4.088 \text{ ohms.}}$$

Example 30.15. *A 220 V d.c. shunt motor has an armature resistance of 0.4 ohm and a field circuit resistance of 200 ohms. When the motor is driving a constant-torque load, the armature-current is 20 A, the speed being 600 r.p.m. It is desired to run the motor at 900 r.p.m. by inserting a resistance in the field circuit. Find its value, assuming that the magnetic circuit is not saturated.* **(Nagpur Univ., November 1996)**

Solution. (*i*) At 600 r.p.m. $i_{f1} = 220/200 = 1.1$ amp

$$E_{b1} = 220 - (20 \times 0.4) = 212 \text{ volts}$$

$$T_L = K_1 \times 1.1 \times 20$$

$$\text{The back e.m.f.} = 212 = K_2 \times 1.1 \times 600$$

or $\quad K_2 = 212/660 = 0.3212$

(*ii*) At 900 r.p.m. : $\quad T_L = K_1 \times i_{f2} \times I_{a2}$

Due to constant load torque,

$$i_{f2} \times I_{a2} = 1.1 \times 20 = 22$$

$$E_{b2} = 220 - (0.4\, I_{a2}) = K_2 \times i_{f2} \times 900 = 289\, i_{f2}$$

$$220 - (0.4 \times 22/i_{f2}) = 289\, i_{f2}$$

Guess for approximate value of i_{f2} : Neglecting armature -resistance drop and saturation, 50% rise in speed is obtained with proportional decrease in i_{f2} related by

$$\frac{600}{900} = \frac{i_{f2}}{i_{f1}} \text{ giving } i_{f2} \equiv 0.73 \text{ amp}$$

[In place of i_{f2}, I_{a2} can be evaluated first. Its guess-work will give $I_{a2} \equiv 1.5 \times 20 \equiv 30$ amp]

Continuing with the solution of the equation to evaluate a value of i_{f2}, accepting that value which is near 0.73 amp, we have $i_{f2} = 0.71865$. [Note that other value of i_{f2}, which is 0.04235, is not acceptable.]. Corresponding $I_{a2} = 30.6$ amp. Previous shunt field current, $i_{f1} = 1.1$, $R_{f1} = 200\ \Omega$. New shunt field current, $i_{f2} = 0.71865$, $R_{f2} = 220/0.71865 = 306\ \Omega$. Final answer is that a resistor of 106 ohms is to be added to the field circuit to run the motor at 900 r.p.m. at constant torque.

Example 30.16. *A 220 V d.c. shunt motor has an armature resistance of 0.40 ohm and field-resistance of 200 ohms. It takes an armature current of 22 A and runs at 600 r.p.m. It drives a load whose torque is constant. Suggest a suitable method to raise the speed to 900 r.p.m. Calculate the value of the controllable parameter.* **(Nagpur Univ., April 1998)**

Solution. At 600 r.p.m. $I_{a1} = 22$ amp, $N_1 = 600$ r.p.m., $i_{f1} = 220/200 = 1.1$ amp.

$$E_{b1} = 220 - (22 \times 0.40) = 211.2 \text{ volts.}$$

Let the Load-torque be denoted by T_2, k_1 and k_2 in the equations below represent machine constants appearing in the usual emf-equation and torque-equation for the d.c. shunt motor.

$$E_{b1} = 211.2 = k_1 \times i_{f1} \times N_1 = k_1 \times 1.10 \times 600 \text{ or } k_1 = 211.2/660 = 0.32$$

$$T_L = k_2 \times i_{f1} \times I_{a1} = k_2 \times 1.10 \times 22$$

Since the load torque will remain constant at 900 r.p.m. also, the corresponding field current $(= i_{f2})$ and armature current $(= i_{a2})$ must satisfy the following relationships :

$$T_L = k_2\, i_{f2}\, I_{a2} = k_2 \times 1.10 \times 22$$

or $$i_{f2} \times I_{a2} = 24.2$$

And $$E_{b2} = 220 - (I_{a2} \times 0.40) = k_1 \times i_{f2} \times 900$$

$$220 - (I_{a2} \times 0.40) = 0.32 \times (24.2 \times I_{a2}) \times 900$$

(Alternatively, the above equation can also lead to a quadratic in i_{f2}.)

This leads to a quadratic equation in I_{a2}.

Guess for i_{f2} : Approximately, speed of a d.c. shunt motor is inversely proportional to the field current. Comparing the two speeds of 600 and 900 r.p.m., the value of i_{f2} should be approximately given by

$$i_{f2} \equiv i_{f1} \times (600/900) = 0.733 \text{ amp}$$

Guess for I_{a2} : For approximate conclusions, armature-resistance drop can be ignored. With constant load-torque, armature-power must be proportional to the speed.

$$\frac{\text{Armature-power at 900 r.p.m.}}{\text{Armature-power at 600 r.p.m.}} = \frac{900}{600} = 1.5$$

$$E_{b2}\, I_{a2} = 1.5 \times E_{b1}\, I_{a1}$$

Neglecting armature-resistance drops, $E_{b1} - V$ and $E_{b2} - V$.

This gives $$I_{a2} = 1.5 \times 22 = 33 \text{ amps}$$

Thus, out of the two roots for i_{f2}, that which is close to 0.733 is acceptable. If quadratic equation for I_{a2} is being handled, that root which is near 33 amp is acceptable.

Continuing with the solution to quadratic equation for I_{a2}, we have

$$220 - 0.40\, I_{a2} = 0.32 \times (24.2 \times I_{a2}) \times 900$$

$$220 - 0.40\, I_{a2} = 6969/I_{a2}$$

$$I_{a2}^2 - 550\, I_{a2} + 17425 = 0$$

This gives I_{a2} as ***either*** 33.75 amp *or* 791.25 amp.

From the reasoning given above, acceptable root corresponds to $I_{a2} = 33.75$ amp.

Corresponding field current, $i_{f2} = 24.2/33.75 = 0.717$ amp

Previous field circuit resistance = 200 ohms

New field circuit resistance = 220/0.717 = 307 ohms

Hence, additional resistance of 107 ohms must be added to the shunt field circuit to run the motor at 900 r.p.m. under the stated condition of constant Load torque.

Additional Check : Exact calculations for proportions of armature-power in two cases will give the necessary check.

$$E_{b2} = 220 - (33.75 \times 0.40) = 206.5$$

As mentioned above, while guessing the value of I_{a2}, the proportion of armature-power should be 1.5.

$$\frac{E_{b2}\, I_{a2}}{E_{b1}\, I_{a1}} = \frac{206.56 \times 33.75}{211.2 \times 22} = 1.50$$

Thus, the results obtained are confirmed.

Example 30.17. *A 250 V, 25 kW d.c. shunt motor has an efficiency of 85% when running at 1000 r.p.m. on full load. The armature resistance is 0.1 ohm and field resistance is 125 ohms. Find the starting resistance required to limit the starting current to 150% of the rated current.*

(Amravati Univ., 1999)

Solution.

$$\text{Output power} = 25 \text{ kW, at full-load.}$$

$$\text{Input power} = \frac{25{,}000}{0.85} = 29412 \text{ watts}$$

$$\text{At Full load, Input Current} = 29412/250 = 117.65 \text{ amp}$$

$$\text{Field Current} = 250/125 = 2 \text{ amp}$$

$$\text{F.L. Armature Current} = 117.65 - 2 = 115.65 \text{ amp}$$

$$\text{Limit of starting current} = 1.50 \times 115.65 = 173.5 \text{ amp}$$

Total resistance in armature circuit at starting

$$= \frac{250}{173.5} = 1.441 \text{ ohms}$$

External resistance to be added to armature circuit

$$= 1.441 - 0.1 = 1.341 \text{ ohm.}$$

Tutorial Problems 30.1

1. A d.c. shunt motor runs at 900 r.p.m. from a 460 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230-V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 460 V. **[595 r.p.m.]**
2. A 250 V shunt motor has an armature resistance of 0.5 Ω and runs at 1200 r.p.m. when the armature current is 80 A. If the torque remains unchanged, find the speed and armature current when the field is strengthened by 25%. **[998 r.p.m. ; 64 A]**
3. When on normal full-load, a 500 V, d.c. shunt motor runs at 800 r.p.m. and takes an armature current 42 A. The flux per pole is reduced to 75% of its normal value by suitably increasing the field circuit resistance. Calculate the speed of the motor if the total torque exerted on the armature is (*a*) unchanged (*b*) reduced by 20%.

   The armature resistance is 0.6 Ω and the total voltage loss at the brushes is 2 V.

   [(*a*) 1,042 r.p.m. (*b*) 1,061 r.p.m.]
4. The following data apply to d.c. shunt motor.

   Supply voltage = 460 V ; armature current = 28 A ; speed = 1000 r.p.m. ; armature resistance = 0.72 Ω. Calculate (*i*) the armature current and (*ii*) the speed when the flux per pole is increased to 120% of the initial value, given that the total torque developed by the armature is unchanged. **[(*i*) 23.33 A (*ii*) 840 r.p.m.]**
5. A 100-V shunt motor, with a field resistance of 50 Ω and armature resistance of 0.5 Ω runs at a speed of 1,000 r.p.m. and takes a current of 10 A from the supply. If the total resistance of the field circuit is reduced to three quarters of its original value, find the new speed and the current taken from the supply. Assume that flux is directly proportional to field current. **[1,089 r.p.m. ; 8.33 A]**

6. A 250 V d.c. shunt motor has armature circuit resistance of 0.5 Ω and a field circuit resistance of 125 Ω. It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to 150 Ω. If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current. **[1186 r.p.m. 33.6 A]**
7. A 250 V, shunt motor with an armature resistance of 0.5 Ω and a shunt field resistance of 250 Ω drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be affected in the shunt field resistance ? Assume that the magnetization curve of the motor is a straight line. **[88 Ω]**
8. A 240 V, d.c. shunt motor runs at 800 r.p.m. with no extra resistance in the field or armature circuit, on no-load. Determine the resistance to be placed in series with the field so that the motor may run at 950 r.p.m. when taking an armature current of 20 A. Field resistance = 160 Ω. Armature resistance = 0.4 Ω. It may be assumed that flux per pole is proportional to field current. **[33.6 Ω]**
9. A shunt-wound motor has a field resistance of 400 Ω and an armature resistance of 0.1 Ω and runs off 240 V supply. The armature current is 60 A and the motor speed is 900 r.p.m.; Assuming a straight line magnetization curve, calculate (*a*) the additional resistance in the field to increase the speed to 1000 r.p.m. for the same armature current and (*b*) the speed with the original field current of 200 A. **[(*a*) 44.4 Ω (*b*) 842.5 r.p.m.]**
10. A 230 V d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of $76\,^2/_3$ Ω. The motor draws a line current of 13 A while running light at 1000 r.p.m. At a certain load, the field circuit resistance is increased by $38\,^1/_3$ Ω. What is the new speed of the motor if the line current at this load is 42 A ? **[1400 r.p.m.] (*Electrical Engg. ; Grad I.E.T.E. Dec. 1986*)**
11. A 250 V d.c. shunt motor runs at 1000 r.p.m. and takes an armature current of 25 amp. Its armature resistance is 0.40 ohm. Calculate the speed with increased load with the armature current of 50 amp. Assume that the increased load results into flux-weakening by 3%, with respect to the flux in previous loading condition. **(*Nagpur Univ., April 1996*)**

Hint : (*i*) First Loading condition :

$$E_{b1} = 250 - 25 \times 0.40 = K_1 \times 1000$$

(*ii*) Second Loading condition :

$$E_{b2} = 250 - 50 \times 0.40 = 230\ K_1 \times (0.97\ \phi) \times N_2.\ \text{This gives } N_2.$$

[988 r.p.m.]

(*ii*) Armature or Rheostatic Control Method

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 30.4 (*a*). As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed. For a load constant torque, speed is approximately proportional to the p.d. across the armature. From the speed/armature current characteristic [Fig. 30.4 (*b*)], it is seen that greater the resistance in the armature circuit, greater is the fall in the speed.

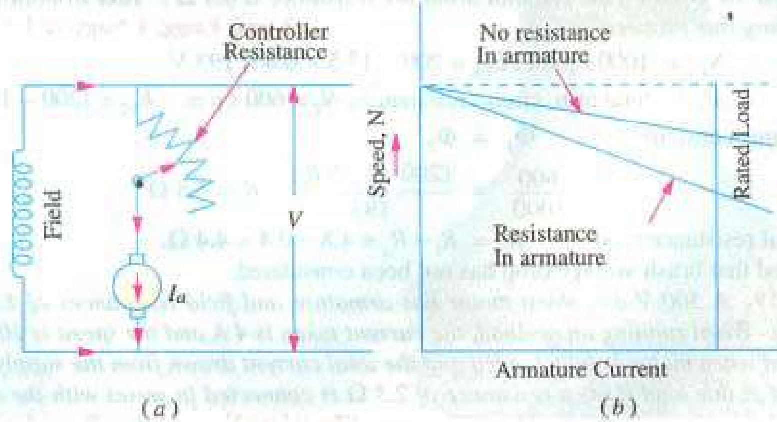


Fig. 30.4

Let I_{a1} = armature current in the first case

I_{a2} = armature current in the second case

(If $I_{a1} = I_{a2}$, then the load is of constant torque.)

N_1, N_2 = corresponding speeds, V = supply voltage

Then $N_1 \propto V - I_{a1} R_a \propto E_{b1}$

Let some controller resistance of value R be added to the armature circuit resistance so that its value becomes $(R + R_a) = R_t$

Then $N_2 \propto V - I_{a2} R_t \propto E_{b2}$ $\therefore$ $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$

(In fact, it is a simplified form of relation given in Art. 27.9 because here $\Phi_1 = \Phi_2$.)

Considering no-load speed, we have $\dfrac{N}{N_0} = \dfrac{V - I_a R_t}{V - I_{a0} R_a}$

Neglecting $I_{a0} R_a$ with respect to V, we get

$$N = N_0 \left(1 - \frac{I_a R_t}{V}\right)$$

It is seen that for a given resistance R_t the speed is a linear function of armature current I_a as shown in Fig. 30.5 (*a*).

The load current for which the speed would be zero is found by putting $N = 0$ in the above relation.

$$\therefore \quad 0 = N_0 \left(1 - \frac{I_a R_t}{V}\right) \quad \text{or} \quad I_a = \frac{V}{R_t}$$

This is the maximum current and is known as *stalling* current.

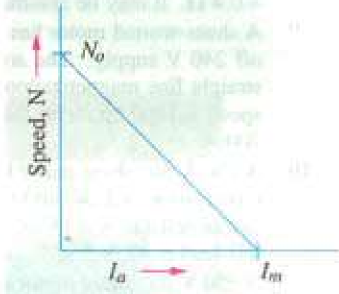


Fig. 30.5 (*a*)

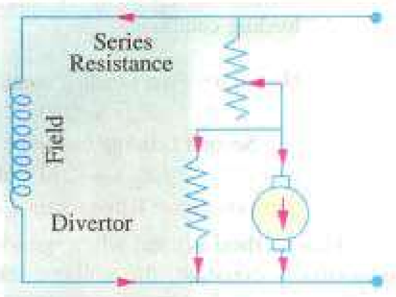


Fig. 30.5 (*b*)

As will be shown in Art. 30.5 (*a*), this method is very wasteful, expensive and unsuitable for rapidly changing loads because for a given value of R_t, speed will change with load. A more stable operation can be obtained by using a divertor across the armature in addition to armature control resistance (Fig. 30.5 (*b*)). Now, the changes in armature current (due to changes in the load torque) will not be so effective in changing the p.d. across the armature (and hence the armature speed).

Example 30.18. *A 200 V d.c. shunt motor running at 1000 r.p.m. takes an armature current of 17.5 A. It is required to reduce the speed to 600 r.p.m. What must be the value of resistance to be inserted in the armature circuit if the original armature resistance is 0.4 Ω ? Take armature current to be constant during this process.* **(Elect. Engg. I Nagpur Univ. 1993)**

Solution. $N_1 = 1000$ r.p.m. ; $E_{b1} = 200 - 17.5 \times 0.4 = 193$ V

R_t = total arm. circuit resistance ; $N_2 = 600$ r.p.m. ; $E_{b2} = (200 - 17.5\, R_t)$

Since I_{sh} remains constant ; $\Phi_1 = \Phi_2$

$$\therefore \quad \frac{600}{1000} = \frac{(200 - 17.5\, R_t)}{193} ; \quad R_t = 4.8\ \Omega$$

$\therefore$ Additional resistance reqd. $R = R_t - R_a = 4.8 - 0.4 = \mathbf{4.4\ \Omega}$.

It may be noted that brush voltage drop has not been considered.

Example 30.19. *A 500 V d.c. shunt motor has armature and field resistances of 1.2 Ω and 500 Ω respectively. When running on no-load, the current taken is 4 A and the speed is 1000 r.p.m. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 26 A. Estimate the speed at this load if* (*a*) *a resistance of 2.3 Ω is connected in series with the armature and* (*b*) *the shunt field current is reduced by 15%.* **(Electrical Engg. I, Sd. Patel Univ. 1985)**

Solution. $I_{sh} = 500/500 = 1$ A ; $I_{a1} = 4 - 1 = 3$ A

$E_{b1} = 500 - (3 \times 1.2) = 496.4$ V $\quad I_{a2} = 26 - 1 = 25$ A

$E_{b2} = 500 - (25 \times 1.2) = 470$ V $\quad \therefore \dfrac{N_2}{1000} = \dfrac{470}{496.4}$; $N_2 =$ **947 r.p.m.**

(*a*) In this case, total armature circuit resistance = $1.2 + 2.3 = 3.5\ \Omega$

$\therefore \quad E_{b2} = 500 - (25 \times 3.5) = 412.5$ V $\quad \therefore \dfrac{N_2}{1000} = \dfrac{412.5}{496.4}$; $N_2 =$ **831 r.p.m.**

(*b*) When shunt field is reduced by 15%, $\Phi_2 = 0.85\ \Phi_1$ assuming straight magnetisation curve.

$$\frac{N_2}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85} \ ; \ N_2 = \textbf{977.6 r.p.m.}$$

Example 30.20. *A 250-V shunt motor (Fig. 30.6) has an armature current of 20 A when running at 1000 r.p.m. against full load torque. The armature resistance is 0.5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 r.p.m. at the same torque and what will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout and neglect brush contact drop.*

(Elect. Machines AMIE Sec. B Summer 1991)

Solution. $E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240$ V

Let R_t to be total resistance in the armature circuit *i.e.* $R_t = R_a + R$, where R is the additional resistance.

$\therefore \quad E_{b2} = V - I_{a2} R_t = 250 - 20\ R_t$

It should be noted that $I_{a1} = I_{a2} = 20$ A because torque remains the same and $\Phi_1 = \Phi_2$ in both cases.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \text{ or } \frac{500}{1000} = \frac{250 - 20\ R_t}{240}$$

$\therefore \quad R_t = 6.5\ \Omega$; hence, $R = 6.5 - 0.5 =$ **6 Ω**

Since the load is halved, armature current is also halved because flux remains constant. Hence, $I_{a3} = 10$ A.

$$\therefore \frac{N_3}{1000} = \frac{250 - 10 \times 6.5}{240} \quad \text{or} \quad N_3 = \textbf{771 r.p.m.}$$

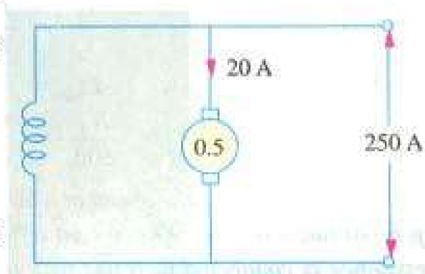


Fig. 30.6

Example 30.21. *A 250-V shunt motor with armature resistance of 0.5 ohm runs at 600 r.p.m. on full-load and takes an armature current of 20 A. If resistance of 1.0 ohm is placed in the armature circuit, find the speed at (i) full-load torque (ii) half full-load torque.*

(Electrical Machines-II, Punjab Univ. May 1991)

Solution. Since flux remains constant, the speed formula becomes $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$.

(*i*) In the first case, full-load torque is developed.

$$N_1 = 600 \text{ r.p.m.}; E_{b1} = V - I_{a1} R_{a1} = 250 - 20 \times 0.5 = 240 \text{ V}$$

Now, $\quad T \propto \Phi I_a \propto I_a \quad$ ($\because$ Φ is constant)

$\therefore \quad \dfrac{T_2}{T_1} = \dfrac{I_{a2}}{I_{a1}} \quad$ Since $T_2 = T_1$; $I_{a2} = I_{a1} = 20$ A

$$E_{b2} = V - I_{a2} R_{a2} = 250 - 20 \times 1.5 = 220 \text{ V},$$

$$N_2 = \frac{N_2}{600} = \frac{220}{240} \ ; N_2 = \frac{600 \times 220}{240} = \textbf{550 r.p.m.}$$

(*ii*) In this case, the torque developed is half the full-load torque.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{T_1/2}{T_1} = \frac{I_{a2}}{20} \;;\; I_{a2} = 10\text{ A} \;;\; E_{b2} = 250 - 10 \times 1.5 = 235\text{ V}$$

$$\frac{N_2}{600} = \frac{235}{240} \;;\; N_2 = 600 \times \frac{235}{240} = \mathbf{587.5\ r.p.m.}$$

Example 30.22. *A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs of 500 rev. per minute and takes an armature current of 30 A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at (a) full-load torque (b) double full-load torque.* **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Since flux remains constant, the speed formula becomes $N_2/N_1 = E_{b2}/E_{b1}$.

(a) Full-load torque

With no additional resistance in the armature circuit,

$$N_1 = 500\text{ r.p.m.} \;;\; I_{a1} = 30\text{ A} \;;\; E_{b1} = 220 - 30 \times 0.5 = 205\text{ V}$$

Now, $T \propto I_a$ (since Φ is constant.) $\therefore \quad \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$ Since $T_2 = T_1$; $I_{a2} = I_{a1} = 30$ A

When additional resistance of 1 Ω is introduced in the armature circuit,

$$E_{b2} = 220 - 30\,(1 + 0.5) = 175\text{ V} \;;\; N_2 = ? \quad \frac{N_2}{500} = \frac{175}{205} \;;\; N_2 = \mathbf{427\ r.p.m.}$$

(b) Double Full-load Torque

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{2T_1}{T_1} = \frac{I_{a2}}{30} \;;\; I_{a2} = 60\text{ A}$$

$$\therefore \quad E_{b2} = 220 - 60\,(1 + 0.5) = 130\text{ V}$$

$$\therefore \quad \frac{N_2}{500} = \frac{130}{205} \;;\; N_2 = \mathbf{317\ r.p.m.}$$

Example 30.23. *The speed of a 50 h.p (37.3 kW) series motor working on 500 V supply is 750 r.p.m. at full-load and 90 per cent efficiency. If the load torque is made 350 N-m and a 5 ohm resistance is connected in series with the machine, calculate the speed at which the machine will run. Assume the magnetic circuit to be unsaturated and the armature and field resistance to be 0.5 ohm.* **(Electrical Machinery I, Madras Univ. 1986)**

Solution. Load torque in the first case is given by

$$T_1 = 37{,}300/2\pi\,(750/60) = 474.6\text{ N-m}$$

Input current, $I_{a1} = 37{,}300/0.9 \times 500 = 82.9$ A

Now, $T_2 = 250$ N-m ; $I_{a2} = ?$

In a series motor, before magnetic saturation,

$$T \propto \Phi I_a \propto I_{a2} \quad \therefore \quad T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2$$

$$\therefore \quad \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \frac{T_2}{T_1} \quad \therefore \quad I_{a2} = 82.9 \times \sqrt{250/474.6} = 60.2\text{ A}$$

Now, $E_{b1} = 500 - (82.9 \times 0.5) = 458.5$ V ;

$$E_{b2} = 500 - 60.2\,(5 + 0.5) = 168.9\text{ V}$$

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a2}}{I_{a1}}$, we get $\frac{N_2}{750} = \frac{168.9}{458.5} \times \frac{82.9}{60.2}$ $\quad \therefore N_2 = \mathbf{381\ r.p.m.}$

Example 30.24. *A 7.46 kW, 220 V, 900 r.p.m. shunt motor has a full-load efficiency of 88 per cent, an armature resistance of 0.08 Ω and shunt field current of 2 A. If the speed of this motor is*

reduced to 450 r.p.m. by inserting a resistance in the armature circuit, the load torque remaining constant, find the motor output, the armature current, the external resistance and the overall efficiency.
(Elect. Machines, Nagpur Univ. 1993)

Solution. Full-load motor input current $I = 7460/220 \times 0.88 = 38.5$ A

$\therefore \quad I_{a1} = 38.5 - 2 = 36.5$ A

Now, $\quad T \propto \Phi I_a$. Since flux remains constant.

$\therefore \quad T \propto I_a \quad \therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2}$ or $\dfrac{T_{a2}}{T_{a1}} = \dfrac{I_{a2}}{I_{a1}}$

It is given that $\quad T_{a1} = T_{a2}$; hence $I_{a1} = I_{a2} =$ **36.5 A**

$E_{b1} = 220 - (36.5 \times 0.08) = 217.1\ \Omega$

$E_{b2} = 220 - 36.5\ R_t$; $N_1 = 900$ r.p.m.; $N_2 = 450$ r.p.m.

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \qquad (\because \quad \Phi_1 = \Phi_2)$$

$$\therefore \quad \frac{450}{900} = \frac{200 - 36.5\ R_t}{217.1}\ ; \ R_t = 3.05\ \Omega$$

$\therefore \quad$ External resistance $R = 3.05 - 0.08 =$ **2.97 Ω**

For calculating the motor output, it will be assumed that all losses except copper losses vary directly with speed.

Since motor speed is halved, stray losses are also halved in the second case. Let us find their value.

In the first case, motor input $= 200 \times 38.5 = 8,470$ W; Motor output $= 7,460$ W

Total Cu losses + stray losses $= 8470 - 7460 = 1010$ W

Arm. Cu loss $= I_{a1}^2 R_a = 36.5^2 \times 0.08 = 107$ W ; Field Cu loss $= 220 \times 2 = 440$ W

Total Cu loss $= 107 + 440 = 547$ W $\quad \therefore \quad$ Stray losses in first case $= 1010 - 547 = 463$ W

Stray losses in the second case $= 463 \times 450/900 = 231$ W

Field Cu loss $= 440$ W, as before ; Arm. Cu loss $= 36.5^2 \times 3.05 = 4,064$ W

Total losses in the 2nd case $= 231 + 440 + 4,064 = 4,735$ W

Input $= 8,470$ W $\quad$ – as before

Output in the second case $= 8,470 - 4,735 =$ **3,735 W**

$\therefore \quad$ Overall $\eta = 3,735/8,470 = 0.441$ or **44.1 per cent***

Example 30.25. *A 240 V shunt motor has an armature current of 15 A when running at 800 r.p.m. against F.L. torque. The arm. resistance is 0.6 ohms. What resistance must be inserted in series with the armature to reduce the speed to 400 r.p.m., at the same torque ?*

What will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout. **(Elect. Machines-I Nagpur Univ. 1993)**

Solution. Here, $\quad N_1 = 800$ r.p.m., $E_{b1} = 240 - 15 \times 0.6 = 231$ V

Flux remaining constant, $T \propto I_a$. Since torque is the same in both cases, $I_{a2} = I_{a1} = 15$ A. Let R be the additional resistance inserted in series with the armature. $E_{b2} = 240 - 15\,(R + 0.6)$; $N_2 = 400$ r.p.m.

$$\therefore \quad \frac{400}{800} = \frac{240 - 15\,(R + 0.6)}{231}\ ; \ R = 7.7\ \Omega$$

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* It may be noted that efficiency is reduced almost in the ratio of the two speeds.

When load torque is halved :

With constant flux when load torque is halved, I_a is also halved. Hence, $I_{a3} = I_{a1}/2 = 15/2 = 7.5$ A

$$\therefore \quad E_{b3} = 240 - 7.5\,(7.7 + 0.6) = 177.75 \text{ V} \; ; N_3 = ?$$

$$\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b1}} \quad \text{or} \quad \frac{N_3}{800} = \frac{177.75}{231} \; ; N_3 = \mathbf{614.7 \text{ r.p.m.}}$$

Example 30.26. (*a*) *A 400 V shunt connected d.c. motor takes a total current of 3.5 A on no load and 59.5 A at full load. The field circuit resistance is 267 ohms and the armature circuit resistance is 0.2 ohms (excluding brushes where the drop may be taken as 2 V). If the armature reaction effect at 'full-load' weakens the flux per pole by 2 percentage change in speed from no-load to full-load.*

(*b*) *What resistant must be placed in series with the armature in the machine of (a) if the full-load speed is to be reduced by 50 per cent with the gross torque remaining constant ? Assume no change in the flux.* **(Electrical Machines, AMIE Sec. B, 1989)**

Solution. (*a*) Shunt current $I_{sh} = 400/267 = 1.5$ A. At no load, $I_{a1} = 3.5 - 1.5 = 2$ A, $E_{b1} = V - I_{a1} R_{ai}$ – brush drop $= 400 - 2 \times 0.2 - 2 = 397.6$ V. On full-load, $I_{a2} = 59.5 - 1.5 = 58$ A, $E_{b2} = 400 - 58 \times 0.2 - 2 = 386.4$ V.

$$\therefore \quad \frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \quad \text{or} \quad \frac{397.6}{386.4} = \frac{\Phi_1 N_1}{0.98\,\Phi_1 N_2} ; \frac{N_1}{N_2} = 1.0084$$

$$\% \text{ change in speed} = \frac{N_1 - N_2}{N_1} \times 100$$

$$= \left(1 - \frac{1}{1.0084}\right) \times 100 = 0.833$$

(*b*) Since torque remains the same, I_a remains the same, hence $I_{a3} = I_{a2}$. Let R be the resistance connected in series with the armature.

$$E_{b3} = V - I_{a2}\,(R_a + R) - \text{brush drop}$$

$$= 400 - 58\,(0.2 + R) - 2 = 386.4 - 58\,R$$

$$\therefore \quad \frac{E_{b2}}{E_{b3}} = \frac{\Phi_2 N_2}{\Phi_3 N_3} = \frac{N_2}{N_3} \qquad (\because \quad \Phi_2 = \Phi_3)$$

$$\frac{386.4}{386.4 - 58R} = \frac{1}{0.5} \; ; \; R = \mathbf{3.338\ \Omega}$$

Example 30.27. *A d.c. shunt drives a centrifugal pump whose torque varies as the square of the speed. The motor is fed from a 200 V supply and takes 50 A when running at 1000 r.p.m. What resistance must be inserted in the armature circuit in order to reduce the speed to 800 r.p.m. ? The armature and field resistance of the motor are 0.1 Ω and 100 Ω respectively.*

(Elect. Machines, Allahabad Univ. 1992)

Solution. In general, $T \propto \Phi I_a$

For shunt motors whose excitation is constant,

$$T \propto I_a \propto N^2, \text{ as given.}$$

$$\therefore \quad I_a \propto N^2. \text{ Now } I_{sh} = 200/100 = 2 \text{ A} \quad \therefore \quad I_{a1} = 50 - 2 = 48 \text{ A}$$

Let I_{a2} = new armature current at 800 r.p.m.

then $48 \propto N_1^2 \propto 1000^2$ and $I_{a2} \propto N_2^2 \propto 800^2$

$$\therefore \quad \frac{I_{a2}}{48} = \left(\frac{800}{1000}\right)^2 = 0.8^2 \quad \therefore \quad I_{a2} = 48 \times 0.64 = 30.72 \text{ A}$$

$\therefore \quad E_{b1} = 200 - (48 \times 0.1) = 195.2 \text{ V} ; E_{b2} = (200 - 30.72 R_t) \text{ V}$

Now, $\quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \quad \therefore \quad \dfrac{800}{1000} = \dfrac{200 - 30.72 R_t}{195.2}, R_t = 1.42 \ \Omega$

Additional resistance $= 1.42 - 0.1 =$ **1.32 Ω**

Example 30.28. *A 250 V, 50 h.p. (373 kW) d.c. shunt motor has an efficiency of 90% when running at 1,000 r.p.m. on full-load. The armature and field resistances are 0.1 Ω and 115 Ω respectively. Find*

(*a*) *the net and developed torque on full-load.*

(*b*) *the starting resistance to have the line start current equal to 1.5 times the full-load current.*

(*c*) *the torque developed at starting.* **(Elect. Machinery-I, Kerala Univ. 1987)**

Solution. (*a*) $\quad T_{sh} = 9.55 \times 37,300/1000 =$ **356.2 N-m**

Input current $= \dfrac{37,300}{250 \times 0.9} = 165.8 \text{ A} ; I_{sh} = \dfrac{250}{125} = 2 \text{ A}$

$\therefore \quad I_a = 165.8 - 2 = 163.8 \text{ A} ; E_b = 250 - (163.8 \times 0.1) = 233.6 \text{ V}$

$\therefore \quad T_a = 9.55 \ \dfrac{233.6 \times 163.8}{1000} =$ **365.4 N-m**

(*b*) F.L. input line $I = 165.8$ A ; Permissible input $I = 165.8 \times 1.5 = 248.7$ A

Permissible armature current $= 248.7 - 2 = 246.7$ A

Total armature resistance $= 250/246.7 = 1.014 \ \Omega$

$\therefore$ Starting resistance required $= 1.014 - 0.1 =$ **0.914 Ω**

(*c*) Torque developed with 1.5 times the F.L. current would be practically 1.5 times the F.L. torque.

i.e. $\quad 1.5 \times 365.4 =$ **548.1 N-m.**

Example 30.29. *A 200 V shunt motor with a shunt resistance of 40 Ω and armature resistance of 0.02 Ω takes a current of 55 A and runs at 595 r.p.m. when there is a resistance of 0.58 Ω in series with armature. Torque remaining the same, what change should be made in the armature circuit resistance to raise the speed to 630 r.p.m. ? Also find*

(*i*) *At what speed will the motor run if the load torque is reduced such that armature current is 15 A.*

(*ii*) *Now, suppose that a divertor of resistance 5 Ω is connected across the armature and series resistance is so adjusted that motor speed is again 595 r.p.m., when armature current is 50 A. What is the value of this series resistance ? Also, find the speed when motor current falls of 15 A again.*

Solution. The circuit is shown in Fig. 30.7.

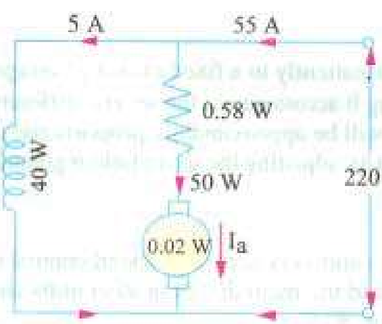


Fig. 30.7

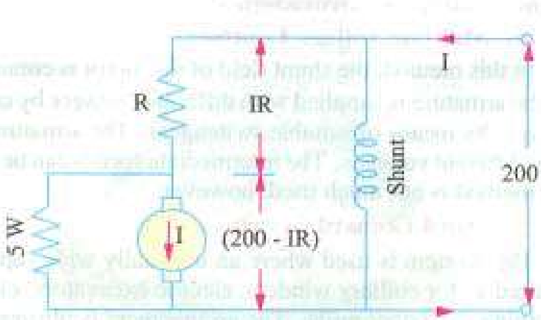


Fig. 30.8

$$I_{sh} = 200/40 = 5\text{ A} \quad \therefore \quad I_{a1} = 55 - 5 = 50\text{ A}$$

$$\text{Armature circuit resistance} = 0.58 + 0.02 = 0.6\ \Omega$$

$$\therefore \quad E_{b1} = 200 - (50 \times 0.6) = 170\text{ V}$$

Since torque is the same in both cases, $I_{a1}\,\Phi_1 = I_{a2}\,\Phi_2$

Moreover, $\Phi_1 = \Phi_2 \quad \therefore \quad I_{a1} = I_{a2} \quad \therefore \quad I_{a2} = 50\text{ A}$

Now $E_{b1} = 170\text{ V}, \quad N_1 = 595\text{ r.p.m.}, \ N_2 = 630\text{ r.p.m.}, E_{b2} = ?$

Using $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \qquad (\because \ \Phi_1 = \Phi_2)$$

we get $E_{b2} = 170 \times (630/595) = 180\text{ V}$

Let R_2 be the new value of armature circuit resistance for raising the speed from 595 r.p.m. to 630 r.p.m.

$$\therefore \quad 180 = 200 - 50\,R_2 \quad \therefore \quad R = 0.4\ \Omega$$

Hence, armature circuit resistance should be reduced by $0.6 - 0.4 =$ **$0.2\ \Omega$.**

(*i*) We have seen above that

$$I_{a1} = 50\text{ A}, E_{b1} = 170\text{ V}, N_1 = 595\text{ r.p.m.}$$

If $I_{a2} = 15\text{ A}, \ E_{b2} = 200 - (15 \times 0.6) = 191\text{ V}$

$$\therefore \quad \frac{N_2}{595} = \frac{191}{170} \quad \therefore \quad N_2 = \mathbf{668.5\ r.p.m.}$$

(*ii*) When armature divertor is used (Fig. 30.8).

Let R be the new value of series resistance

$$\therefore \quad E_{b3} = 200 - IR - (50 \times 0.02) = 199 - IR$$

Since speed is 595 r.p.m., E_{b3} must be equal to 170 V

$\therefore \quad 170 = 199 - IR \quad \therefore \quad IR = 29$ V; P.D. across divertor $= 200 - 29 = 171$ V

Current through divertor $I_d = 171/5 = 34.2\text{ A} \quad \therefore \quad I = 50 + 34.2 = 84.2\text{ A}$

As $IR = 29\text{ V} \quad \therefore \quad R = 29/84.2 =$ **0.344 W**

When $I_a = 15$ A, then $I_d = (I - 15)$ A

P.D. across divertor $= 5\,(I - 15) = 200 - 0.344\,I \quad \therefore \quad I = 51.46\text{ A}$

$$E_{b4} = 200 - 0.344\,I - (15 \times 0.02)$$
$$= 200 - (0.344 \times 51.46) - 0.3 = 182\text{ V}$$

$$\therefore \quad \frac{N_4}{N_1} = \frac{E_{b4}}{E_{b1}} \quad \text{or} \quad \frac{N_4}{595} = \frac{182}{170} \quad \therefore \quad N_4 = \mathbf{637\ r.p.m.}$$

The effect of armature divertor is obvious. The speed without divertor is 668.5 r.p.m. and with armature divertor, it is 637 r.p.m.

(*iii*) Voltage Control Method

(*a*) Multiple Voltage Control

In this method, the shunt field of the motor is connected permanently to a fixed exciting voltage, but the armature is supplied with different voltages by connecting it across one of the several different voltages by means of suitable switchgear. The armature speed will be approximately proportional to these different voltages. The intermediate speeds can be obtained by adjusting the shunt field regulator. The method is not much used, however.

(*b*) Ward-Leonard System

This system is used where an unusually wide (upto 10 : 1) and very sensitive speed control is required as for colliery winders, electric excavators, elevators and the main drives in steel mills and blooming and paper mills. The arrangement is illustrated in Fig. 30.9.

M_1 is the main motor whose speed control is required. The field of this motor is permanently connected across the d.c. supply lines. By applying a variable voltage across its armature, any desired speed can be obtained. This variable voltage is supplied by a motor-generator set which consists of either a d.c. or an a.c. motor M_2 directly coupled to generator G.

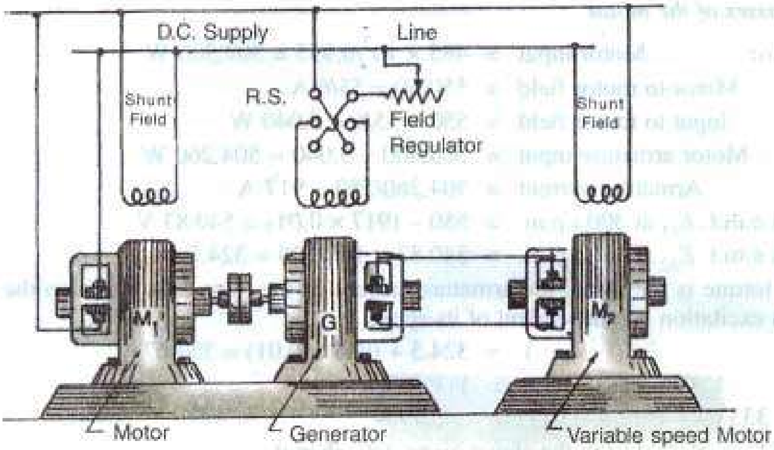


Fig. 30.9

The motor M_2 runs at an approximately constant speed. The output voltage of G is directly fed to the main motor M_1. The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator. By reversing the direction of the field current of G by means of the reversing switch RS, generated voltage can be reversed and hence the direction of rotation of M_1. It should be remembered that motor generator set always runs in the same direction.

Despite the fact that capital outlay involved in this system is high because *(i)* a large output machine must be used for the motor generator set and *(ii)* that two extra machines are employed, still it is used extensively for elevators, hoist control and for main drive in steel mills where motor of ratings 750 kW to 3750 kW are required. The reason for this is that the almost unlimited speed control in either direction of rotation can be achieved entirely by field control of the generator and the resultant economies in steel production outwiegh the extra expenditure on the motor generator set.

A modification of the Ward-Leonard system is known as Ward-Leonard-Ilgner system which uses a smaller motor-generator set with the addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit. When main motor M_1 becomes suddenly overloaded, the driving motor M_2 of the motor generator set slows down, thus allowing the inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor M_1, then M_2 speeds up, thereby again storing energy in the flywheel.

When the Ilgner system is driven by means of an a.c. motor (whether induction or synchronous) another refinement in the form of a 'slip regulator' can be usefully employed, thus giving an additional control.

The chief disadvantage of this system is its low overall efficiency especially at light loads. But as said earlier, it has the outstanding merit of giving wide speed control from maximum in one direction through zero to the maximum in the opposite direction and of giving a smooth acceleration.

Example 30.30. *The O.C.C. of the generator of a Ward-Leonard set is*

Field amps :	*1.4*	*2.2*	*3*	*4*	*5*	*6*	*7*	*8*
Armature volts :	*212*	*320*	*397*	*472*	*522*	*560*	*586*	*609*

The generator is connected to a shunt motor, the field of which is separately-excited at 550 V. If the speed of motor is 300 r.p.m. at 550 V, when giving 485 kW at 95.5% efficiency, determine the excitation of the generator to give a speed of 180 r.p.m. at the same torque. Resistance of the motor *armature circuit = 0.01 Ω, resistance of the motor field = 60 Ω, resistance of generator armature circuit = 0.01 Ω. Ignore the effect of armature reaction and variation of the core factor and the windage losses of the motor.*

Solution. Motor input $= 485 \times 10^3/0.955 = 509,300$ W

Motor to motor field $= 550/60 = 55/6$ A

Input to motor field $= 550 \times 55/6 = 5,040$ W

$\therefore$ Motor armature input $= 509,300 - 5,040 = 504,260$ W

$\therefore$ Armature current $= 504,260/550 = 917$ A

Back e.m.f. E_{b1} at 300 r.p.m. $= 550 - (917 \times 0.01) = 540.83$ V

Back e.m.f. E_{b2} at 180 r.p.m. $= 540.83 \times 180/300 = 324.5$ V

Since torque is the same, the armature current of the main motor is also the same *i.e.* 917 A because its excitation is independent of its speed.

$\therefore$ $V = 324.5 + (917 \times 0.01) = 333.67$ V

Generated e.m.f. $= V + I_a R_a$ ...for generator

$333.67 + (917 \times 0.011) = 343.77$ V.

If O.C.C. is plotted from the above given data, then it would be found that the excitation required to give 343.77 V is 2.42 A.

$\therefore$ Generator exciting current = **2.42 A**

30.3. Speed Control of Series Motors

1. Flux Control Method

Variations in the flux of a series motor can be brought about in any one of the following ways :

(*a*) Field Divertors

The series winding are shunted by a variable resistance known as field divertor (Fig. 30.10). Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.

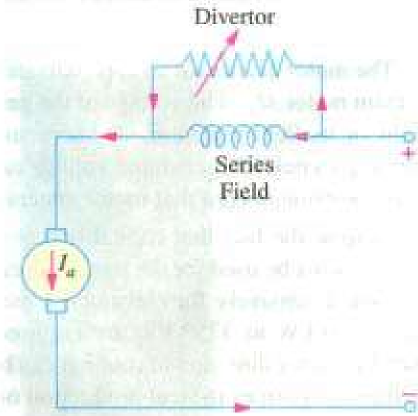


Fig. 30.10

(*b*) Armature Divertor

A divertor across the armature can be used for giving speeds lower than the normal speed (Fig. 30.11). For a ***given constant load torque***, if I_a is reduced due to armature divertor, the Φ must increase.

($\because$ $T_a \propto \Phi I_a$). This results in an increase in current taken from the supply (which increases the flux and a fall in speed ($N \propto 1/\Phi$)). The variation in speed can be controlled by varying the divertor resistance.

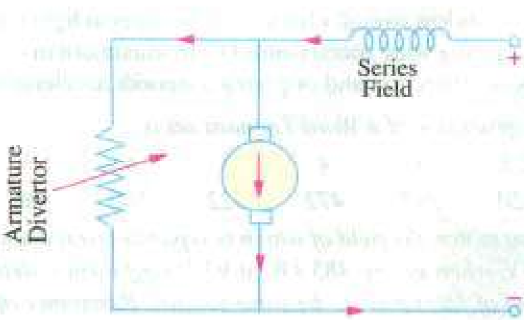


Fig. 30.11

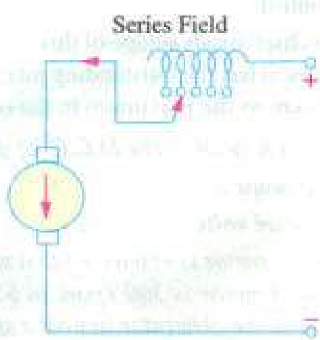


Fig. 30.12

(c) Trapped Field Control Field

This method is often used in electric traction and is shown in Fig. 30.12.

The number of series filed turns in the circuit can be changed at will as shown. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

(d) Paralleling Field coils

In this method, used for fan motors, several speeds can be obtained by regrouping the field coils as shown in Fig. 30.13. It is seen that for a 4-pole motor, three speeds can be obtained easily. (Ex.30.35)

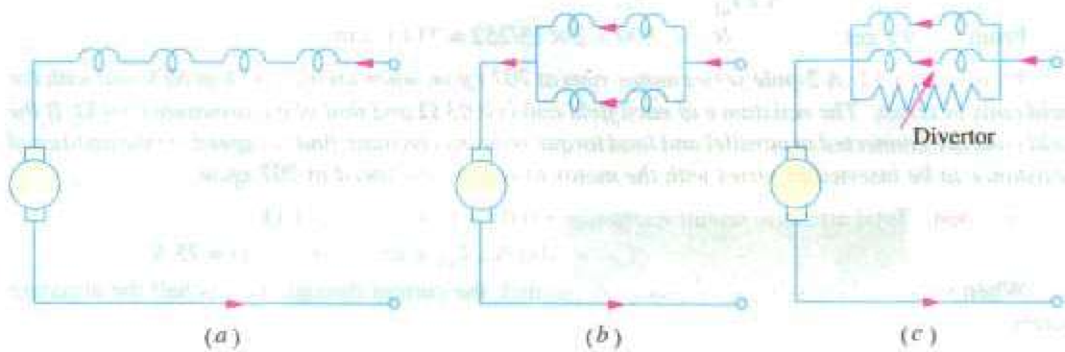


Fig. 30.13

2. Variable Resistance in Series with Motor

By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

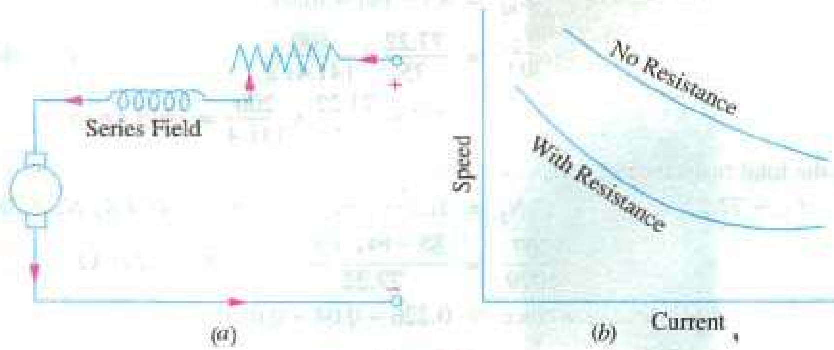


Fig. 30.14

Example 30.31. *A d.c. series motor drives a load the torque of which varies as the square of the speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and the current when the motor field winding is shunted by a divertor of the same resistance as that of the field winding. Mention the assumptions made, if any.* **(Elect. Machines, AMIE Sec B, 1993)**

Solution.

$$T_{a1} \propto N_1^2, T_{a2} \propto N_2^2 \quad \therefore \quad T_{a2}/T_{a1} = N_2^2/N_1^2$$

Also, $\quad T_{a1} \propto \Phi_1 I_{a1} \propto I_{a1}^2, T_{a2} \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$

It is so because in the second case, field current is half the armature current.

$$\therefore \quad \frac{N_2^2}{N_1^2} = \frac{I_{a2}^2/2}{I_{a1}^2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{I_{a2}}{\sqrt{2}\, I_{a1}} \quad \text{... (i)}$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

If we neglect the armature and series winding drops as well as brush drop, then $E_{b1} = E_{b2} = V$

$$\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}/2} = \frac{2I_{a1}}{I_{a2}} \qquad ...(ii)$$

From (i) and (ii), $$\frac{I_{a2}}{\sqrt{2}\,I_{a1}} = \frac{2I_{a1}}{I_{a2}} \quad \text{or} \quad I_{a2}^2 = 2\sqrt{2}\,I_{a1}^2 = 2\sqrt{2} \times 15^2 \quad \text{or} \quad I_{a2} = 25.2 \text{ A}$$

From (ii), we get, $$N_2 = 600 \times 2 \times 15/252 = \textbf{714 r.p.m.}$$

Example 30.32. *A 2-pole series motor runs at 707 r.p.m. when taking 100 A at 85 V and with the field coils in series. The resistance of each field coil is 0.03 Ω and that of the armature 0.04 Ω. If the field coils are connected in parallel and load torque remains constant, find (a) speed (b) the additional resistance to be inserted in series with the motor to restore the speed to 707 r.p.m.*

Solution. Total armature circuit resistance = 0.04 + (2 × 0.03) = 0.1 Ω

$$I_{a1} = 100 \text{ A} ; E_{b1} = 85 - (100 \times 0.1) = 75 \text{ V}$$

When series field windings are placed in parallel, the current through each is half the armature current.

If $$I_{a2} = \text{new armature current ; then } \Phi_2 \propto I_{a2}/2.$$

As torque is the same in the two cases,

$$\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \text{or} \quad I_{a1}^2 = \frac{I_{a2}}{2} \times I_{a2} = \frac{I_{a2}^2}{2}$$

$$\therefore \quad 100^2 = I_{a2}^2 \qquad \therefore \quad I_{a2} = 100\sqrt{2} = 141.4 \text{ A}$$

In this case, series field resistance = 0.03/2 = 0.015 Ω

$$\therefore \quad E_{b2} = 85 - 141.4\,(0.04 + 0.015) = 77.22 \text{ V}$$

$$\frac{N_2}{707} = \frac{77.22}{75} \times \frac{100}{141.4/2} \qquad (\because \quad \Phi_2 \propto I_{a2}/2)$$

(a) $$\therefore \quad N_2 = 707 \times \frac{77.22}{75} \times \frac{200}{141.4} = \textbf{1029 r.p.m.}$$

(b) Let the total resistance of series circuit be R_t.

Now, $E_{b1} = 77.22$ V, $N_1 = 1029$ r.p.m. ; $E_{b2} = 85 - 141.4\,R_t$, $N_2 = 707$ r.p.m.

$$\frac{707}{1029} = \frac{85 - 141.4\,R_t}{77.22} \qquad \therefore \quad R_t = 0.226\ \Omega$$

$\therefore$ Additional resistance = 0.226 − 0.04 − 0.015 = **0.171 Ω**

Example 30.33. *A 240 V series motor takes 40 amperes when giving its rated output at 1500 r.p.m. Its resistance is 0.3 ohm. Find what resistance must be added to obtain rated torque (i) at starting (ii) at 1000 r.p.m.* **(Elect. Engg., Madras Univ. 1987)**

Solution. Since torque remains the same in both cases, it is obvious that current drawn by the motor remains constant at 40 A. $(\because \quad T_a \propto I_a^2)$

(i) If R is the series resistance added, then $40 = 240/(R + 0.3) \quad \therefore \quad R = 5.7\ \Omega$

(ii) Current remaining constant, $T_a \propto E_b/N$ — Art. 29.7

$$\frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2}$$

Now, $$E_{b1} = 240 - 40 \times 0.3 = 228 \text{ V} ; N_1 = 1500 \text{ r.p.m.}$$

$$E_{b2} = 240 - 40\,(R + 0.3) \text{ V} ; N_2 = 1000 \text{ r.p.m.}$$

$$\therefore \quad \frac{228}{1500} = \frac{240 - 40(R + 0.3)}{1000} \; ; \; R = \mathbf{1.9\ \Omega}$$

Example 30.34. *A 4-pole, series-wound fan motor runs normally at 600 r.p.m. on a 250 V d.c. supply taking 20 A. The field coils are connected at in series. Estimate the speed and current taken by the motor if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.* **(Elect. Machines, AMIE, Sec B. 1990)**

Solution. When coils are connected in two parallel groups, current through each becomes $I_{a2}/2$ where I_{a2} is the new armature current.

Hence, $\Phi_2 \propto I_{a2}/2$

Now $T_a \propto \Phi I_a$

$\propto N^2$ – given

$$\therefore \quad \Phi_1 I_{a1} \propto N_1^2 \text{ and } \Phi_2 I_{a2} \propto N_2^2 \quad \therefore \quad \left(\frac{N_2}{N_1}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad ...(i)$$

Since losses are negligible, field coil resistance as well as armature resistance are negligible. It means that armature and series field voltage drops are negligible. Hence, back e.m.f. in each case equals the supply voltage.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ becomes } \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} \quad ...(ii)$$

Putting this value in *(i)* above, we get

$$\left(\frac{\Phi_1}{\Phi_2}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \text{or} \quad \frac{I_{a2}}{I_{a1}} = \left(\frac{\Phi_1}{\Phi_2}\right)^3$$

Now, $\Phi_1 \propto 20$ and $\Phi_2 \propto I_{a2}/2 \quad \therefore \quad \dfrac{I_{a2}}{20} = \left(\dfrac{20}{I_{a2}/2}\right)^3$ or $I_{a2} = 20 \times 2^{3/4} = \mathbf{33.64\ A}$

From *(ii)* above, we get $\dfrac{N_2}{N_1} = \dfrac{\Phi_1}{\Phi_2} = \dfrac{I_{a1}}{I_{a2}/2} = \dfrac{2 I_{a1}}{I_{a2}}$; $N_2 = 600 \times 2 \times 20/33.64 = \mathbf{714\ r.p.m.}$

Example 30.35. *A d.c. series motor having a resistance of 1 Ω drives a fan for which the torque varies as the square of the speed. At 220 V, the set runs at 350 r.p.m. and takes 25 A. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated.*

(Electrical Engg., Banaras Hindu Univ. 1998)

Solution. Since $\Phi \propto I_a$, hence $T_a \propto \Phi I_a \propto I_a^2$. Also $T_a \propto N^2$ ...(given)

$$\therefore \quad I_a^2 \propto N^2 \quad \text{or} \quad I_a \propto N \quad \text{or} \quad I_{a1} \propto N_1 \text{ and } I_{a2} \propto N_2$$

$$\therefore \quad \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{500}{350} ; \; I_{a2} = 25 \times \frac{500}{350} = \frac{250}{7} \text{ A}$$

$$E_{b1} = 220 - 25 \times 1 = 195 \text{ V} ; E_{b1} = \text{V} - (250/7) \times 1, \; \frac{\Phi_1}{\Phi_2} = \frac{25}{250/7} = \frac{7}{10}$$

$$\text{Now} \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} ; \frac{500}{350} = \frac{V - (250/7)}{195} \times \frac{7}{10} ; \; V = \mathbf{433.7\ V}$$

Example 30.36. *A d.c. series motor runs at 1000 r.p.m. when taking 20 A at 200 V. Armature resistance is 0.5 Ω. Series field resistance is 0.2 Ω. Find the speed for a total current of 20 A when a divertor of 0.2 Ω resistance is used across the series field. Flux for a field current of 10 A is 70 per cent of that for 20 A.*

Solution. $E_{b1} = 200 - (0.5 + 0.2) \times 20 = 186$ V ; $N_1 = 1000$ r.p.m.

Since divertor resistance equals series field resistance, the motor current of 20 A is divided equally between the two. Hence, a current of 10 A flows through series field and produces flux which is 70% of that corresponding to 20 A. In other words, $\Phi_2 = 0.7$ or $\Phi_1/\Phi_2 = 1/0.7$

Moreover, their combined resistance $= 0.2/2 = 0.1\ \Omega$

Total motor resistance becomes $= 0.5 + 0.1 = 0.6\ \Omega$

$$\therefore \quad E_{b2} = 200 - 0.6 \times 20 = 188 \text{ V} ; N_2 = ?$$

$$\therefore \quad \frac{N_2}{1000} = \frac{188}{186} \times \frac{1}{0.7} ; N_2 = \textbf{1444 r.p.m.}$$

Example 30.37. *A 200 V, d.c. series motor takes 40 A when running at 700 r.p.m. Calculate the speed at which the motor will run and the current taken from the supply if the field is shunted by a resistance equal to the field resistance and the load torque is increased by 50%.*

Armature resistance = 0.15 Ω, field resistance = 0.1 Ω

It may be assumed that flux per pole is proportional to the field.

Solution. In a series motor, prior to magnetic saturation

$$T \propto \Phi I_a \propto I_a^2 \quad \therefore \quad T_1 \propto I_{a1}^2 \propto 40^2 \qquad \ldots(i)$$

If I_{a2} is the armature current (or motor current) in the second case when divertor is used, then only $I_{a2}/2$ passes through the series field winding.

$$\therefore \quad \Phi_2 \propto I_{a2}/2 \text{ and } T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) \times I_{a2} \propto I_{a2}^2/2 \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get $\dfrac{T_2}{T_1} = \dfrac{I_{a2}^2}{2 \times 40^2}$

Also $T_2/T_1 = 1.5 \quad \therefore \quad 1.5 = I_{a2}/2 \times 40^2$

$$\therefore \quad I_{a2} = \sqrt{1.5 \times 2 \times 40^2} = 69.3 \text{ A} ;$$

Now
$$E_{b1} = 220 - (40 \times 0.25) = 210 \text{ V}$$
$$E_{b2} = 220 - (69.3 \times 0.2^*) = 206.14 \text{ V} ; N_1 = 700 \text{ r.p.m.} ; N_2 = ?$$
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \therefore \quad \frac{N_2}{700} = \frac{206.14}{210} \times \frac{40}{69.3/2} \quad \therefore N_2 = \textbf{794 r.p.m.}$$

Example 30.38. *A 4-pole, 250 V d.c. series motor takes 20 A and runs at 900 r.p.m. Each field coil has resistance of 0.025 ohm and the resistance of the armature is 0.1 ohm. At what speed will the motor run developing the same torque if :*

(*i*) *a divertor of 0.2 ohm is connected in parallel with the series field*

(*ii*) *rearranging the field coils in two series and parallel groups*

Assume unsaturated magnetic operation.

(Electric Drives and their Control, Nagpur Univ. 1993)

Solution. The motor with its field coils all connected in series is shown in Fig. 30.15 (*a*). Here, $N_1 = 900$ r.p.m., $E_{b1} = 250 - 20 \times (0.1 + 4 \times 0.025) = 246$ V.

In Fig. 30.15 (*b*), a divertor of resistance 0.2 Ω has been connected in parallel with the series field coils. Let I_{a2} be the current drawn by the motor under this condition. As per current-divider rule, part of the current passing through the series fields is $I_{a2} \times 0.2/(0.1 + 0.2) = 2\, I_{a2}/3$. Obviously, $\Phi_2 \propto 2\, I_{a2}/3$.

* The combined resistance of series field winding and the divertor is $0.1/2 = 0.05\ \Omega$. Hence, the total resistance $= 0.15 + 0.05 = 0.2\ \Omega$, in example 30.37.

Now, $T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2$; $T_2 \propto \Phi_2 I_{a2} \propto (2 I_{a2}/3) I_{a2} \propto 2 I_{a2}^2/3$

Since $T_1 = T_2$; $\therefore$ $I_{a1}^2 = 2 I_{a2}^2/3$ or $20^2 = 2 I_{a2}^2/3$; $\therefore$ $I_{a2} = 24.5$ A.

Combined resistance of the field and divertor $= 0.2 \times 0.1/0.3 = 0.667\ \Omega$; Arm. circuit resistance $= 0.1 + 0.0667 = 0.1667\ \Omega$; $E_{b2} = 250 - 24.5 \times 0.1667 = 250 - 4.1 = 245.9$ V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} ; \frac{N_2}{900} = \frac{245.9}{246} \times \frac{20}{(2/3)\, 24.5} ; N_2 = 1102 \text{ r.p.m.} \qquad ...(\because\ \Phi_2 \propto 2 I_{a2}/3)$$

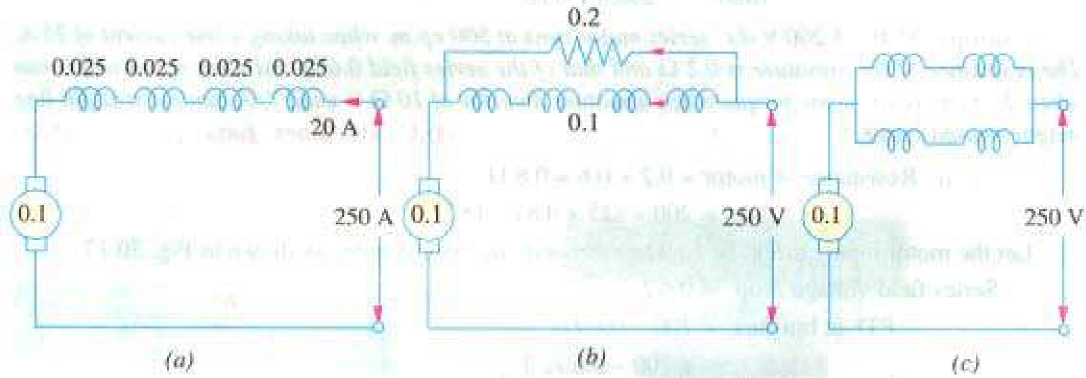


Fig. 30.15

(*ii*) In Fig. 30.15 (*c*), the series field coils have been arranged in two parallel groups. If the motor current is I_{a2}, then it is divided equally between the two parallel paths. Hence, $\Phi_2 \propto I_{a2}/2$.

Since torque remains the same,

$T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \propto 20^2$; $T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$

Since $T_1 = T_2$; $\therefore$ $20^2 = I_{a2}^2/2$; $I_{a2} = 28.28$ A

Combined resistance of the two parallel paths $= 0.05/2 = 0.025\ \Omega$

Total arm. circuit resistance $= 0.1 + 0.025 = 0.125\ \Omega$

$\therefore$ $E_{b2} = 250 - 28.28 \times 0.125 = 246.5$ V

$$\frac{N_2}{900} = \frac{246.5}{246} \times \frac{20}{28.28/2} ; \ N_2 = \mathbf{1275 \text{ r.p.m.}}$$

Example 30.39. *A 4-pole, 230 V series motor runs at 1000 r.p.m., when the load current is 12 A. The series field resistance is 0.8 Ω and the armature resistance is 1.0 Ω. The series field coils are now regrouped from all in series to two in series with two parallel paths. The line current is now 20 A. If the corresponding weakening of field is 15%, calculate the speed of the motor.*

(Electrotechnology-I, Gauhati Univ. 1987)

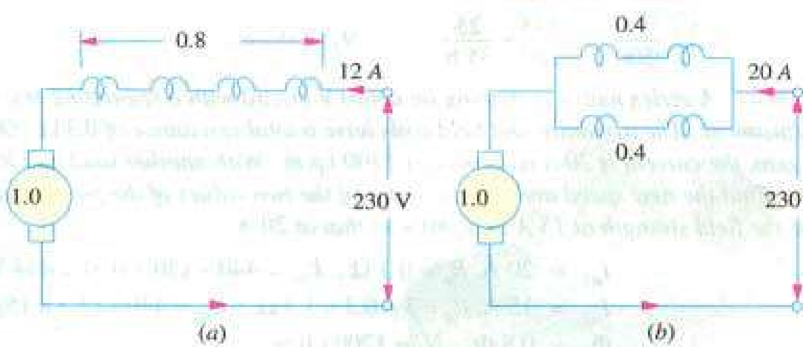


Fig. 30.16

Solution. $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$, $E_{b1} = 230 - 12 \times 1.8 = 208.4$ V, as in Fig. 30.16(*a*)

For circuit in Fig. 30.16 (*b*),

$$E_{b2} = 230 - 20\,(1 + 0.4/2) = 206 \text{ V};$$

$$\Phi_2 = 0.85\,\Phi_1 \text{ or } \Phi_1/\Phi_2 = 1/0.85$$

$$\therefore \quad \frac{N_2}{1000} = \frac{206}{208.4} \times \frac{1}{0.85} \quad \therefore \quad N_2 = \textbf{1163 r.p.m.}$$

Example 30.40. *A 200 V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0.2 Ω and that of the series field 0.6 Ω. At what speed will it run when developing the same torque when armature divertor of 10 Ω is used ? Assume a straight line magnetisation curve.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Resistance of motor = 0.2 + 0.6 = 0.8 Ω

$$\therefore \quad E_{b1} = 200 - (25 \times 0.8) = 180 \text{ V}$$

Let the motor input current be I_2, when armature divertor is used, as shown in Fig. 30.17.

$$\text{Series field voltage drop} = 0.6\,I_2$$

$$\therefore \quad \text{P.D. at brushes} = 200 - 0.6\,I_2$$

$$\therefore \quad \text{Arm. divertor current} = \left(\frac{200 - 0.6\,I_2}{10}\right) \text{A}$$

$$\therefore \quad \text{Armature current} = I_2\left(\frac{200 - 0.6\,I_2}{10}\right)$$

$$\therefore \quad I_{a2} = \frac{10.6\,I_2 - 200}{10}$$

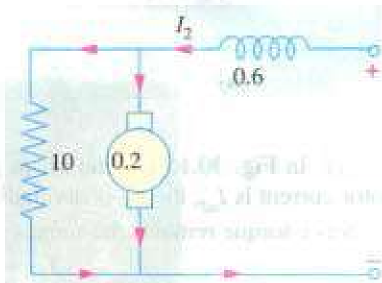


Fig. 30.17

As torque in both cases is the same, $\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$\therefore \quad 25 \times 25 = I_2\left(\frac{10.6\,I_2 - 200}{10}\right) \quad \text{or} \quad 6{,}250 = 10.6\,I_2^2 - 200\,I_2$$

or $10.6\,I_2^2 - 200\,I_2 - 6250 = 0$ or $I_2 = 35.6$ A

$$\text{P.D. at brushes in this case} = 200 - (35.6 \times 0.6) = 178.6 \text{ V}$$

$$\therefore \quad I_{a2} = \frac{10.6 \times 35.6 - 200}{10} = 17.74 \text{ A};$$

$$E_{b2} = 178.6 - (17.74 \times 0.2) = 175 \text{ V}$$

Now $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$ or $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{I_1}{I_2}$ $(\because \; \Phi \propto I)$

$$\therefore \quad \frac{N_2}{500} = \frac{175}{180} \times \frac{25}{35.6} \quad \therefore \quad N_2 = \textbf{314 r.p.m.}$$

Example 30.41. *A series motor is running on a 440 V circuit with a regulating resistance of R Ω for speed adjustment. The armature and field coils have a total resistance of 0.3 Ω. On a certain load with R = zero, the current is 20 A and speed is 1200 r.p.m. With another load and R = 3 Ω, the current is 15 A. Find the new speed and also the ratio of the two values of the power outputs of the motor. Assume the field strength at 15 A to be 80% of that at 20 A.*

Solution. $I_{a1} = 20$ A, $R_a = 0.3$ Ω ; $E_{b1} = 440 - (20 \times 0.3) = 434$ V

$I_{a2} = 15$ A, $R_a = 3 + 0.3 = 3.3$ Ω $\therefore E_{b2} = 440 - (3.3 \times 15) = 390.5$ V

$\Phi_2 = 0.8\,\Phi_1$, $N_1 = 1200$ r.p.m.

Using $\quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$, we get $N_2 = 1200 \times \dfrac{390.5}{434} \times \dfrac{1}{0.8}$ = **1350 r.p.m.**

Now, in a series motor,

$$T \propto \Phi I_a \text{ and power } P \propto T \times N \quad \text{or} \quad P \propto \Phi N I_a$$

$$\therefore \quad P_1 \propto \Phi_1 \times 1200 \times 20 \quad \text{and} \quad P_2 \propto 0.8\,\Phi_1 \times 1350 \times 15$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{1200 \times 20\,\Phi_1}{1350 \times 15 \times 0.8\,\Phi_1} = \mathbf{1.48}$$

Hence, power in the first case is 1.48 times the power in the second case.

Example 30.42. *A d.c. series with an unsaturated field and negligible resistance, when running at a certain speed on a given load takes 50 A at 460 V. If the load-torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.*

(Nagpur, Univ. November 1999, Madras Univ. 1987)

Solution. Let the speed be ω radians/sec and the torque, T_{em} Nw-m developed by the motor. Hence power handled = $T.\ \omega$ watts

Let load torque be T_L, $\quad T_L \propto \omega^3$

$$T_L = T_{em},\ T_{em} \propto (50)^2$$

Hence $\quad$ Load power $= T_L\,\omega$

Since no losses have to be taken into account, $50^2 \propto \omega^3$

Armature power, $460 \times 50 \propto \omega^4$

Based on back e.m.f. relationship, $E_b \propto \omega I_a$

$$460 \propto \omega \times 50$$

To reduce the speed by 25%, operating speed = 0.75 ω rad/sec

Let the new current be I.

From Load side torque $\propto (0.75\,\omega)^3$

From electro-mech side, $\quad T \propto I^2$

$$I^2 \propto (0.75\,\omega)^3$$

Comparing similar relationship in previous case,

$$\frac{I^2}{50^2} = \frac{(0.75\,\omega)^3}{\omega^3} = 0.75^3$$

$$I^2 = 50^2 \times 0.422 = 1055$$

$$I = 32.48 \text{ amp}$$

$$E_{b2} \propto I \times \text{speed}$$

$$\propto I \times 0.75\,\omega$$

$$\propto 32.48 \times 0.75\,\omega$$

$$\frac{E_{b2}}{460} = \frac{32.48 \times 0.75}{50}$$

$$E_{b2} = 224 \text{ volts}$$

If R is the resistance externally connected in series with the motor,

$$E_{b2} = 460 - 32.48 \times R = 224$$

$$\therefore \quad R = 7.266 \text{ ohms}$$

Previous armature power $= 460 \times 50 \times 10^{-3} = 23$ kW

New armature power should be

$$23 \times (0.75)^4 = 7.28 \text{ kW}$$

With E_b as 224 V and current as 32.48 amp

$$\text{Armature power} = 224 \times 32.48 \text{ watts} = 7.27 \text{ kW}$$

Thus, the final answer is checked by this step, since the results agree.

Example 30.43. *A d.c. series motor drives a load, the torque of which varies as the square of speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and current when the motor-field-winding is shunted by a divertor of equal resistance as that of the field winding. Neglect all motor losses and assume the magnetic circuit as unsaturated.*

(Bharathithasan Univ. April 1997)

Solution. Let the equations governing the characteristics of series motor be expressed as follows, with no losses and with magnetic circuit unsaturated.

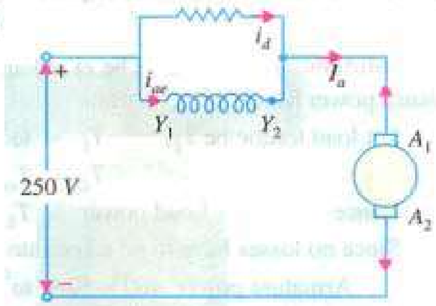


Fig. 30.18 (Divertor for speed control

$$\text{Torque developed by motor} = \text{Load torque}$$

$$k_m \times i_{se} \times I_a = k_L \times (600)^2$$

where k_m, k_L are constants

I_{se} = series field current

$$I_a = \text{Armature current}$$

In the first case $\quad i_{se} = I_a$

With divertor, $\quad i_{se} = i_d = 0.5\ I_a$,

Since, the resistances of divertor and series field are equal.

In the first case, $k_m \times 15 \times 15 = k_L \times (600)^2$ ...(*a*)

Let the supply voltage be V_L volts.

Since Losses are to be neglected, armature receives a power of $(V_L \times 15)$ watts.

Case (*ii*) Let the new speed $= N_2$ r.p.m. and the new armature current $= I_{a2}$ amp

So that new series-field current $= 0.5\ I_{a2}$

$$\text{Torque developed by motor} = \text{Load torque}$$

$$k_m \times (0.5\ I_{a2}) \times I_{a2} = k_L \times (N_2)^2 \quad ...(b)$$

From equations (*a*) and (*b*) above,

$$\frac{0.5\ I_{a2}^2}{225} = \frac{N_2^2}{600^2}$$

or

$$\left(\frac{N_2^2}{I_{a2}^2}\right) = \frac{600^2}{450} = 800 \quad ...(c)$$

or

$$N_2/I_{a2} = 28.28 \quad ...(d)$$

Now armature receives a power of $I_{a2}\ V_L$ watts. Mechanical outputs in the two cases have to be related with these electrical-power-terms.

$$k_L = (600)^2 \times 2600/60 = 15\ V_L \quad ...(e)$$

$$k_L = N^2 \times 2\ N/60 = I_{a2}\ V_L \quad ...(f)$$

From these two equations,

$$N^3/600^3 = I_{a2}/15 \quad ...(g)$$

From (*c*) and (*g*), $\quad N_2\ I_{a2} = 18{,}000$ ...(*h*)

From (*d*) and (*h*), $N_2 = 713.5$ r.p.m.

And $I_{a2} = 25.23$ amp

Additional Correlation : Since the load-torque is proportional to the square of the speed, the mechanical output power is proportional to the cube of the speed. Since losses are ignored, electrical power (input) must satisfy this proportion.

$$(15\ V_L) / (25.23\ V_L) = (600/713.5)^3$$

$$\text{L.H.S.} = 0.5945,\ \text{R.H.S.} = 0.5947$$

Hence, correlated and checked.

30.4. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also. This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.
2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.
3. Maximum power developed is diminished in the same ratio as speed.
4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.
5. It gives speeds ***below*** the normal, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method is, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where motor is continually started and stopped.

Advantages of Field Control Method

This method is economical, more efficient and convenient though it can give speeds *above* (not below) the normal speed. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

30.5. Series-parallel Control

In this system of speed control, which is widely used in electric traction, two or more similar mechanically-coupled series motors are employed. At low speeds, the motors are joined in series Fig. 30.19 (*a*) and for high speeds, are joined in parallel Fig. 30.19 (*b*).

When in series, the two motors have the same current passing through them, although the voltage across each motor is $V/2$ *i.e.*, half the supply voltage. When joined in parallel, voltage across each machines is V, though current drawn by each motor is $I/2$.

When in Parallel

Now speed $\propto E_b / \phi \propto E_b$ / current (being series motors)

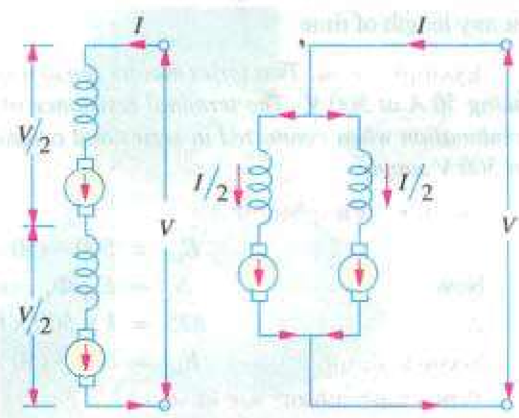


Fig. 30.19 (*a*) Fig. 30.19 (*b*)

Since E_b is approximately equal to the applied voltage V :

$$\therefore \qquad \text{speed} \propto \frac{V}{I/2} \propto \frac{2V}{I}$$

Also, $\qquad \text{torque} \propto \Phi I \propto I^2 \qquad (\because \ \Phi \propto I) \qquad ...(i)$

$$\therefore \qquad T \propto (I/2)^2 \propto I^2/4 \qquad ...(ii)$$

When in Series

Here $\qquad \text{speed} \propto \frac{E_b}{\Phi} \propto \frac{V/2}{I} \propto \frac{V}{2I} \qquad ...(iii)$

This speed is one-fourth of the speed of the motors when in parallel.

Similarly $\qquad T \propto \Phi I \propto I^2$

The torque is four times that produced by motors when in parallel.

This system of speed control is usually combined with the variable resistance method of control described in Art. 30.3 (2).

The two motors are started up in series with each other and with variable resistance which is cut out in sections to increase the speed. When all the variable series resistance is cut out, the motors are connected in parallel and at the same time, the series resistance is reinserted. The resistance is again reduced gradually till full speed is attained by the motors. The switching sequence is shown in Fig.30.20. As the variable series controller resistance is not continuously rated, it has to be cut out of the circuit fairly quickly although in the four running positions A, B, C and D, it may be left in circuit for any length of time.

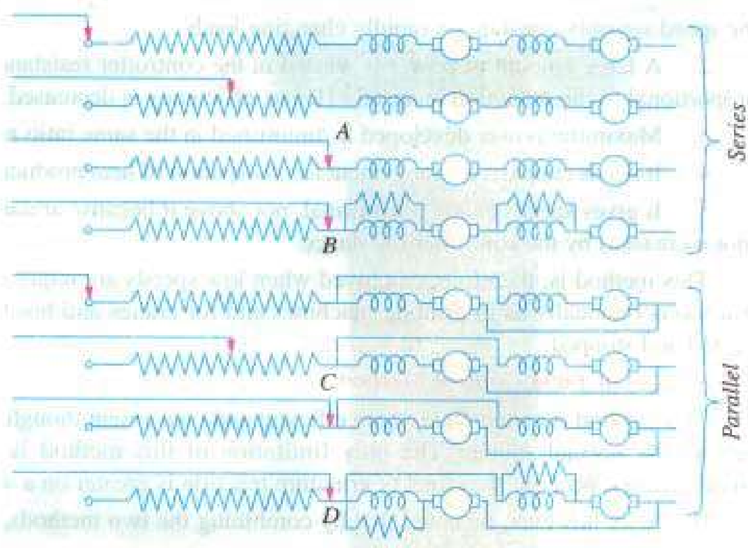


Fig. 30.20

Example 30.44. *Two series motors run at a speed of 500 r.p.m. and 550 r.p.m. respectively when taking 50 A at 500 V. The terminal resistance of each motor is 0.5 Ω. Calculate the speed of the combination when connected in series and coupled mechanically. The combination is taking 50 A on 500 V supply.* **(Electrical Machinery-I, Mysore Univ. 1985)**

Solution. First Motor

$$E_{b1} = 500 - (50 \times 0.5) = 475 \text{ V} ; I = 50 \text{ A}$$

Now, $\qquad N_1 \propto E_{b1}/\Phi_1 \quad \text{or} \quad E_{b1} \propto N_1 \Phi_1 \quad \text{or} \quad E_{b1} = k N_1 \Phi_1$

$$\therefore \qquad 475 = k \times 500 \times \Phi_1 \quad \therefore \quad k\Phi_1 = 475/500$$

Second Motor $\qquad E_{b2} = 500 - (50 \times 0.5) = 475 \text{ V. Similarly, } k\Phi_2 = 475/550$

When both motors are in series

$$E_b' = 500 - (50 \times 2 \times 0.5) = 450 \text{ V}$$

Now, $E_b' = E_{b1} + E_{b2} = k\,\Phi_1\,N + k\,\Phi_2\,N$

where N is the common speed when joined in series.

$$\therefore \quad 450 = \frac{475}{500}N + \frac{475}{550}N \quad \therefore \quad N = \textbf{248 r.p.m.}$$

Example 30.45. *Two similar 20 h.p. (14.92 kW), 250 V, 1000 r.p.m. series motors are connected in series with each other across a 250 V supply. The two motors drive the same shaft through a reduction gearing 5 : 1 and 4 : 1 respectively. If the total load torque on the shaft is 882 N-m, calculate (i) the current taken from the supply main (ii) the speed of the shaft and (iii) the voltage across each motor. Neglect all losses and assume the magnetic circuits to be unsaturated.*

(Elect. Machines, Punjab Univ., 1991)

Solution. (*i*) Rated current of each motor = 14,920/250 = 59.68 A

Back e.m.f. $E_b = 250$ V (neglecting $I_a R_a$ drop)

Now, $E_b \propto N\Phi$ As $\Phi \propto I$ $\therefore$ $E_b \propto NI$ or $E_b = kNI$

$$250 = k \times (1000/60) \times 59.68 \qquad \therefore \quad k = 0.25$$

Let N_{sh} be the speed of the shaft.

Speed of the first motor $N_1 = 5\,N_{sh}$; Speed of the second motor $N_2 = 4\,N_{sh}$

Let I be the new current drawn by the series set, then

$$E_b' = E_{b1} + E_{b2} = kI \times 5\,N_1 + kI \times N_2 = kI \times 5\,N_{sh} + kI \times 4\,N_{sh}$$

$$250 = 9 \times kI\,N_{sh} \qquad ...(i)$$

Now, $$\text{torque } T = 0.159\frac{E_b I}{N} = 0.159 \times \frac{kI\,N_{sh} \times I}{N_{sh}} = 0.159\,k\,I^2$$

Shaft torque due to gears of 1st motor = $5 \times 0.159\,kI^2$

Shaft torque due to gears of 2nd motor = $4 \times 0.159\,kI^2$

$$\therefore \quad 882 = kI^2\,(5 \times 0.159 + 4 \times 0.159) = 1.431\,kI^2$$

$$\therefore \quad I^2 = 882/1.431 \times 0.25 = 2{,}449 \text{ A} \quad \therefore \quad I = \textbf{49.5 A}$$

(*ii*) From equation (*i*), we get

$$250 = 9 \times 0.25 \times 49.5 \times N_{sh} \quad \therefore \quad N_{sh} = 2.233 \text{ r.p.s.} = \textbf{134 r.p.m.}$$

(*iii*) Voltage across the armature of Ist motor is

$$E'_{b1} = 5\,kI\,N_{sh} = 5 \times 0.25 \times 49.5 \times 2.233 = \textbf{139 V}$$

Voltage across the armature of 2nd motor

$$E_{b2} = 4\,kI\,N_{sh} = 4 \times 0.25 \times 49.5 \times 2.233 = \textbf{111 V}$$

Note that E_{b1} and E_{b2} are respectively equal to the applied voltage across each motor because $I_a R_a$ drops are negligible.

30.6. Electric Braking

A motor and its load may be brought to rest quickly by using either (*i*) Friction Braking or (*ii*) Electric Braking. The commonly-used mechanical brake has one drawback : it is difficult to achieve a smooth stop because it depends on the condition of the braking surface as well as on the skill of the operator.

Strong electric brake for airwheels

The excellent electric braking methods are available which eliminate the need of brake lining levers and other mechanical gadgets. Electric braking, both for shunt and series motors, is of the following three types: (*i*) rheostatic or dynamic braking (*ii*) plugging *i.e.*, reversal of torque so that armature tends to rotate in the opposite direction and (*iii*) regenerative braking.

Obviously, friction brake is necessary for holding the motor even after it has been brought to rest.

30.7. Electric Braking of Shunt Motors

(*a*) Rheostatic or Dynamic Braking

In this method, the armature of the shunt motor is disconnected from the supply and is connected across a variable resistance R as shown in Fig. 30.21 (*b*). The field winding is, however, left connected across the supply undisturbed. The braking effect is controlled by varying the series resistance R. Obviously, this method makes use of generator action in a motor to bring it to rest.* As seen from Fig. 30.21 (*b*), armature current is given by

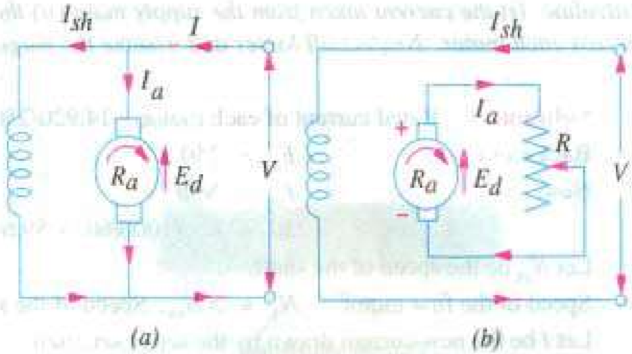


Fig. 30.21

$$I_a = \frac{E_b}{R+R_a} = \frac{\Phi ZN(P/A)}{R+R_a} = \frac{k_1\Phi N}{R+R_a}$$

Braking torque is given by

$$T_B = \frac{1}{2\pi}\Phi Z I_a\left(\frac{P}{A}\right) \text{N-m}$$

$$= \frac{1}{2\pi}\Phi Z\left(\frac{P}{A}\right).\frac{\Phi ZN(P/A)}{R+R_a} = \frac{1}{2\pi}\left(\frac{ZP}{A}\right)^2\frac{\Phi^2 N}{R+R_a} = k_2\Phi^2 N \quad \therefore\ T_B \propto N$$

Obviously, T_B decreases as motor slows down and disappear altogether when it comes to a stop.

(*b*) Plugging or Reverse Current Braking

This method is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

In this method, connections to the armature terminals are reversed so that motor tends to run in the opposite direction (Fig. 30.22). Due to the reversal of armature connections, applied voltage V and E_b start acting ***in the same direction*** around the circuit. In order to limit the armature current to a reasonable value, it is necessary to insert a resistor in the circuit while reversing armature connections.

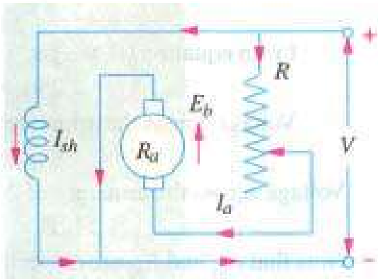


Fig. 30.22

$$I_a = \frac{V+E_b}{R+R_a} = \frac{V}{R+R_a} + \frac{E_b}{R+R_a}$$

$$= \frac{V}{R+R_a} + \frac{\Phi ZN(P/A)}{R+R_a} = \frac{V}{R+R_a} + \frac{k_1\Phi Z}{R+R_a}$$

$$T_B = \frac{1}{2\pi}.\Phi Z I_a\left(\frac{P}{A}\right) = \frac{1}{2\pi}.\left(\frac{\Phi ZP}{A}\right).I_a = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left[\frac{V}{R+R_a} + \frac{\Phi ZN(P/A)}{R+R_a}\right]$$

* The motor while acting as a generator feeds current to the resistor dissipating heat at the rate of I^2R. The current I_a produced by dynamic braking flow in the opposite direction, thereby producing a counter torque that slows down the machine.

$$= \frac{1}{2\pi}\left(\frac{ZP}{A}\right)\left(\frac{V}{R+R_a}\right).\Phi + \frac{1}{2\pi}.\left(\frac{ZP}{A}\right)^2.\frac{\Phi^2 N}{R+R_a} = k_2\Phi + k_3\Phi^2 N$$

or $$T_b = k_4 + k_5 N, \text{ where } k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R+R_a}\right)$$

and $$k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \times \frac{1}{(R+R_a)}.$$

Plugging gives greater braking torque than rheostatic braking. Obviously, during plugging, power is drawn from the supply and is dissipated by R in the form of heat. It may be noted that even when motor is reaching zero speed, there is some braking torque $T_B = k_4$ (see Ex. 30.47).

(c) Regenerative Braking

This method is used when the load on the motor has overhauling characteristic as in the lowering of the cage of a hoist or the downgrade motion of an electric train. Regeneration takes place when E_b becomes grater than V. This happens when the overhauling load acts as a prime mover and so drives the machines as a generator. Consequently, direction of I_a and hence of armature torque is reversed and speed falls until E_b becomes lower than V. It is obvious that during the slowing down of the motor, power is returned to the line which may be used for supplying another train on an upgrade, thereby relieving the powerhouse of part of its load (Fig. 30.23).

Regenerative braking demonstrations

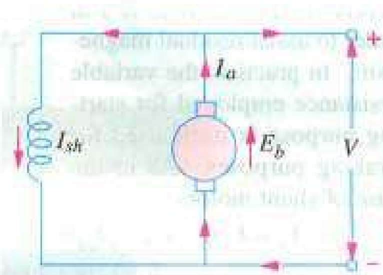


Fig. 30.23

For protective purposes, it is necessary to have some type of mechanical brake in order to hold the load in the event of a power failure.

Example 30.46. *A 220 V compensated shunt motor drives a 700 N-m torque load when running at 1200 r.p.m. The combined armature compensating winding and interpole resistance is 0.008 Ω and shunt field resistance is 55 Ω. The motor efficiency is 90%. Calculate the value of the dynamic braking resistor that will be capable of 375 N-m torque at 1050 r.p.m. The windage and friction losses may be assumed to remain constant at both speeds.*

Solution. Motor output $= \omega T = 2\pi NT = 2\pi\,(1200/60) \times 700 = 87{,}965$ W

Power drawn by the motor $= 87{,}965/0.9 = 97{,}740$ W

Current drawn by the motor $= 97{,}740/220 = 444$ A.

$$I_{sh} = 220/55 = 4 \text{ A} ;\; I_{a1} = 444 - 4 = 440 \text{ A}$$

$$E_{b1} = 220 - 440 \times 0.008 = 216.5 \text{ V}$$

Since field flux remains constant, T_1 is proportional to I_{a1} and T_2 to I_{a2}.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad I_{a2} = 440 \times \frac{375}{100} = 2650 \text{ A}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{1050}{1200} = \frac{E_{b2}}{216.5} ;\; E_{b2} = 189.4 \text{ V}$$

With reference to Fig. 30.23, we have

$$189.4 = 2650\,(0.008 + R) ;\; R = 0.794\ \Omega$$

30.8. Electric Braking of Series Motor

The above-discussed three methods as applied to series motors are as follows :

(a) Rheostatic (or Dynamic) Braking

The motor is disconnected from the supply, the field connections *are reversed* and the motor is connected in series with a variable resistance R as shown in Fig. 30.24. Obviously, now, the machine is running as a generator. The field connections are reversed to make sure that current through field winding flows in the *same* direction as before (*i.e.*, from M to N) in order to assist residual magnetism. In practice, the variable resistance employed for starting purpose is itself used for braking purposes. As in the case of shunt motors,

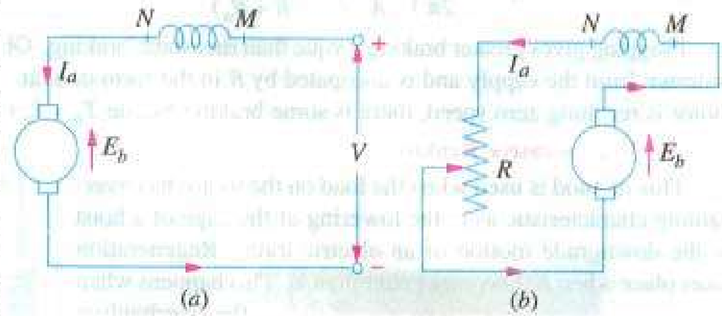


Fig. 30.24

$$T_B = k_2\,\Phi^2\,N = k_3\,I_{a2}\,N \qquad \text{... prior to saturation}$$

(b) Plugging or Reverse Current Braking

As in the case of shunt motors, in this case also the connections of the armature are reversed and a variable resistance R is put in series with the armature as shown in Fig. 30.25. As found in Art. 30.7 (*b*),

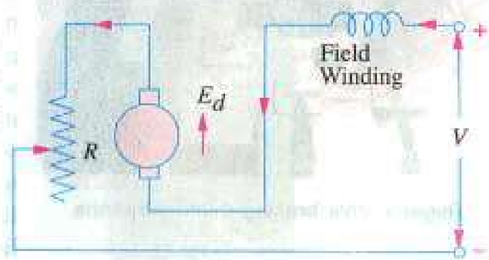


Fig. 30.25

$$T_B = k_2\,\Phi + k_3\Phi^2 N$$

(c) Regenerative Braking

This type of braking of a sereis motor is not possible without modification because reversal of I_a would also mean reversal of the field and hence of E_b. However, this method is sometimes used with traction motors, special arrangements being necessary for the purpose.

Example 30.47. *A 400 V, 25 h.p. (18.65 kW), 45 r.p.m., d.c. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full-load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2 Ω.* **(Electrical Technology, M.S. Univ. Baroda, 1988)**

Solution.

F.L. Motor input current $\quad I = 18{,}650/0.746 \times 400 = 62.5$ A

$I_a = 62.5$ A (neglecting I_{sh}); $\quad E_b = 400 - 62.5 \times 0.2 = 387.5$ V

Total voltage around the circuit is $= 400 + 387.5 = 787.5$ V

Max. braking current $= 2 \times 62.5 = 125$ A

Total resistance required in the circuit $= 787.5/125 = 6.3\ \Omega$

Braking resistance $\quad R = 6.3 - 0.2 =$ **6.1 Ω**

Maximum braking torque will be produced initially when the motor speed is maximum *i.e.*, 450 r.p.m. or 7.5 r.p.s.

$$\text{Maximum value of } T_B = k_4 + k_5\,N \qquad \text{– Art. 25.7}(b)$$

Now, $$k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R+R_a}\right) \text{ and } k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \cdot \frac{1}{(R+R_a)}$$

Now, $E_b = \Phi ZN\ (P/A)$; also $N = 450/60 = 7.5$ r.p.s.

$\therefore$ $387.5 = 7.5\ (\Phi ZP/A)$ or $(\Phi ZP/A) = 51.66$

$\therefore$ $$k_4 = \frac{1}{2\pi} \times 51.66 \times \frac{400}{6.3} = 522 \text{ and } k_5 = \frac{1}{2\pi} \times (51.66)^2 \times \frac{1}{6.3} = 67.4$$

$\therefore$ Maximum $T_B = 522 + 67.4 \times 7.5 =$ **1028 N-m**

When speed is also zero *i.e.*, $N = 0$, the value of torque is $T_B = K_4 =$ **522 N-m.**

30.9. Electronic Speed Control Method for DC Motors

Of late, solid-state circuits using semiconductor diodes and thyristors have become very popular for controlling the speed of a.c. and d.c. motors and are progressively replacing the traditional electric power control circuits based on thyratrons, ignitrons, mercury arc rectifiers, magnetic amplifiers and motor-generator sets etc. As compared to the electric and electromechanical systems of speed control, the electronic methods have higher accuracy, greater reliability, quick response and also higher efficiency as there are no I^2R losses and moving parts. Moreover, full 4-quadrant speed control is possible to meet precise high-speed standards.

All electronic circuits control the motor speed by adjusting either (*i*) the voltage applied to the motor armature or (*ii*) the field current or (*iii*) both.

DC motors can be run from d.c. supply if available or from a.c. supply after it has been converted into d.c. supply with the help of rectifiers which can be either half-wave or full-wave and either controlled (by varying the conduction angle of the thyristors used) or uncontrolled.

AC motors can be run on the a.c. supply or from d.c. supply after it has been converted into a.c. supply with the help of inverters (opposite of rectifiers).

As stated above, the average output voltage of a thyristor-controlled rectifier can be changed by changing its conduction angle and hence the armature voltage of the d.c. motor can be adjusted to control its speed.

When run on a d.c. supply, the armature d.c. voltage can be changed with the help of a thyristor chopper circuit which can be made to interrupt d.c. supply at different rates to give different average values of the d.c. voltage. If d.c. supply is not available, it can be obtained from the available a.c. supply with the help of uncontrolled rectifiers (using only diodes and not thyristors). The d.c. voltages so obtained can be then chopped with the help of a thyristor chopper circuit.

A brief description of rectifiers, inverters * and d.c. choppers would now be given before discussing the motor speed control circuits.

30.10. Uncontrolled Rectifiers

As stated earlier, rectifiers are used for a.c. to d.c. conversion *i.e.*, when the supply is alternating but the motor to be controlled is a d.c. machine.

Fig. 30.26 (*a*) shows a half-wave uncontrolled rectifier. The diode D conducts only during positive half-cycles of the single-phase a.c. input *i.e.*, when its anode A is positive with respect to its cathode K. As shown, the average voltage available across the load (or motor) is 0.45 V where V is the r.m.s. value of the a.c. voltage (in fact, $V = V_m/\sqrt{2}$). As seen it is a pulsating d.c. voltage.

In Fig. 30.26 (*b*) a single-phase, full-wave bridge rectifier which uses four semiconductor diodes and provides double the voltage *i.e.*, 0.9 V is shown. During positive input half-cycles when end A is positive with respect to end B, diodes D_1 and D_4 conduct (*i.e.* opposite diodes) whereas during negative input half-cycles, D_2 and D_3 conduct. Hence, current flows through the load during both half-cycles in the *same* direction. As seen, the d.c. voltage supplied by a bridge rectifier is much less pulsating than the one supplied by the half-wave rectifier.

* Rectifiers convert a.c. power into d.c power, whereas inverters convert d.c. power into a.c. power. However, converter is a general term embracing both rectifiers and inverters.

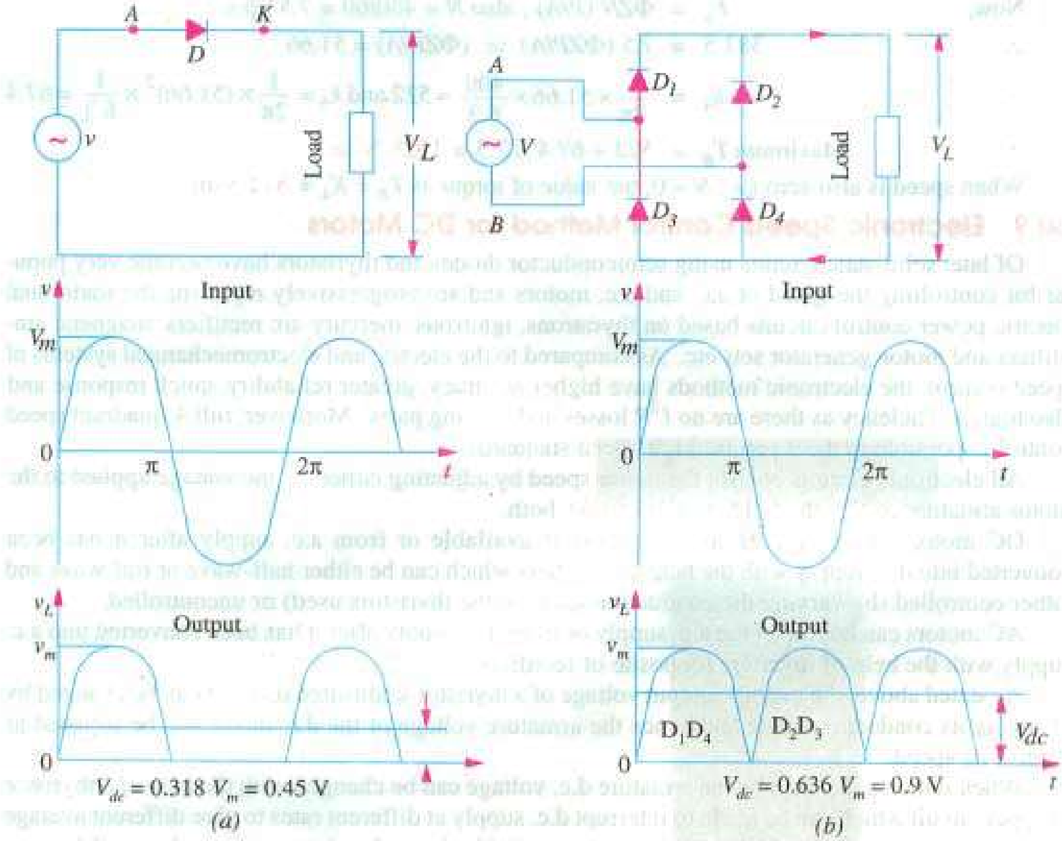


Fig. 30.26

30.11. Controlled Rectifiers

In these rectifiers, output load current (or voltage) can be varied by controlling the point in the input a.c. cycle at which the thyristor is turned ON with the application of a suitable low-power gate pulse. Once triggered (or fired) into conduction, the thyristor remains in the conducting state for the rest of the half-cycle *i.e.*, upto 180°. The firing angle α can be adjusted with the help of a control circuit. When conducting, it offers no resistance *i.e.*, it acts like a short-circuit.

Fig. 30.27 (*a*) shows an elementary half-wave rectifier in which thyristor triggering is delayed by angle α with the help of a phase-control circuit. As shown, the thyristor starts conducting at point *A* and not at point *O* because its gate pulse is applied after a delay of α. Obviously, the conduction angle is reduced from 180° to (180° – α) with a consequent decrease in output voltage whose value is given by

$$V_L = \frac{V_m}{2\pi}(1+\cos\alpha) = 0.16\ V_m(1+\cos\alpha) = 0.32\ V_m \cos^2\frac{\alpha}{2}$$

where V_m is the peak value of a.c. input voltage. Obviously, V_L is maximum when $\alpha = 0$ and is zero when $\alpha = 180°$.

Fig. 30.27 (*b*) shows the arrangement where a thyristor is used to control current through a load connected in series with the a.c. supply line.

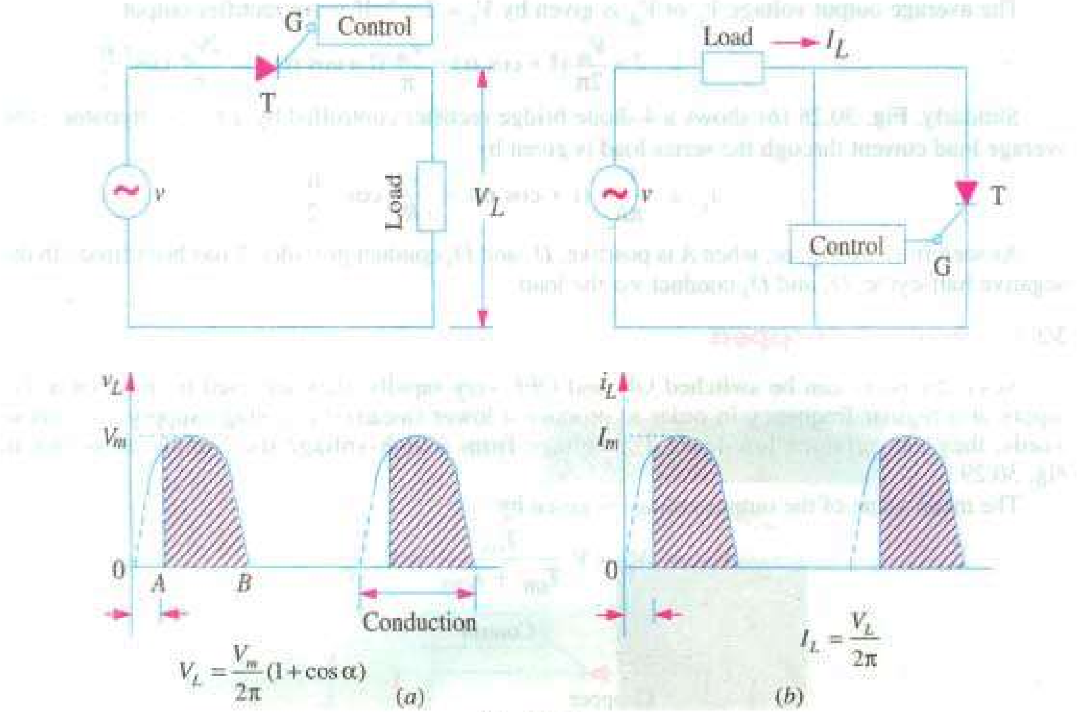


Fig. 30.27

The load current is given by

$$I_L = \frac{V_L}{R_L} = \frac{V_m}{2\pi R_L}(1+\cos\alpha) = \frac{V_m}{\pi R_L}\cos^2\frac{\alpha}{2}$$

Fig. 30.28 (*a*) shows a single-phase, full-wave half-controlled rectifier. It is called half-controlled because it uses two thyristors and two diodes instead of four thyristors. During positive input half-cycle when *A* is positive, conduction takes place via T_1, load and D_1. During the negative half-cycle when *B* becomes positive, conduction route is via T_2, load and D_2.

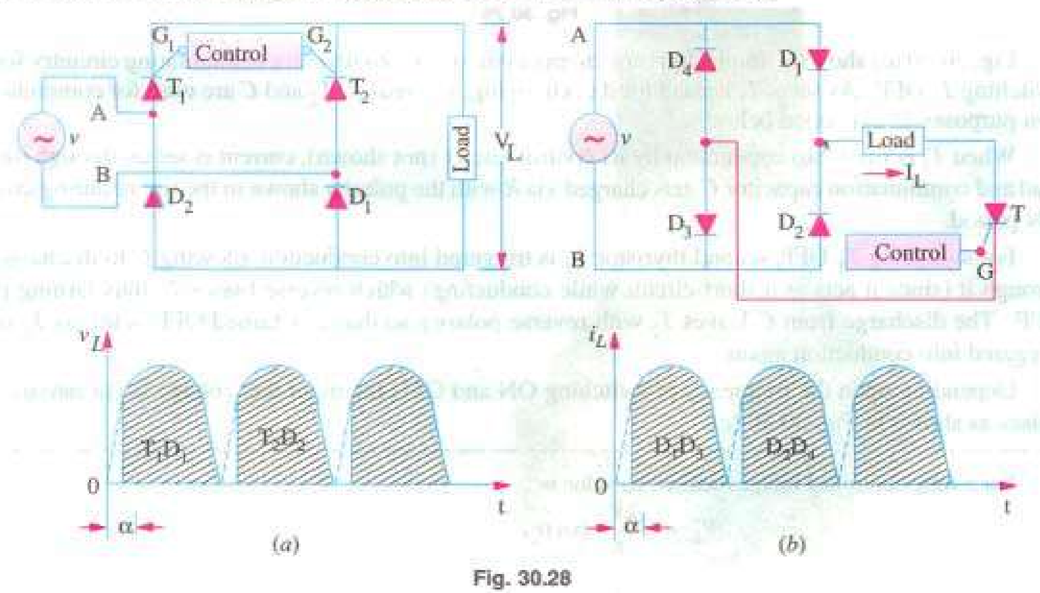


Fig. 30.28

The average output voltage V_L or V_{dc} is given by $V_L = 2 \times$ half-wave rectifier output

$$\therefore \quad V_L = 2 \times \frac{V_m}{2\pi}(1 + \cos\alpha) = \frac{V_m}{\pi}(1 + \cos\alpha)^* = \frac{2V_m}{\pi}\cos^2\frac{\alpha}{2}$$

Similarly, Fig. 30.28 (*b*) shows a 4-diode bridge rectifier controlled by a single thyristor. The average load current through the series load is given by

$$I_L = \frac{V_m}{\pi R_L}(1 + \cos\alpha) = \frac{2V_m}{\pi R_L}\cos^2\frac{\alpha}{2}$$

As seen from the figure, when A is positive, D_1 and D_3 conduct provided T has been fired. In the negative half-cycle, D_2 and D_4 conduct via the load.

30.12. Thyristor Choppers

Since thyristors can be switched ON and OFF very rapidly, they are used to interrupt a d.c. supply at a regular frequency in order to produce a lower (mean) d.c. voltage supply. In simple words, they can produce low-level d.c. voltage from a high-voltage d.c. supply as shown in Fig. 30.29.

The mean value of the output voltage is given by

$$V_{dc} = V_L = V\frac{T_{ON}}{T_{ON} + T_{OFF}} = V\frac{T_{ON}}{T}$$

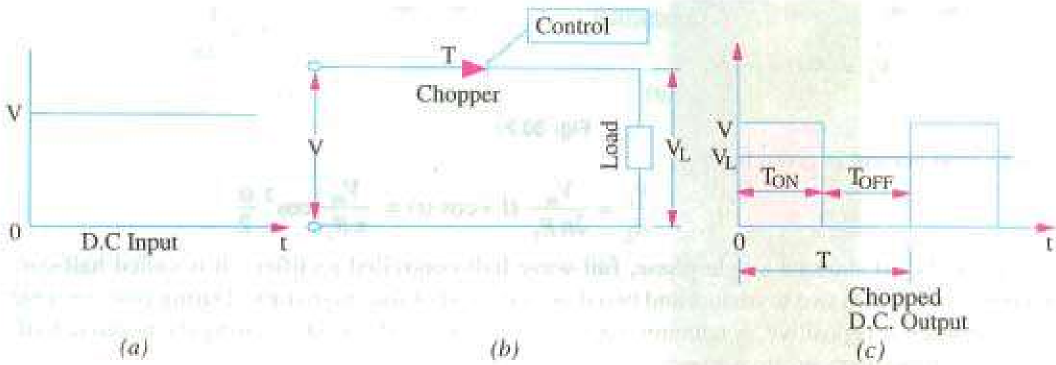


Fig. 30.29

Fig. 30.30 (*a*) shows a simple thyristor chopper circuit alongwith extra commutating circuitry for switching T_1 OFF. As seen, T_1 is used for d.c. chopping, whereas R, T_2 and C are used for commutation purposes as explained below.

When T_1 is fired into conduction by its control circuit (not shown), current is set up through the load and commutation capacitor C gets charged via R with the polarity shown in the figure during this ON period.

For switching T_1 OFF, second thyristor T_2 is triggered into conduction allowing C to discharge through it (since it acts as a short-circuit while conducting) which reverse-biases T_1 thus turning it OFF. The discharge from C leaves T_2 with reverse polarity so that it is turned OFF, whereas T_1 is triggered into conduction again.

Depending upon the frequency of switching ON and OFF, the input d.c. voltage is cut into d.c. pulses as shown in Fig. 30.30 (*c*).

* For a fully-controlled bridge rectifier, its value is

$$V_L = \frac{2V_m}{\pi}\cos\alpha.$$

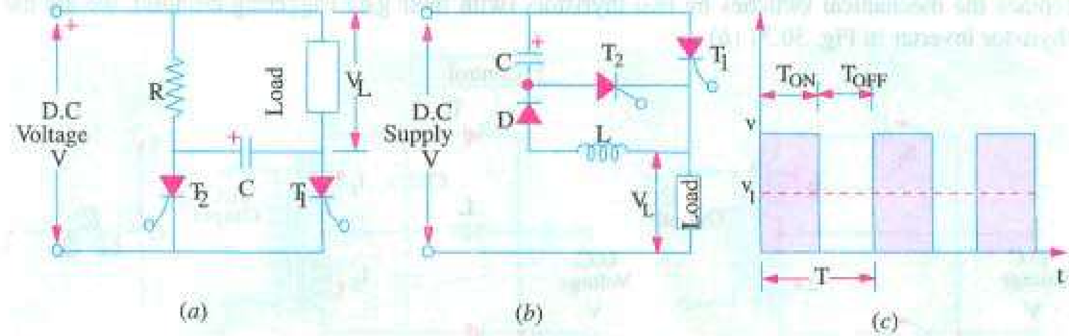


Fig. 30.30

In Fig. 30.30 (*b*), T_1 is the chopping thyristor, whereas *C, D*, T_2 and *L* constitute the commutation circuitry for switching T_1 OFF and ON at regular intervals.

When T_2 is fired, *C* becomes charged via the load with the polarity as shown. Next, when T_1 is fired, *C* reverse-biases T_2 to OFF by discharging via T_1, *L* and *D* and then recharges with reverse polarity. T_2 is again fired and the charge on *C* reverse-biases T_1 to non-conducting state.

It is seen that output (or load) voltage is present only when T_1 is ON and is absent during the interval it is OFF. The mean value of output d.c. voltage depends on the relative values of T_{ON} and T_{OFF}. In fact, output d.c. voltage is given by

$$V_{dc} = V_L \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

Obviously, by varying thyristor ON/OFF ratio, V_L can be made any percentage of the input d.c. voltage *V*.

Example 30.48. *The speed of a separately excited d.c. motor is controlled by a chopper. The supply voltage is 120 V, armature circuit resistance = 0.5 ohm, armature circuit inductance = 20 mH and motor constant = 0.05 V/r.p.m. The motor drives a constant load torque requiring an average current of 20 A. Assume motor current is continuous. Calculate* **(a)** *the range of speed control* **(b)** *the range of duty cycle.* **(Power Electronics-I, Punjab Univ. Nov. 1990)**

Solution. The minimum speed is zero when $E_b = 0$

$$V_t = E_b + I_a R_a = I_a \times R_a = 200 \times 0.5 = 10 \text{ V}$$

Now, $$V_t = \frac{T_{ON}}{T} V = \alpha V, \therefore 10 = 120, \alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ when $V_t = V = 120$ V

$\therefore$ $$E_b = 120 - 20 \times 0.5 = 110 \text{ V}$$

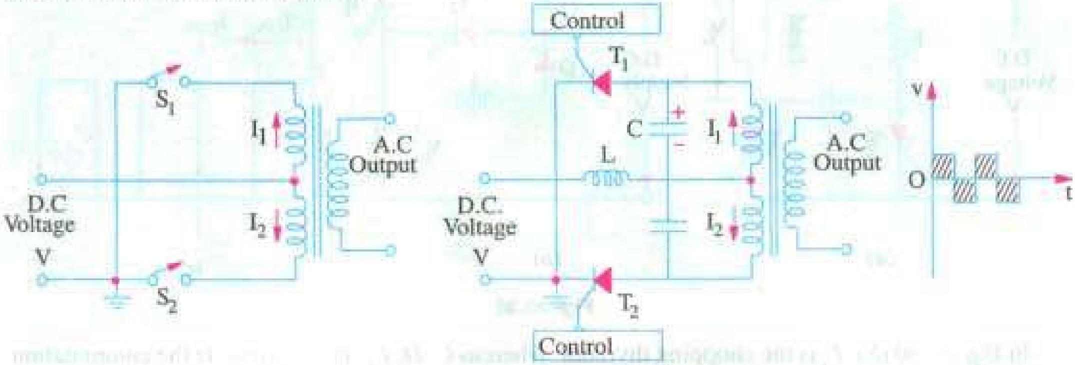
Now, $$N = E_b/K_a \Phi = 110/0.05 = 2200 \text{ r.p.m.}$$

(*a*) Hence, speed range is from 0 to 2200 r.p.m.

(*b*) Range of duty cycle is from $\frac{1}{12}$ to 1.

30.13. Thyristor Inverters

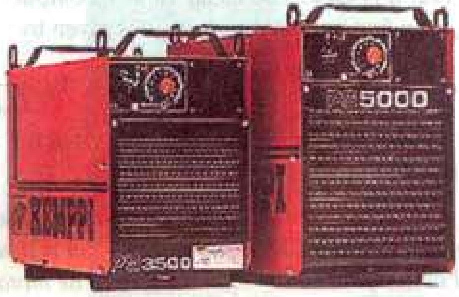
Such inverters provide a very efficient and economical way of converting direct current (or voltage) into alternating current (or voltage). In this application, a thyristor serves as a controlled switch alternately opening and closing a d.c. circuit. Fig. 30.31 (*a*), shows a basic inverter circuit where an a.c. output is obtained by alternately opening and closing switches S_1 and S_2. When we replace the mechanical switches by two thyristors (with their gate triggering circuits), we get the thyristor inverter in Fig. 30.31 (*b*).



(*a*) (*b*) (*c*)

Fig. 30.31

Before discussing the actual circuit, it is worthwhile to recall that thyristor is a latching device which means that once it starts conducting, gate loses control over it and cannot switch it OFF whatever the gate signal. A separate ***commutating*** circuitry is used to switch the thyristor OFF and thus enable it to perform ON-OFF switching function.



Second generation thyristor inverter

Suppose T_1 is fired while T_2 is still OFF. Immediately I_1 is set up which flows through *L*, one half of transformer primary and T_1. At the same time, *C* is charged with the polarity as shown.

Next when T_2 is fired into condition, I_2 is set up and *C* starts discharging through T_1 thereby reverse-biasing it to CUT-OFF.

When T_1 is again pulsed into condition, I_1 is set up and *C* starts discharging thereby reverse-biasing T_2 to *OFF* and the process just described repeats. As shown in Fig. 30.31 (*c*), the output is an alternating voltage whose frequency depends on the switching frequency to thyristors T_1 and T_2.

30.14. Thyristor Speed Control of Separately-excited D.C. Motor

In Fig. 30.32, the bridge rectifier converts a voltage into d.c. voltage which is then applied to the armature of the separately-excited d.c. motor *M*.

As we know, speed of a motor is given by

$$N = \frac{V - I_a R_a}{\Phi}\left(\frac{A}{ZP}\right)$$

If Φ is kept constant and also if $I_a R_a$ is neglected, then, $N \propto V \propto$ voltage across the armature. The value of this voltage furnished by the rectifier can be changed by varying the firing angle α of the thyristor *T* with the help of its contol circuit. As α is increased *i.e.*, thyristor firing is delayed

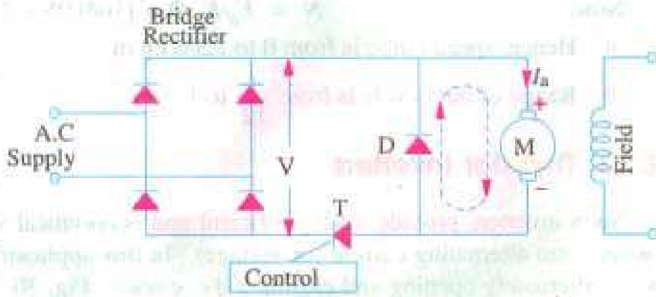


Fig. 30.32

more, its conduction period is reduced and, hence, armature voltage is decreased which, in turn, decreases the motor speed. When α is decreased *i.e.*, thyristor is fired earlier, conduction period is increased which increases the mean value of the voltage applied across the motor armature. Consequently, motor speed is increased. In short, as α increases, V decreases and hence N decreases. Conversely, as α decreases, V increases and so, N increases. The free-wheeling diode D connected across the motor provides a circulating current path (shown dotted) for the energy stored in the inductance of the armature winding at the time T turns OFF. Without D, current will flow through T and bridge rectifier, prohibiting T from turning OFF.

30.15. Thyristor Speed Control of a D.C. Series Motor

In the speed control circuit of Fig. 30.33, an RC network is used to control the diac voltage that triggers the gate of a thyristor. As the a.c. supply is switched ON, thyristor T remains OFF but the capacitor C is charged through motor armature and R towards the peak value of the applied a.c. voltage. The time it takes for the capacitor voltage V_C to reach the breakover voltage of the diac* depends on the setting of the variable resistor T. When V_C becomes equal to the breakover voltage of diac, it conducts and a triggering pulse is applied to the thyristor gate G. Hence, T is turned ON and allows current to pass through the motor. Increasing R delays the rise of V_C and hence the breakover of diac so that thyristor is fired later in each positive half cycle of the a.c. supply. It reduces the conduction angle of the thyristor which, consequently, delivers less power to the motor. Hence, motor speed is reduced.

If R is reduced, time-constant of the RC network is decreased which allows V_C to rise to the breakover voltage of diac more quickly. Hence, it makes the thyristor fire early in each positive input half-cycle of the supply. Due to increase in the conduction angle of the thyristor, power delivered to the motor is increased with a subsequent increase in its speed. As before D is the free-wheeling diode which provides circulating current path for the energy stored in the inductance of the armature winding.

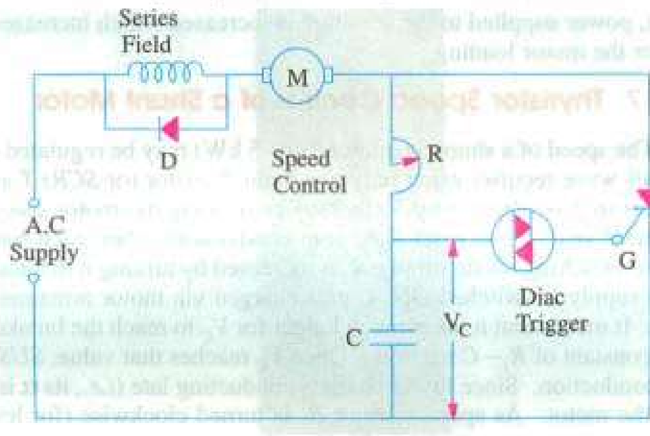


Fig. 30.33

30.16. Full-wave Speed Control of a Shunt Motor

Fig. 30.34 shows a circuit which provides a wide range of speed control for a fractional kW shunt d.c. motor. The circuit uses a bridge circuit for full-wave rectification of the a.c. supply. The shunt field winding is permanently connected across the d.c. output of the bridge circuit. The armature voltage is supplied through thyristor T. The magnitude of this

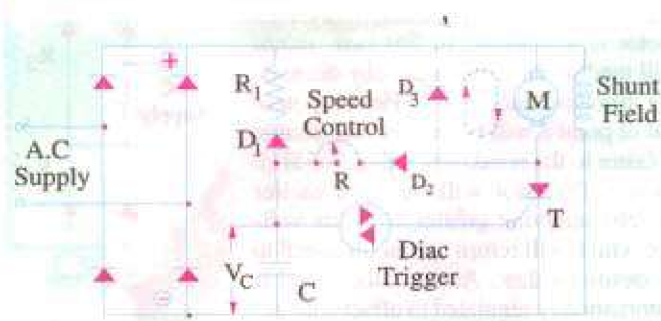


Fig. 30.34

* Like a triac, it has directional switching characteristics.

voltage (and hence, the motor speed) can be changed by turning T_{ON} at different points in each half-cycle with the help of R. The thyristor turns OFF only at the end of each half-cycle. Free-wheeling diode D_3 provides a circulating current path (shows dotted) for the energy stored in the armature winding at the time T turns OFF. Without D_3. This current would circulate through T and the bridge rectifier thereby prohibiting T from turning OFF.

At the beginning of each half-cycle, T is the OFF state and C starts charging up via motor armature, diode D_2 and speed-control variable resistor R (it cannot charge through R_1 because of reverse-biased diode D_1). When voltage across C *i.e.*, V_C builds up to the breakover voltage of diac, diac conducts and applies a sudden pulse to T thereby turning it ON. Hence, power is supplied to the motor armature for the remainder of that half-cycle. At the end of each half-cycle, C is discharged through D_1, R_1 and shunt field winding. The delay angle α depends on the time it takes V_C to become equal to the breakover voltage of the diac. This time, in turn, depends on the time-constant of the R-C circuit and the voltage available at point A. By changing R, V_C can be made to build-up either slowly or quickly and thus change the angle α at will. In this way, the average value of the d.c. voltage across the motor armature can be controlled. It further helps to control the motor speed because it is directly proportional to the armature voltage.

Now, when load is increased, motor tends to slow down. Hence, E_b is reduced. The voltage of point A is increased because it is equal to the d.c. output voltage of the bridge rectifier ***minus*** back e.m.f. E_b. Since V_A increases *i.e.*, voltage across the R-C charging circuit increases, it builds up V_C more quickly thereby decreasing which leads to early switching ON of T in each half-cycle. As a result, power supplied to the armature is increased which increases motor speed thereby compensating for the motor loading.

30.17. Thyristor Speed Control of a Shunt Motor

The speed of a shunt d.c.motor (upto 5 kW) may be regulated over a wide range with the help of the full-wave rectifier using only one main thyristor (or *SCR*) T as shown in Fig. 30.35. The firing angle α of T is adjusted by R_1 thereby controlling the motor speed. The thyristor and *SUS* (silicon unilateral switch) are reset (*i.e.*, stop conduction) when each half-wave of voltage drops to zero. Before switching on the supply, R_1 is increased by turning it in the counter-clockwise direction. Next, when supply is switched ON, C gets charged via motor armature and diode D_1 (being forward biased). It means that it takes much longer for V_C to reach the breakdown voltage of *SUS** due to large time constant of R_1– C network. Once V_C reaches that value, *SUS* conducts suddenly and triggers T into conduction. Since thyristor starts conducting late (*i.e.*, its α is large), it furnishes low voltage to start the motor. As speed selector R_1 is turned clockwise (for less resistance), C charges up more rapidly (since time constant is decreased) to the breakover voltage of *SUS* thereby firing T into conduction earlier. Hence, average value of the d.c. voltage across the motor armature increases thereby increasing its speed.

While the motor is running at the speed set by R_1, suppose that load on the motor is increased. In that case, motor will tend to slow down thereby decreasing armature back e.m.f. Hence, potential of point 3 will rise which will charge C faster to the breakover voltage of *SUS*. Hence, thyristor will be fired earlier thereby applying greater armature voltage which will return the motor speed to its desired value. As seen, the speed is automatically regulated to offset changes in load.

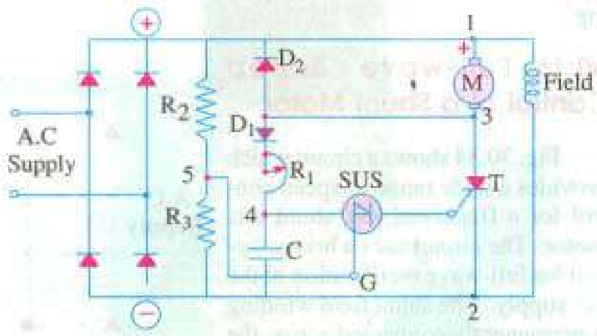


Fig. 30.35

The function of free-wheeling diode D_2 is to allow dissipation of energy stored in motor

* It is a four-layer semiconductor diode with a gate terminal. Unlike diac, it conducts in one direction only.

armature during the time the full-wave rectified voltage drops to zero between half-cycles. If D_2 is not there, then decreasing armature current during those intervals would be forced to flow through T thereby preventing its being reset. In that case, T would not be ready to be fired in the next half-cycle.

Similarly, towards the end of each half cycle as points 1 and 5 decrease towards zero potential, the negative going gate G turns SUS on thereby allowing C to discharge completely through SUS and thyristor gate-cathode circuit so that it can get ready to be charged again in the next half-cycle.

30.18. Thyristor Speed Control of a Series D.C. Motor

Fig. 30.36 shows a simple circuit for regulating the speed of a d.c. motor by changing the average value of the voltage applied across the motor armature by changing the thyristor firing angle α. The trigger circuit $R_1 - R_2$ can give a firing range of almost 180°. As the supply is switched on, full d.c. voltage is applied across $R_1 - R_2$. By changing the variable resistance R_2, drop across it can be made large enough to fire the SCR at any desired angle from 0° – 180°. In this way, output voltage of the bridge rectifier can be changed considerably, thus enabling a wide-range control of the motor speed. The speed control can be made somewhat smoother by joining a capacitor C across R_2 as shown in the figure.

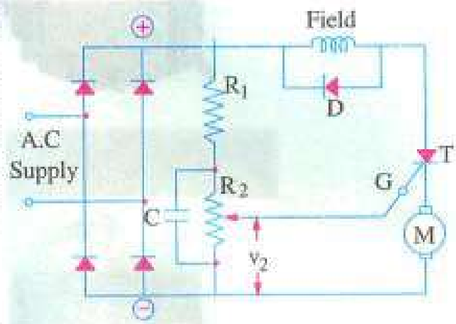


Fig. 30.36

30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = (V - E_b)/R_a$$

where V is the supply voltage, E_b the back e.m.f. and R_a the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resitance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of 0.25 Ω and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of 440/0.25 = 1760 A which is 1760/50 = 35.2 times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

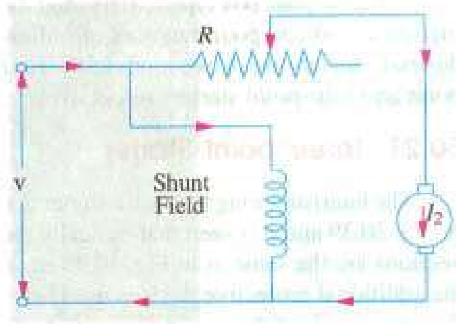


Fig. 30.37

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons :

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance R used for starting a shunt motor is shown. It will be seen that the starting resistance R is in series with the ***armature*** and not with the ***motor*** as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the resistance R. If R wes introduced in the motor circuit, then I_{sh} will be small at the start, hence starting torque T_a would be small ($\because\ T_a \propto \Phi I_a$) and there would be experienced some difficulty in starting the motor. Such a simple starter is shown diagramatically in Fig. 30.38.

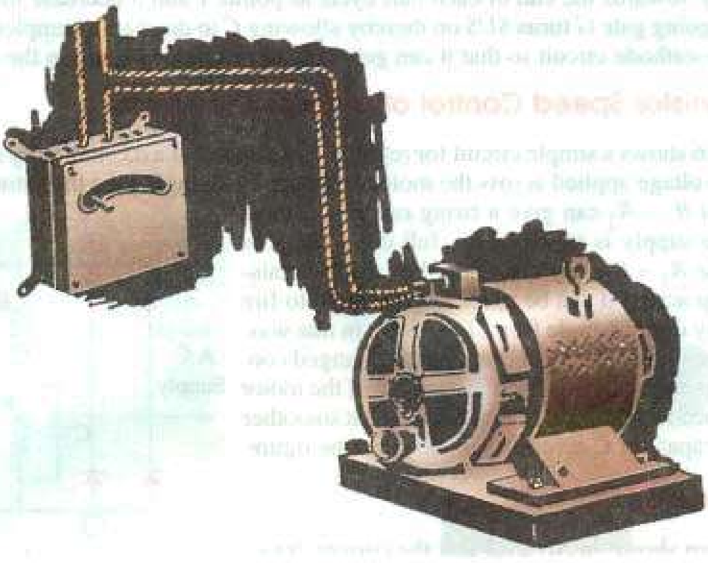


Fig. 30.38

30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked A, B and C. One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point A which is further connected to the starting arm L, through the overcurrent (or overload) release M.

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance R_s is placed in series with the armature. The starting current drawn by the armature $= V/(R_a + R_s)$ where R_s is the starting

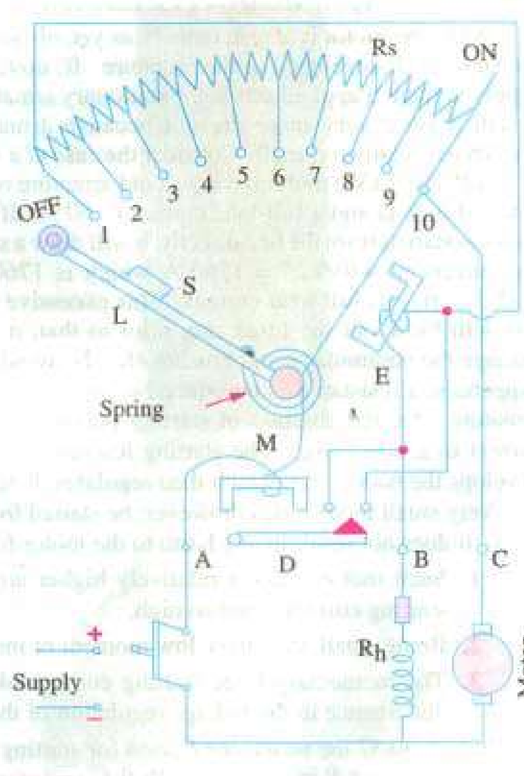


Fig. 30.39

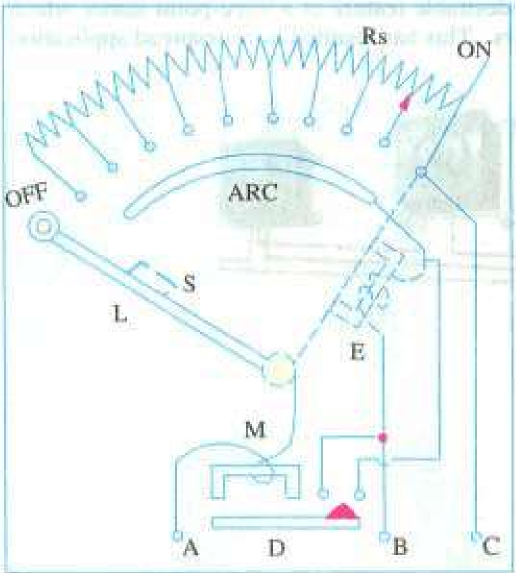


Fig. 30.40

resistance. As the arm is further moved, the starting resistance is gradually cut out till, when the arm reaches the running position, the resistance is all cut out. The arm moves over the various studs against a strong spring which tends to restore it to OFF position. There is a soft iron piece *S* attached to the arm which in the full 'ON' or running position is attracted and held by an electromagnet *E* energised by the shunt current. It is variously known as 'HOLD-ON' coil, LOW VOLTAGE (or NO-VOLTAGE) release.

It will be seen that as the arm is moved from stud NO. 1 to the last stud, the field current has to travel back through that portion of the starting resistance that has been cut out of the armature circuit. This results is slight decrease of shunt current. But as the value of starting resistance is very small as compared to shunt field resistance, this slight decreases in I_{sh} is negligible. This defect can, however, be remedied by using a brass arc which is connected to stud No. 1 (Fig. 30.40). The field circuit is completed through the starting resistance as it did in Fig. 30.39.

Now, we will discuss the action of the two protective devices shown in Fig. 30.39. The normal function of the HOLD-ON coil is to hold on the arm in the full running position when the motor is in normal operation. But, in the case of failure or disconnection of the supply or a break in the field circuit, it is de-energised, thereby releasing the arm which is pulled back by the spring to the OFF position. This prevents the stationary armature from being put across the lines again when the supply is restored after temporary shunt down. This would have happened if the arm were left in the full ON position. One great advantage of connecting the HOLD-ON coil in series with the shunt field is that, should the field circuit become open, the starting arm immediately springs back to the OFF position thereby preventing the motor from running away.

The overcurrent release consists of an electromagnet connected in the supply line. If the motor becomes overloaded beyond a certain predetermined value, then *D* is lifted and short circuits the electromagnet. Hence, the arm is released and returns to OFF position.

The form of overload protection described above is becoming obsolete, because it cannot be made either as accurate or as reliable as a separate well-designed circuit breaker with a suitable time element attachment. Many a times a separated magnetic contactor with an overload relay is also used.

Often the motors are protected by thermal overload relays in which a bimetallic strip is heated by the motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contactor, thereby isolating the motor from the supply.

If it is desired to control the speed of the motor in addition, then a field rheostat is connected in the filed circuit as indicated in Fig.30.39.The motor speed can be increased by weakening the flux ($N \propto I/\Phi$). Obviously, there is a limit to the speed increase obtained in this way, although speed ranges of three to four are possible. The connections of a starter and speed regulator with the motor are shown diagrammatically in Fig. 30.41. But there is one difficulty with such an arrangement for speed control. If too much resistance is 'cut in' by the field rheostat, then field current is reduced very much so that it is unable to create enough electromagnetic pull to overcome the spring tension. Hence, the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-speed motors. This has resulted in widespread application of four-point starter discussed below.

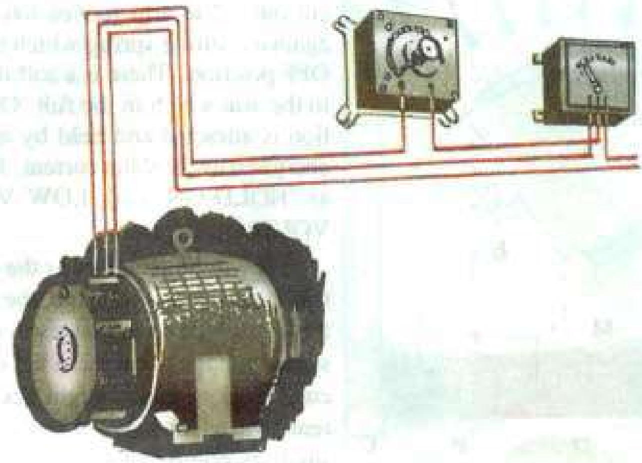


Fig. 30.41

30.22. Four-point Starter

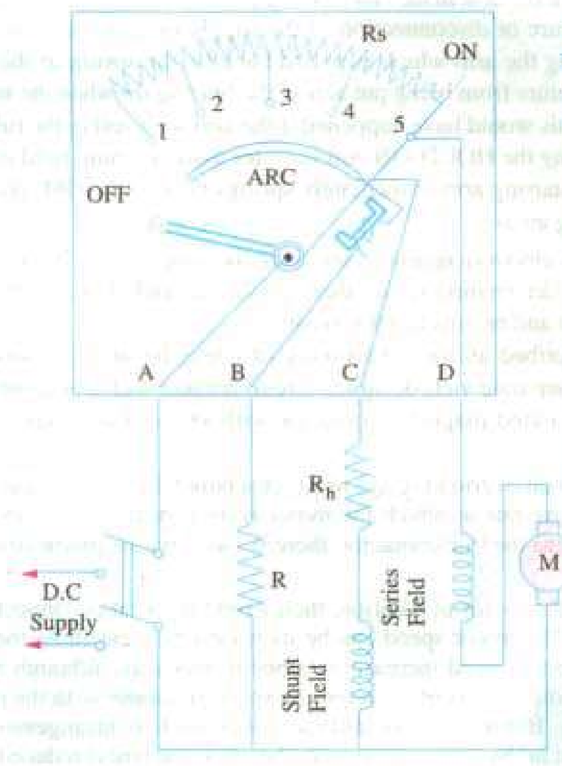


Fig. 30.42

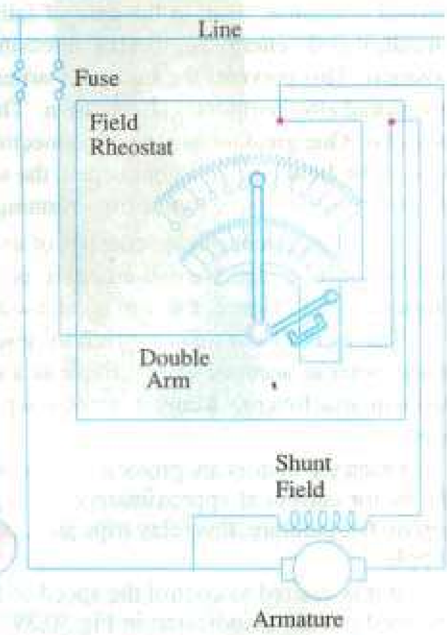


Fig. 30.43

Such a starter with its internal wiring is shown, connected to a long-shunt compound motor in Fig. 30.42. When compared to the three-point starter, it will be noticed that one important change has been made *i.e.*, the HOLD-ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a protecting resistance as shown. When the arm touches stud No. 1, then the line current divides into three parts (*i*) one part passes through starting resistance R_s, series field and motor armature (*ii*) the second part passes through the shunt field and its field rheostat R_h and (*iii*) the third part passes through the HOLD-ON coil and current-protecting resistance R. It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not at all affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means that the electromagnetic pull exerted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting arm to OFF position no matter how the field rheostat or regulator is adjusted.

30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor, whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.

Speed-control Rheostats

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

30.24. Starting and Speed Control of Series Motor

For starting and speed control of series motor either a face-plate type or drum-type controller is used which usually has the reversing feature also. A face-plate type of reversing controller is shown in Fig. 30.44.

Except for a separate overload circuit, no inter-locking or automatic features are required because the operator watches the performance continuously.

As shown, the regulating lever consists of three pieces separated by strips of insulation. The outside parts form the electrical connections and the middle one is insulated from them. By moving the regulating lever, resistance can be cut in and out of the motor circuit. Reversing is obtained by moving the lever in the opposite direction as shown, because in that case, connections to the armature are reversed. Such an arrangement is employed where series motors are used as in the case of cranes, hoists and streetcars etc.

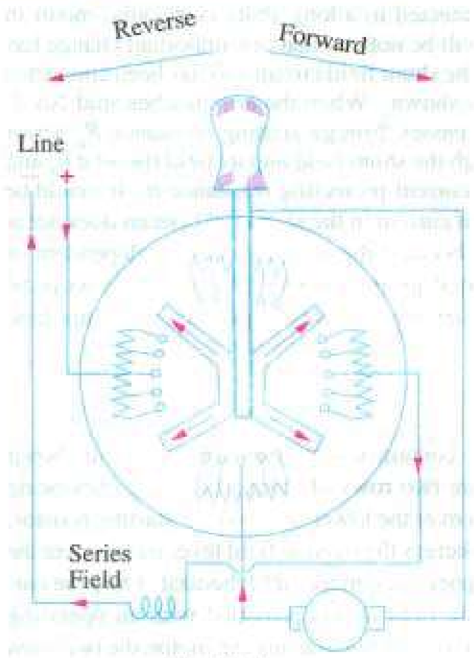


Fig. 30.44

However, for adjustable speed service in connection with the operation of machine tools, a drum controller is preferred. It is called 'controller' because in addition to accelerating the motor to its normal speed, it provides the means for reversing the direction of the motor. Other desirable features such as safety protection against an open field or the temporary failure of power supply and overloads are frequently provided in this type of controller.

The controller consists of armature resistance grids of cross-section sufficient to carry the full-load operating current continuously and are used for adjusting the motor speed to values lower than the base speed obtained with no external resistance in the armature of field circuit. As the operating handle is gradually turned, the resistance is cut out of the armature circuit–there being as yet no resistance in the field circuit at this stage, then when resistance in the armature circuit is completely cut out, further rotation of the handle inserts resistance into the field circuit. Turning of the handle in the opposite direction starts and speeds up the motor in the reverse direction.

30.25. Grading of Starting Resistance for Shunt Motors

T_{st} would be small in designing shunt motor starters, it is usual to allow an overload of 50% for starting and to advance the starter a step when armature current has fallen to definite lower value. Either this lower current limit may be fixed or the number of starter steps may be fixed. In the former case, the number of steps are so chosen as to suit the upper and lower current limits whereas in the latter case, the lower current limit will depend on the number of steps specified. It can be shown that the resistances in the circuit on successive studs from geometrical progression, having a common ratio equal to lower current limit/upper current limit *i.e.*, I_2/I_1.

In Fig. 30.45 the starter connected to a shunt motor is shown. For the sake of simplicity, four live studs have been taken. When arm *A* makes contact with stud No. 1, full shunt field is

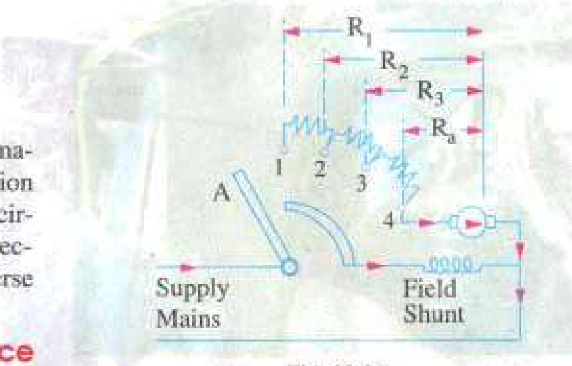


Fig. 30.45

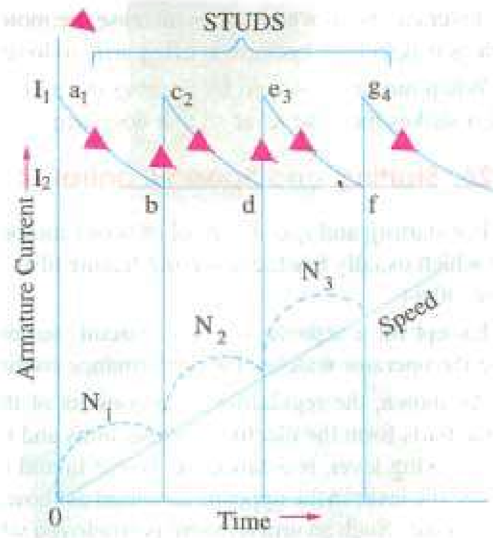


Fig. 30.46

established and at the same time the armature current immediately jumps to a maximum value I_1 given by $I_1 = V/R_1$ where R_1 = armature and starter resistance (Fig. 30.45).

I_1 the maximum permissible armature current at the start (I_{max}) and is, as said above, usually limited to 1.5 times the full-load current of the motor. Hence, the motor develops 1.5 times its full-load torque and accelerates very rapidly. As the motor speeds up, its back e.m.f. grows and hence decreases the armature current as shown by curve *ab* in Fig. 30.46.

When the armature current has fallen to some predetermined value I_2 (also called I_{min}) arm *A* is moved to stud No. 2. Let the value of back e.m.f. be E_{b1} at the time of leaving stud No. 1. Then

$$I_2 = \frac{V - E_{b1}}{R_1} \quad \text{...(i)}$$

It should be carefully noted that I_1 and I_2 [(I_{max}) and (I_{min})] are respectively the maximum and minimum currents of the motor. When arm *A* touches stud No. 2, then due to diminution of circuit resistance, the current again jumps up to its previous value I_1. Since speed had no time to change, the back e.m.f. remains the same as initially.

$$\therefore \quad I_1 = \frac{V - E_{b1}}{R_2} \quad \text{...(ii)}$$

From (*i*) and (*ii*), we get $$\frac{I_1}{I_2} = \frac{R_1}{R_2} \quad \text{... (iii)}$$

When arm *A* is held on stud No. 2 for some time, then speed and hence the back e.m.f. increases to a value E_{b2}, thereby decreasing the current to previous value I_2, so that

$$I_2 = \frac{V - E_{b3}}{R_2} \quad \text{...(iv)}$$

Similarly, on first making contact with stud No. 3, the current is

$$I_1 = \frac{V - E_{b2}}{R_3} \quad \text{...(v)}$$

From (*iv*) and (*v*), we again get $$\frac{I_1}{I_2} = \frac{R_2}{R_3} \quad \text{...(vi)}$$

When arm *A* is held on stud No. 3 for some time, the speed and hence back e.m.f. increases to a new value E_{b3}, thereby decreasing the armature current to a value I_2 such that

$$I_2 = \frac{V - E_{b3}}{R_3} \quad \text{...(vii)}$$

On making contact with stud No. 4, current jumps to I_1 given by

$$I_1 = \frac{V - E_{b3}}{R_a} \quad \text{...(viii)}$$

From (*vii*) and (*viii*), we get $$\frac{I_1}{I_2} = \frac{R_3}{R_a} \quad \text{...(ix)}$$

From (*iii*), (*vi*) and (*ix*), it is seen that

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K \text{ (say)} \quad \text{...(x)}$$

Obviously, $$R_3 = KR_a;\ R_2 = KR_3 = K^2R_a$$
$$R_1 = KR_2 = K.K^2R_a = K^3R_a$$

In general, if n is the number of live studs and therefore $(n-1)$ the number of sections in the starter resistance, then

$$R_1 = K^{n-1}.R_a \text{ or } \frac{R_1}{R_a} = K^{n-1} \text{ or } \left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_a} \quad \text{...from } (x)$$

Other variations of the above formula are

(*a*) $$K^{n-1} = \frac{R_1}{R_a} = \frac{V}{I_1 R_a} = \frac{V}{I_{max}\, R_a}$$

(*b*) $$K^{n} = \frac{V}{I_1 R_a} \cdot \frac{I_1}{I_2} = \frac{V}{I_2 R_a} = \frac{V}{I_{min} R_a} \text{ and}$$

(*c*) $$n = 1 + \frac{\log (V / R_a . I_{max})}{\log K} \text{ from } (a) \text{ above.}$$

Since $R_1 = V/I_1$ and R_a are usually known and K is known from the given values of maximum and minimum currents (determined by the load against which motor has to start), the value of n can be found and hence the value of different starter sections.

When Number of Sections is Specified.

Since I_1 would be given, R_1 can be found from $R_1 = V/I_1$.

Since n is known, K can be found from $R_1/R_a = K^{n-1}$ and the lower current limit I_2 from $I_1/I_2 = K$.

Example 30.49. *A 10 b.h.p. (7.46 kW) 200-V shunt motor has full-load efficiency of 85%. The armature has a resistance of 0.25 Ω. Calculate the value of the starting resistance necessary to limit the starting current to 1.5 times the full-load current at the moment of first switching on. The shunt current may be neglected. Find also the back e.m.f. of the motor, when the current has fallen to its full-load value, assuming that the whole of the starting resistance is still in circuit.*

Solution. Full-load motor current $= 7{,}460/200 \times 0.85 = 43.88$ A

Starting current, $I_1 = 1.5 \times 43.88 = 65.83$ A

$R_1 = V/I_1 = 200/65.83 = 3.038\ \Omega$; $R_a = 0.25\ \Omega$

∴ Starting resistance $= R_1 - R_a = 3.038 - 0.25 = 2.788\ \Omega$

Now, full-load current $I_2 = 43.88$ A

Now, $$I_2 = \frac{V - E_{b1}}{R_1}$$

∴ $E_{b1} = V - I_2 R_1 = 200 - (43.88 \times 3.038) =$ **67 V**

Example 30.50. *A 220-V shunt motor has an armature resistance of 0.5 Ω. The armature current at starting must not exceed 40 A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter.* **(Elect. Machines, AMIE Sec. B, 1992)**

Solution. Since the number of starter sections is specified, we will use the relation.

$$R_1/R_a = K^{n-1} \quad \text{or} \quad R_1 = R_a K^{n-1}$$

Now, $R_1 = 220/40 = 5.5\,\Omega$, $R_a = 0.4\ \Omega$; $n-1 = 6$, $n = 7$

∴ $5.5 = 0.4\,K^6$ or $K^6 = 5.5/0.4 = 13.75$

$6 \log_{10} K = \log_{10} 13.75 = 1.1383$; $K = 1.548$

Now, $R_2 = R_1/K = 5.5/1.548 = 3.553\ \Omega$

$R_3 = R_2/K = 3.553/1.548 = 2.295\ \Omega$

$R_4 = 2.295/1.548 = 1.482\ \Omega$

$R_5 = 1.482/1.548 = 0.958\ \Omega$

$R_6 = 0.958/1.548 = 0.619\ \Omega$

Resistance of Ist section $= R_1 - R_2 = 5.5 - 3.553 =$ **1.947 Ω**

" 2nd " $= R_2 - R_3 = 3.553 - 2.295 =$ **1.258 Ω**

" 3rd " $= R_3 - R_4 = 2.295 - 1.482 =$ **0.813 Ω**

" 4th " $= R_4 - R_5 = 1.482 - 0.958 =$ **0.524 Ω**

,, 5th " $= R_5 - R_6 = 0.958 - 0.619 = \mathbf{0.339\ \Omega}$

,, 6th " $= R_6 - R_a = 0.619 - 0.4 = \mathbf{0.219\ \Omega}$

Example 30.51. *Find the value of the step resistance in a 6-stud starter for a 5 h.p. (3.73 kW), 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The total Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is found to be 88%.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Output $= 3,730$ W

Total loss $= 4,238 - 3,730 = 508$ W

Armature Cu loss alone $= 508/2 = 254$ W

Input current $= 4,238/200 = 21.19$ A

Armature current $= 21.19 - 0.6 = 20.59$ A

$\therefore\ 20.59^2 R_a = 254 \quad \therefore\ R_a = 254/20.59^2 = 0.5989\ \Omega$

Permissible input current $= 21.19 \times 2 = 42.38$ A

Permissible armature current $= 42.38 - 0.6 = 41.78$ A

$\therefore\ R_1 = 200/41.78 = 4.787\ \Omega;\ n = 6;\ n - 1 = 5$

$\therefore\ 4.787 = K^5 \times 0.5989 \qquad \therefore\ K^5 = 4.787/0.5989 = 7.993$

or $5 \log K = \log 7.993 = 0.9027$

$\therefore\ \log K = 0.1805;\ K = 1.516$

Now $R_2 = R_1/K = 4.789/1.516 = 3.159\ \Omega$

$R_3 = 2.084\ \Omega;\ R_4 = 1.376\ \Omega;\ R_5 = 0.908\ \Omega$

Resistance in 1st step $= R_1 - R_2 = 4.787 - 3.159 = \mathbf{1.628\ \Omega}$

Resistance in 2nd step $= R_2 - R_3 = 3.159 - 2.084 = \mathbf{1.075\ \Omega}$

Resistance in 3rd step $= R_3 - R_4 = 2.084 - 1.376 = \mathbf{0.708\ \Omega}$

Resistance in 4th step $= R_4 - R_5 = 1.376 - 0.908 = \mathbf{0.468\ \Omega}$

Resistance in 5th step $= R_5 - R_a = 0.908 - 0.5989 = \mathbf{0.309\ \Omega}$

The various sections are shown in Fig. 28.47.

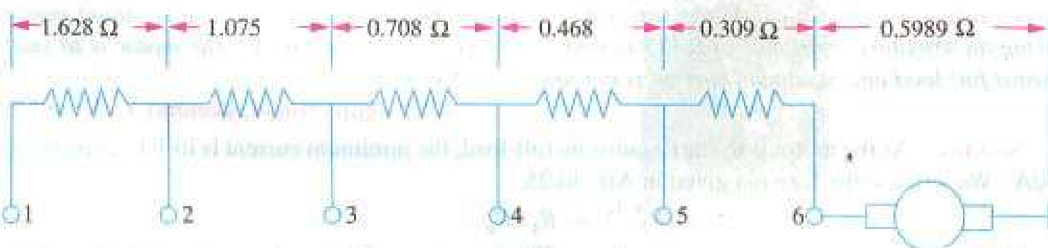


Fig. 30.47

Example 30.52. *Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, d.c. shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.*

(Elec. Machines, Gujarat Univ. 1987)

Solution. Output $= 36,775$ W; Input $= 36,775/0.92 = 39,980$ W

Total Cu loss $= 0.05 \times 39,980 = 1,999$W

Shunt Cu loss $= V^2/R_{sh} = 400^2/200 = 800$ W

$$\text{Armature Cu loss} = 1{,}999 - 800 = 1199 \text{ W}$$

$$\text{F.L. input current} = 39{,}980/400 = 99.95 \text{ A}$$

$$I_{sh} = 400/200 = 2\text{A}\ ; \quad I_a = 99.95 - 2 = 97.95 \text{ A}$$

$$\therefore \quad 97.95^2\, R_a = 1199 \text{ W} \quad \text{or} \quad R_a = 0.125\ \Omega$$

Now, minimum armature current equals full-load current *i.e.* $I_a = 97.95$ A. As seen from Art. 30.25 in its formula given in (*b*), we have

$$K^n = \frac{V}{I_a R_a}$$

or $$K^7 = 400/97.95 \times 0.125 = 32.68$$

$$\therefore \quad K = 32.68^{1/7} = 1.645$$

$$I_1 = \text{maximum permissible armature current}$$

$$= KI_2 = 1.645 \times 97.94 = 161 \text{ A}$$

$$\therefore \quad R_1 = V/I_1 = 400/161 = 2.483\ \Omega$$

$$R_2 = R_1/K = 2.483/1.645 = 1.51\ \Omega$$

$$R_3 = 1.51/1.645 = 0.917\ \Omega$$

$$R_4 = 0.917/1.645 = 0.557\ \Omega$$

$$R_5 = 0.557/1.645 = 0.339\ \Omega$$

$$R_6 = 0.339/1.645 = 0.206\ \Omega$$

$$R_7 = 0.206/1.645 = 0.125\ \Omega$$

Resistance in 1st step $= R_1 - R_2 =$ **0.973 Ω**

Resistance in 2nd step $= R_2 - R_3 =$ **0.593 Ω**

Resistance in 3rd step $= R_3 - R_4 =$ **0.36 Ω**

Resistance in 4th step $= R_4 - R_5 =$ **0.218 Ω**

Resistance in 5th step $= R_5 - R_6 =$ **0.133 Ω**

Resistance in 6th step $= R_6 - R_a =$ **0.081 Ω**

The various starter sections are shown in Fig 30.48.

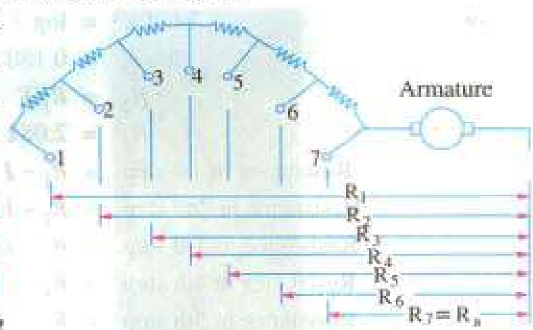


Fig. 30.48

Example 30.53. *Calculate the resistance steps for the starter of a 250-V, d.c. shunt motor having an armature resistance of 0.125 Ω and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A*

(Elect.Engineering-I, Bombay Univ. 1989)

Solution. As the motor is to start against its full-load, the minimum current is its F.L. current *i.e.* 150A. We will use the formula given in Art. 30.25.

$$(I_1/I_2^{n-1}) = R_1 / R_a$$

Here $$I_1 = 200 \text{ A}\ ; \quad I_2 = 150 \text{ A}\ ; \quad R_1 = 250/200 = \mathbf{1.25\ \Omega}$$

$$R_a = 0.125\ \Omega\ ; \quad n = \text{No. of live studs}$$

$$\therefore \quad (200/150)^{n-1} = 1.25/0.125 = 10 \quad \text{or} \quad (4/3)^{n-1} = 10$$

$$(n-1)\log 4/3 = \log 10 \quad \text{or} \quad (n-1) \times 0.1249 = 1$$

$$\therefore \quad (n-1) = 1/0.1249 = 8$$

Hence, there are 9 studs and 8 steps.

Now $$R_2 = R_1 \times I_2/I_1 = 1.25 \times 3/4 = 0.938\ \Omega$$

$$R_3 = 0.938 \times 3/4 = 0.703\ \Omega$$

$$R_4 = 0.703 \times 3/4 = 0.527\ \Omega$$

$$R_5 = 0.527 \times 3/4 = 0.395\ \Omega$$
$$R_6 = 0.395 \times 3/4 = 0.296\ \Omega$$
$$R_7 = 0.296 \times 3/4 = 0.222\ \Omega$$
$$R_8 = 0.222 \times 3/4 = 0.167\ \Omega$$
$$R_a = 0.167 \times 3/4 = 0.125\ \Omega$$

∴ Resistance of 1st element $= 1.25 - 0.938 =$ **0.312 Ω**

„ 2nd „ $= 0.938 - 0.703 =$ **0.235 Ω**

„ 3rd „ $= 0.703 - 0.527 =$ **0.176 Ω**

„ 4th „ $= 0.527 - 0.395 =$ **0.132 Ω**

„ 5th „ $= 0.395 - 0.296 =$ **0.099 Ω**

„ 6th „ $= 0.296 - 0.222 =$ **0.074 Ω**

„ 7th „ $= 0.222 - 0.167 =$ **0.055 Ω**

„ 8th „ $= 0.167 - 0.125 =$ **0.042 Ω**

Example 30.54. *The 4-pole, lap-wound armature winding of a 500-V, d.c. shunt motor is housed in a total number of 60 slots each slot containing 20 conductors. The armature resistance is 1.31 Ω. If during the period of starting, the minimum torque is required to be 218 N-m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.*

(Elect. Machinery-II, Bangalore Univ. 1991)

Solution. From the given minimum torque, we will be able to find the minimum current required during starting. Now

$$T_a = 0.159\ \Phi Z I_a (P/A)$$

∴ $218 = 0.159 \times 23 \times 10^{-3} \times (60 \times 20)\, I_a \times (4/4)$ ∴ $I_a = 50$ A (approx.)

Maximum current $= 50 \times 1.5 = 75$ A

∴ $I_1 = 75$ A ; $I_2 = 50$ A ∴ $I_1/I_2 = 75/50 = 1.5$

$$R_1 = 500/75 = 6.667\ \Omega$$

If n is the number of stater studs, then

$$(I_1/I_2)^{n-1} = R_1/R_a \qquad \text{or} \qquad 1.5^{n-1} = 6.667/1.31 = 5.09$$

∴ $(n-1)\log_{10} 1.5 = \log_{10} 5.09$ ∴ $(n-1) \times 0.1761 = 0.7067$ ∴ $(n-1) = 4$ or $n = 5$

Hence, there are five studs and four sections.

$$R_2 = R_1 \times I_2/I_1 = 6.667 \times 2/3 = 4.44\ \Omega$$
$$R_3 = 4.44 \times 2/3 = 2.96\ \Omega; \quad R_4 = 2.96 \times 2/3 = 1.98\ \Omega$$

Resistance of 1st section $= R_1 - R_2 = 6.67 - 4.44 =$ **2.23 Ω**

„ 2nd „ $= R_2 - R_3 = 4.44 - 2.96 =$ **1.48 Ω**

„ 3rd „ $= R_3 - R_4 = 2.96 - 1.98 =$ **0.98 Ω**

„ 4th „ $= R_4 - R_a = 1.98 - 1.31 =$ **0.67 Ω**

30.26. Series Motor Starters

The basic principle employed in the design of a starter for series motor is the same as for a shunt motor *i.e.*, the motor current is not allowed to exceed a certain upper limit as the starter arm moves from one stud to another. However, there is one significant difference. In the case of a series motor, the flux does not remain constant but varies with the current because armature current is also the exciting current. The determination of the number of steps is rather complicated as illustrated in Example 30.55. It may however, be noted that the section resistances form a geometrical progression.

The face-plate type of starter formerly used for d.c. series motor has been almost entirely replaced by automatic starter in which the resistance steps are cut out automatically by means of a contactor operated by electromagnets. Such starters are well-suited for remote control.

However, for winch and crane motors where frequent starting, stopping, reversing and speed variations are necessary, drum type controllers are used. They are called controllers because they can be left in the circuit for any length of time. In addition to serving their normal function of starters, they also used as speed controllers.

Example 30.55. *(a) Show that, in general, individual resistances between the studs for a rheostat starter for a series d.c. motor with constant ratio of maximum to minimum current at starting, are in geometrical progression, stating any assumptions made.*

(b) Assuming that for a certain d.c. series motor the flux per pole is proportional to the starting current, calculate the resistance of the each rheostat section in the case of a 50 b.h.p. (37.3 kW) 440-V motor with six sections.

The total armature and field voltage drop at full-load is 2% of the applied voltage, the full-load efficiency is 95% and the maximum starting current is 130% of full-load current.

Solution. (a) Let I_1 = maximum current, I_2 = minimum current

Φ_1 = flux/pole for I_1 ; Φ_2 = flux/pole for I_2

$$\frac{I_1}{I_2} = K \text{ and } \frac{\Phi_1}{\Phi_2} = \alpha.$$

Let us now consider the conditions when the starter arm is on the nth and $(n+1)$th stud. When the current is I_2, then $E_b = V - I_2 R_n$

If, now, the starter is moved up to the $(n+1)$th stud, then

$$E_b' = \frac{\Phi_1}{\Phi_2}.E_b = \alpha E_b$$

$$\therefore \quad R_{n+1} = \frac{V - E'_{b1}}{I_1} = \frac{V - \alpha E_b}{I_1} = \frac{V - \alpha(V - I_2 R_n)}{I_1} = \frac{V}{I_1}(1-\alpha) + \alpha\frac{I_2}{I_1}\cdot R_n$$

Now, $V/I_1 = R_1$–the total resistance in the circuit when the starter arm is on the first stud.

$$\therefore \quad R_{n+1} = R_1(1-\alpha) + \frac{\alpha}{K}R_n$$

Similarly, by substituting $(n-1)$ for n, we get $R_n = R_1(1-\alpha) + \dfrac{\alpha}{K}R_{n-1}$

Therefore, the resistance between the nth and $(n+1)$th studs is

$$r_n = R_n - R_{n+1} = \frac{\alpha}{K}R_{n-1} - \frac{\alpha}{K}R_n = \frac{\alpha}{K}(R_{n-1} - R_n) = \frac{\alpha}{K}r_{n-1}$$

$$\therefore \quad \frac{r_n}{r_{n-1}} = \frac{\alpha}{K} = \frac{\Phi_1}{\Phi_2}\times\frac{I_2}{I_1} = b \qquad \text{– constant}$$

Obviously, the resistance elements form a geometrical progression series.

(b) Full-load input current $= \dfrac{37,300}{440\times 0.95} = 89.2$ A

Max. starting current $I_1 = 1.3 \times 89.2 = 116$ A

Arm. and field voltage drop on full-load = 2% of 440 $= 0.02 \times 440 = 8.8$ V

Resistance of motor $= 8.8/89.2 = 0.0896\ \Omega$

Total circuit resistance on starting, $R_1 = V/I_1 = 440/116 = 3.79\ \Omega$

Assuming straight line magnetisation, we have $I_1 \propto \Phi_1$ and $I_2 \propto \Phi_2$

$\therefore \quad I_1/I_2 = \Phi_1/\Phi_2 \qquad \therefore \quad \alpha = K$ and $b = \alpha/K = 1 \qquad \therefore \quad r_n = b \times r_{n-1}$

In other words, all sections have the same resistance.

$$\therefore \qquad r = \frac{R_1 - R_{motor}}{\text{No. of sections}} = \frac{3.79 - 0.0896}{6} = \mathbf{0.6176\ \Omega}$$

Example 30.56. *A 75 h.p. (55.95 kW) 650-V, d.c. series tractions motor has a total resistance of 0.51 Ω. The starting current is to be allowed to fluctuate between 140 A and 100 A the flux at 140 A being 20 % greater than at 100 A. Determine the number of steps required in the controller and the resistance of each step.*

Solution. Let R_1 = total resistance on the first stud = 650/140 = 4.65 Ω

When motor speeds up, then back e.m.f. is produced and current falls to I_2.

$$V = E_{b1} + I_2R_1 \qquad \text{...}(i)$$

When the starter moves to the next stud, the speed is still the same, but since current rises to I_1 for which flux is 1.2 times greater than for I_2, hence back e.m.f. becomes 1.2 E_{b1}. Since resistance in the circuit is now R_2.

$$\therefore \qquad V = I_1R_2 + 1.2\,E_{b1} \qquad \text{...}(ii)$$

From (*i*) and (*ii*) we get, $0.2\,V = 1.2\,I_2R_1 - I_1R_2$

$$\therefore \qquad R_2 = (1.2I_2/I_1)\,R_1 - 0.2\,V/I_1 = (1.2I_2/I_1)\,R_1 - 0.2\,R_1 = (1.2I_2/I_1 - 0.2)\,R_1$$

Similarly $\qquad R_3 = (1.2I_2/I_1)\,R_2 - 0.2\,R_1$

In this way, we continue till we reach the value of resistance equal to the armature resistance. Hence, we obtain R_1, R_2 etc. and also the number of steps.

In the present case, $I_1 = 140$ A, $I_2 = 100$A, $V = 650$ V and $I_1/I_2 = 100/140 = 1.14$

$$\therefore \qquad R_1 = 4.65\ \Omega;\ R_2 = (1.2/1.4 - 0.2)\,4.65 = 3.07\ \Omega$$

$$R_3 = (1.2/1.4) \times 3.07 - 0.2 \times 4.65 = 1.70\ \Omega$$

$$R_4 = (1.2/1.4) \times 1.7 - 0.2 \times 4.65 = 0.53\ \Omega$$

We will stop here because R_4 is very near the value of the motor resistance. Hence, there are 4 studs and 3 sections or steps.

$$R_1 - R_2 = 4.65 - 3.07 = 1.58\ \Omega,\ R_2 - R_3 = 3.07 - 1.7 = 1.37\ \Omega$$

$$R_3 - R_4 = 1.70 - 0.53 = 1.17\ \Omega$$

Note. It will be seen that

$$R_2 - R_3 = (1.2\,I_2/I_1)\,(R_1 - R_2) \text{ and } R_3 - R_4 = (1.2\,I_2/I_1)\,(R_2 - R_3) \text{ and so on.}$$

or $$\frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} = 1.2\,\frac{I_2}{I_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = \frac{\alpha}{K} = b$$

It is seen that individual resistances of various sections decrease in the ratio of $\alpha/K = b$.

30.27. Thyristor Controller Starters

The moving parts and metal contacts etc., of the resistance starters discussed in Art. 30.21 can be eliminated by using thyristors which can short circuit the resistance sections one after another. A thyristor can be switched on to the conducting state by applying a suitable signal to its gate terminal. While conducting, it offers zero resistance in the forward (*i.e.*, anode-to-cathode) direction and thus acts as a short-circuit for the starter resistance section across which it is connected. It can be switched off (*i.e.*, brought back to the non-conducting state) by reversing the polarity of its anode-cathode voltage. A typical thyristor-controlled starter for d.c. motors is shown in Fig. 30.49.

After switching on the main supply, when switch S_1 is pressed, positive signal is applied to gate G of thyristor T_1 which is, therefore, turned ON. At the same time, shunt field gets established since it is directly connected across the d.c. supply. Consequently, motor armature current I_a flows *via* T_1,

R_2, R_3 and R_4 because T_2, T_3 and T_4 are, as yet in the non-conducting state. From now onwards, the starting procedure is automatic as detailed below :

1. As S_1 is closed, capacitor C starts charging up with the polarity as shown when I_a starts flowing.
2. The armature current and field flux together produce torque which accelerates the motor and load.

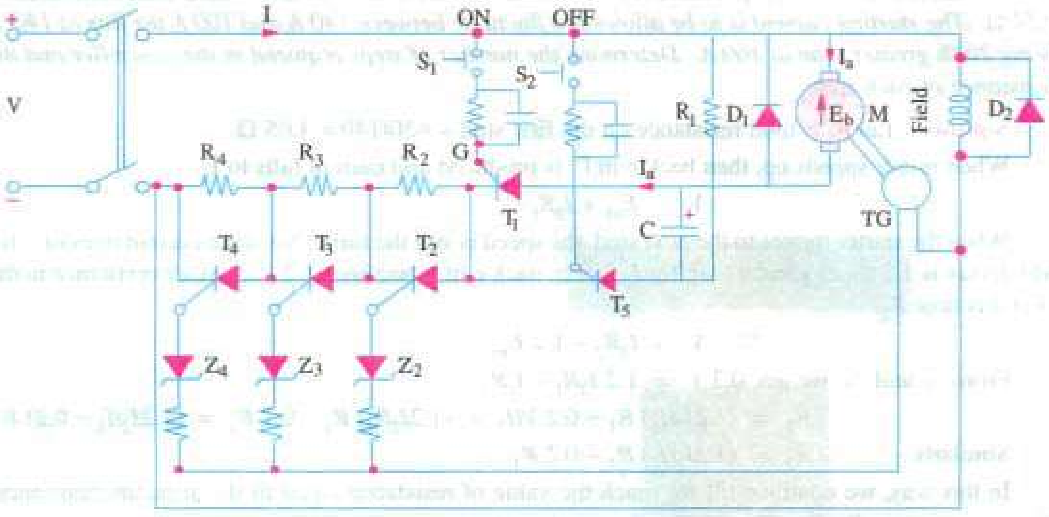


Fig. 30.49

3. As motor speeds up, voltage provided by tachogenerator (*TG*) is proportionately increased because it is coupled to the motor.
4. At some motor speed, the voltage provided by *TG* becomes large enough to breakdown Zener diode Z_2 and hence trigger T_2 into conduction. Consequently, R_2 is shorted out and now I_a flows via motor armature, T_1, T_2, R_3 and R_4 and back to the negative supply terminal.
5. As R_2 is cut out, I_a increases, armature torque increases, motor speed increases which further increases the voltage output of the tachogenerator. At some speed, Z_3 breaks down, thereby triggering T_3 into conduction which cuts out R_3.
6. After sometime, R_4 is cut out as Z_4 breaks down and triggers T_4 into conduction. In fact, Zener diodes Z_2, Z_3 and Z_4 can be rated for 1/3, 1/2 and 3/4 full speed respectively.

For stopping the motor, switch S_2 is closed which triggers T_5 into conduction, thereby establishing current flow via R_1. Consequently, capacitor C starts discharging thereby reverse-biasing T_1 which stops conducting. Hence I_a ceases and, at the same time, T_2, T_3 and T_4 also revert back to their non-conducting state.

Incidentally, it may be noted that the function of C is to switch T_1, ON and OFF. Hence, it is usually called ***commutating capacitor.***

The function of the diodes D_1 and D_2 is to allow the decay of inductive energy stored in the motor armature and field when supply is disconnected. Supply failure will cause the thyristors to block because of this current decay, thereby providing protection usually given by no-voltage release coil.

Recently, thyristor starting circuits have been introduced which use no starting resistance at all, thereby making the entire system quite efficient and optimized as regards starting time. These are based on the principle of 'voltage chopping' (Art. 30.12). By varying the chopping frequency, the ratio of the time the voltage is ON to the time it is OFF can be varied. By varying this ratio, the average voltage applied to the motor can be changed. A low average voltage is needed to limit the armature current while the motor is being started and gradually the ratio is increased to reach the maximum at the rated speed of the motor.

Tutorial Problems 32.2

1. A shunt-wound motor runs at 600 r.p.m. from a 230-V supply when taking a line current of 50 A. Its armature and field resistances are 0.4 Ω and 104.5 Ω respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (*a*) the no-load speed if the no-load line current is 5A (*b*) the resistance to be placed in armature circuit in order to reduce the speed to 500 r.p.m. when motor is taking a line current of 50 A (*c*) the percentage reduction in the flux per pole in order that the speed may be 750 r.p.m. when the armature current is 30 A with no added resistance in the armature circuit. **[(*a*) 652 r.p.m. (*b*) 0.73 Ω (*c*) 1.73 %]**
2. The resistance of the armature of a 250-V shunt motor is 0.3 Ω and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run ? Neglect armature reaction. **[0.94 Ω; 932 r.p.m.]**
3. A 230-V d.c. shunt motor takes an armature current of 20 A on a certain load. Resistance of the armature is 0.5 Ω. Find the resistance required in series with the armature to half the speed if (*a*) the load torque is constant (*b*) the load torque is proportional to the square of the speed. **[(*a*) 5.5 Ω (*b*) 23.5 Ω]**
4. A 230-V series motor runs at 1200 r.p.m. at a quarter full-load torque, taking a current of 16 A. Calculate its speed at half and full-load torques. The resistance of the armature brushes, and field coils is 0.25 Ω. Assume the flux per pole to be proportional to the current. Plot torque/speed graph between full and quarter-load. **[842 r.p.m. ; 589 r.p.m.]**
5. A d.c. series motor drives a load the torque of which is proportional to the square of the speed. The motor current is 20 A when speed is 500 r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistance of the same value as the field winding. Neglect all motor losses and assume that the magnetic field is unsaturated. **[595 r.p.m. ; 33.64 A]**
(Electrical Machines-I, Aligarh Muslim Univ. 1979)
6. A d.c. series motor, with unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load takes 50 A at 500 V. If the load torque varies as the cube of the speed, find the resistance which should be connected in series with machine to reduce the speed by 25 per cent. **[7.89 Ω]**
(Electrical Engg-I, M.S. Univ. Baroda 1980)
7. A series motor runs at 500 r.p.m. on a certain load. Calculate the resistance of a divertor required to raise the speed to 650 r.p.m. with the same load current, given that the series field resistance is 0.05 Ω and the field is unsaturated. Assume the ohmic drop in the field and armature to be negligible. **[0.1665 Ω]**
8. A 230-V d.c. series motor has armature and field resistances of 0.5 Ω and 0.3 Ω respectively. The motor draws a line current of 40 A while running at 400 r.p.m. If a divertor of resistance 0.15 W is used, find the new speed of the motor for the same armature current.
It may be assumed that flux per pole is directly proportional to the field current. **[1204 r.p.m.]**
(Electrical Engineering Grad. I.E.T.E. June 1986)
9. A 250-V, d.c. shunt motor runs at 700 r.p.m. on no-load with no extra resistance in the field and armature circuit. Determine :
(*i*) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 A.
(*ii*) the resistance to be placed in series with the field to produce a speed of 1,000 r.p.m. when taking an armature current of 18 A.

Assume that the useful flux is proportional to the field. Armature resistance = 0.35 Ω, field resistance = 125 Ω. **[(*i*) 7.3 Ω (*ii*) 113 Ω] (Elect. Engg. Grad. I.E.T.E., June 1984)**

10. A d.c. series motor is operating from a 220-V supply. It takes 50 A and runs at 1000 r.p.m. The resistance of the motor is 0.1 Ω. If a resistance of 2 Ω is placed in series with the motor, calculate the resultant speed if the load torque is constant. **[534 r.p.m.]**

11. A d.c. shunt motor takes 25 A when running at 1000 r.p.m. from a 220-V supply.

    Calculate the current taken form the supply and the speed if the load torque is halved, a resistance of 5 Ω is placed in the armature circuit and a resistance of 50 Ω is placed in the field circuit.

    Armature resistance = 0.1 Ω ; field resistance = 100 Ω

    Assume that the field flux per pole is directly proportional to the field current. **[17.1 A; 915 r.p.m.]**

    (Elect. Technology, Gwalior Univ. Nov. 1977)

12. A 440-V shunt motor takes an armature current of 50 A and has a flux/pole of 50 mWb. If the flux is suddenly decreased to 45 mWb, calculate (*a*) instantaneous increase in armature current (*b*) percentage increase in the motor torque due to increase in current (*c*) value of steady current which motor will take eventually (*d*) the final percentage increase in motor speed. Neglect brush contact drop and armature reaction and assume an armature resistance of 0.6 Ω.

    [(*a*) 118 A (*b*) 112 % (*c*) 5.55 A (*d*) 10%]

13. A 440-V shunt motor while running at 1500 r.p.m. takes an armature current of 30 A and delivers a mechanical output of 15 h.p. (11.19 kW). The load torque varies as the square of the speed. Calculate the value of resistance to be connected in series with the armature for reducing the motor speed to 1300 r.p.m. and the armature current at that speed. **[2.97 Ω, 22.5 A]**

14. A 460-V series motor has a resistance of 0.4 Ω and takes a current of 25 A when there is no additional controller resistance in the armature circuit. Its speed is 1000 r.p.m. The control resistance is so adjusted as to reduce the field flux by 5%. Calculate the new current drawn by the motor and its speed. Assume that the load torque varies as the square of the speed and the same motor efficiency under the two conditions of operation.

    [22.6 A; 926 r.p.m.] (Elect. Machines, South Gujarat Univ. Oct. 1977)

15. A 460-V, series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change and torque if the load is reduced so that the motor is taking 30 A. Total resistance of armature and field circuit is 0.8 Ω. Assume flux proportional to the field current.

    [680 r.p.m. 43.75%]

16. A 440-V, 25 h.p (18.65 kW) motor has an armature resistance of 1.2 Ω and full-load efficiency of 85%. Calculate the number and value of resistance elements of a starter for the motor if maximum permissible current is 1.5 times the full-load current. **[1.92 Ω, 1.30 Ω, 0.86 Ω; 0.59 Ω]**

    (Similar example in JNTU, Hyderabad, 2000)

17. A 230-V, d.c. shunt motor has an armature resistance of 0.3 Ω. Calculate (*a*) the resistance to be connected in series with the armature to limit the armature current to 75 A at starting and (*b*) value of the generated e.m.f. when the armature current has fallen to 50 A with this value of resistance still in circuit. **[(*a*) 2.767 Ω (*b*) 76.7 A]**

18. A 200-V, d.c. shunt motor takes full-load current of 12 A. The armature circuit resistance is 0.3 Ω and the field circuit resistance is 100 Ω. Calculate the value of 5 steps in the 6-stud starter for the motor. The maximum starting current is not to exceed 1.5 times the full-load current.

    [6.57 Ω, 3.12 Ω, 1.48 Ω, 0.7 Ω, 0.33 Ω]

19. The resistance of a starter for a 200-V, shunt motor is such that maximum starting current is 30 A. When the current has decreased to 24 A, the starter arm is moved from the first to the second stud. Calculate the resistance between these two studs if the maximum current in the second stud is 34 A. The armature resistance of the motor is 0.4 Ω. **[1.334 Ω]**

20. A totally-enclosed motor has thermal time constant of 2 hr. and final temperature rise at no-load and 40° on full load.

Determine the limits between which the temperature fluctuates when the motor operates on a load cycle consisting of alternate period of 1 hr. on full-load and 1 hr. on no-load, steady state conditions having been established. **[28.7° C, 21.3°C]**

21. A motor with a thermal time constant of 45 min. has a final temperature rise of 75°C on continuous rating (*a*) What is the temperature rise after one hour at this load ? (*b*) If the temperature rise on one-hour rating is 75°C, find the maximum steady temperature at this rating (*c*) When working at its one-hour rating, how long does it take the temperature to increase from 60°C to 75°C ? **[(*a*) 55 °C (*b*) 102 °C (*c*) 20 min]**

(Electrical Technology, M.S. Univ. Baroda. 1976)

OBJECTIVE TESTS – 30

1. The speed of a d.c. motor can be controlled by varying
   (*a*) its flux per pole
   (*b*) resistance of armature circuit
   (*c*) applied voltage
   (*d*) all of the above
2. The most efficient method of increasing the speed of a 3.75 kW d.c. shunt motor would be the ....method.
   (*a*) armature control
   (*b*) flux control
   (*c*) Ward-Leonard
   (*d*) tapped-field control
3. Regarding Ward-Leonard system of speed control which statement is **false** ?
   (*a*) It is usually used where wide and very sensitive speed control is required.
   (*b*) It is used for motors having ratings from 750 kW to 4000 kW
   (*c*) Capital outlay involved in the system is right since it uses two extra machines.
   (*d*) It gives a speed range of 10 : 1 but in one direction only.
   (*e*) It has low overall efficiency especially at light loads.
4. In the rheostatic method of speed control for a d.c. shunt motor, use of armature divertor makes the method
   (*a*) less wasteful
   (*b*) less expensive
   (*c*) unsuitable for changing loads
   (*d*) suitable for rapidly changing loads
5. The chief advantage of Ward-Leonard system of d.c. motor speed control is that it
   (*a*) can be used even for small motors
   (*b*) has high overall efficiency at all speeds
   (*c*) gives smooth, sensitive and wide speed control
   (*d*) uses a flywheel to reduce fluctuations in power demand
6. The flux control method using paralleling of field coils when applied to a 4-pole series d.c. motor can give .... speeds.
   (*a*) 2 (*b*) 3
   (*c*) 4 (*d*) 6
7. The series-parallel system of speed control of series motors widely used in traction work gives a speed range of about
   (*a*) 1 : 2 (*b*) 1 : 3
   (*c*) 1 : 4 (*d*) 1 : 6
8. In practice, regenerative braking is used when
   (*a*) quick motor reversal is desired
   (*b*) load has overhauling characteristics
   (*c*) controlling elevators, rolling mills and printing presses etc.
   (*d*) other methods can not be used.
9. **Statement 1.** A direct-on-line (**DOL**) starter is used to start a small d.c. motor because
   Statement 2. it limits initial current drawn by the armature circuit.
   (*a*) both statement 1 and 2 are incorrect
   (*b*) both statement 1 and 2 are correct
   (*c*) statement 1 is correct but 2 is wrong
   (*d*) statement 2 is corract but 1 is wrong
10. Ward-Leonard system of speed control is **NOT** recommended for
    (*a*) wide speed range
    (*b*) constant-speed operation
    (*c*) frequent motor reversals
    (*d*) very low speeds
11. Thyristor chopper circuits are employed for
    (*a*) lowering the level of a d.c. voltage
    (*b*) rectifying the a.c. voltage
    (*c*) frequency conversion
    (*d*) providing commutation circuitry

12. An invertor circuit is employed to convert
    (a) a.c. voltage into d.c. voltage
    (b) d.c.voltage into a.c. voltage
    (c) high frequency into low frequency
    (d) low frequency into high frequency
13. The phase-control rectifiers used for speed of d.c. motors convert fixed a.c. supply voltage into
    (a) variable d.c. supply voltage
    (b) variable a.c. supply voltage
    (c) full-rectified a.c. voltage
    (d) half-rectified a.c. voltage
14. If some of the switching devices in a convertor are controlled devices and some are diodes, the convertor is called
    (a) full convertor (b) semiconvertor
    (c) solid-state chopper
    (d) d.c. convertor
15. A solid-state chopper converts a fixed-voltage d.c. supply into a
    (a) variable-voltage a.c. supply
    (b) variable-voltage d.c. supply
    (c) higher-voltage d.c. supply
    (d) lower-voltage a.c. supply
16. The d.c. motor terminal voltage supplied by a solid-state chopper for speed control purposes varies..........with the duty ratio of the chopper
    (a) inversely (b) indirectly
    (c) linearly (d) parabolically

ANSWERS

1. d	2. b	3. d	4. d	5. c	6. b	7. c	8. b	9. c	10. b	11. a
12. b	13. a	14. b	15. b	16. c						

CHAPTER 31

Learning Objectives

- Brake Test
- Swinburne's Test
- Advantages of Swinburne's Test
- Main Disadvantages
- Regenerative or Hopkinson's Test
- Alternative Connections for Hopkinson's Test
- Merits of Hopkinson's Test
- Retardation or Running Down Test
- Field's Test for Series Motors
- Objective Test
- Questions and Answers on D.C. Motors.

TESTING OF D.C. MACHINES

Testing is performed on D.C. machines to determine efficiency and power losses

31.1 Brake Test

It is a direct method and consists of applying a brake to a water-cooled pulley mounted on the motor shaft as shown in Fig. 31.1. The brake band is fixed with the help of wooden blocks gripping the pulley. One end of the band is fixed to earth via a spring balance S and the other is connected to a suspended weight W_1. The motor is running and the load on the motor is adjusted till it carries its full load current.

Motor shaft

Let W_1 = suspended weight in kg

W_2 = reading on spring balance in kg-wt

The net pull on the band due to friction at the pulley is $(W_1 - W_2)$ kg. wt. or $9.81\,(W_1 - W_2)$ newton.

If R = radius of the pulley in metre

and N = motor or pulley speed in r.p.s.

Then, shaft torque T_{sh} developed by the motor

$$= (W_1 - W_2)\,R \text{ kg-m} = 9.81\,(W_1 - W_2)\,R \text{ N-m}$$

Motor output power $= T_{sh} \times 2\pi N$ watt

$$= 2\pi \times 9.81\,N\,(W_1 - W_2)\,R \text{ watt}$$

$$= 61.68\,N\,(W_1 - W_2)\,R \text{ watt}$$

Let V = supply voltage ; I = full-load current taken by the motor.

Then, input power = VI watt

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Input}} = \frac{61.68\,N(W_1 - W_2)R}{VI}$$

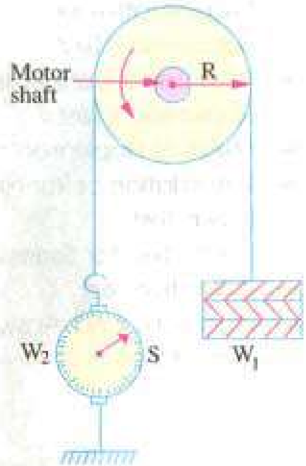


Fig. 31.1

The simple brake test described above can be used for small motors only, because in the case of large motors, it is difficult to dissipate the large amount of heat generated at the brake.

Another simple method of measuring motor output is by the use of poney brake one form of which is shown in Fig. 31.2 (*a*). A rope is wound round the pulley and its two ends are attached to two spring balances S_1 and S_2. The tension of the rope can be adjusted with the help of swivels. Obviously, the force acting tangentially on the pulley is equal to the difference between the readings of the two spring balances. If R is the pulley radius, the torque at the pulley is $T_{sh} = (S_1 - S_2)R$. If $\omega\,(= 2\pi N)$ is the angular velocity of the pulley, then

motor output $= T_{sh} \times \omega = 2\pi N\,(S_1 - S_2)R$ m-kg. wt. $= 9.81 \times 2\pi N\,(S_1 - S_2)\,R$ watt.

The motor input may be measured as shown in Fig. 31.2 (*b*). Efficiency may, as usual, be found by using the relation η = output/input.

Example 31.1. *In a brake test the effective load on the branch pulley was 38.1 kg, the effective diameter of the pulley 63.5 cm and speed 12 r.p.s. The motor took 49 A at 220 V. Calculate the output power and the efficiency at this load.*

Solution. Effective load $(W_1 - W_2) = 38.1$ kg. wt ; radius $= 0.635/2 = 0.3175$ m

Shaft torque $= 38.1 \times 0.3175$ kg-m $= 9.81 \times 38.1 \times 0.3175 = 118.6$ N-m

Power output = torque × angular velocity in rad/s $= 118.6 \times 2\pi \times 12 =$ **8,945 W**

Now, motor input $= 49 \times 220$ W $\quad \therefore \quad$ Motor $\eta = \dfrac{8,945}{49 \times 220} =$ **0.83** or **83%**

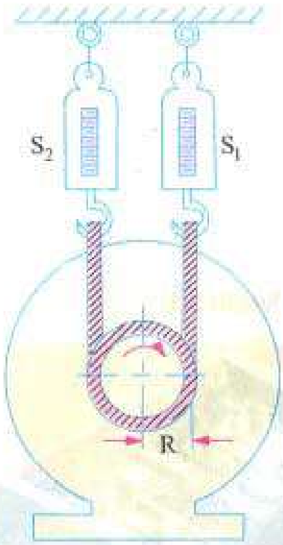


Fig. 31.2 (*a*)

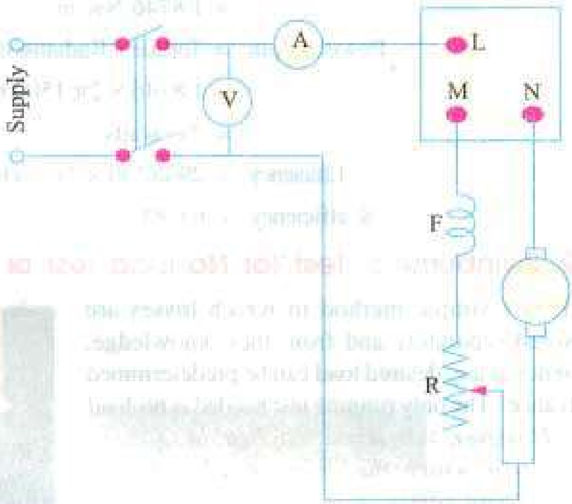


Fig. 31.2 (*b*)

Example 31.2(*a*). *The following readings are obtained when doing a load test on a d.c. shunt motor using a brake drum :*

Spring balance reading	*10 kg and 35 kg*	*Diameter of the drum*	*40 cm*
Speed of the motor	*950 r.p.m.*	*Applied voltage*	*200 V*
Line current	*30 A*		

Calculate the output power and the efficiency. **(Electrical Engineering, Madras Univ. 1986)**

Solution. Force on the drum surface $F = (35 - 10) = 25 \text{ kg wt} = 25 \times 9.8 \text{ N}$

$$\text{Drum radius} \quad R = 20 \text{ cm} = 0.2 \text{ m} ; \text{Torque } T_{sh} = F \times R = 25 \times 9.8 \times 0.2 = 49 \text{ N}$$

$$N = 950/60 = 95/6 \text{ r.p.s.} ; \omega = 2\pi\,(95/6) = 99.5 \text{ rad/s}$$

$$\text{Motor output} = T_{sh} \times \omega \text{ watt} = 49 \times 99.5 = 4{,}876 \text{ W}$$

$$\text{Motor input} = 200 \times 30 = 6000 \text{ W} ; \eta = 4876/6000 = 0.813 \text{ or } \mathbf{81.3\%}$$

Example 31.2(*b*). *In a brake-test, on a d.c. shunt motor, the tensions on the two sides of the brake were 2.9 kg and 0.17 kg. Radius of the pulley was 7 cm. Input current was 2 amp at 230 volts. The motor speed was 1500 rpm. Find the torque, power-output and efficiency.*

(Bharathiar Univ. April 1998)

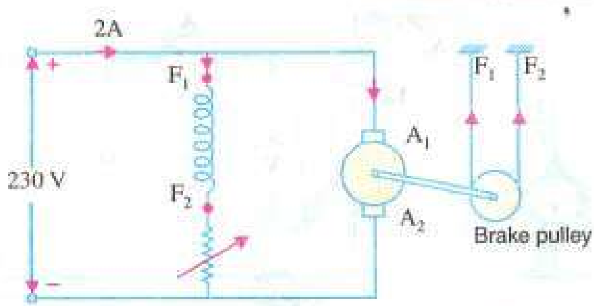


Fig. 31.3. D.C. Shunt Motor Brake Test

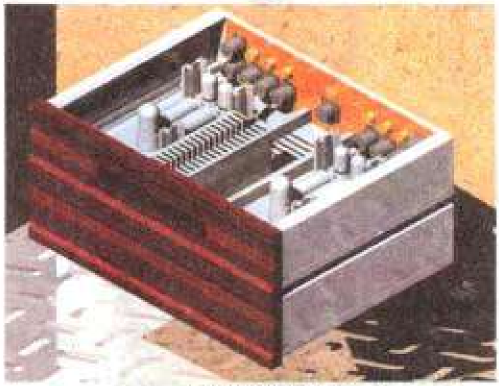
Solution. Net force on pulley $= 2.90 - 0.17 = 2.73 \text{ kg}$

$$= 2.73 \times 9.81 = 26.78 \text{ Nw}$$

$$\text{Net torque} = \text{Force} \times \text{Radius} = 26.78 \times 7/100$$
$$= 1.8746 \text{ Nw-m}$$
$$\text{Power output} = \text{Torque} \times \text{Radians/sec.}$$
$$= 1.8746 \times 2\pi\ 1500/60$$
$$= 294 \text{ watts}$$
$$\text{Efficiency} = 294/(230 \times 2) = 0.639$$
$$\%\ \text{efficiency} = 63.9\%$$

31.2. Swinburne's* Test (or No-load Test or Losses Method)

It is a simple method in which losses are measured separately and from their knowledge, efficiency at any desired load can be predetermined in advance. The only running test needed is no-load test. *However, **this test is applicable to those machines in which flux is practically constant i.e. shunt and compound-wound machines.***



Shunt regulator

The machine is running as a motor on no-load at its rated voltage *i.e.* voltage stamped on the name-plate. The speed is adjusted to the rated speed with the help of shunt regulator as shown in Fig. 31.4.

The no-load current I_0 is measured by the ammeter A_1 whereas shunt field current I_{sh} is given by ammeter A_2. The no-load armature current is $(I_0 - I_{sh})$ or I_{a0}.

Let, supply voltage $= V$; no-load input $= VI_0$ watt

$\therefore$ Power input to armature $= V(I_0 - I_{sh})$; Power input to shunt $= VI_{sh}$

No-load power input to armature supplies the following :

(*i*) Iron losses in core (*ii*) friction loss (*iii*) windage loss and

(*iv*) armature Cu loss, $(I_0 - I_{sh})^2 R_a$ or $I_{a0}^2 R_a$

In calculating armature Cu loss, 'hot' resistance of armature should be used. A stationary measurement of armature circuit resistance at the room-temperature of, say, 15°C is made by passing current through the armature from a low voltage d.c. supply [Fig. 31.5 (*a*)].

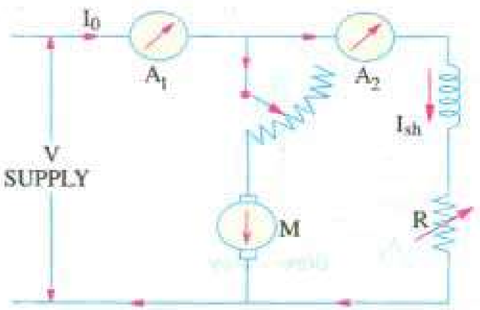


Fig. 31.4

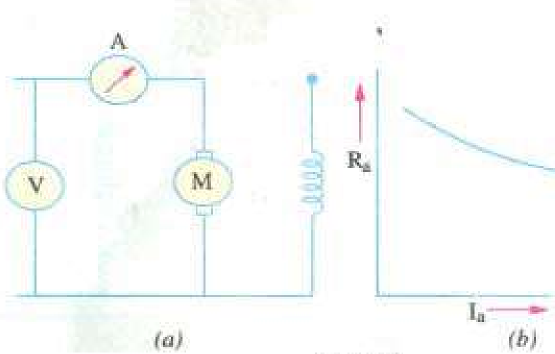


Fig. 31.5

* Sir James Swinburne (1858-1958) made outstanding contributions to the development of electric lamps, electric machines and synthetic resins.

Then, the 'hot' resistance, allowing a temperature rise of 50°C is found thus :

$$R_{15} = R_0(1 + 15\alpha_0) \; ; \; R_{65} = (1 + 65\alpha_0), \; R_{65} = R_{15} \times \frac{1 + 65\alpha_0}{1 + 15\alpha_0}$$

Taking $\alpha_0 = 1/234.5$, we have $\quad R_{65} = R_{15} \times \dfrac{234.5 + 65}{234.5 + 15} = 1.2\, R_{15}$ (approx.*)

If we subtract from the total input the no-load armature Cu loss, then we get constant losses.

∴ Constant losses $\quad W_c = VI_0 - (I_0 - I_{sh})^2 R_a$

Knowing the constant losses of the machine, its efficiency at any other load can be determined as given below. Let I = load current at which efficiency is required.

Then, armature current is $\quad I_a = I - I_{sh}$ ...if machine is motoring

$\quad = I + I_{sh}$ ...if machine is generating

Efficiency when running as a motor

Input = VI, Armature Cu loss = $I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses = W_c ...found above

$$\therefore \text{ Total losses} = (I - I_{sh})^2 R_a + W_c \; ; \; \eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{VI - (I - I_{sh})^2 R_a - W_c}{VI}$$

Efficiency when running as a generator

Output = VI ; Armature, Cu loss = $(I + I_{sh})^2 R_a$; Constant loss = W_c ...found above

$$\therefore \text{ Total losses} = (I + I_{sh})^2 R + W_c \; ; \; \eta_g = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + W_c}$$

31.3. Advantages of Swinburne's Test

1. It is convenient and economical because power required to test a large machine is small *i.e.* only no-load input power.
2. The efficiency can be predetermined at any load because constant-losses are known.

31.4. Main Disadvantages

1. No account is taken of the change in iron losses from no-load to full-load. At full-load, due to armature reaction, flux is distorted which increases the iron losses in some cases by as much as 50%.
2. As the test is on no-load, it is impossible to know whether commutation would be satisfactory at full-load and whether the temperature rise would be within the specified limits.

Example 31.3. *A 220 V, d.c. shunt motor at no load takes a current of 2.5 A. The resistances of the armature and shunt field are 0.8 Ω and 200 Ω respectively. Estimate the efficiency of the motor when the input current is 20 A. State precisely the assumptions made.*

(Electrical Technology, Kerala Univ. 1986)

Solution. No-load input = 220 × 2.5 = 550 W

This input meets all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

$I_{sh} = 220/200 = 1.1$ A. No-load arm current, $I_{a0} = 2.5 - 1.1 = 1.4$ A

---

* The armature resistance is found to decrease slightly with increasing armature current as shown in Fig. 31.5 (*b*). This is due to the fact that brush contact resistance is inversely proportional to the armature current.

No-load armature Cu loss = $I_{a0}^2 R_a = 1.4^2 \times 0.8 = 1.6$ W

Constant losses = 550 − 1.6 = 548.4 W

When input current is 20 A

$I_a = 32 - 1.1 = 30.9$ A ; Armature Cu loss = $30.9^2 \times 0.8 = 764$ W

Total loss = 764 + 548.4 = 1312 W (approx.) ; Input = 220 × 20 = 4,400 W

Output = 4,400 − 1,312 = 3,088 W ; Efficiency = (3088/4400) × 100 = **70.2%**

In the above calculations, it has been assumed that :

1. mechanical losses remain constant even through motor speed changes from no-load to the given load.

2. effect of armature reaction on main pole flux with a consequent change in iron losses has been neglected.

3. decrease in flux due to increase in shunt resistance by heating has been neglected.

Example 31.4. *When running on no-load, a 400-V shunt motor takes 5 A. Armature resistance is 0.5 Ω and field resistance 200 Ω. Find the output of the motor and efficiency when running on full-load and taking a current of 50 A. Also, find the percentage change in speed from no-load to full-load.* **(Electro Mechanics, Allahabad Univ. 1991)**

Solution. No-load input = 400 × 5 = 2,000 W

This input goes to meet all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

I_{sh} = 400/200 = 2 A ; No-load $I_a = 5 - 2 = 3$ A

No-load arm. Cu loss = $3^2 \times 0.5 = 4.5$ W ; Constant losses = 2,000 − 4.5 = **1,995.5 W**

When line current is 50 A

I_a = 50 − 2 = 48 A ; Arm. Cu loss = $48^2 \times 0.5 = 1{,}152$ W

Total loss on F.L. = 1,152 + 1,995.5 = 3,147.5 W ; Input = 50 × 400 = 20,000 W

Output = 20,000 − 3,147.5 = **16,852.5 W = 16.8 kW**

F.L. efficiency = 16,852.5/20,000 = 0.8426 or **84.26%**

Now, $E_{b1} = 400 - (3 \times 0.5) = 398.5$ V ; $E_{b2} = 400 - (48 \times 0.5) = 376$ V

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} = \frac{398.5}{376} \quad \therefore \quad \frac{N_1 - N_2}{N_2} = \frac{22.5}{376} = 0.0598$$

∴ percentage change in speed = **5.98**

Example 31.5. *The no-load test of a 44.76 kW, 220-V, d.c. shunt motor gave the following figures:*

Input current = 13.25 A ; field current = 2.55 A ; resistance of armature at 75°C = 0.032 Ω and brush drop = 2 V. Estimate the full-load current and efficiency.

(Electrical Engineering, Madras Univ. 1987)

Solution. No-load Condition

No-load input = 220 × 13.25 = 2915 W ; Armature current = 13.25 − 2.55 = 10.7 A

Armature Cu loss = $10.7^2 \times 0.032 = 3.6$ W

Loss due to brush drop = 2 × 10.7 = 21.4 W

Variable loss = 21.4 + 3.6 = 25 W_e . Constant losses W_c = 2915 − 25 = 2890 W

Full-load Condition

If I_a is the full-load armature current, then full-load motor input current is (I_a + 2.55) A.

F.L. motor power input = 220 (I_a + 2.55) W

This input must be equal to the sum of

(*i*) output = 44.76 kW = 44,760 W (*ii*) W_c = 2,890 W

(*iii*) brush loss = $2I_a$ watt (*iv*) Arm. Cu loss = $0.032\, I_a^2$

$\therefore \quad 220\,(I_a + 2.55) = 44{,}750 + 2{,}890 + 2I_a + 0.032\,I_a^2$

or $\quad 0.032I_a^2 - 218\,I_a + 47{,}090 = 0$

or $$I_a = \frac{218 \pm \sqrt{218^2 - 4 \times 0.032 \times 47{,}090}}{2 \times 0.032} = 223.5 \text{ A}$$

Line input current $I = I_a + I_{sh} = 223.5 + 2.55 =$ **226 A**

F.L. power input $= 226 \times 220 = 49{,}720$ W

$\therefore$ F.L. efficiency $= 44{,}760/49{,}720 = 0.9$ or **90%**.

Example 31.6. *A 200-V, shunt motor develops an output of 17.158 kW when taking 20.2 kW. The field resistance is 50 Ω and armature resistance 0.06 Ω. What is the efficiency and power input when the output is 7.46 kW ?* **(Elect. Machines-I, Aligarh Muslim Univ. 1989)**

Solution. In the first case :

Output $= 17{,}158$ W $\qquad$ Input $= 20{,}200$ W

Total losses $= 20{,}200 - 17{,}158 = 3{,}042$ W ; Input current $= 20{,}200/200 = 101$ A

$I_{sh} = 200/50 = 4$ A ; $I_a = 101 - 4 = 97$ A

$\therefore$ Armature Cu loss $= 97^2 \times 0.06 = 564.5$ W

$\therefore$ Constant losses $= 3{,}042 - 564.5 = 2{,}477.5 = 2478$ W (approx.)

In the second case :

Let, $\quad I_a =$ armature current $\qquad$ Input current $= (I_a + 4)$ A

Now, input power $=$ output $+ I_a^2 R_a +$ constant losses

$\therefore \quad 200(I_a + 4) = 7{,}460 + 0.06\,I_a^2 + 2{,}478$

or $\quad 0.06I_a^2 - 200I_a + 9{,}138 = 0$

$$\therefore \quad I_a = \frac{200 \pm \sqrt{200^2 - 4 \times 0.06 \times 9{,}138}}{2 \times 0.06} = \frac{200 \pm 194}{0.12} = 3{,}283.3 \text{ A or } 46 \text{ A}$$

We will reject the larger value because it corresponds to unstable operation of the motor. Hence, take $I_a = 46$ A.

$\therefore$ Input current $\quad I = I_a + I_{sh} = 46 + 4 = 50$ A

$$\text{Power input} = \frac{50 \times 200}{1000} = \mathbf{10\ kW} \quad \therefore \quad \eta = \frac{7{,}460 \times 100}{10{,}000} = \mathbf{74.6\%}$$

Example 31.7. *A 200-V, 14.92 kW dc shunt motor when tested by the Swinburne method gave the following results :*

Running light : armature current was 6.5 A and field current 2.2 A. With the armature locked, the current was 70 A when a potential difference of 3 V was applied to the brushes. Estimate the efficiency of the motor when working under full-load conditions.

(Electrical Engg.-I, Bombay Univ. 1985)

Solution. No-load input current $= 6.5 + 2.2 = 8.7$ A

No-load power input $= 200 \times 8.7 = 1{,}740$ W

No-load input equals Cu losses and stray losses.

Field Cu loss $= 200 \times 2.2 = 440$ W

Armature Cu loss $= 6.5^2 \times 0.04286 = 1.8$ W $\qquad (R_a = 3/70 = 0.04286\ \Omega)$

$\therefore$ Constant losses $= 1{,}740 - 1.8 = 1738$ W

We will assume that constant losses are the same at full-load also.

Let, $\quad I_a =$ full-load armature current

$$\text{F.L. armature Cu loss} = 0.04286\, I_a^2 \text{ W} ; \text{Constant losses} = 1{,}738 \text{ W}$$
$$\text{F.L. total loss} = 1{,}738 + 0.04286\, I_a^2$$
$$\text{F.L. output} = 14{,}920 \text{ W} ; \text{F.L. input} = 200\,(I_a + 2.2) \text{ W}$$

We know, $\quad \text{input} = \text{output} + \text{losses}$

or $\quad 200\, I_a + 440 = 14{,}920 + 1{,}738 + 0.04286\, I_a^2$

or $\quad 0.04286\, I_a^2 - 200\, I_a + 16{,}218 = 0 \quad \therefore \quad I_a = 82.5 \text{ A}$

$\therefore \quad \text{Input current} = 82.5 + 2.2 = 84.7 \text{ A}$

$$\text{F.L. power input} = 200 \times 84.7 \text{ A} = \mathbf{16{,}940 \text{ W}}$$

$\therefore \quad \eta = 14{,}920 \times 100/16{,}940 = \mathbf{88\%}$

Example 31.8. *In a test on a d.c. shunt generator whose full-load output is 200 kW at 250 V, the following figures were obtained :*

(a) When running light as a motor at full speed, the line current was 36 A, the field current 12 A, and the supply voltage 250.

(b) With the machine at rest, a p.d. of 6 V produced a current of 400 A through the armature circuit. Explain how these results may be utilised to obtain the efficiency of generator at full-load and half-load. Neglect brush voltage drop.

Solution. At no-load :

$$I_a = 36 - 12 = 24 \text{ A} ; R_a = 6/400 = 0.015\, \Omega$$

$\therefore \quad \text{Armature Cu loss} = 24^2 \times 0.015 = 8.64 \text{ watt}$

$$\text{No-load input} = \text{total losses in machine} = 250 \times 36 = 9{,}000 \text{ W}$$
$$\text{Constant losses} = 9{,}000 - 8.64 = \mathbf{8{,}991.4 \text{ W}}$$

At full-load :

$$\text{Output} = 200{,}000 \text{ W} ; \text{Output current} = 200{,}000/250 = 800 \text{ A} ; I_{sh} = 12 \text{ A}$$

$\therefore \quad \text{F.L. armature current} = 800 + 12 = 812 \text{ A}$

$\therefore \quad \text{F.L. armature Cu losses} = 812^2 \times 0.015 = 9890 \text{ W}$

$\therefore \quad \text{F.L. total losses} = 9890 + 8{,}991.4 = 18881 \text{ W} \quad \therefore \quad \eta = \dfrac{200{,}000 \times 100}{200{,}000 + 18{,}881} = \mathbf{91.4\%}$

At half-load :

$$\text{Output} = 100{,}000 \text{ W} ; \text{Output current} = 100{,}000/250 = 400 \text{ A}$$
$$I_a = 400 + 12 = 412 \text{ A} \quad \therefore \quad I_a^2 R_a = 412^2 \times 0.015 = 2{,}546 \text{ W}$$
$$\text{Total losses} = 8{,}991.4 + 2{,}546 = 11{,}537 \text{ W} \quad \therefore \quad \eta = \frac{100{,}000 \times 100}{111{,}537} = \mathbf{89.6\%}$$

Example 31.9. *A 250-V, 14.92 kW shunt motor has a maximum efficiency of 88% and a speed of 700 r.p.m. when delivering 80% of its rated output. The resistance of its shunt field is 100 Ω. Determine the efficiency and speed when the motor draws a current of 78 A from the mains.*

Solution. $\quad \text{Full-load output} = 14{,}920 \text{ W}$

$$80\% \text{ of F.L. output} = 0.8 \times 14{,}920 = 11{,}936 \text{ W} ; \eta = 0.88$$
$$\text{Input} = 11{,}936/0.88 = 13{,}564 \text{ W}$$
$$\text{Total losses} = 13{,}564 - 11{,}936 = 1{,}628 \text{ W}$$

As efficiency is maximum at this load, the variable loss is equal to constant losses.

$\therefore \quad W_c = I_a^2 R_a = 1{,}628/2 \quad \therefore \quad I_a^2 R_a = 814 \text{ W}$

Now, $\quad \text{input current} = 13{,}564/250 = 54.25 \text{ A}$

$\therefore \quad I_{sh} = 250/100 = 2.5 \text{ A} \quad \therefore \quad I_a = 54.25 - 2.5 = 51.75 \text{ A}$

$$\therefore \quad 51.75^2 R_a = 814 \quad \therefore \quad R_a = 814/51.75^2 = 0.3045\ \Omega$$

When input current is 78 A

$$I_a = 78 - 2.5 = 75.5 \text{ A} \quad \therefore \quad I_a^2 R_a = 75.5^2 \times 0.3045 = 1{,}736 \text{ W}$$

$$\text{Total losses} = 1{,}736 + 814 = 2{,}550 \text{ W} \;;\; \text{Input} = 250 \times 78 = 19{,}500 \text{ W}$$

$$\eta = \frac{19,500 - 2,550}{19,550} \times 100 = \mathbf{86.9\%}$$

Speed :

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{N_2}{700} = \frac{250 - (75.5 \times 0.3045)}{250 - (51.75 \times 0.3045)} = \frac{227}{234.25} \;;\; N_2 = \mathbf{680 \text{ r.p.m.}}$$

31.5. Regenerative or Hopkinson's Test (Back-to-Back Test)

By this method, *full-load test* can be carried out on two shunt machines, preferably identical ones, without wasting their outputs. The two machines are mechanically coupled and are so adjusted electrically that one of them runs as a motor and the other as a generator. The mechanical output of the motor drives the generator and the electrical output of generator is used in supplying the greater part of input to the motor. If there were no losses in the machines, they would have run without any external power supply. But due to these losses, generator output is not sufficient to drive the motor and *vice-versa*. The losses are supplied either by an extra motor which is belt-connected to the motor-generator set or as suggested by Kapp, electrically from the supply mains.

Essential connections for the test are shown in Fig. 31.6. The two shunt machines are connected in parallel. They are, to begin with, started as unloaded motors. Then, the field of one is weakened and that of the other is strengthened so that the former runs as a motor and the latter as a generator. The usual method of procedure is as follows :

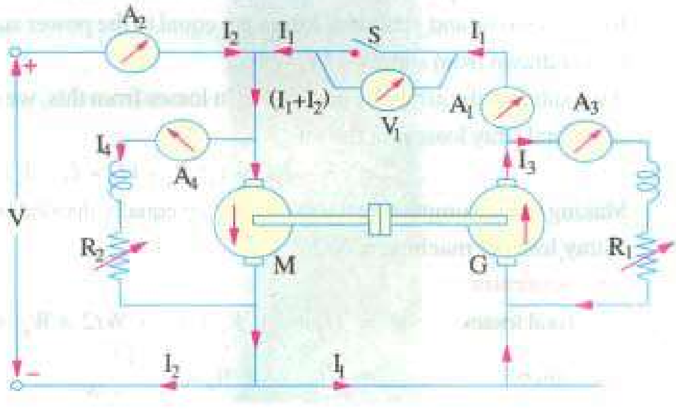
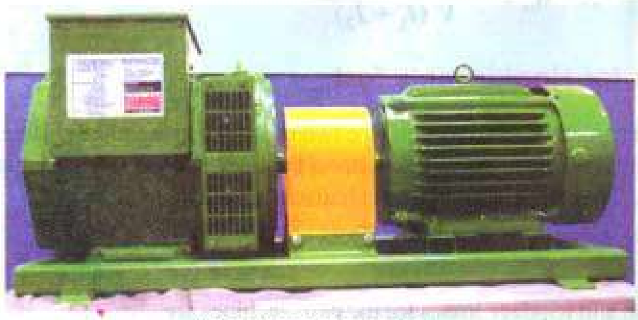


Fig. 31.6

Machine M is started up from the supply mains with the help of a starter (not shown) whereas main switch S of the other machine is kept open. Its speed is adjusted to normal value by means of its shield regulator. Machine M drives machine G as a generator and its voltage is read on voltmeter V_1. The voltage of G is adjusted by its field regulator until voltmeter V_1 reads zero, thereby showing that its voltage is the same, both in polarity and magnitude as that of the main supply. Thereafter, S is closed to parallel the machines. By adjusting the respective field regulators, any load can now be thrown on to the machines. Generator current I_1 can be adjusted to any desired value by increasing the excitation of G or by reducing the excitation of M and the corresponding values of different ammeters are read.



Motor-cum generator set

The electrical output of the generator ***plus*** the small power taken from the supply, is taken by the motor and is given out as a mechanical power ***after*** supplying the motor losses.

If supply voltage is V, then

Motor input $= V(I_1 + I_2)$, where I_2 is the current taken from the supply.

Generator output $= VI_1$ ...(*i*)

Assuming that both machines have the same efficiency η,

Output of motor $= \eta \times \text{input} = \eta\, V(I_1 + I_2) = \text{generator input}$

Output of generator $= \eta \times \text{input} = \eta \times \eta\, V(I_1 + I_2) = \eta^2\, V(I_1 + I_2)$ ...(*ii*)

Hence, from (*i*) and (*ii*), we get

$$\eta^2 V(I_1 + I_2) = VI_1 \quad \text{or} \quad \eta = \sqrt{\frac{I_1}{I_1 + I_2}}$$

However, it is not quite correct to assume equal efficiencies for two machines because their armature currents as well as excitations are different. We will not find the efficiencies separately.

Let R_a = armature resistance of each machine

I_3 = exciting current of the generator

I_4 = exciting current of the motor

Armature Cu loss in generator $= (I_1 + I_3)^2 R_a$; Armature Cu loss in motor $= (I_1 + I_2 - I_4)^2 R_a$

Shunt Cu loss in generator $= VI_3$; Shunt Cu loss in motor $= VI_4$

But total motor and generator losses are equal to the power supplied by the mains.

Power drawn from supply $= VI_2$

If we subtract the armature and shunt Cu losses from this, we get the stray losses of *both* machines.

$\therefore$ Total stray losses for the set

$$= VI_2 - [(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4] = \text{W (say)}$$

Making one assumption that stray losses are equally divided between the two machines, we have

Stray loss per machine = W/2

For Generator

Total losses $= (I_1 + I_3)^2\, R_a + VI_3 + \text{W}/2 = W_g$ (say)

Output $= VI_1 \quad \therefore \quad \eta_g = \dfrac{VI_1}{VI_1 + W_g}$

Total losses $= (I_1 + I_2 - I_4)^2 R_a + VI_d + \text{W}/2 = W_m$ (say)

Input $= V(I_1 + I_2) \quad \therefore \quad \eta_m = \dfrac{V\,(I_1 + I_2) - W_m}{V\,(I_1 + I_2)}$

31.6. Alternative Connections for Hopkinson's Test

In Fig. 31.7 is shown in slightly different method of connecting the two machines to the supply. Here, the main difference is that the shunt windings are directly connected across the lines. Hence, the line input current is I_1 ***excluding the field currents***. The efficiencies can be calculated as detailed below :

Motor armature Cu loss $= (I_1 + I_2)^2 R_a$; Generator armature Cu loss $= I_2^2 R_a$

Power drawn from the supply $= V I_1$

$\therefore$ Total stray losses *i.e.* iron, friction and windage losses for the two machines are

$$= VI_1 - [(I_1 + I_2)^2\, R_a - I_2^2\, R_a] = \text{W (say)}$$

∴ stray loss for each machine = W/2

Motor Efficiency

$$\text{Motor input} = \text{armature input} + \text{shunt field input} = V(I_1 + I_2) + VI_3 + W_{input}$$

$$\text{Motor losses} = \text{armature Cu loss} + \text{shunt Cu loss} + \text{stray losses}$$

$$= (I_1 + I_2)^2 R_a + VI_3 + W/2 = W_m \text{ (say)}$$

$$\text{Motor} \quad \eta = \frac{W_{input} - W_m}{W_{input}} \times 100$$

Generator Efficiency

$$\text{Generator output} = VI_2 \text{ ; Generator losses} = I_2^2 R_a + VI_4 + W/2 = W_g \text{ (say)}$$

$$\therefore \text{Generator} \quad \eta = \frac{VI_2}{VI_2 + W_g}$$

31.7. Merits of Hopkinson's Test

1. Power required for the test is small as compared to the full-load powers of the two machines.

2. As machines are being tested under full-load conditions, the temperature rise and the commutation qualities of the machines can be observed.

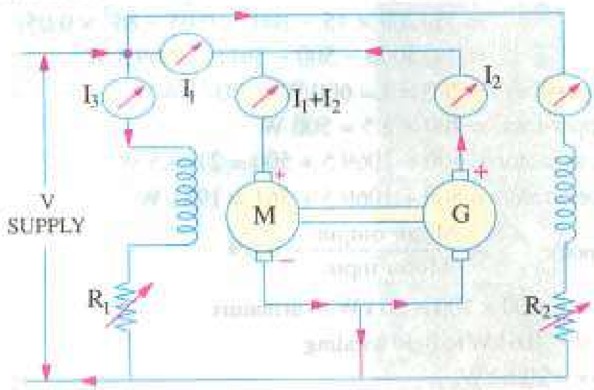


Fig. 31.7

3. Because of full-load conditions, any change in iron loss due to flux distortion at full-load, is being taken into account.

The only disadvantage is with regard to the availability of two identical machines.

Example 31.10 (*a*). *In a Hopkinson's test on two 220-V, 100-kW generators, the circulating current is equal to the full-load current and, in addition, 90 A are taken from the supply. Obtain the efficiency of each machine.*

Solution. Output current of the generator

$$I_1 = \frac{100,000}{220} = \frac{5,000}{11} = 454.4 \text{ A}, I_2 = 90 \text{ A}$$

Assuming equal efficiencies, from Art. 29.5, we have

$$\eta = \sqrt{\frac{I_1}{I_1 + I_2}} = \sqrt{\frac{454.5}{454.5 + 90}} = 0.914 \text{ or } \mathbf{91.4\%}$$

Example 31.10 (*b*). *In the Hopkinson's test on two d.c. machines, machine A has a field current of 1.4 A and machine B has a field current of 1.3 A. Which machine acts as a generator ?*

(Bharathithasan University April 1997)

Solution. In Hopkinson's test (on two identical d.c. shunt machines), since the two machines are coupled, the speed is common and is decided by the field current of the motor. The field windings of both the machines are in parallel with a separate D.C. source.

Since the machines are identical and are running at the *same speed*, their e.m.fs are in proportion to their field currents.

$$\frac{\text{E.M.F. induced in the armature of machine A}}{\text{E.M.F. induced in the armature of machine B}} = \frac{1.4}{1.3}$$

$$E_A = (1.4/1.3) \times E_B = 1.077\ E_B$$

Since E_A is larger than E_B, Machine A supplies power to Machine B. It means, A is working as a generator, and B is motoring.

Example 31.11. *Two shunt machines loaded for the Hopkinson's test take 15 A at 200 V from the supply. The motor current is 100 A and the shunt currents are 3 A and 2.5 A. If the armature resistance of each machine is 0.05 ohm, calculate the efficiency of each machine for this particular load-condition.* **(Bharathithasan Univ. April 1997)**

Solution. Line current into armature circuits = 15 A, Motor armature copper-loss = 500 W

Motor-armature-current = 100 A, Generator armature copper loss = 361 W

Hence generator-armature-current = 85 A

For each machine, No load Mechanical losses + Core-loss + Stray losses

$$= ½(VI_a - I^2_{am}\, r_{am} - I^2_{ag}\, r_{ag})$$
$$= ½\,(200 \times 15 - 100^2 \times 0.05 - 85^2 \times 0.05)$$
$$= ½\,(3000 - 500 - 361) = 1069.5\ \text{W}$$

Motor field copper-loss $= 200 \times 3 = 600\ \text{W} \equiv 1.07\ \text{kW}$

Generator field copper-loss $= 200 \times 2.5 = 500\ \text{W}$

Total Losses in motor $= 600 + 1069.5 + 500 = 2169.5\ \text{W}$

Total Losses in Generator $= 500 + 1069.5 + 361 = 1931\ \text{W}$

$$\text{Efficiency of motor} = \frac{\text{Motor output}}{\text{Motor input}} \times 100\%$$

Motor Input : (*a*) $200 \times 100 = 20$ kW to armature

(*b*) 0.6 kW to field winding

Total Input to motor = 20.6 kW

From armature side, losses to be catered are :

(*i*) Stray losses + No Load Mech. Losses + Core Losses = 1.07 kW

(*ii*) Armature copper-loss = 0.5 kW

Motor Output from armature = 20 − 0.5 − 1.07 = 18.43 kW

$$\text{Motor efficiency} = \frac{18.43}{20.6} \times 100\% = 89.47\%$$

Generator armature output $= 200 \times 85 \times 10^{-3} = 17$ kW

Generator losses : (*a*) Field wdg : 0.5 kW

(*b*) Total no-load-losses : 1.07 kW

(*c*) armature copper-loss = 0.36 kW

Total losses in Generator = 1.93 kW

$$\text{Generator efficiency} = \frac{17}{17 + 1.93} \times 100\% \ = \mathbf{89.80\%}$$

Special Note: 15 A current for d.c. supply is related here to armature-input for two machines which are under back-to-back regenerative tests. There are different variations in handling and giving the test data. It is always desirable to draw the circuit diagram according to which the calculations are being related.

Example 31.12. *The Hopkinson's test on two similar shunt machines gave the following full-load data :*

Line voltage	= *110 V*	*Field currents are 3 A and 3.5 A*
Line current	= *48 A*	*Arm. resistance of each is 0.035 Ω*
Motor arm. current	= *230 A*	

Calculate the efficiency of each machine assuming a brush contact drop of 1 volt per brush.

(Electrical Machines, Nagpur Univ. 1992)

Solution. The motor-generator set is shown in Fig. 31.8. It should also be noted that the machine with lesser excitation is motoring. We will find the total armature Cu losses and brush contact loss for both machines.

Motor

Arm. Cu loss $= 230^2 \times 0.035 = 1{,}851.5$ W

Brush contact loss $= 230 \times 2 = 460$ W

Total arm. Cu loss $= 1851.5 + 460 = 2{,}312$ W

Shunt Cu loss $= 110 \times 3 = 330$ W

Total Cu loss $= 2{,}312 + 330 = 2{,}642$ W

Generator

Generator arm. current $= 233 - 48 + 3.5$

$= 188.5$ W

Arm. Cu loss $= 188.5^2 \times 0.035 = 1{,}244$ W

Brush contact Cu loss $= 188.5 \times 2 = 377$ W

Total arm. Cu loss $= 1{,}244 + 377 = 1{,}621$ W

Shunt Cu loss $= 110 \times 3.5 = 385$ W ;

Total Cu loss $= 1{,}621 + 385 = 2{,}006$ W

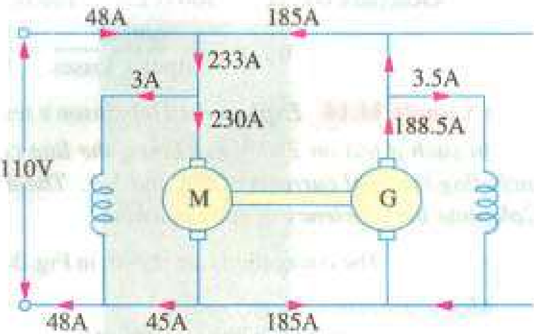


Fig. 31.8

For the Set

Total arm. and shunt Cu loss for the set

$= 2{,}642 + 2{,}006 = 4{,}648$ W

Total input $= 110 \times 48 = 5{,}280$ W ; Stray losses for the set $= 5{,}280 - 4{,}648 = 632$ W

Stray losses per machine $= 632/2 = 316$ W

Motor Efficiency

Arm. Cu + brush drop loss $= 2{,}312$ W   Shunt Cu loss $= 330$ W

Stray losses $= 316$ W   Total loss $= 2{,}312 + 330 + 316 = 2{,}958$ W

Motor input $= 110 \times 233 = 25{,}630$ W ; Motor output $= 25{,}630 - 2{,}958 = 22{,}672$

$\therefore \quad \eta = 22{,}672 \times 100/25{,}630 =$ **88.8%**

Generator Efficiency

Total losses $= 2{,}006 + 316 = 2{,}322$ W ; Output $= 110 \times 185 = 20{,}350$ W

Generator input $= 20{,}350 + 2{,}322 = 22{,}672$ W = motor input

$\eta = 20{,}350/22{,}672 = 0.894$ or **89.4%**

Example 31.13. *In a Hopkinson's test on a pair of 500-V, 100-kW shunt generators, the following data was obtained :*

Auxiliary supply, 30 A at 500 V : Generator output current, 200 A

Field currents, 3.5 A and 1.8 A

Armature circuit resistances, 0.075 Ω each machine. Voltage drop at brushes, 2 V (each machine).

Calculate the efficiency of the machine acting as a generator.

(Elect. Technology-1, Gwalior Univ. 1986)

Solution. Motor arm. current = 200 + 30 = 230 A, as shown in Fig. 31.9.

Motor arm. Cu loss $= 230^2 \times 0.075 + 230 \times 2 = 4{,}428$ W

Motor field Cu loss $= 500 \times 1.8 = 900$ W

Generator arm. Cu loss $= 200^2 \times 0.075 + 200 \times 2 = 3{,}400$ W

Geneator field Cu loss $= 500 \times 3.5 = 1{,}750$ W

Total Cu loss for two machines

$= 4{,}428 + 900 + 3400 + 1750$

$= 10{,}478$ W

Power taken from auxiliary supply

$= 500 \times 30 = 15{,}000$ W

Stray losses for the two machines

$= 15{,}000 - 10{,}478 = 4{,}522$ W

Stray loss per machine = 4,522/2 = 2,261 W

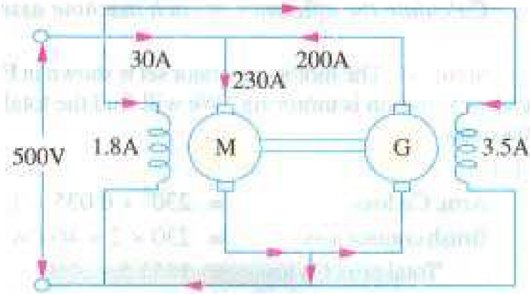


Fig. 31.9

Total losses in generator = 3400 + 1750 + 2261 = 7,411 W

Generator output = 500 × 200 = 100,000 W

$$\therefore \quad \eta_g = \frac{\text{output}}{\text{output + losses}} = \frac{100{,}000}{107{,}411} \times 100 = \mathbf{93.09\%}$$

Example 31.14. *Explain the Hopkinson's test on a pair of shunt motors.*

In such a test on 250-V machines, the line current was 50 A and the motor current 400 A not including the field currents of 6 A and 5 A. The armature resistance of each machine was 0.015 Ω. Calculate the efficiency of each machine. **(Adv. Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. The connections are shown in Fig. 31.10.

Motor armature Cu loss

$= 400^2 \times 0.015 = 2{,}400$ W

Generator armature Cu loss

$= 350^2 \times 0.015 = 1{,}838$ W

Power drawn from supply

$= 250 \times 50 = 12{,}500$ W

∴ Iron, friction and windage losses for the two machines

$= 12{,}500 - (2{,}400 + 1{,}838)$

$= 8{,}262$ W

Iron, friction and windage loss per machine

$= 8.262/2 = 4{,}130$ W* (approx.)

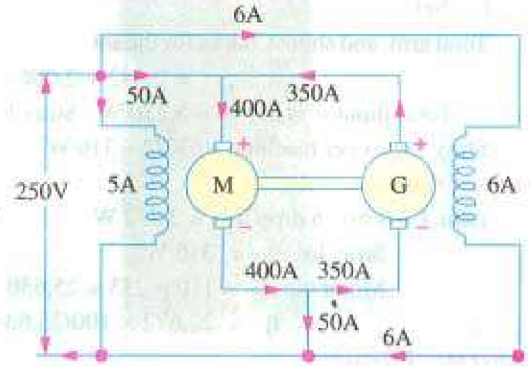


Fig. 31.10

Motor Losses and Efficiency

Motor arm. Cu loss = 2,400 W ; Motor field Cu loss = 250 × 5 = 1,250 W

Iron, friction and windage losses = 4,130 W

Total motor losses = 2,400 + 1,250 + 4,130 = 7,780 W

Motor input = 250×400 + 250×5 = 101,250 W

∴ Motor efficiency = (101,250 − 7,780)/101,250 = 0.923 or **92.3%**

* We could also get this value as follows :

Total supply input = 250 × 61 = 15,250 W ; Gen. and motor field Cu loss = 250 × 6 + 250 × 5 = 2,750 W

Iron, friction and windage losses for both machines

= 15,250 − (2,400 + 1,838 + 2,750) = **8,262 W** – as before

Generator Losses and Efficiency

Generator arm. Cu loss = 1,838 W ; Generator field Cu loss = $250 \times 6 = 1{,}500$ W

Iron, friction and windage loss = 4,130 W

Total losses = 1,838 + 1,500 + 4,130 = 7.468 W

Generator output = $250 \times 350 = 87{,}500$ W

$\therefore$ Generator efficiency = (87,500 − 7.468)/87,500 = 0.915 or **91.5%**

Example 31.15. *The Hopkinson's test on two shunt machines gave the following results for full-load :*

Line voltage = 250 V ; current taken from supply system excluding field currents = 50 A ; motor armature current = 380 A ; field currents 5 A and 4.2 A. Calculate the efficiency of the machine working as a generator. Armature resistance of each machine is 0.2 Ω.

(Electrical Machinery-I Mysore Univ. 1988)

Solution. The connections are shown in Fig. 31.11.

Motor arm. Cu loss = $380^2 \times 0.02 = 2{,}888$ W

Generator arm. Cu loss = $330^2 \times 0.02 = 2{,}178$ W

Power drawn from supply = $250 \times 50 = 12{,}500$ W

Stray losses for the two machines

= 12,500 − (2,888 + 2,178) = 7.434 W

Stray losses per machine = 7,434/2 = 3,717 W

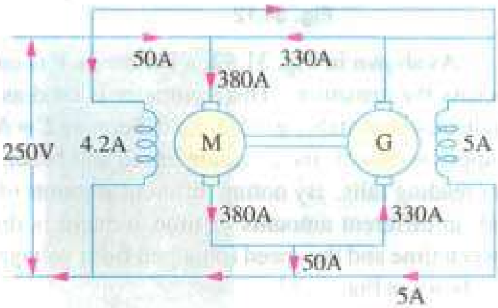


Fig. 31.11

Motor Efficiency

Arm. Cu loss = 2,888 W

Field Cu loss = $250 \times 4.2 = 1050$ W

Stray losses = 3,717 W

Total loss = 2,888 + 1050 + 3,717

= 7,655 W

Motor input = $250 \times 380 + 250 \times 4.2$

= 96,050 W

Motor output = 96,050 − 7,655

= 88,395 W

$\therefore$ η = 88,395/96,050 = 0.9203 or **92.03%**

Generator Efficiency

Arm. Cu loss = 2,178 W ; Field Cu loss = $250 \times 5 = 1250$ W

Stray losses = 3,717 W ; Total losses = 7,145 W

Generator output = $250 \times 330 = 82{,}500$ W

Generator input = 82,500 + 7,145 = 89,645 W

η = 82,500/89,645 = 0.9202 or **92.02%**

31.8. Retardation or Running Down Test

This method is applicable to shunt motors and generators and is *used for finding stray losses*. Then, knowing the armature and shunt Cu losses at a given load current, efficiency can be calculated.

The machine under test is speeded up slightly beyond its normal speed and then supply is cut off from the armature while keeping the field excited. Consequently, the armature slows down and its kinetic energy is used to meet the *rotational losses i.e.* friction, windage and iron losses.*

Kinetic energy of the armature is $K.E. = \frac{1}{2} I\omega^2$

where I = moment of inertia of the armature and ω = angular velocity

$\therefore$ Rotational losses, W = Rate of loss of $K.E.$

---

* If armature slows down with no excitation, then energy of the armature is used to overcome mechanical losses only, there being no iron losses (see Ex. 31.19).

$$\therefore \qquad W = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega.\frac{d\omega}{dt}$$

Two quantities need be known (*i*) moment of inertia (*I*) of the armature and (*ii*) $\frac{d\omega}{dt}$ or $\frac{dN}{dt}$ because $\omega \propto N$. These are found as follows :

(*a*) Finding $\frac{d\omega}{dt}$

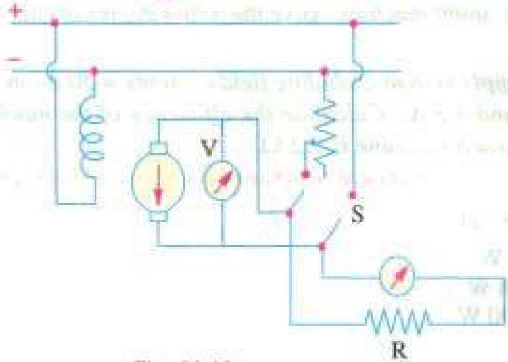


Fig. 31.12

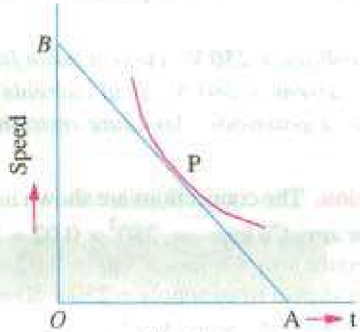


Fig. 31.13

As shown in Fig. 31.12, a voltmeter *V* is connected across the armature. This voltmeter is used as a speed indicator by suitably graduating it, because $E \propto N$. When supply is cut off, the armature speed and hence voltmeter reading falls. By noting different amounts of voltage fall in different amounts of time, a curve is drawn between time and the speed (obtained from voltage values) as shown in Fig. 31.13.

From any point *P* which corresponds to normal speed, a tangent *AB* is drawn.

$$\text{Then} \qquad \frac{dN}{dt} = \frac{OB \text{ (in r.p.m.)}}{OA \text{ (in seconds)}}$$

Shunt wound generator

$$\text{From } (i), \text{ above} \qquad W = I\,\omega\frac{d\omega}{dt}$$

$$\text{Now} \qquad \omega = \frac{2\pi N}{60} \qquad \text{...(}N\text{ in r.p.m.)}$$

$$W = I\left(\frac{2\pi N}{60}\right)\times\frac{d}{dt}\left(\frac{2\pi N}{60}\right);\; W = \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt} = 0.011\,I.N.\frac{dN}{dt} \qquad \text{...(ii)}$$

(*ii*) Finding Moment of Inertia (I)

(*a*) First Method–*where I is calculated.*

First, slowing down curve is drawn with armature alone. Next, a fly-wheel of known moment of inertia I_1 is keyed onto the shaft and slowing down curve is drawn again. Obviously, slowing down time will be longer due to combined increased moment of inertia of the two. For any given speed, (dN/dt_1) and (dN/dt_2) are determined as before. It should be noted that the losses in both cases would be almost the same, because addition of a fly-wheel will not make much difference to the losses.

Hence, from equation (*ii*) above

$$\text{In the first case,} \qquad W = \left(\frac{2\pi}{60}\right)^2 IN\left(\frac{dN}{dt_1}\right)$$

In the second case, $W = \left(\frac{2\pi}{60}\right)^2 (I + I_1)N \times \left(\frac{dN}{dt_2}\right)$

$$\therefore \quad (I + I_1)\left(\frac{dN}{dt_2}\right) = I\left(\frac{dN}{dt_1}\right) \text{ or } \left(\frac{I + I_1}{I}\right) = \left(\frac{dN}{dt_1}\right) \Big/ \left(\frac{dN}{dt_2}\right)$$

$$\therefore \quad I = I_1 \times \frac{(dN/dt_2)}{(dN/dt_1) - (dN/dt_2)} = I_1 \frac{dt_1}{dt_2 - dt_1}; \; I = I_1 \frac{t_1}{t_2 - t_1}$$

(b) Second Method–*where I is eliminated.*

In this method, first, time taken to slow down, say by 5%, is noted with armature alone. Next, a retarding torque–mechanical or preferably electrical, is applied to the armature and again time is noted. The method using electrical torque is shown in Fig. 31.12. The double-throw switch S while cutting off the armature from supply, automatically joins it to a non-inductive resistance R as shown. The power drawn by this resistance acts as a retarding torque on the armature, thereby making it slow down comparatively quickly. The additional loss is $I_a^2 (R_a + R)$ or VI_a,

where I_a = *average* current through R ; V = *average* voltage across R.

Let W' be this power. Then from (*i*) above

$$\left.\begin{aligned} W &= \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt_1} \\ W + W' &= \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt_2} \end{aligned}\right\} \text{If } dN \text{ is the same}$$

$$\therefore \quad \frac{W + W'}{W} = \frac{dt_1}{dt_2} \qquad \therefore \quad W = W' \times \frac{dt_2}{dt_1 - dt_2} \text{ or } W = W' \times \frac{t_2}{t_1 - t_2}$$

where $\frac{dN}{dt_1}$ = rate of change of speed ***without*** extra load

$\frac{dN}{dt_2}$ = rate of change of speed ***with*** extra electrical load.

Example 31.16. *In a retardation test on a separately-excited motor, the induced e.m.f. in the armature falls from 220 V to 190 V in 30 seconds on disconnecting the armature from the supply. The same fall takes place in 20 seconds if, immediately after disconnection, armature is connected to a resistance which takes 10 A (average) during this fall. Find stray losses of the motor.*

(Adv. Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution. Let W = stray losses (mechanical and magnetic losses)

Average voltage across resistance = (200 + 190)/2 = 195 V, Average current = 10 A

$\therefore$ Power absorbed W' = 1950 W

Using the relation $\frac{W}{W'} = \frac{t_2}{t_1 - t_2}$; we get $W = 1950 \times \frac{20}{30 - 20}$ = **3,900 watt**

Example 31.17. *In a retardation test on a d.c. motor, with its field normally excited, the speed fell from 1525 to 1475 r.p.m. in 25 seconds. With an average load of 1.0 kW supplied by the armature, the same speed drop occurred in 20 seconds. Find out the moment of inertia of the rotating parts in kg.m*^{2}.

(Electrical Machines-III, Gujarat Univ. 1984)

Solution. As seen from Art. 31.8 (*ii*) (*b*).

$$W' = \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt}. \text{ Also } W = W' \times \frac{t_2}{t_1 - t_2}$$

Here, W' = 1 kW = 1000 W, t_1 = 25 second, t_2 = 20 second

$\therefore \quad W = 1000 \times 20/(25 - 20) = 4000$ W

Now, $\quad N = 1500$ r.p.m. (average speed) ; $dN = 1525 - 1475 = 50$ r.p.m. ; $dt = 25$

$\therefore \quad 4000 = (2\pi/60)^2 I.1500 \times 50/25 \quad \therefore \quad I = \mathbf{121.8\ kg.m^2}$.

Example 31.18. *A retardation test is made on a separately-excited d.c. machine as a motor. The induced voltage falls from 240 V to 225 V in 25 seconds on opening the armature circuit and 6 seconds on suddenly changing the armature connection from supply to a load resistance taking 10 A (average). Find the efficiency of the machines when running as a motor and taking a current of 25 A on a supply of 250 V. The resistance of its armature is 0.4 Ω and that of its field winding is 250 Ω.*

(Elect. Technology, Allahabad Univ. 1991)

Solution. Average voltage across load

$$= (240 + 225)/2 = 232.5 \text{ V} ; I_{av} = 10 \text{ A}$$

$\therefore$ Power absorbed $W' = 232.5 \times 10 = 2{,}325$ W

and $\quad t_1 = 30$ second, $t_2 = 6$ second ; W = stray loss

Using $\quad \dfrac{W}{W'} = \dfrac{t_2}{t_1 - t_2} = \mathbf{734.1\ W}$, we get

Stray losses $\quad W = 2325 \times \dfrac{6}{25 - 6} = \mathbf{734.1\ W}$

Input current $= 25$ A ; $I_{sh} = 250/250 = 1$ A ; $I_a = 25 - 1 = 24$ A

Armature Cu loss $= 24^2 \times 0.4 = 230.4$ W ; Shunt Cu loss $= 250 \times 1 = 250$ W

$\therefore$ Total losses $= 734.1 + 230.4 + 250 = 1{,}215$ W (approx.)

Input $= 250 \times 25 = 6{,}250$ W ; Output $= 6{,}250 - 1{,}215 = 5{,}035$ W

$\therefore \quad \eta = 5{,}035/6{,}250 = 0.806$ or **80.6%**

Example 31.19. *A retardation test is carried out on a 1000 r.p.m. d.c. machine. The time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m. is :*

(a) 36 seconds with no excitation

(b) 15 seconds with full excitation and

(c) 9 seconds with full excitation and the armature supplying an extra load of 10 A at 219 V.

Calculate (i) the moment of inertia of the armature in kg. m² (ii) iron losses and (iii) the mechanical losses at the mean speed of 1000 r.p.m.

Solution. It should be noted that

(*i*) when armature slows down with no excitation, its kinetic energy is used to overcome mechanical losses only ; because due to the absence of flux, there is no iron loss.

(*ii*) with excitation, kinetic energy is used to supply mechanical and iron losses collectively known as stray losses.

(*iii*) If I is taken in kg-m² unit, then rate of loss of energy is in watts.

Mechanical loss $\quad W_m = \left(\dfrac{2\pi}{60}\right)^2 I.N.\dfrac{dN}{dt}$ ...Art. 31.8

Here $\quad dN = 1030 - 970 = 60$ r.p.m., $dt = 36$ seconds, $N = 1000$ r.p.m.

$$W_m = \left(\frac{2\pi}{60}\right)^2 I.N.\frac{60}{36} \quad \text{...(i)}$$

Similarly $\quad W_s = \left(\dfrac{2\pi}{60}\right)^2 I.N.\dfrac{60}{15}$ ...(*ii*)

Also $W_s \quad = W'\dfrac{t_2}{t_1 - t_2} = 219 \times 10 \times \dfrac{9}{15 - 9} = 3{,}285$ W

Using equation (*ii*), we get

$$3{,}285 = \left(\frac{2\pi}{60}\right)^2 \times I \times 1000 \times \frac{60}{15}$$

(*i*) $\therefore$ $I = 75$ **kg.m²**

Dividing (*i*) by (*ii*), we get $\frac{W_m}{W_s} = \frac{15}{36}$

(*ii*) $\therefore$ $W_m = 3{,}285 \times 15/36 =$ **1,369 W**

(*iii*) $\therefore$ Iron losses $= W_s - W_m = 3{,}285 - 1{,}369 =$ **1,916 W**

31.9. Field's Test for Series Motor

This test is applicable to two similar *series motors*. Series motors which are mainly used for traction work are easily available in pairs. The two machines are coupled mechanically.

One machine runs normally as a motor and drives generator whose output is wasted in a variable load R (Fig. 31.14). Iron and friction losses of two machines are made equal (*i*) by joining the series field winding of the generator in the motor armature circuit so that both machines are equally excited and (*ii*) by running them at equal speed. Load resistance R is varied till the motor current reaches its full-load value indicated by ammeter A_1. After this adjustment for full-load current, different ammeter and voltmeter readings are noted.

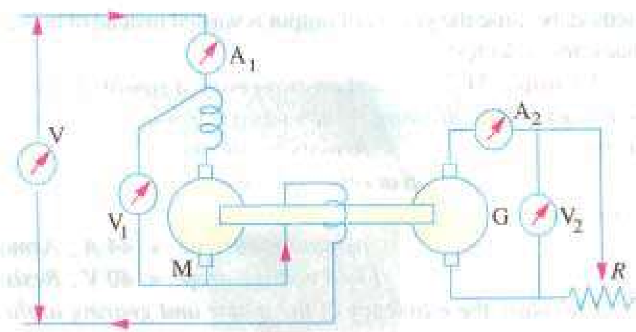


Fig. 31.14

Micro series motors

Let V = supply voltage ; I_1 = motor current ; V_2 = terminal p.d. of generator ; I_2 = load current.

$\therefore$ Intake of the whole set $= VI_1$; output $= V_2I_2$.

Total losses in the set, $W_t = VI_1 - V_2I_2$

Armature and field Cu losses $W_{cu} = (R_a + 2R_{se})I_1^2 + I_2^2R_a$

where R_a = hot armature resistance of each machine

R_{se} = hot series field resistance of each machine

$\therefore$ Stray losses for the set $= W_t - W_{cu}$

Stray losses per machine $W_s = \frac{W_t - W_{cu}}{2}$

Stray losses are equally divided between the machines because of their equal excitation and speed.

Motor Efficiency

Motor input $= V_1I_1$

Motor losses = armature + field Cu losses + stray losses

$= (R_a + R_{se})I_1^2 + W_s = W_m$ (say)

$$\eta_m = \frac{V_1I_1 - W_m}{V_1I_1}$$

Generator Efficiency

The generator efficiency will be of little use because it is running under abnormal conditions of *separate* excitation. However, the efficiency under these unusual conditions can be found if desired.

Generator output $= V_2I_2$

Field Cu loss $= I_1^2R_{se}$ (∵ Motor current is passing through it.)

Armature Cu loss $= I_2^2R_a$; Stray losses $= W_s$

Total losses $= I_1^2R_{se} + I_2^2R_a + W_s = W_g$ (say)

$$\eta_g = \frac{V_2 I_2}{V_2I_2 + W_g}$$

It should be noted that although the two machines are mechanically coupled yet it is not a regenerative method, because the generator output is wasted instead of being fed back into the motor as in Hopkinson's (back-to-back) test.

Example 31.20. *A test on two coupled similar tramway motors, with their fields connected in series, gave the following results when one machine acted as a motor and the other as a generator.*

Motor : *Armature current = 56 A ; Armature voltage = 590 V*

Voltage drop across field winding = 40 V

Generator :

Armature current = 44 A ; Armature voltage = 400 V

Field voltage drop = 40 V ; Resistance of each armature = 0.3 Ω

Calculate the efficiency of the motor and gearing at this load.

(Elect. Machinery-II, Nagpur Univ. 1992 & JNTU, Hyderabad, 2000)

Solution. The connection for the two machines are shown in Fig. 31.15.

Total input $= 630 \times 56 = 35{,}280$ W

Output $= 400 \times 44 = 17{,}600$ W

Total losses in the two machines are

$= 35{,}280 - 17{,}600 = 17{,}680$ W

Series field resistance

$R_{se} = 40/56$

$= 0.714\ \Omega$

Total Cu loss $= (0.3 + 2 \times 0.714) \times 56^2 + 44^2 \times 0.3 = 5{,}425 + 581 = 6{,}006$ W

Stray losses of the set $= 17{,}680 - 6{,}006$

$= 11{,}674$ W

∴ Stray losses/machine $= 11{,}674/2 = 5{,}837$ W

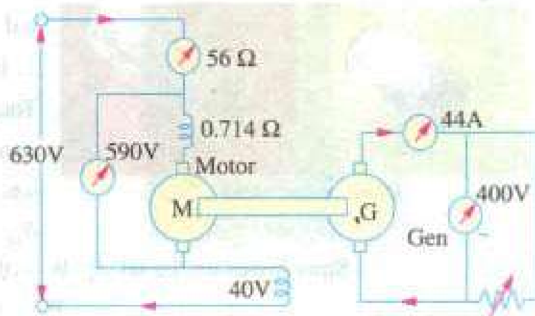


Fig. 31.15

Motor Efficiency

Motor armature input = arm. voltage × motor current $= 590 \times 56 = 33{,}040$ W

Armature circuit Cu loss

$= (0.3 + 0.714) \times 56^2 = 3{,}180$ W

Stray loss $= 5{,}837$ W – found above

Total losses $= 3{,}180 + 5{,}837 = 9{,}017$ W, Output $= 33{,}040 - 9{,}017 = 24{,}023$ W

∴ $\eta_m = 24{,}023/33{,}040 = 0.727$ or **72.7%**

Generator Efficiency

$$\text{Armature Cu loss} = 44^2 \times 0.3 = 581 \text{ W, Series field Cu loss} = 40 \times 56 = 2{,}240 \text{ W}$$

$$\text{Stray losses} = 5{,}837 \text{ W ; Total losses} = 581 + 2{,}240 + 5{,}837 = 8{,}658 \text{ W}$$

$$\text{Output} = 400 \times 44 = 17{,}600 \text{ W}$$

$$\therefore \quad \eta_g = 17{,}600/(17{,}600 + 8{,}658) = 0.67 \text{ or } 67\%$$

Tutorial Problem No. 31.1

1. A 500-V, shunt motor takes a total current of 5 A when running unloaded. The resistance of armature circuit is 0.25 Ω and the field resistance is 125 Ω. Calculate the efficiency and output when the motor is loaded and taking a current of 100 A. **[90.4% ; 45.2 kW]**

2. A d.c. shunt motor rated at 12.5 kW output runs at no-load at 1000 r.p.m. from a 250-V supply consuming an input current of 4 A. The armature resistance is 0.5 Ω and shunt field resistance is 250 Ω. Calculate the efficiency of the machine when delivering full-load output of 12.5 kW while operating at 250 V. **[81.57%]** (*Elect. Technology-I Madras Univ. 1979*)

3. The following results were obtained during Hopkinson's test on two similar 230-V machines; armature currents 37 A and 30 A; field currents 0.85 A and 0.8 A. Calculate the efficiencies of machines if each has an armature resistance of 0.33 Ω. **[Generator 87.9%, Motor 87.7%]**

4. In a Field's test on two 230-V, 1.492 kW mechanically-coupled similar series motors, the following figures were obtained. Each has armature and compole winding resistance of 2.4 Ω, series field resistance of 1.45 Ω and total brush drop of 2 V. The p.d. across armature and field was 230 V with a motor current of 10.1 A. The generator supplied a current of 8.9 A at a terminal p.d. of 161 V. Calculate the efficiency and output of the motor for this load. **[76.45%, 1.775 kW]**

5. Describe the Hopkinson's test for obtaining the efficiency of two similar shunt motors. The readings obtained in such a test were as follows ; line voltage 100 V ; motor current 30 A ; generator current 25 A ; armature resistance of each machine 0.25 Ω. Calculate the efficiency of each machine from these results, ignoring the field currents and assuming that their iron and mechanical losses are the same. **[Motor 90.05%, Generator 92.5%]**

6. The Hopkinson's test on two similar d.c. shunt machines gave the following results :

Line voltage = 220 V ; line current excluding field currents = 40 A ; the armature current of motoring machine = 200 A ; field currents 6 A and 7 A. Calculate the efficiency of each of the machines at the given load conditions. The armature resistance of each of the machines is 0.05 Ω.

[η_m = 86.58% ; η_g = 86.3%] (***Electrical Engg-I, M.S. Univ. Baroda 1980***)

OBJECTIVE TEST – 31

1. One of the main advantages of Swinburne's test is that it
   (*a*) is applicable both to shunt and compound motors
   (*b*) needs one running test
   (*c*) is very economical and convenient
   (*d*) ignores any charge in iron loss

2. The main disadvantage of Hopkinson's test for finding efficiency of shunt d.c. motors is that it
   (*a*) requires full-load power
   (*b*) ignores any change in iron loss
   (*c*) needs one motor and one generator
   (*d*) requires two identical shunt machines

3. The most economical method of finding no-load losses of a large d.c. shunt motor is—test.
   (*a*) Hopkinson's
   (*b*) Swinburne's
   (*c*) retardation
   (*d*) Field's

4. Retardation test on a d.c. shunt motor is used for finding—losses.
   (*a*) stray
   (*b*) copper
   (*c*) friction
   (*d*) iron

5. The main thing common between Hopkinson's test and Field's test is that both
   (*a*) require two electrically-coupled series motors
   (*b*) need two similar mechanically-coupled motors
   (*c*) use negligible power
   (*d*) are regenerative tests

6. The usual test for determining the efficiency of a traction motor is the .... test.
   (*a*) Field's
   (*b*) retardation
   (*c*) Hopkinson's
   (*d*) Swinburne's

ANSWERS

1. *c* **2.** *d* **3.** *b* **4.** *a* **5.** *b* **6.** *a*

QUESTIONS AND ANSWERS ON D.C. MOTORS

Q. 1. How may the direction of rotation of a d.c. motor be reversed ?

Ans. By reversing either the field current or current through the armature. Usually, reversal of current through the armature is adopted.

Q. 2. What will happen if both currents are reversed ?

Ans. The motor will run in the original direction.

Q. 3. What will happen if the field of a d.c. shunt motor is opened ?

Ans. The motor will achieve dangerously high speed and may destroy itself.

Q. 4. What happens if the direction of current at the terminals of a series motor is reversed ?

Ans. It does not reverse the direction of rotation of motor because current flows through the armature in the same direction as through the field.

Q. 5. Explain what happens when a d.c. motor is connected across an a.c. supply ?

Ans. 1. Since on a.c. supply, reactance will come into the picture, the a.c. supply will be offered impedance (not resistance) by the armature winding. Consequently, with a.c. supply, current will be much less. The motor will run but it would not carry the same load as it would on d.c. supply.

2. There would be more sparking at the brushes.

3. Though motor armature is laminated as a rule, the field poles are not. Consequently, eddy currents will cause the motor to heat up and eventually burn on a.c. supply.

Q. 6. What will happen if a shunt motor is directly connected to the supply line ?

Ans. Small motors up to 1 kW rating may be line-started without any adverse results being produced. High rating motors must be started through a suitable starter in order to avoid the huge starting current which will

(*i*) damage the motor itself and (*ii*) badly affect the voltage regulation of the supply line.

Q. 7. What is the function of interpoles and how are interpole windings connected ?

Ans. Interpoles are small poles placed in between the main poles. Their function is to assist commutation by producing the auxiliary or commutating flux. Consequently, brush sparking is practically eliminated. Interpole windings are connected in series with the armature windings.

Q. 8. In rewinding the armature of a d.c. motor, progressive connections are changed to retrogressive ones. Will it affect the operation in any way ?

Ans. Yes. Now, the armature will rotate in the opposite direction.

Q. 9. A d.c. motor fails to start when switched on. What could be the possible reasons and remedies ?

Ans. Any one of the following reasons could be responsible :

1. Open-circuit in controller–should be checked for open starting resistance or open switch or open fuse.
2. Low terminal voltage–should be adjusted to name-plate value.
3. Overload–should be reduced if possible otherwise larger motor should be installed.
4. Excessive friction–bearing lubrication should be checked.

Q. 10. A d.c. motor is found to stop running after a short period of time. What do you think could be the reasons ? How would you remedy each ?

Ans. Possible causes are as under :

1. Motor not getting enough power–check voltage at motor terminals as well as fuses, clups and overload relay.
2. Weak or no field–in the case of adjustable-speed motors, check if rheostat is correctly set. Also, check field winding for any 'open'. Additionally, look for any loose winding or broken connection.
3. Motor torque insufficient for driving the given load–check line voltage with name-plate voltage. If necessary, use larger motor to match the load.

Q. 11. What are the likely causes if a d.c. motor is found to run too slow under load ? And the remedy ?

Ans.
1. Supply line voltage too low–remove any excessive resistance in supply line, connections or controller.
2. Brushes ahead of neutral–set them on neutral.
3. Overload–reduce it to allowable value or use larger motor.

Q. 12. Why does a d.c. motor sometime run too fast when under load ? Give different possible causes and their remedies.

Ans. Different possible causes are as under :

1. Weak field–remove any extra resistance in shunt field circuit. Also, check for 'grounds'.
2. Line voltage too high–reduce it to name-plate value.
3. Brushes back of neutral–set them on neutral.

Q. 13. Under what conditions is sparking produced at the brushes of a d.c. motor ? How would you remedy it ?

Ans.
1. Commutator in bad condition–clean and reset brushes.
2. commutator either eccentric or rough–grind and true the commutator. Also, undercut mica.
3. Excessive vibration–balance armature. Make sure that brushes ride freely in holders.
4. Brush-holding spring broken or sluggish–replace spring and adjust pressure to recommended value.
5. Motor overloaded–reduce load or install motor of proper rating.
6. Short-circuit in armature circuit–remove any metallic particles between commutator segments and check for short between adjacent commutator risers. Locate and repair internal armature short if any.

Q. 14. Sometimes a hissing noise (or brush chatter) is heard to emanate from the commutator end of a running d.c. motor. What could it be due to and how could it be removed ?

Ans. Any one of the following causes could produce brush chatter :

1. Excessive clearance of brush holders–adjust properly
2. Incorrect angle of brushes–adjust to correct value
3. Unsuitable brushes–replace them
4. High mica–undercut it
5. Wrong brush spring pressure–adjust to correct value.

Q. 15. What are the possible causes of excessive sparking at brushes in a d.c. motor ?

Ans.
1. Poor brush fit on commutator–sand-in the brushes and polish commutator.
2. Brushes binding in the brush holders–clean holders and brushes and remove any irregularities on surfaces of brush holders or rough spots on brushes.
3. Excessive or insufficient pressure on brushes–adjust pressure.
4. Brushes off neutral–set them on neutral.

Q. 16. Why does a d.c. motor sometime spark on light load ?

Ans. Due to the presence of paint spray, chemical, oil or grease etc. on commutator.

Q. 17. When is the armature of a d.c. motor likely to get over-heated ?

Ans.
1. When motor is over-loaded.
2. When it is installed at a place having restricted ventilation.
3. When armature winding is shorted.

Q. 18. What causes are responsible for over-heating of commutator in a d.c. motor ?

Ans. It could be due either to the brushes being off neutral or being under excessive spring pressure. Accordingly, brushes should be adjusted properly and the spring pressure should be reduced but not to the point where sparking is introduced.

C H A P T E R 32

TRANSFORMER

Learning Objectives

- Working Principle of Transformer
- Transformer Construction
- Core-type Transformers
- Shell-type Transformers
- E.M.F. Equation of Transformer
- Voltage Transformation Ratio
- Transformer with losses
- Equivalent Resistance
- Magnetic Leakage
- Transformer with Resistance and Leakage Reactance
- Total Approximate Voltage Drop in Transformer
- Exact Voltage Drop
- Separation of Core Losses
- Short-Circuit or Impedance Test
- Why Transformer Rating in KVA?
- Regulation of a Transformer
- Percentage Resistance, Reactance and Impedance
- Kapp Regulation Diagram
- Sumpner or Back-to-back-Test
- Efficiency of a Transformer
- Auto-transformer

To overcome losses, the electricity from a generator is passed through a step-up transformer, which increases the voltage. Throughout the distribution system, the voltages are changed using step-down transformers to voltages suitable to the applications at industry and homes.

32.1. Working Principle of a Transformer

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is ***mutual induction*** between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig. 32.1. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction $e = MdI/dt$). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called ***primary*** winding and the other from which energy is drawn out, is called ***secondary*** winding. In brief, a transformer is a device that

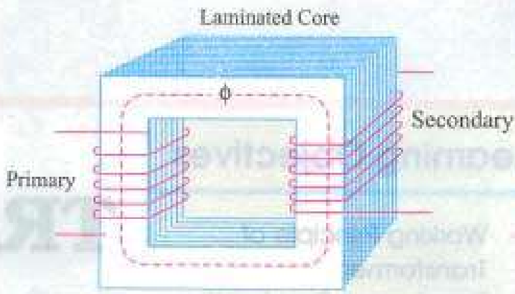
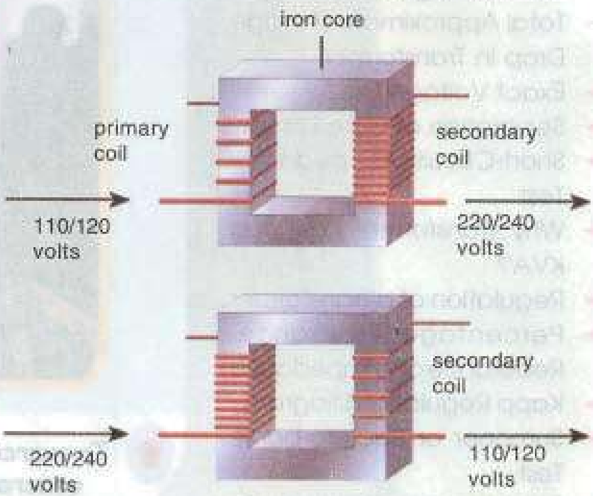


Fig. 32.1

1. transfers electric power from one circuit to another
2. it does so without a change of frequency
3. it accomplishes this by electromagnetic induction and
4. where the two electric circuits are in mutual inductive influence of each other.

32.2. Transformer Construction

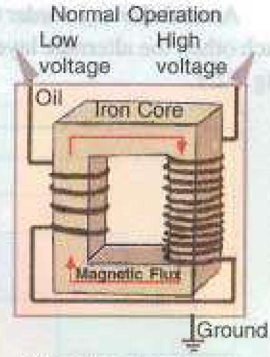
The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are : some suitable container for assembled core and windings ; a suitable medium for insulating the core and its windings from its container ; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.



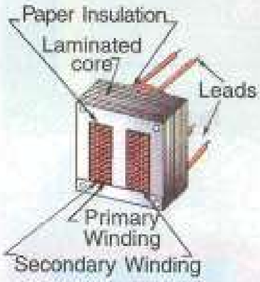
Principle of transformer

Fig. 32.2

In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum of air-gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz. The core laminations (in the form of strips) are joined as shown in Fig. 32.2. It is seen that the joints in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross-section of the core. Such staggered joints are said to be 'imbricated'.



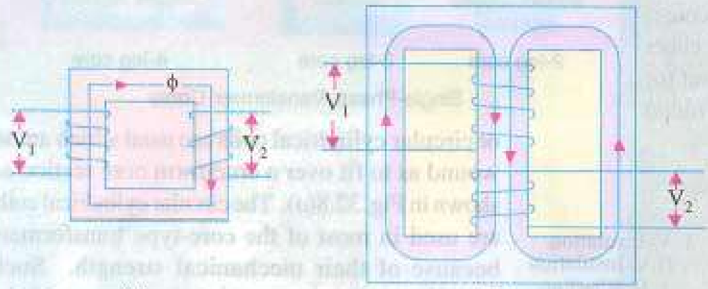
Core-type transformer



Shell-Type transformer

Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core. The two types are known as ***(i)* core-type** and ***(ii)* shell-type.** Another recent development is ***spiral-core or wound-core type***, the trade name being *spirakore* transformer.

In the so-called core type transformers, the ***windings surround a considerable part of the core*** whereas in shell-type transformers, the ***core surrounds a considerable portion of the windings*** as shown schematically in Fig. 32.3 (*a*) and (*b*) respectively.



(*a*) (*b*)

Fig. 32.3

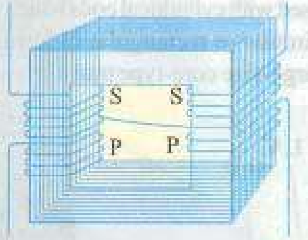


Fig. 32.4

In the simplified diagram for the core type transformers [Fig. 32.3 (*a*)], the primary and secondary winding are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux. As shown in Fig. 32.4, half the primary and half the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or leg) and the secondary on the other.


Fig. 32.5

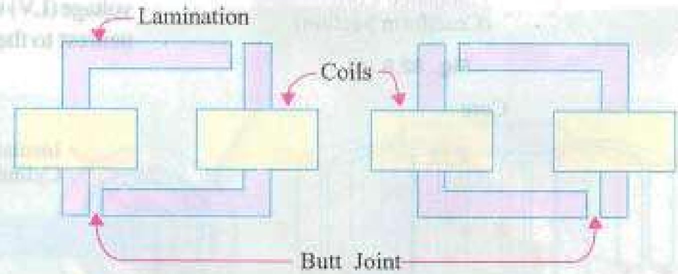


Fig. 32.6

In both core and shell-type transformers, the individual laminations are cut in the form of long strips of *L*'s, *E*'s and *I*'s as shown in Fig. 32.5. The assembly of the complete core for the two types of transformers is shown in Fig. 32.6 and Fig. 32.7.

As said above, in order to avoid high reluctance at the joints where the laminations are butted against each other, the alternate layers are stacked differently to eliminate these joints as shown in Fig. 32.6 and Fig. 32.7.

Fig. 32.7

32.3. Core-type Transformers

The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval or rectangular. In small size core-type transformers, a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large-size core-type transformers, round or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 32.8(*a*). The circular cylindrical coils are used in most of the core-type transformers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from each other by paper, cloth, micarta board or cooling ducts. Fig. 32.8(*c*) shows the general arrangement of these coils with respect to the core. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. Since the low-voltage (LV) winding is easiest to insulate, it is placed nearest to the core (Fig. 32.8).

Single-Phase Transformer Cores

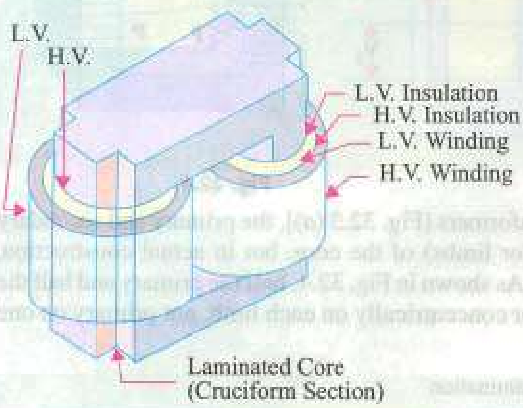


Fig. 32.8 (a)

Fig. 32.8

Because of laminations and insulation, the net or effective core area is reduced, due allowance for which has to be made (Ex. 32.6). It is found that, in general, the reduction in core sectional area due to the presence of paper, surface oxide etc. is of the order of 10% approximately.

As pointed out above, rectangular cores with rectangular cylindrical coils can be used for small-size core-type transformers as shown in Fig. 32.9 (*a*) but for large-sized transformers, it becomes wasteful to use rectangular cylindrical coils and so circular cylindrical coils are preferred. For such purposes, square cores may be used as shown in Fig. 32.9 (*b*) where circles represent the tubular former carrying the coils. Obviously, a considerable amount of useful space is still wasted. A common improvement on square core is to employ cruciform core as in Fig. 32.9 (*c*) which demands, at least, two sizes of core strips. For very large transformers, further core-stepping is done as in Fig. 32.9 (*d*) where at least three sizes of core plates are necessary. Core-stepping not only gives high space factor but also results in reduced length of the mean turn and the consequent I^2R loss. Three stepped core is the one most commonly used although more steps may be used for very large transformers as in Fig. 32.9 (*e*). From the geometry of Fig. 32.9, it can be shown that maximum gross core section for Fig. 32.9 (*b*) is 0.5 d^2 and for Fig. 32.9 (*c*) it is 0.616 d^2 where *d* is the diameter of the cylindrical coil.

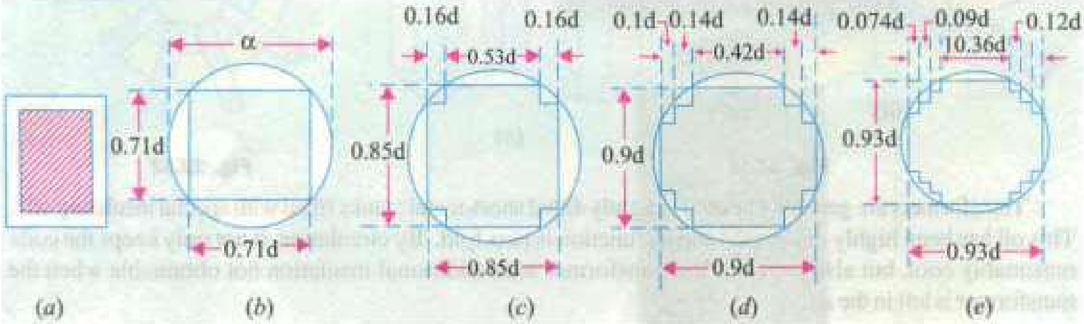


Fig. 32.9

32.4. Shell-type Transformers

In these case also, the coils are form-would but are multi-layer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper. The complete winding consists of stacked discs with insulation space between the coils–the spaces forming horizontal cooling and insulating ducts. A shell-type transformer may have a simple rectangular form as shown in Fig. 32.10 or it may have distributed form as shown in Fig. 32.11.

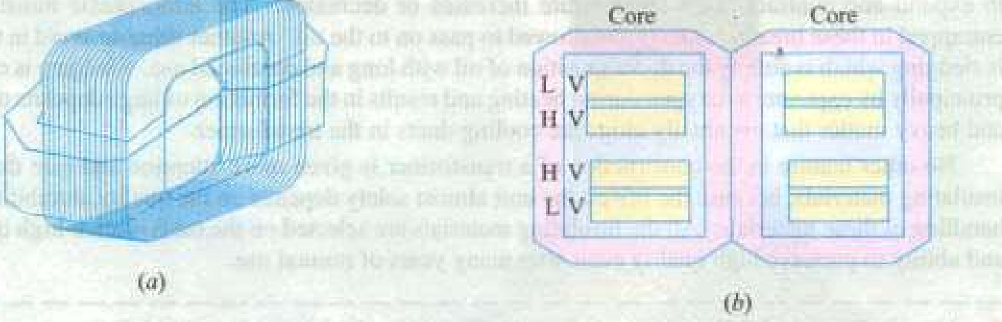
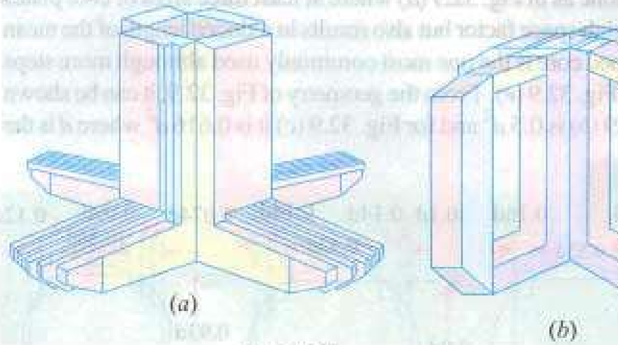


Fig. 32.10

A very commonly-used shell-type transformer is the one known as Berry Transformer–so called after the name of its designer and is cylindrical in form. The transformer core consists of laminations arranged in groups which radiate out from the centre as shown in section in Fig. 32.12.

It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise–a humming sound–during operation.

The spiral-core transformer employs the newest development in core construction. The core is assembled of a continuous strip or ribbon of transformer steel wound in the form of a circular or elliptical cylinder. Such construction allows the core flux to follow the grain of the iron. Cold-rolled steel of high silicon content enables the designer to use considerably higher operating flux densities with lower loss per kg. The use of higher flux density reduces the weight per kVA. Hence, the advantages of such construction are (*i*) a relatively more rigid core (*ii*) lesser weight and size per kVA rating (*iii*) lower iron losses at higher operating flux densities and (*iv*) lower cost of manufacture.



(*a*) (*b*)

Fig. 32.11

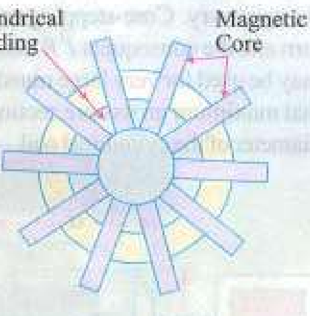


Fig. 32.12

Transformers are generally housed in tightly-fitted sheet-metal ; tanks filled with special insulating oil*. This oil has been highly developed and its function is two-fold. By circulation, it not only keeps the coils reasonably cool, but also provides the transformer with additional insulation not obtainable when the transformer is left in the air.

In cases where a smooth tank surface does not provide sufficient cooling area, the sides of the tank are corrugated or provided with radiators mounted on the sides. Good transformer oil should be absolutely free from alkalies, sulphur and particularly from moisture. The presence of even an extremely small percentage of moisture in the oil is highly detrimental from the insulation viewpoint because it lowers the dielectric strength of the oil considerably. The importance of avoiding moisture in the transformer oil is clear from the fact that even an addition of 8 parts of water in 1,000,000 reduces the insulating quality of the oil to a value generally recognized as below standard. Hence, the tanks are sealed air-tight in smaller units. In the case of large-sized transformers where complete air-tight construction is impossible, chambers known as ***breathers*** are provided to permit the oil inside the tank to expand and contract as its temperature increases or decreases. The atmospheric moisture is entrapped in these breathers and is not allowed to pass on to the oil. Another thing to avoid in the oil is sledging which is simply the decomposition of oil with long and continued use. Sledging is caused principally by exposure to oxygen during heating and results in the formation of large deposits of dark and heavy matter that eventually clogs the cooling ducts in the transformer.

No other feature in the construction of a transformer is given more attention and care than the insulating materials, because the life on the unit almost solely depends on the quality, durability and handling of these materials. All the insulating materials are selected on the basis of their high quality and ability to preserve high quality even after many years of normal use.

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* Instead of natural mineral oil, now-a-days synthetic insulating fluids known as ASKARELS (trade name) are used. They are non-inflammable and, under the influence of an electric arc, do not decompose to produce inflammable gases. One such fluid commercially known as PYROCLOR is being extensively used because it possesses remarkable stability as a dielectric and even after long service shows no deterioration through sledging, oxidation, acid or moisture formation. Unlike mineral oil, it shows no rapid burning.

All the transformer leads are brought out of their cases through suitable bushings. There are many designs of these, their size and construction depending on the voltage of the leads. For moderate voltages, porcelain bushings are used to insulate the leads as they come out through the tank. In general, they look almost like the insulators used on the transmission lines. In high voltage installations, oil-filled or capacitor-type bushings are employed.

The choice of core or shell-type construction is usually determined by cost, because similar characteristics can be obtained with both types. For very high-voltage transformers or for multiwinding design, shell-type construction is preferred by many manufacturers. In this type, usually the mean length of coil turn is longer than in a comparable core-type design. Both core and shell forms are used and the selection is decided by many factors such as voltage rating, kVA rating, weight, insulation stress, heat distribution etc.

Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use :

(a) oil-filled self-cooled *(b) oil-filled water-cooled* *(c) air-blast type*

Small and medium size distribution transformers–so called because of their use on distribution systems as distinguished from line transmission–are of type *(a)*. The assembled windings and cores of such transformers are mounted in a welded, oil-tight steel tank provided with steel cover. After putting the core at its proper place, the tank is filled with purified, high quality insulating oil. The oil serves to convey the heat from the core and the windings to the case from where it is radiated out to the surroundings. For small size, the tanks are usually smooth-surfaced, but for larger sizes, the cases are frequently corrugated or fluted to get greater heat radiation area without increasing the cubical capacity of the tank. Still larger sizes are provided with radiators or pipes.

Construction of very large self-cooled transformers is expensive, a more economical form of construction for such large transformers is provided in the oil-immersed, water-cooled type. As before, the windings and the core are immersed in the oil, but there is mounted near the surface of oil, a cooling coil through which cold water is kept circulating. The heat is carried away by this water. The largest transformers such as those used with high-voltage transmission lines, are constructed in this manner.

Oil-filled transformers are built for outdoor duty and as these require no housing other than their own, a great saving is thereby effected. These transformers require only periodic inspection.

For voltages below 25,000 V, transformers can be built for cooling by means of an air-blast. The transformer is not immersed in oil, but is housed in a thin sheet-metal box open at both ends through which air is blown from the bottom to the top by means of a fan or blower.

32.5. Elementary Theory of an Ideal Transformer

An ideal transformer is one which has no losses *i.e.* its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted *that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer*.

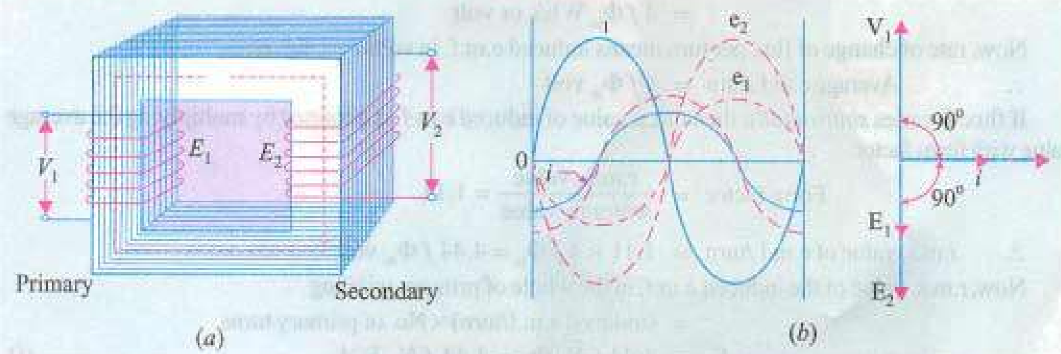
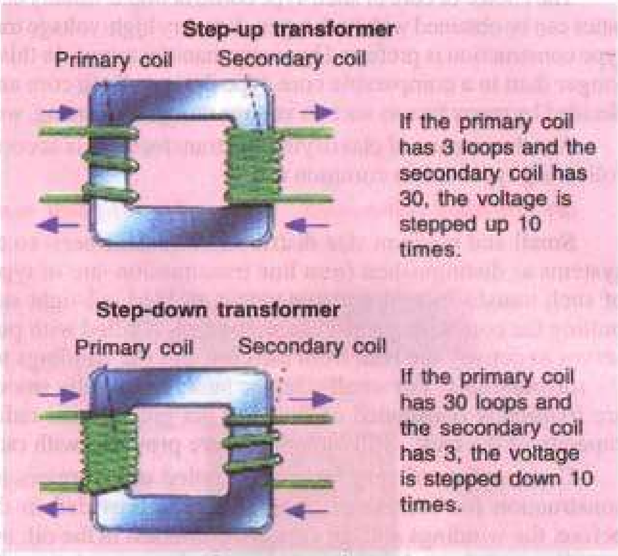


Fig. 32.13

Consider an ideal transformer [Fig. 32.13 (*a*)] whose secondary is open and whose primary is connected to sinusoidal alternating voltage V_1. This potential difference causes an alternating current to flow in the primary. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current I_μ only. The function of this current is merely to magnetise the core, it is small in magnitude and lags V_1 by 90°. This alternating current I_μ produces an alternating flux ϕ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary. This *self-induced* e.m.f. E_1 is, at every instant, equal to and in opposition to V_1. It is also known as counter e.m.f. or back e.m.f. of the primary.



Step-up transformer

Similarly, there is produced in the secondary an induced e.m.f. E_2 which is known as *mutually* induced e.m.f. This e.m.f. is antiphase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

The instantaneous values of applied voltage, induced e.m.fs, flux and magnetising current are shown by sinusoidal waves in Fig. 32.13 (*b*). Fig. 32.13 (*c*) shows the vectorial representation of the effective values of the above quantities.

32.6. E.M.F. Equation of a Transformer

Let N_1 = No. of turns in primary

N_2 = No. of turns in secondary

Φ_m = Maximum flux in core in webers

$= B_m \times A$

f = Frequency of a.c. input in Hz

As shown in Fig. 32.14, flux increases from its zero value to maximum value Φ_m in one quarter of the cycle *i.e.* in 1/4 *f* second.

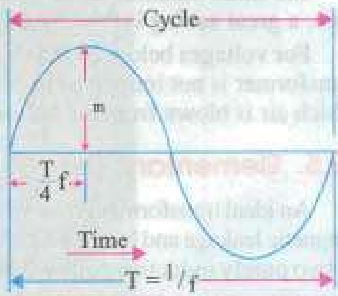


Fig. 32.14

$$\therefore \quad \text{Average rate of change of flux} = \frac{\Phi_m}{1/4f}$$

$$= 4f\Phi_m \text{ Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\therefore \quad \text{Average e.m.f./turn} = 4f\Phi_m \text{ volt}$$

If flux Φ varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$$\therefore \quad \text{r.m.s. value of e.m.f./turn} = 1.11 \times 4f\Phi_m = 4.44f\Phi_m \text{ volt}$$

Now, r.m.s. value of the induced e.m.f. in the whole of primary winding

$$= (\text{induced e.m.f/turn}) \times \text{No. of primary turns}$$

$$E_1 = 4.44fN_1\Phi_m = 4.44fN_1B_mA \quad \text{...(i)}$$

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E_2 = 4.44\, f\, N_2\, \Phi_m = 4.44\, f\, N_2\, B_m A \qquad \ldots(ii)$$

It is seen from (*i*) and (*ii*) that $E_1/N_1 = E_2/N_2 = 4.44 f \Phi_m$. It means that e.m.f./turn is the *same* in both the primary and secondary windings.

In an ideal transformer on no-load, $V_1 = E_1$ and $E_2 = V_2$ where V_2 is the terminal voltage (Fig. 32.15).

32.7 Voltage Transformation Ratio (K)

From equations (*i*) and (*ii*), we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This constant K is known as voltage transformation ratio.

(*i*) If $N_2 > N_1$ *i.e.* $K > 1$, then transformer is called ***step-up*** transformer.

(*ii*) If $N_2 < N_1$ *i.e.* $K < 1$, then transformer is known as ***step-down*** transformer.

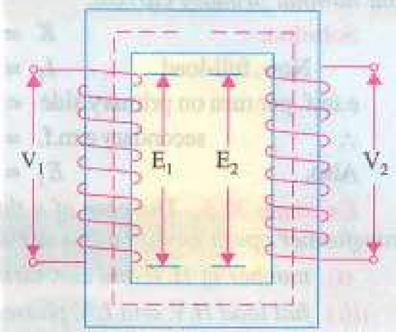


Fig. 32.15

Again, for an *ideal* transformer, input *VA* = output *VA*.

$$V_1 I_1 = V_2 I_2 \text{ or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Example 32.1. *The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m². If the e.m.f. per turn is 8 volt, determine*

(*i*) *primary and secondary turns* (*ii*) *area of the core.*

(Electrical Engg.-I, Nagpur Univ. 1991)

Solution. (*i*) $E_1 = N_1 \times \text{e.m.f. induced/turn}$

$N_1 = 250/8 = \mathbf{32}$; $N_2 = 3000/8 = \mathbf{375}$

(*ii*) We may use $E_2 = -\,4.44\, f\, N_2\, B_m A$

$\therefore \quad 3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$; $\mathbf{A = 0.03 m^2}$.

Example 32.2. *The core of a 100-kVA, 11000/550 V, 50-Hz, 1-ph, core type transformer has a cross-section of 20 cm × 20 cm. Find* (*i*) *the number of H.V. and L.V. turns per phase and* (*ii*) *the e.m.f. per turn if the maximum core density is not to exceed 1.3 Tesla. Assume a stacking factor of 0.9.*

What will happen if its primary voltage is increased by 10% on no-load ?

(Elect. Machines, A.M.I.E. Sec. B, 1991)

Solution. (*i*) $B_m = 1.3\ T$, $A = (0.2 \times 0.2) \times 0.9 = 0.036\ m^2$

$\therefore \quad 11{,}000 = 4.44 \times 50 \times N_1 \times 1.3 \times 0.036$, $N_1 = 1060$

$550 = 4.44 \times 50 \times N_2 \times 1.3 \times 0.036$; $N_2 = 53$

or, $N_2 = KN_1 = (550/11{,}000) \times 1060 = 53$

(*ii*) $\text{e.m.f./turn} = 11{,}000/1060 = 10.4\ V$ or $550/53 = 10.4\ V$

Keeping supply frequency constant, if primary voltage is increased by 10%, magnetising current will increase by much more than 10%. However, due to saturation, flux density will increase only marginally and so will the eddy current and hysteresis losses.

Example 32.3. *A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm². If the primary winding be connected to a 50-Hz supply at 520 V, calculate* (*i*) *the peak value of flux density in the core* (*ii*) *the voltage induced in the secondary winding.* **(Elect. Engg-I, Pune Univ. 1989)**

Solution. $K = N_2/N_1 = 1000/400 = 2.5$

(*i*) $E_2/E_1 = K \quad \therefore \quad E_2 = KE_1 = 2.5 \times 520 =$ **1300 V**

(*ii*) $E_1 = 4.44\, f N_1 B_m A$

or $520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4}) \therefore B_m =$ **0.976 Wb/m²**

Example 32.4. *A 25-kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core. Neglect leakage drops and no-load primary current.* **(Elect. & Electronic Engg., Madras Univ. 1985)**

Solution. $K = N_2/N_1 = 50/500 = 1/10$

Now, full-load $I_1 = 25{,}000/3000 =$ **8.33 A.** F.L. $I_2 = I_1/K = 10 \times 8.33 =$ **83.3 A**

e.m.f. per turn on primary side = 3000/500 = 6 V

$\therefore$ secondary e.m.f. = 6 × 50 = **300 V** (or $E_2 = KE_1 = 3000 \times 1/10 = 300\ V$)

Also, $E_1 = 4.44\, f N_1 \Phi_m$; $3000 = 4.44 \times 50 \times 500 \times \Phi_m \therefore \Phi_m =$ **27 mWb**

Example 32.5. *The core of a three phase, 50 Hz, 11000/550 V delta/star, 300 kVA, core-type transformer operates with a flux of 0.05 Wb. Find*

(*i*) *number of H.V. and L.V. turns per phase.* (*ii*) *e.m.f. per turn*

(*iii*) *full load H.V. and L.V. phase-currents.* **(Bharathithasan Univ. April 1997)**

Solution. Maximum value of flux has been given as 0.05 Wb.

(*ii*) e.m.f. per turn

$$= 4.44\, f\, \phi_m$$
$$= 4.44 \times 50 \times 0.05 = 11.1 \text{ volts}$$

(*i*) Calculations for number of turns on two sides :

Voltage per phase on delta-connected primary winding = 11000 volts

Voltage per phase on star-connected secondary winding = 550/1.732 = 317.5 volts

$$T_1 = \text{number of turns on primary, per phase}$$
$$= \text{voltage per phase/e.m.f. per turn}$$
$$= 11000/11.1 = 991$$
$$T_2 = \text{number of turns on secondary, per phase}$$
$$= \text{voltage per phase/e.m.f. per turn}$$
$$= 317.5/11.1 = 28.6$$

Note : (*i*) Generally, Low-voltage-turns are calculated first, the figure is rounded off to next higher even integer. In this case, it will be 30. Then, number of turns on primary side is calculated by turns-ratio.

In this case, $T_1 = T_2 (V_1/V_2) = 30 \times 11000/317.5 = 1040$

This, however, reduces the flux and results into less saturation. This, in fact, is an elementary aspect in Design-calculations for transformers. (Explanation is added here only to overcome a doubt whether a fraction is acceptable as a number of L.V. turns).

(*ii*) Full load H.V. and L.V. phase currents :

Output per phase = (300/3) = 100 kVA

$$\text{H.V. phase-current} = \frac{100 \times 1000}{11{,}000} = 9.1 \text{ Amp}$$

$$\text{L.V. phase-current} = (100 \times 1000/317.5) = 315 \text{ Amp}$$

Example 32.6. *A single phase transformer has 500 turns in the primary and 1200 turns in the secondary. The cross-sectional area of the core is 80 sq. cm. If the primary winding is connected to a 50 Hz supply at 500 V, calculate* (*i*) *Peak flux-density, and* (*ii*) *Voltage induced in the secondary.*

(Bharathiar University November 1997)

Solution. From the e.m.f. equation for transformer,

$$500 = 4.44 \times 50 \times \phi_m \times 500$$

$$\phi_m = 1/222 \text{ Wb}$$

(*i*) Peak flux density, $B_m = \phi_m / (80 \times 10^{-4}) = 0.563 \text{ wb/m}^2$

(*ii*) Voltage induced in secondary is obtained from transformation ratio or turns ratio

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

or $V_2 = 500 \times 1200/500 = 1200$ volts

Example 32.7. *A 25 kVA, single-phase transformer has 250 turns on the primary and 40 turns on the secondary winding. The primary is connected to 1500-volt, 50 Hz mains. Calculate (i) Primary and Secondary currents on full-load. (ii) Secondary e.m.f., (iii) maximum flux in the core.* **(Bharathiar Univ. April 1998)**

Solution. (*i*) If V_2 = Secondary voltage rating, = secondary e.m.f.,

$$\frac{V_2}{1500} = \frac{40}{250}, \text{ giving } V_2 = 240 \text{ volts}$$

(*ii*) Primary current $= 25000/1500 = 16.67$ amp

Secondary current $= 25000/240 = 104.2$ amp

(*iii*) If ϕ_m is the maximum core-flux in *Wb*,

$$1500 = 4.44 \times 50 \times \phi_m \times 250, \text{ giving } \phi_m = 0.027 \text{ Wb or 27 mWb}$$

Example 32.8. *A single-phase, 50 Hz, core-type transformer has square cores of 20 cm side. Permissible maximum flux-density is 1 Wb/m^2. Calculate the number of turns per Limb on the High and Low-voltage sides for a 3000/220 V ratio.* **(Manonmaniam Sundaranar Univ. April 1998)**

Solution. E.M.F. equation gives the number of turns required on the two sides. We shall first calculate the L.V.-turns, round the figure off to the next higher even number, so that given maximum flux density is not exceeded. With the corrected number of L.V. turns, calculate H.V.-turns by transformation ratio. Further, there are two Limbs. Each Limb accommodates half-L.V. and half H.V. winding from the view-point of reducing leakage reactance.

Starting with calculation for L.V. turns, T_2,

$$4.44 \times 50 \times [(20 \times 20 \times 10^{-4}) \times 1] \times T_2 = 220$$

$$T_2 = 220/8.88 = 24.77$$

Select $T_2 = 26$

$$T_1/T_2 = V_1/V_2$$

$$T_1 = 26 \times 3000/220 = 354, \text{ selecting the nearest even integer.}$$

Number of H.V. turns on each Limb = 177

Number of L.V. turns on each Limb = 13

32.8. Transformer with Losses but no Magnetic Leakage

We will consider two cases (*i*) when such a transformer is on no load and (*ii*) when it is loaded.

32.9. Transformer on No-load

In the above discussion, we assumed an ideal transformer *i.e.* one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing theory. When an ***actual*** transformer is put on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Even when the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply *(i)* iron losses in the core *i.e.* hysteresis loss and eddy current loss and *(ii)* a very small amount of copper loss in primary (there being no Cu loss in secondary as it is open). Hence, the no-load primary input current I_0 is not at 90° behind V_1 but lags it by an angle $\phi_0 <$ 90°. No-load input power

$$W_0 = V_1 I_0 \cos \phi_0$$

where $\cos \phi_0$ is primary power factor under no-load conditions. No-load condition of an actual transformer is shown vectorially in Fig. 32.16.

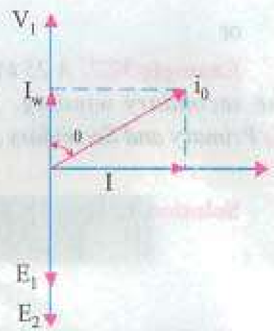


Fig. 32.16

As seen from Fig. 32.16, primary current I_0 has two components :

(i) One in phase with V_1. This is known as ***active*** or ***working*** or ***iron*** loss component I_w because it mainly supplies the iron loss plus small quantity of primary Cu loss.

$$I_w = I_0 \cos \phi_0$$

(ii) The other component is in quadrature with V_1 and is known as ***magnetising*** component I_μ because its function is to sustain the alternating flux in the core. It is wattless.

$$I_\mu = I_0 \sin \phi_0$$

Obviously, I_0 is the vector sum of I_w and I_μ, hence $I_0 = (I_\mu^2 + I_\omega^2)$.

The following points should be noted carefully :

1. The no-load primary current I_0 is very small as compared to the full-load primary current. It is about 1 per cent of the full-load current.

2. Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting current, the wave of the exciting or magnetising current is not truly sinusoidal. As such it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors. But, in practice, it makes no appreciable difference.

3. As I_0 is very small, the no-load primary Cu loss is negligibly small which means ***that no-load primary input is practically equal to the iron loss in the transformer.***

4. As it is principally the core-loss which is responsible for shift in the current vector, angle ϕ_0 is known as ***hysteresis angle of advance.***

Example 32.9. *(a) A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetising and iron loss currents.*

(b) A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetising and working components of no-load primary current.

Solution. *(a)* Iron-loss current

$$= \frac{\text{no-load input in watts}}{\text{primary voltage}} = \frac{400}{2,200} = \mathbf{0.182\ A}$$

Now $\qquad I_0^2 = I_w^2 + I_\mu^2$

Magnetising component $\qquad I_\mu = \sqrt{(0.6^2 - 0.182)^2} = \mathbf{0.572\ A}$

The two components are shown in Fig. 29.15.

(b) $$I_0 = 0.5\text{ A}, \cos\phi_0 = 0.3 \therefore I_w = I_0\cos\phi_0 = 0.5\times0.3 = 0.15\text{ A}$$

$$I_\mu = \sqrt{0.5^2 - 0.15^2} = 0.476\text{ A}$$

Example 32.10. *A single-phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the iron core is 150 cm and the joints are equivalent to an air-gap of 0.1 mm. When a p.d. of 3,000 V is applied to the primary, maximum flux density is 1.2 Wb/m². Calculate (a) the cross-sectional area of the core (b) no-load secondary voltage (c) the no-load current drawn by the primary (d) power factor on no-load. Given that AT/cm for a flux density of 1.2 Wb/m² in iron to be 5, the corresponding iron loss to be 2 watt/kg at 50 Hz and the density of iron as 7.8 gram/cm³.*

Solution. (a) $$3{,}000 = 4.44\times50\times500\times1.2\times A \therefore A = 0.0225\text{ m}^2 = \mathbf{225\text{ cm}^2}$$

This is the net cross-sectional area. However, the gross area would be about 10% more to allow for the insulation between laminations.

(b) $$K = N_2/N_1 = 40/500 = 4/50$$

$\therefore$ N.L. secondary voltage $= KE_1 = (4/50)\times3000 = \mathbf{240\text{ V}}$

(c) AT per cm $= 5 \therefore AT$ for iron core $= 150\times5 = \mathbf{750}$

$$AT\text{ for air-gap} = Hl = \frac{B}{\mu_0}\times l = \frac{1.2}{4\pi\times10^{-7}}\times0.0001 = 95.5$$

Total AT for given $B_{max} = 750 + 95.5 = 845.5$

Max. value of magnetising current drawn by primary $= 845.5/500 = 1.691$ A

Assuming this current to be sinusoidal, its r.m.s. value is $I_\mu = 1.691/\sqrt{2} = 1.196$ A

Volume of iron $=$ length $\times$ area $= 150\times225 = 33{,}750\text{ cm}^3$

Density $= 7.8\text{ gram/cm}^3 \therefore$ Mass of iron $= 33{,}750\times7.8/1000 = 263.25$ kg

Total iron loss $= 263.25\times2 = 526.5$ W

Iron loss component of no-load primary current I_0 is $I_w = 526.5/3000 = 0.176$ A

$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{1.196^2 + 0.176^2} = \mathbf{0.208\text{ A}}$$

(d) Power factor, $\cos\phi_0 = I_w/I_0 = 0.176/1.208 = \mathbf{0.1457}$

Example 32.11. *A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 amp. at a p.f. of 0.2 lagging. Calculate the primary current and power-factor when the secondary current is 280 Amp at a p.f. of 0.80 lagging.*

(Nagpur University, November 1997)

Solution. V_2 is taken as reference. $\cos^{-1}0.80 = 36.87°$

$$I_2 = 280\angle-36.87°\text{ amp}$$

$$I'_2 = (280/5)\angle-36.87°\text{ amp}$$

$$\phi = \cos^{-1}0.20 = 78.5°, \sin\phi = 0.98$$

$$I_1 = I_0 + I'_2 = 3(0.20 - j0.98) + 56(0.80 - j0.60)$$

$$= 0.6 - j2.94 + 44.8 - j33.6$$

$$= 45.4 - j2.94 + 44.8 - j33.6$$

$$= 45.4 - j36.54 = 58.3\angle38.86°$$

Thus I lags behind the supply voltage by an angle of 38.86°.

Tutorial Problems 32.1

1. The number of turns on the primary and secondary windings of a 1−φ transformer are 350 and 35 respectively. If the primary is connected to a 2.2 kV, 50-Hz supply, determine the secondary voltage on no-load. **[220 V]** *(Elect. Engg.-II, Kerala Univ. 1980)*
2. A 3000/200-V, 50-Hz, 1-phase transformer is built on a core having an effective cross-sectional area of 150 cm^2 and has 80 turns in the low-voltage winding. Calculate
   (*a*) the value of the maximum flux density in the core
   (*b*) the number of turns in the high-voltage winding. **[(*a*) 0.75 Wb/m^2 (*b*) 1200]**
3. A 3,300/230-V, 50-Hz, 1-phase transformer is to be worked at a maximum flux density of 1.2 Wb/m^2 in the core. The effective cross-sectional area of the transformer core is 150 cm^2. Calculate suitable values of primary and secondary turns. **[830; 58]**
4. A 40-kVA, 3,300/240-V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine
   (*a*) the number of turns on the secondary
   (*b*) the maximum value of flux in the core
   (*c*) the approximate value of primary and secondary full-load currents.
   Internal drops in the windings are to be ignored. **[(*a*) 48 (*b*) 22.5 mWb (*c*) 12.1 A; 166.7 A]**
5. A double-wound, 1-phase transformer is required to step down from 1900 V to 240 V, 50-Hz. It is to have 1.5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively.
   The peak value of flux density is required to be not more than 1.2 Wb/m^2. Calculate the required cross-sectional area of the steel core. If the output is 10 kVA, calculate the secondary current. **[1,267; 160; 56.4 cm^2; 41.75 A]**
6. The no-load voltage ratio in a 1-phase, 50-Hz, core-type transformer is 1,200/440. Find the number of turns in each winding if the maximum flux is to be 0.075 Wb. **[24 and 74 turns]**
7. A 1-phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is 75 cm^2. If the primary winding be connected to a 400-V, 50 Hz supply, calculate.
   (*i*) the peak value of flux density in the core and (*ii*) voltage induced in the secondary winding. **[0.48 Wb/m^2; 60 V]**
8. A 10-kVA, 1-phase transformer has a turn ratio of 300/23. The primary is connected to a 1500-V, 60 Hz supply. Find the secondary volts on open-circuit and the approximate values of the currents in the two windings on full-load. Find also the maximum value of the flux. **[115 V; 6.67 A; 87 A; 11.75 mWb]**
9. A 100-kVA, 3300/400-V, 50 Hz, 1 phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns.
   How does the core flux vary with load ? **[30.3 A; 250 A; 16.4 mWb; 907]**
10. The no-load current of a transformer is 5.0 A at 0.3 power factor when supplied at 230-V, 50-Hz. The number of turns on the primary winding is 200. Calculate (*i*) the maximum value of flux in the core (*ii*) the core loss (*iii*) the magnetising current. **[5.18 mWb; 345 W; 4.77 A]**
11. The no-load current of a transformer is 15 at a power factor of 0.2 when connected to a 460-V, 50-Hz supply. If the primary winding has 550 turns, calculate
    (*a*) the magnetising component of no-load current
    (*b*) the iron loss
    (*c*) the maximum value of the flux in the core. **[(*a*) 14.7 A (*b*) 1,380 W (*c*) 3.77 mWb]**
12. The no-load current of a transformer is 4.0 A at 0.25 p.f. when supplied at 250-V, 50 Hz. The number of turns on the primary winding is 200. Calculate
    (*i*) the r.m.s. value of the flux in the core (assume sinusoidal flux)
    (*ii*) the core loss
    (*iii*) the magnetising current. **[(*i*) 3.96 mWb (*ii*) 250 W (*iii*) 3.87 A]**

13. The following data apply to a single- phase transformer:
output : 100 kVA, secondary voltage; 400 V; Primary turns: 200; secondary turns: 40; Neglecting the losses, calculate: (*i*) the primary applied voltage (*ii*) the normal primary and secondary currents (*iii*) the secondary current, when the load is 25 kW at 0.8 power factor.
(*Rajiv Gandhi Technical University, Bhopal 2000*) [(*i*) 2000 V, (*ii*) 50 amp, (*iii*) 78.125 amp]

32.10. Transformer on Load

When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. ($=N_2I_2$) and hence its own flux Φ_2 which is in opposition to the main primary flux Φ which is due to I_0. The secondary ampere-turns $N_2\ I_2$ are known as ***demagnetising*** amp-turns. The opposing secondary flux Φ_2 weakens the primary flux Φ momentarily, hence primary back e.m.f. E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 and hence causes more current to flow in primary.

Let the additional primary current be I_2'. It is known as ***load component of primary current***. This current is antiphase with I_2'. The additional primary m.m.f. $N_1\ I_2$ sets up its own flux Φ_2' which is in opposition to Φ_2 (but is in the same direction as Φ) and is equal to it in magnitude. Hence, the two cancel each other out. So, we find that the magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current I_2' which is brought into existence exactly at the same instant as I_2. The whole process is illustrated in Fig. 32.17.

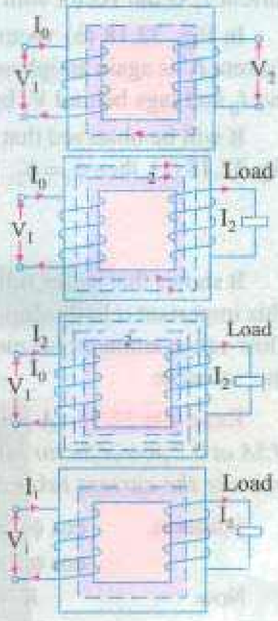


Fig. 32.17

Hence, whatever the load conditions, ***the net flux passing through the core is approximately the same as at no-load***. An important deduction is that due to the constancy of core flux at all loads, ***the core loss is also practically the same under all load conditions***.

As $$\Phi_2 = \Phi_2' \quad \therefore \quad N_2I_2 = N_1I_2' \quad \therefore \quad I_2' = \frac{N_2}{N_1} \times I_2 = KI_2$$

Hence, when transformer is on load, the primary winding has two currents in it; one is I_0 and the other is I_2' which is anti-phase with I_2 and K times in magnitude. ***The total primary current is the vector sum of I_0 and I_2'***

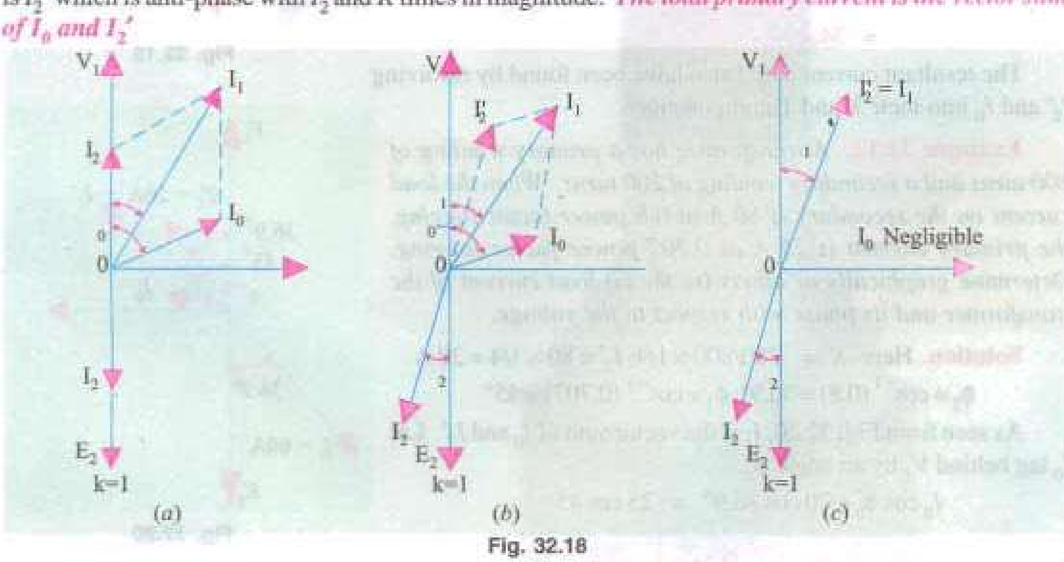


Fig. 32.18

In Fig. 32.18 are shown the vector diagrams for a load transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Fig. 32.18 (*a*), I_2 is secondary current in phase with E_2 (strictly speaking it should be V_2). It causes primary current I_2' which is anti-phase with it and equal to it in magnitude ($K = 1$). Total primary current I_1 is the vector sum of I_0 and I_2' and lags behind V_1 by an angle ϕ_1.

In Fig. 32.18 (*b*) vectors are drawn for an inductive load. Here I_2 lags E_2 (actually V_2) by ϕ_2. Current I_2' is again antiphase with I_2 and equal to it in magnitude. As before, I_1 is the vector sum of I_2' and I_0 and lags behind V_1 by ϕ_1.

It will be observed that ϕ_1 is slightly greater than ϕ_2. But if we neglect I_0 as compared to I_2' as in Fig. 32.18 (*c*), then $\phi_1 = \phi_2$. Moreover, under this assumption

$$N_1 I_2' = N_2 I_1 = N_1 I_2 \quad \therefore \quad \frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant. This important relationship is made the basis of current transformer–a transformer which is used with a low-range ammeter for measuring currents in circuits where the direct connection of the ammeter is impracticable.

Example 32.12. *A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. $\cos\phi_2 = 0.8, \phi_2 = \cos^{-1}(0.8) = 36°54'$

$\cos\phi_0 = 0.2 \quad \therefore \quad \phi_0 = \cos^{-1}(0.2) = 78°\,30'$

Now $K = V_2/V_1 = 110/440 = 1/4$

$\therefore \quad I_2' = KI_2 = 120 \times 1/4 = 30$ A

$I_0 = 5$ A.

Angle between I_0 and I_2'

$= 78°\,30' - 36°\,54' = 41°\,36'$

Using parallelogram law of vectors (Fig. 32.19) we get

$$I_1 = \sqrt{(5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41°\,36')}$$

$= $ **34.45 A**

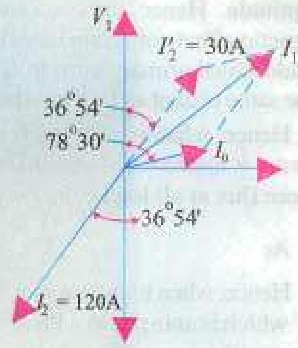


Fig. 32.19

The resultant current could also have been found by resolving I_2' and I_0 into their X and Y-components.

Example 32.13. *A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage.*

Solution. Here $K = 200/800 = 1/4; I_2' = 80 \times 1/4 = 20$ A

$\phi_2 = \cos^{-1}(0.8) = 36.9°; \phi_1 = \cos^{-1}(0.707) = 45°$

As seen from Fig. 32.20, I_1 is the vector sum of I_0 and I_2'. Let I_0 lag behind V_1 by an angle ϕ_0.

$$I_0 \cos\phi_0 + 20 \cos 36.9° = 25 \cos 45°$$

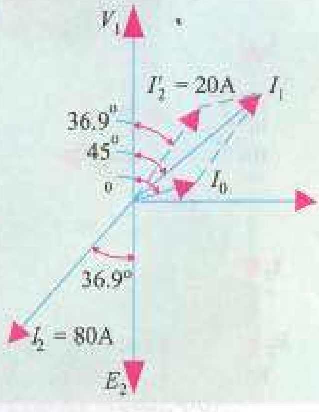


Fig. 32.20

$$\therefore \quad I_0 \cos \phi_2 = 25 \times 0.707 - 20 \times 0.8$$
$$= 1.675 \text{ A}$$
$$I_0 \sin \phi_0 + 20 \sin 36.9° = 25 \sin 45°$$
$$\therefore \quad I_0 \sin \phi_0 = 25 \times 0.707 - 20 \times 0.6$$
$$= 5.675 \text{ A}$$
$$\therefore \quad \tan \phi_0 = 5.675/1.675 = 3.388$$
$$\therefore \quad \phi_0 = \mathbf{73.3°}$$
$$\text{Now,} \quad I_0 \sin \phi_0 = 5.675$$
$$\therefore \quad I_0 = 5.675/\sin 73.3° = \mathbf{5.93\ A}$$

Example 32.14. *A single phase transformer takes 10 A on no load at p.f. of 0.2 lagging. The turns ratio is 4 : 1 (step down). If the load on the secondary is 200 A at a p.f. of 0.85 lagging. Find the primary current and power factor.*

Neglect the voltage-drop in the winding. **(Nagpur University November 1999)**

Solution. Secondary load of 200 A, 0.85 lag is reflected as 50 A, 0.85 lag in terms of the primary equivalent current.

$$I_0 = 10 \angle -\phi_0, \text{ where } \phi_0 = \cos^{-1} 0.20 = 78.5° \text{ lagging}$$
$$= 2 - j9.8 \text{ amp}$$
$$I_2' = 50 \angle -\phi_L \text{ where } \phi_L = \cos^{-1} 0.85 = 31.8°, \text{ lagging}$$
$$I_2' = 42.5 - j26.35$$

Hence primary current I_1

$$= I_0 + I_2'$$
$$= 2 - j9.8 + 42.5 - j26.35$$
$$= 44.5 - j36.15$$
$$|I_1| = 57.333 \text{ amp}, \phi = 0.776 \text{ Lag.}$$
$$\phi = \cos^{-1} \frac{44.5}{57.333} = 39.10° \text{ lagging}$$

The phasor diagram is shown in Fig. 32.21.

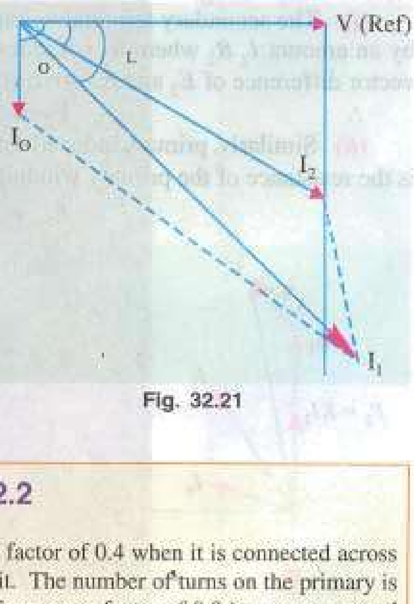


Fig. 32.21

Tutorial Problems 32.2

1. The primary of a certain transformer takes 1 A at a power factor of 0.4 when it is connected across a 200-V, 50-Hz supply and the secondary is on open circuit. The number of turns on the primary is twice that on the secondary. A load taking 50 A at a lagging power factor of 0.8 is now connected across the secondary. What is now the value of primary current ? **[25.9 A]**
2. The number of turns on the primary and secondary windings of a single-phase transformer are 350 and 38 respectively. If the primary winding is connected to a 2.2 kV, 50-Hz supply, determine
   (*a*) the secondary voltage on no-load,
   (*b*) the primary current when the secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging,
   (*c*) the power factor of the primary current. **[239 V; 25-65 A; 0.715 lag]**
3. A 400/200-V, 1-phase transformer is supplying a load of 25 A at a p.f. of 0.866 lagging. On no-load the current and power factor are 2 A and 0.208 respectively. Calculate the current taken from the supply. **[13.9 A lagging V1 by 36.1°]**
4. A transformer takes 10 A on no-load at a power factor of 0.1. The turn ratio is 4 : 1 (step down). If

a load is supplied by the secondary at 200 A and p.f. of 0.8, find the primary current and power factor (internal voltage drops in transformer are to be ignored). **[57.2 A; 0.717 lagging]**

5. A 1-phase transformer is supplied at 1,600 V on the h.v. side and has a turn ratio of 8 : 1. The transformer supplies a load of 20 kW at a power factor of 0.8 lag and takes a magnetising current of 2.0 A at a power factor of 0.2. Calculate the magnitude and phase of the current taken from the h.v. supply. **[17.15 A ; 0.753 lag]** *(Elect. Engg. Calcutta Univ. 1980)*
6. A 2,200/200-V, transformer takes 1 A at the H.T. side on no-load at a p.f. of 0.385 lagging. Calculate the iron losses.

   If a load of 50 A at a power of 0.8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its power factor. **[847 W; 5.44 A; 0.74 lag]**
7. A 400/200-V, 1-phase transformer is supplying a load of 50 A at a power factor of 0.866 lagging. The no-load current is 2 A at 0.208 p.f. lagging. Calculate the primary current and primary power factor. **[26.4 A; 0.838 lag]** *(Elect. Machines-I, Indore Univ. 1980)*

32.11. Transformer with Winding Resistance but No Magnetic Leakage

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that :

(*i*) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f. E_2 by an amount $I_2 R_2$ where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to the vector difference of E_2 and resistive voltage drop $I_2 R_2$.

$$\therefore \quad V_2 = E_2 - I_2 R_2 \quad \text{...vector difference}$$

(*ii*) Similarly, primary induced e.m.f. E_1 is equal to the vector difference of V_1 and $I_1 R_1$ where R_1 is the resistance of the primary winding.

$$E_1 = V_1 - I_1 R_1 \quad \text{...vector difference}$$

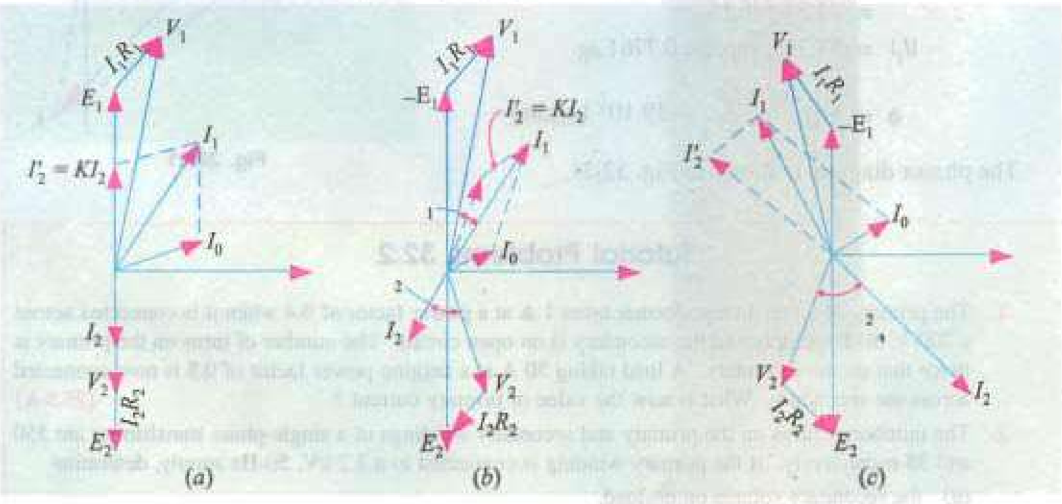


Fig. 32.22

The vector diagrams for non-inductive, inductive and capacitive loads are shown in Fig. 32.22 (*a*), (*b*) and (*c*) respectively.

32.12. Equivalent Resistance

In Fig. 32.23 a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. The resistances have been shown external to the windings.

Fig. 32.23

It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. The value R_2/K^2 will be denoted by R_2'– *the equivalent secondary resistance as referred to primary*.

The copper loss in secondary is $I_2^2 R_2$. This loss is supplied by primary which takes a current of I_1. Hence if R_2' is the *equivalent resistance in primary which would have caused the same loss* as R_2 in secondary, then

$$I_1^2 R_2' = I_2^2 R_2 \text{ or } R_2' = (I_2/I_1)^2 R_2$$

Now, if *we neglect no-load current* I_0, then $I_2/I_1 = I/K^*$. Hence, $R_2' = R_2/K^2$

Similarly, equivalent primary resistance as referred to secondary is $R_1' = K^2 R_1$

In Fig. 32.24, secondary resistance has been transferred to primary side leaving secondary circuit resistanceless. The resistance $R_1 + R_2' = R_1 + R_2/K^2$ is known as the *equivalent or effective resistance of the transformer as referred to primary* and may be designated as R_{01}.

$$\therefore \quad R_{01} = R_1 + R_2' = R_1 + R_2/K^2$$

Similarly, the *equivalent resistance of the transformer as referred to secondary is*

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1.$$

This fact is shown in Fig. 32.25 where all the resistances of the transformer has been concentrated in the secondary winding.

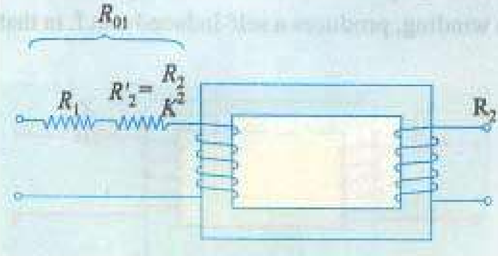


Fig. 32.24

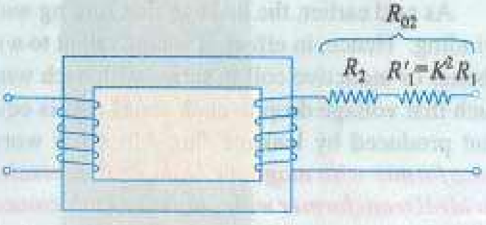


Fig. 32.25

It is to be noted that

1. a resistance of R_1 in primary is equivalent to $K^2 R_1$ in secondary. Hence, it is called *equivalent resistance as referred to secondary* i.e. R_1.

2. a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. Hence, it is called the *equivalent secondary resistance as referred to primary* i.e. R_2'.

3. Total or effective resistance of the transformer as referred to primary is

$$R_{01} = \text{primary resistance + equivalent secondary resistance as referred to primary}$$
$$= R_1 + R_2' = R_1 + R_2/K^2$$

4. Similarly, total transformer resistance as referred to secondary is,

$$R_{02} = \text{secondary resistance + equivalent primary resistance as referred to secondary}$$
$$= R_2 + R_1' = R_2 + K^2 R_1$$

* Actually $I_2 \# 2/I_2' = I/K$ and not $I_2 \# 2/I_1$. However, if I_0 is neglected, then $I_2' = I_1$.

Note : It is important to remember that

(*a*) when shifting any primary resistance to the secondary, ***multiply*** it by K^2 *i.e.* (transformation ratio)^2.

(*b*) when shifting secondary resistance to the primary, ***divide*** it by K^2.

(*c*) however, when shifting any voltage from one winding to another only K is used.

32.13. Magnetic Leakage

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition. It is found, however, that all the flux linked with primary does not link the secondary but part of it *i.e.* Φ_{L_1} completes its magnetic circuit by passing through air rather than around the core, as shown in Fig. 32.26. This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points *a* and *b*, acts along the leakage paths. Hence, this flux is known as ***primary leakage flux*** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of Φ_{L_1}. The flux Φ_{L_1} is in time phase with I_1. It induces an e.m.f. e_{L_1} in primary but not in secondary.

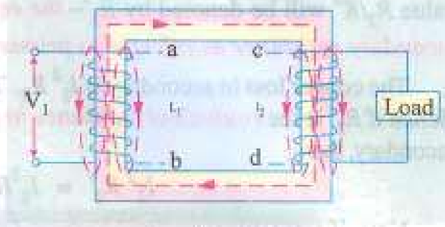


Fig. 32.26

Similarly, secondary ampere-turns (or m.m.f.) acting across points *c* and *d* set up leakage flux Φ_{L_2} which is linked with secondary winding alone (and not with primary turns). This flux Φ_{L_2} is in time phase with I_2 and produces a self-induced e.m.f. e_{L_2} in secondary (but not in primary).

At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.s are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux linking with each winding, produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux. In other words, ***a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits*** as shown in Fig. 32.27 such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

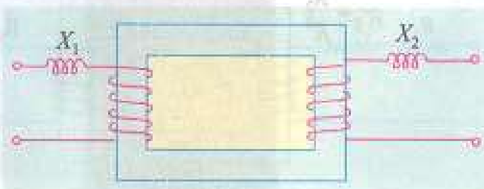


Fig. 32.27

$$X_1 = e_{L1}/I_1 \text{ and } X_2 = e_{L2}/I_2$$

The terms X_1 and X_2 are known as primary and secondary *leakage* reactances respectively.

Following few points should be kept in mind :

1. The leakage flux links one or the other winding but ***not both***, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.

2. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply I_2R_2 and I_2X_2.

3. In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 32.27 because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimised by sectionalizing and interleaving the primary and secondary windings as in Fig. 32.6 or Fig. 32.8.

32.14. Transformer with Resistance and Leakage Reactance

In Fig. 32.28 the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{(R_1^2 + X_1^2)}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{(R_2^2 + X_2^2)}$$

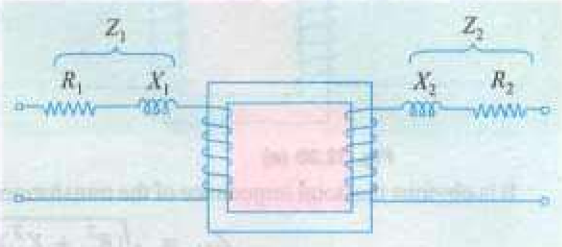


Fig. 32.28

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1). Hence

$$\mathbf{V_1} = \mathbf{E_1} + \mathbf{I_1}(R_1 + jX_1) = \mathbf{E_1} + \mathbf{I_1}\mathbf{Z_1}$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$\mathbf{E_2} = \mathbf{V_2} + \mathbf{I_2}(R_2 + jX_2) = \mathbf{V_2} + \mathbf{I_2}\mathbf{Z_2}$$

The vector diagram for such a transformer for different kinds of loads is shown in Fig. 32.29. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

$\therefore \qquad X_2' = X_2/K^2 \quad \text{and} \quad X_1' = K^2X_1$

and $\qquad X_{01} = X_1 + X_2' = X_1 + X_2/K^2$ and $X_{02} = X_2 + X_1' = X_2 + K^2X_1$

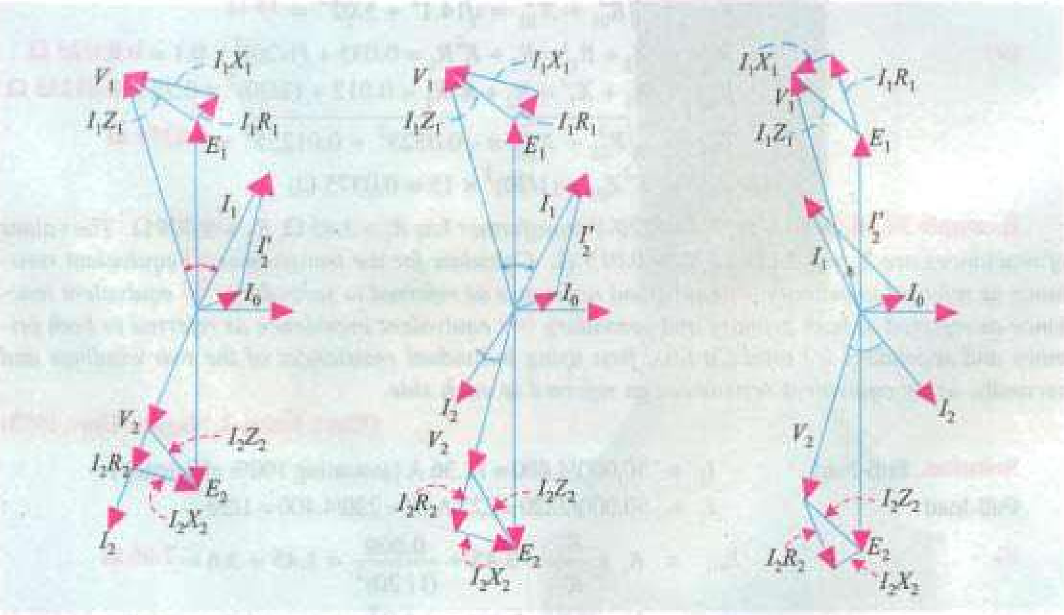


Fig. 32.29

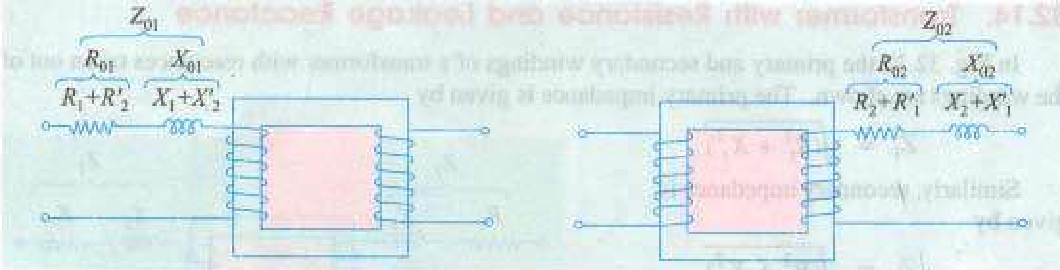


Fig. 32.30 (a) Fig. 32.30 (b)

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} \quad \text{...Fig. 32.30 }(a)$$

and

$$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} \quad \text{...Fig. 32.30 }(b)$$

Example 32.15. *A 30 kVA, 2400/120-V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance referred to the (i) high voltage side and (ii) the low-voltage side.*

(Electrical Machines-I, Bangalore Univ. 1987)

Solution. $K = 120/2400 = 1/20;\ R_1 = 0.1\ \Omega,\ X_1 = 0.22\ \Omega$

$R_2 = 0.035\ \Omega$ and $X_2 = 0.012\ \Omega$

(*i*) Here, high-voltage side is, obviously, the primary side. Hence, values as referred to primary side are

$$R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + 0.035/(1/20)^2 = \mathbf{14.1\ \Omega}$$
$$X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 0.22 + 0.12/(1/20)^2 = \mathbf{5.02\ \Omega}$$
$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15\ \Omega}$$

(*ii*)

$$R_{02} = R_2 + R_1' = R_2 + K^2R_1 = 0.035 + (1/20)^2 \times 0.1 = \mathbf{0.03525\ \Omega}$$
$$X_{02} = X_2 + X_1' = X_2 + K^2X_1 = 0.012 + (1/20)^2 \times 0.22 = \mathbf{0.01255\ \Omega}$$
$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} = \mathbf{0.0374\ \Omega}$$
$$(\text{or } Z_{02} = K^2Z_{01} = (1/20)^2 \times 15 = 0.0375\ \Omega)$$

Example 32.16. *A 50-kVA, 4,400/220-V transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The values of reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer (i) equivalent resistance as referred to primary (ii) equivalent resistance as referred to secondary (iii) equivalent reactance as referred to both primary and secondary (iv) equivalent impedance as referred to both primary and secondary (v) total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.*

(Elect. Engg.-I, Nagpur Univ. 1993)

Solution. Full-load $I_1 = 50{,}000/4{,}400 = 11.36$ A (assuming 100% efficiency)

Full-load $I_2 = 50{,}000/2220 = 227$ A; $K = 220/4{,}400 = 1/20$

(*i*)
$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(1/20)^2} = 3.45 + 3.6 = \mathbf{7.05\ \Omega}$$

(*ii*)
$$R_{02} = R_2 + K^2R_1 = 0.009 + (1/20)^2 \times 3.45 = 0.009 + 0.0086 = \mathbf{0.0176\ \Omega}$$

Also,
$$R_{02} = K^2 R_{01} = (1/20)^2 \times 7.05 = 0.0176\ \Omega \text{ (check)}$$

(iii) $X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 5.2 + 0.015/(1/20)^2 = $ **11.2 Ω**

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.015 + 5.2/20^2 = $ **0.028 Ω**

Also $X_{02} = K^2 X_{01} = 11.2/400 = 0.028\ \Omega$ (check)

(iv) $Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} = \sqrt{(7.05^2 + 11.2)^2} = $ **13.23 Ω**

$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} = \sqrt{(0.0176^2 + 0.028)^2} = $ **0.03311 Ω**

Also $Z_{02} = K^2 Z_{01} = 13.23/400 = 0.0331\ \Omega$ (check)

(v) Cu loss $= I_1^2 R_2 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = $ **910 W**

Also Cu loss $= I_1^2 R_{01} = 11.36^2 \times 7.05 = $ **910 W**

$= I_2^2 R_{02} = 227^2 \times 0.0176 = $ **910 W**

Example 32.17. *A transformer with a 10 : 1 ratio and rated at 50-kVA, 2400/240-V, 50-Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V.*

(a) What load impedance connected to low-tension size will be loading the transformer fully at 0.8 power factor (lag) ?

(b) What is the value of this impedance referred to high tension side ?

(c) What is the value of the current referred to the high tension side ?

(Elect. Engineering-I, Bombay Univ. 1987)

Solution. (a) F. L. $I_2 = 50{,}000/240 = 625/3$ A; $Z_2 = \dfrac{240}{(625/3)} = $ **1.142 Ω**

(b) $K = 240/2400 = 1/10$

The secondary impedance referred to primary side is

$Z_2' = Z_2/K^2 = 1.142/(1/10)^2 = $ **114.2 Ω**

(c) Secondary current referred to primary side is $I_2' = KI_2 = (1/10) \times 625/3 = $ **20.83 A**

Example 32.18. *The full-load copper loss on the h.v. side of a 100-kVA, 11000/317-V, 1-phase transformer is 0.62 kW and on the L.V. side is 0.48 kW.*

(i) Calculate R_1, R_2 and R_3 in ohms (ii) the total reactance is 4 per cent, find X_1, X_2 and X_3 in ohms if the reactance is divided in the same proportion as resistance.

(Elect. Machines A.M.I.E., Sec. B, 1991)

Solution. (i) F.L. $I_1 = 100 \times 10^3/11000 = 9.1$ A. F.L. $I_2 = 100 \times 10^3/317 = 315.5$ A

Now, $I_1^2 R_1 = 0.62$ kW or $R_1 = 620/9.1^2 = 7.5\ \Omega$

$I_2^2 R_2 = 0.48$ kW, $R_2 = 480/315.5^2 = 0.00482\ \Omega$ *

$R_2' = R_2/K^2 = 0.00482 \times (11{,}000/317)^2 = 5.8\ \Omega$

% reactance $= \dfrac{I_1 \times X_{01}}{V_1} \times 100$ or $4 = \dfrac{9.1 \times X_{01}}{11000} \times 100$, $X_{01} = 48.4\ \Omega$

$X_1 + X_2' = 48.4\ \Omega$. Given $R_1/R_2' = X_1/X_2'$

or $(R_1 + R_2')/R_2' = (X_1 + X_2')/X_2'$ $(7.5 + 5.8)/5.8 = 48.4/X_2'$ $\therefore$ $X_2' = 21.1\ \Omega$

$\therefore$ $X_1 = 48.4 - 21.1 = 27.3\ \Omega$, $X_2 = 21.1 \times (317/11000)^2 = $ **0.175 Ω**

Example 32.19. *The following data refer to a 1-phase transformer :*

Turn ratio 19.5 : 1 ; $R_1 = 25\ \Omega$; $X_1 = 100\ \Omega$; $R_2 = 0.06\ \Omega$; $X_2 = 0.25\ \Omega$. No-load current = 1.25 A leading the flux by 30°.

The secondary delivers 200 A at a terminal voltage of 500 V and p.f. of 0.8 lagging. Determine *by the aid of a vector diagram, the primary applied voltage, the primary p.f. and the efficiency.*

(Elect. Machinery-I, Madras Univ. 1989)

Solution. The vector diagram is similar to Fig. 30.28 which has been redrawn as Fig. 32.31. Let us take V_2 as the reference vector.

$$\therefore \qquad \mathbf{V_2} = 500\angle 0° = 500 + j0$$

$$\mathbf{I_2} = 200(0.8 - j0.6) = 160 - j120$$

$$\mathbf{Z_2} = (0.06 + j0.25)$$

$$\mathbf{E_2} = V_2 + \mathbf{I_2 Z_2}$$

$$= (500 + j0) + (160 - j120)(0.06 + j0.25)$$

$$= 500 + (39.6 + j32.8) = 539.6 + j32.8 = 541\angle 3.5°$$

Obviously, $\beta = 3.5°$

$$\mathbf{E_1} = \mathbf{E_2}/K = 19.5\,\mathbf{E_2} = 19.5(539.6 + j32.8)$$

$$= 10{,}520 + j640$$

$$\therefore -\mathbf{E_1} = -10{,}520 - j640 = 10{,}540\angle 183.5°$$

$$\mathbf{I_2'} = -\mathbf{I_2}K = (-160 + j120)/19.5$$

$$= -8.21 + j6.16$$

As seen from Fig. 32.31, $\mathbf{I_0}$ leads $\mathbf{V_2}$ by an angle

$$= 3.5° + 90° + 30° = 123.5°$$

$$\therefore \qquad \mathbf{I_0} = 1.25\angle 123.5°$$

$$= 1.25(\cos 123.5° + j\sin 123.5°)$$

$$= 1.25(-\cos 56.5° + j\sin 56.5°)$$

$$= -0.69 + j1.04$$

$$\mathbf{I_1} = \mathbf{I_2'} + \mathbf{I_0} = (-8.21 + j6.16) + (-0.69 + j1.04)$$

$$= -8.9 + j7.2 = 11.45\angle 141°$$

$$\mathbf{V_2} = -\mathbf{E_1} + \mathbf{I_1 Z_1}$$

$$= -10{,}520 - j640 + (-8.9 + j7.5)(25 + j100)$$

$$= -10{,}520 - j640 - 942 - j710$$

$$= -11{,}462 - j1350$$

$$= 11{,}540\angle 186.7°$$

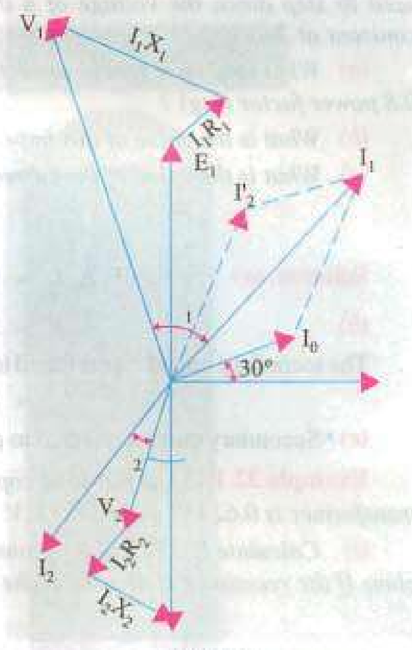


Fig. 32.31

Phase angle between $\mathbf{V_1}$ and $\mathbf{I_1}$ is $= 186.7° - 141° = 45.7°$

$$\therefore \qquad \text{primary p.f.} = \cos 45.7° = \mathbf{0.698\ (lag)}$$

$$\text{No-load primary input power} = V_1 I_0 \sin\phi_0$$

$$= 11{,}540 \times 1.25 \times \cos 60° = 7{,}210\ \text{W}$$

$$R_{02} = R_2 + K^2 R_1 = 0.06 + 25/19.5^2 = 0.1257\ \Omega$$

$$\text{Total Cu loss as referred to secondary} = I_2^2 R_{02} = 200^2 \times 0.1257 = 5{,}030\ \text{W}$$

$$\text{Output} = V_2 I_2 \cos\phi_2 = 500 \times 200 \times 0.8 = 80{,}000\ \text{W}$$

$$\text{Total losses} = 5030 + 7210 = 12{,}240\ \text{W}$$

$$\text{Input} = 80{,}000 + 12{,}240 = 92{,}240\ \text{W}$$

$$\eta = 80{,}000/92{,}240 = 0.8674 \text{ or } \mathbf{86.74\%}$$

Example 32.20. *A 100 kVA, 1100/220 V, 50 Hz, single-phase transformer has a leakage impedance of (0.1 + 0/40) ohm for the H.V. winding and (0.006 + 0.015) ohm for the L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.*

(Bharathiar Univ. Nov. 1997)

Solution. Turns ratio $= (N_1/N_2) = (V_1/V_2) = 1100/220 = 5$

(i) Referred to H.V. side :

Resistance $= r_1 + r_2 = 0.1 + (25 \times 0.006) = 0.25$ ohm

Reactance $= x_1 + x_2' = 0.4 + (25 \times 0.015) = 0.775$ ohm

Impedance $= (0.25^2 + 0.775^2)^{0.5} = 0.8143$ ohm

(ii) Referred to L.V. side :

Resistance $= 0.25/25 = 0.01$

(or resistance $= 0.006 + (0.1/25) = 0.01$ ohm)

Reactance $= 0.775/25 = 0.031$ ohm

Impedance $= 0.8143/25 = 0.0326$ ohm

32.15. Simplified Diagram

The vector diagram of Fig. 32.29 may be considerably simplified if the no-load current I_0 is neglected. Since I_0 is 1 to 3 per cent of full-load primary current I_1, it may be neglected without serious error. Fig. 32.32 shows the diagram of Fig. 32.29 with I_0 omitted altogether.

In Fig. 32.32, V_2, V_1, ϕ_2 are known, hence E_2 can be found by adding vectorially $I_2 R_2$ and $I_2 X_2$ to V_2. Similarly, V_1 is given by the vector addition of $I_1 R_1$ and $I_1 X_1$ to E_1. All the voltages on the primary side can be transferred to the secondary side as shown in figure, where the upper part of the diagram has been rotated through 180°. However, it should be noted that each voltage or voltage drop should be multiplied by transformation ratio K.

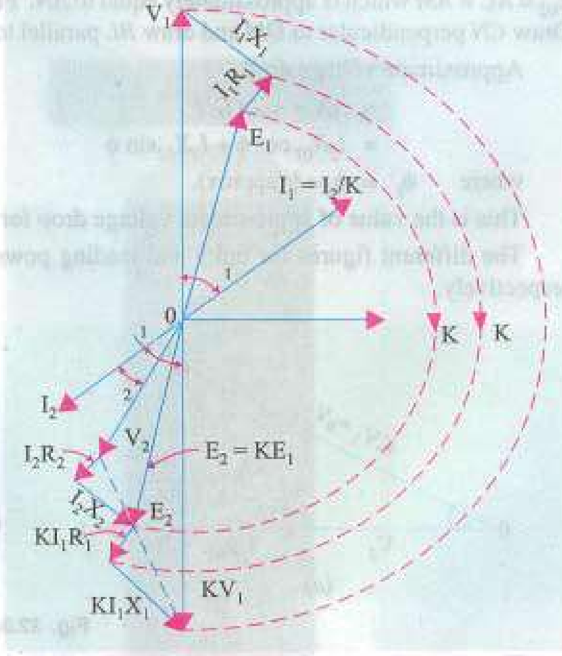


Fig. 32.32

The lower side of the diagram has been shown separately in Fig. 32.34 laid horizontally where vector for V_2 has been taken along X-axis.

It is a simple matter to find transformer regulation as shown in Fig. 32.34 or Fig. 32.35.

It may be noted that $\mathbf{V_2} = K\mathbf{V_1} - \mathbf{I_2}(R_{02} + jX_{02}) = K\mathbf{V_1} - \mathbf{I_2 Z_{02}}$.*

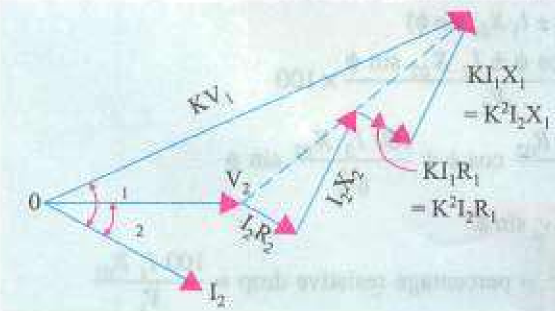


Fig. 32.33

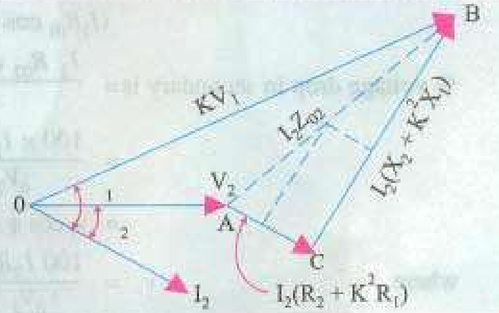


Fig. 32.34

* Also, $\mathbf{V_1} = (\mathbf{V_2} + \mathbf{I_2 Z_{02}})$ #3/K

32.16. Total Approximate Voltage Drop in a Transformer

When the transformer is on *no-load*, then V_1 is approximately equal to E_1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = {}_0V_2$ where ${}_0V_2$ is secondary terminal voltage on *no-load*, hence no-load secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is $I_2 Z_{02}$ as shown in Fig. 32.35. The approximate voltage drop of the transformer *as referred to secondary* is found thus :

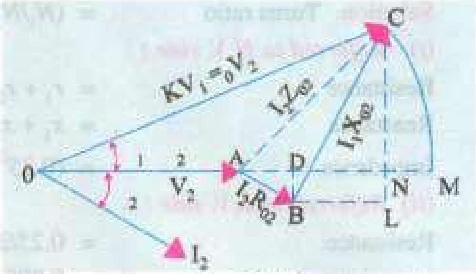


Fig. 32.35

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop $I_2 Z_{02} = AC = AM$ which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

Approximate voltage drop

$$= AN = AD + DN$$

$$= I_2R_{02} \cos\phi + I_2X_{02} \sin\phi$$

where $\quad \phi_1 = \phi_2 = \phi$ (approx).

This is the value of approximate voltage drop for a *lagging* power factor.

The different figures for unity and leading power factors are shown in Fig. 32.36 (*a*) and (*b*) respectively.

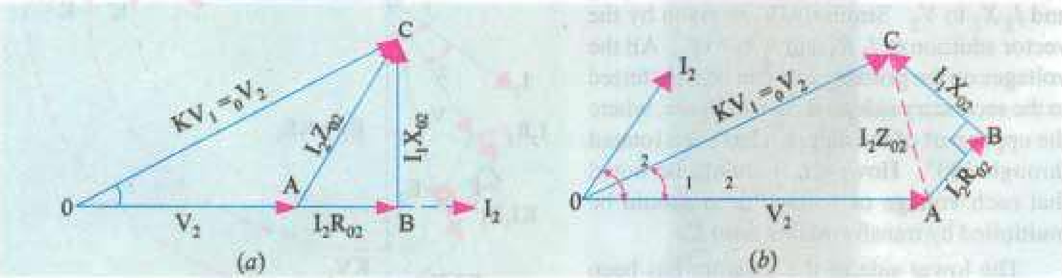


Fig. 32.36

The approximate voltage drop for *leading* power factor becomes

$$(I_2R_{02} \cos\phi \pm I_2 X_{02} \sin\phi)$$

In general, approximate voltage drop is $(I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi)$

It may be noted that approximate voltage drop as referred to primary is

$$(I_1R_{01} \cos\phi \pm I_1 X_{01} \sin\phi)$$

$$\% \text{ voltage drop in secondary is} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{{}_0V_2} \times 100$$

$$= \frac{100 \times I_2 R_{02}}{{}_0V_2} \cos\phi \pm \frac{100\, I_2 X_{02}}{{}_0V_2} \sin\phi$$

$$= v_r \cos\phi \pm v_x \sin\phi$$

where $\quad v_r = \dfrac{100\, I_2R_{02}}{{}_0V_2}$ = percentage resistive drop $= \dfrac{100\, I_1 R_{01}}{V_1}$

$\quad v_x = \dfrac{100\, I_2X_{02}}{{}_0V_2}$ = percentage reactive drop $= \dfrac{100\, I_1 X_{01}}{V_1}$

32.17. Exact Voltage Drop

With reference to Fig. 32.35, it is to be noted that exact voltage drop is AM and not AN. If we add the quantity NM to AN, we will get the exact value of the voltage drop.

Considering the right-angled triangle OCN, we get

$$NC^2 = OC^2 - ON^2 = (OC + ON)(OC - ON) = (OC + ON)(OM - ON) = 2\,OC \times NM$$

$$\therefore \quad NM = NC^2/2.OC \quad \text{Now, } NC = LC - LN = LC - BD$$

$$\therefore \quad NC = I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi \qquad \therefore NM = \frac{(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

$\therefore$ For a ***lagging*** power factor, exact voltage drop is

$$= AN + NM = (I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

For a ***leading*** power factor, the expression becomes

$$= (I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi + I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

In general, the voltage drop is

$$= (I_2 R_{02} \cos\phi \pm I_2 R_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi \pm I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

Percentage drop is

$$= \frac{(I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi) \times 100}{{}_0V_2} + \frac{(I_2 X_{02} \cos\phi \mp I_2 R_{02} \sin\phi)^2 \times 100}{2_0 V_2^2}$$

$$= (v_r \cos\phi \pm v_x \sin\phi) + (1/200)(v_x \cos\phi \mp v_r \sin\phi)^2$$

The upper signs are to be used for a ***lagging*** power factor and the lower ones for a ***leading*** power factor.

Example 32.21. *A 230/460-V transformer has a primary resistance of 0.2 Ω and reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging.*

(Electric. Machines-II, Bangalore Univ. 1991)

Solution.

$$K = 460/230 = 2;\ R_{02} = R_2 + K^2 R_1 = 0.75 + 2^2 \times 0.2 = 1.55\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 1.8 + 2^2 \times 0.5 = 3.8\ \Omega$$

$$\text{Voltage drop} = I_2 (R_{02} \cos\phi + X_{02} \sin\phi) = 10(1.55 \times 0.8 + 3.8 \times 0.6) = 35.2\text{V}$$

$$\therefore \text{Secondary terminal voltage} = 460 - 35.2 = \mathbf{424.8\ V}$$

Example 32.22. *Calculate the regulation of a transformer in which the percentage resistance drops is 1.0% and percentage reactance drop is 5.0% when the power factor is (a) 0.8 lagging (b) unity and (c) 0.8 leading.* **(Electrical Engineering, Banaras Hindu Univ. 1988)**

Soluion. We will use the approximate expression of Art 30.16.

(*a*) **p.f. = cos φ = 0.8 lag** $\quad \mu = v_r \cos\phi + v_x \sin\phi = 1 \times 0.8 + 5 \times 0.6 = \mathbf{3.8\%}$

(*b*) **p.f. = cos φ = 1** $\quad \mu = 1 \times 1 + 5 \times 0 = \mathbf{1\%}$

(*c*) **p.f. = cos φ = 0.8 lead** $\quad \mu = 1 \times 0.8 - 5 \times 0.6 = \mathbf{-2.2\%}$

Example 32.23. *A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. The percentage voltage regulation (μ) is given by

$$\mu = v_r \cos\phi + v_x \sin\phi$$

where v_r is the percentage resistive drop and v_x is the percentage reactive drop.

Differentiating the above equation, we get $\dfrac{d\mu}{d\phi} = -v_r \sin\phi + v_x \cos\phi$

For regulation to be maximum, $d\mu/d\phi = 0 \quad \therefore \quad -v_r \sin\phi + v_x \cos\phi = 0$

or $\tan\phi = v_x/v_r = 5/2.5 = 2 \therefore \phi = \tan^{-1}(2) = 63.5°$ Now, $\cos\phi = 0.45$ and $\sin\phi = 0.892$

Maximum percentage regulation $= (2.5 \times 0.45) + (5 \times 0.892) = 5.585$

Maximum percentage regulation is **5.585** and occurs at a power factor of **0.45** (lag).

Example 32.24. *Calculate the percentage voltage drop for a transformer with a percentage resistance of 2.5% and a percentage reactance of 5% of rating 500 kVA when it is delivering 400 kVA at 0.8 p.f. lagging.* **(Elect. Machinery-I, Indore Univ. 1987)**

Solution.
$$\%\text{ drop} = \frac{(\%R)\, I \cos\phi}{I_f} + \frac{(\%X)\, I \sin\phi}{I_f}$$

where I_f is the full-load current and I the actual current.

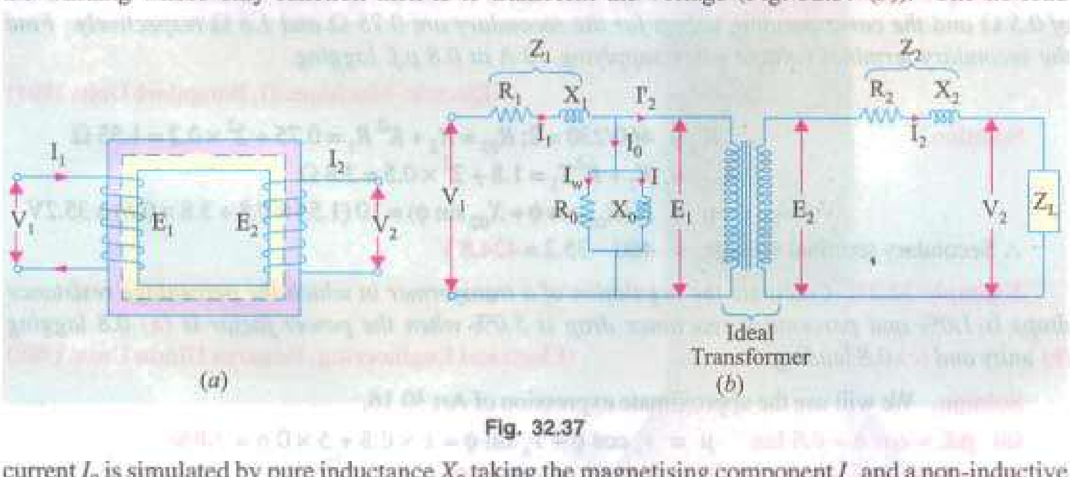
$$\therefore \quad \%\text{ drop} = \frac{(\%R) kW}{kVA \text{ rating}} + \frac{(\%X)\ kVAR}{kVA \text{ rating}}$$

In the present case, $\quad kW = 400 \times 0.8 = 320$ and $kVAR = 400 \times 0.6 = 240$

$$\therefore \quad \%\text{ drop} = \frac{2.5 \times 320}{500} + \frac{5 \times 240}{500} = \mathbf{4\%}$$

32.18. Equivalent Circuit

The transformer shown diagrammatically in Fig. 32.37 (*a*) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage (Fig. 32.37 (*b*)). The no-load



(*a*) (*b*)

Fig. 32.37

current I_0 is simulated by pure inductance X_0 taking the magnetising component I_μ and a non-inductive resistance R_0 taking the working component I_w connected in parallel across the primary circuit. The value of E_1 is obtained by subtracting vectorially $I_1 Z_1$ from V_1. The value of $X_0 = E_1/I_0$ and of $R_0 = E_1/I_w$. It is clear that E_1 and E_2 are related to each other by expression

$$E_2/E_1 = N_2/N_1 = K.$$

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary. In that case, we would have to work in one winding only which is more convenient.

The primary equivalent of the secondary induced voltage is $E_2' = E_2/K = E_1$.

Similarly, primary equivalent of secondary terminal or output voltage is $V_2' = V_2/K$.

Primary equivalent of the secondary current is $I_2' = KI_2$.

For transferring secondary impedance to primary K^2 is used.

$$R_2' = R_2/K^2, X_2' = X_2/K^2, Z_2' = Z_2/K^2$$

The same relationship is used for shifting an external load impedance to the primary.

The secondary circuit is shown in Fig. 32.38 (*a*) and its equivalent primary values are shown in Fig. 32.38 (*b*).

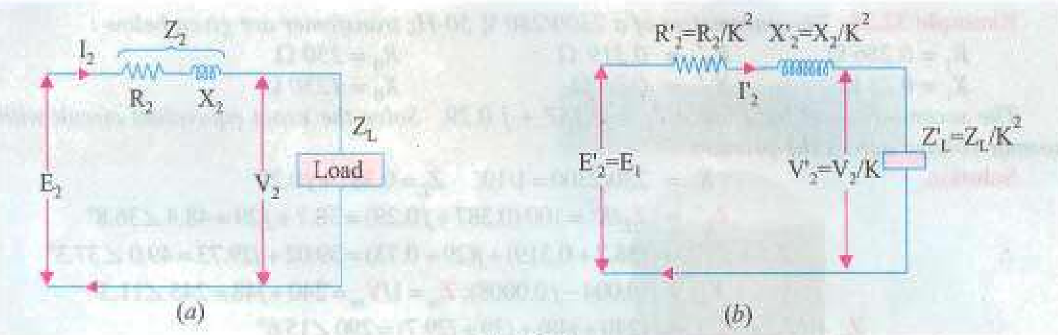


Fig. 32.38

The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in Fig. 32.39. This is known as the exact equivalent circuit but it presents a somewhat harder circuit problem to solve. A simplification can be made by transferring the exciting circuit across the terminals as in Fig. 32.40 or in Fig. 32.41 (*a*). It should be noted that in this case $X_0 = V_1/I_\mu$.

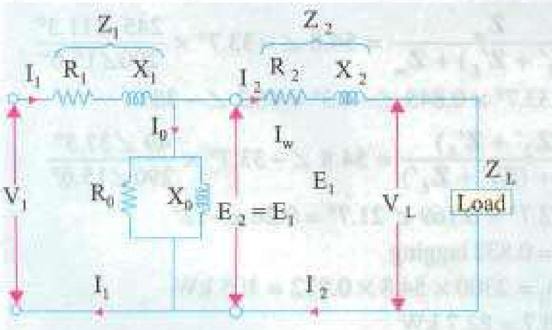


Fig. 32.39

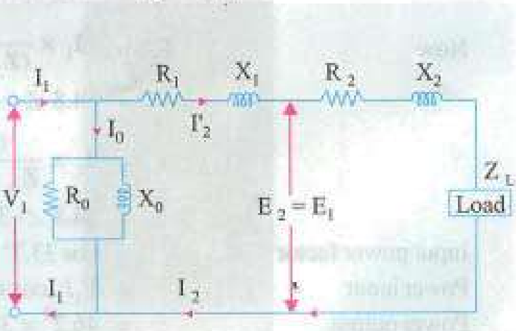


Fig. 32.40

Further simplification may be achieved by omitting I_0 altogether as shown in Fig. 32.41(*b*).

From Fig. 32.39 it is found that total impedance between the input terminal is

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_m \,||\, (\mathbf{Z}_2' + \mathbf{Z}_L') = \left(\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')}\right)$$

where $\mathbf{Z}_2' = R_2' + jX_2'$ and $\mathbf{Z}_m$ = impedance of the exciting circuit.

This is so because there are two parallel circuits, one having an impedance of $\mathbf{Z}_m$ and the other having $\mathbf{Z}_2'$ and $\mathbf{Z}_L'$ in series with each other.

$$\therefore \quad \mathbf{V}_1 = \mathbf{I}_1\left[\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')}\right]$$

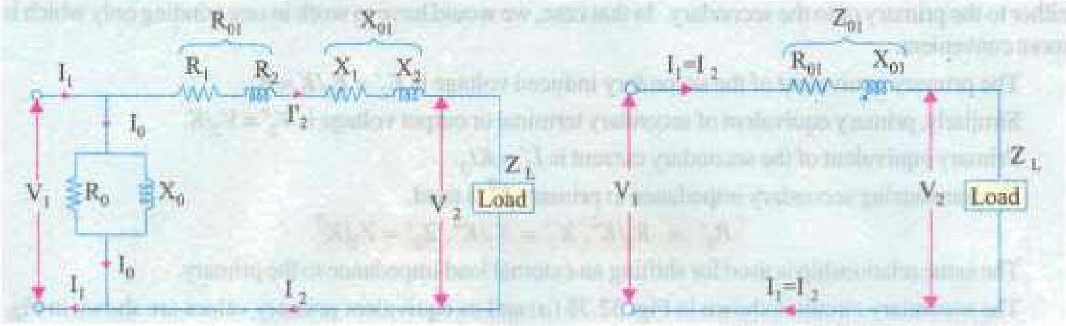


Fig. 32.41 (a) Fig. 32.41 (b)

Example 32.25. *The parametres of a 2300/230 V, 50-Hz transfomer are given below :*

$R_1 = 0.286\ \Omega \qquad R_2' = 0.319\ \Omega \qquad R_0 = 250\ \Omega$

$X_1 = 0.73\ \Omega \qquad X_2' = 0.73\ \Omega \qquad X_0 = 1250\ \Omega$

The secondary load impedance $Z_L = 0.387 + j\,0.29$. Solve the exact equivalent circuit with normal voltage across the primary.

Solution.

$$K = 230/2300 = 1/10; \quad Z_L = 0.387 + j0.29$$

$$\mathbf{Z}_L' = \mathbf{Z}_L/K^2 = 100\,(0.387 + j0.29) = 38.7 + j29 = 48.4\angle 36.8^\circ$$

$$\therefore \quad \mathbf{Z}_2' + \mathbf{Z}_L' = (38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = 49.0\angle 37.3^\circ$$

$$Y_m = (0.004 - j\,0.0008);\ \mathbf{Z}_m = 1/\mathbf{Y}_m = 240 + j48 = 245\angle 11.3^\circ$$

$$\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L') = (240 + j48) + (39 + j29.7) = 290\angle 15.6^\circ$$

$$\therefore \quad \mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1 + \dfrac{\mathbf{Z}_m(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m(\mathbf{Z}_2' + \mathbf{Z}_L')}} = \left[\frac{2300\angle 0^\circ}{0.286 + j0.73 + 41.4\angle 33^\circ}\right]$$

$$= \frac{2300\angle 0^\circ}{42\angle 33.7^\circ} = 54.8\angle -33.7^\circ$$

Now

$$\mathbf{I}_2' = \mathbf{I}_1 \times \frac{\mathbf{Z}_m}{(\mathbf{Z}_2' + \mathbf{Z}'_L) + \mathbf{Z}_m} = 54.8\angle -33.7^\circ \times \frac{245\angle 11.3^\circ}{290\angle 15.6^\circ}$$

$$= 54.8\angle -33.7^\circ \times 0.845\angle -4.3^\circ = 46.2\angle -38^\circ$$

$$\mathbf{I}_0 = \mathbf{I}_1 \times \frac{(\mathbf{Z}_2' + \mathbf{Z}'_L)}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} = 54.8\angle -33.7^\circ \times \frac{49\angle 37.3^\circ}{290\angle 15.6^\circ}$$

$$= 54.8\angle -33.7^\circ \times 0.169\angle 21.7^\circ = 9.26\angle -12^\circ$$

Input power factor $= \cos 33.7^\circ = 0.832$ lagging

Power input $= V_1 I_1 \cos\phi_1 = 2300 \times 54.8 \times 0.832 = 105$ kW

Power output $= 46.2^2 \times 38.7 = 82.7$ kW

Primary Cu loss $= 54.8^2 \times 0.286 = 860$ W

Secondary Cu loss $= 46.2^2 \times 0.319 = 680$ W; Core loss $= 9.26^2 \times 240 = 20.6$ kW

$$\eta = (82.7/105) \times 100 = 78.8\%;\ V_2' = I_2'\,Z_L' = 46.2 \times 48.4 = 2{,}240\text{V}$$

$$\therefore \quad \text{Regulation} = \frac{2300 - 2240}{2240} \times 100 = \mathbf{2.7\%}$$

Example 32.26. *A transformer has a primary winding with a voltage-rating of 600 V. Its secondary-voltage rating is 1080 V with an additional tap at 720 V. An 8 kW resistive load is connected across 1080-V output terminals. A purely inductive load of 10kVA is connected across the tapping point and common secondary terminal so as to get 720 V. Calculate the primary current and its power-factor. Correlate it with the existing secondary loads. Neglect losses and magnetizing current.* **(Nagpur University, Winter 1999)**

Solution. Loads are connected as shown in Fig. 32.42.

$$I_{r2} = \frac{8000}{1080} = 7.41 \text{ at unity p.f.}$$

$$I_{L2} = 10000/720 = 13.89 \text{ at zero lagging p.f.}$$

These are reflected on to the primary sides with appropriate ratios of turns, with corresponding power-factors. If the corresponding transformed currents are represented by the above symbols modified by dashed superscripts,

$$I'_{r2} = 7.41 \times 1080/600 = 13.34 \text{ A at unity p.f.}$$

$$I'_{L2} = 13.89 \times 720/600 = 16.67 \text{ A at zero lag. p.f.}$$

Hence, $$I_{r2} = [I'^2_{r2} + I'^2_{L2}]^{0.5} = 21.35 \text{ A, at } 0.625 \text{ lag p.f.}$$

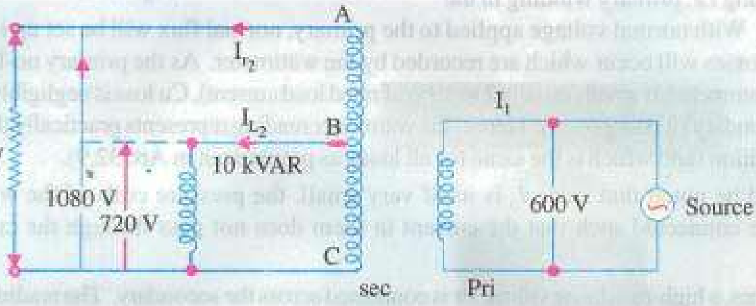


Fig. 32.42

Correlation : Since losses and magnetizing current are ignored, the calculations for primary current and its power-factor can also be made with data pertaining to the two Loads (in kW/kVAR), as supplied by the 600 V source.

S = Load to be supplied : 8 kW at unity p.f. and 10 kVAR lagging

Thus, $$S = P + jQ = 8 - j\ 10 \text{ kVA}$$

$$S = (8^2 + 10^2)^{0.5} = 12.8 \text{ kVA}$$

$$\text{Power} - \text{factor} = \cos\phi = 8/12.8 = 0.625 \text{ lag}$$

$$\text{Primary current} = 12.8 \times 1000/600 = 21.33 \text{ A}$$

32.19. Transformer Tests

As shown in Ex 32.25, the performance of a transformer can be calculated on the basis of its equivalent circuit which contains (Fig. 32.41) four main parameters, the equivalent resistance R_{01} as referred to primary (or secondary R_{02}), the equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02}), the core-loss conductance G_0 (or resistance R_0) and the magnetising susceptance B_0 (or reactance X_0). These constants or parameters can be easily determined by two tests *(i)* **open-circuit test** and *(ii)* **short-circuit test**. These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of very large a.c. machinery consists of running two tests similar to the open and short-circuit tests of a transformer.

Small transformer

32.20. Open-circuit or No-load Test

The purpose of this test is to determine no-load loss or core loss and no-load I_0 which is helpful in finding X_0 and R_0.

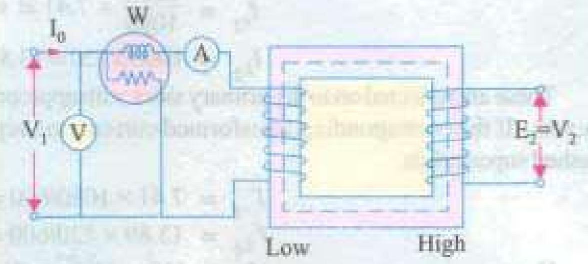


Fig. 32.43

One winding of the transformer – whichever is convenient but usually high voltage winding – is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter W, voltmeter V and an ammeter A are connected in the low-voltage winding *i.e.* primary winding in the present case. With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which are recorded by the wattmeter. As the primary no-load current I_0 (as measured by ammeter) is small (usually 2 to 10% of rated load current), Cu loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition (and which is the same for all loads as pointed out in Art. 32.9).

It should be noted that since I_0 is itself very small, the pressure coils of the wattmeter and the voltmeter are connected such that the current in them does not pass through the current coil of the wattmeter.

Sometimes, a high-resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced e.m.f. in the secondary winding. This helps to find transformation ratio K.

The no-load vector diagram is shown in Fig. 32.16. If W is the wattmeter reading (in Fig. 32.43), then

$$W = V_1 I_0 \cos\phi_0 \quad \therefore \quad \cos\phi_0 = W/V_1 I_0$$

$$\therefore \quad I_\mu = I_0 \sin\phi_0,\ I_w = I_0\cos\phi_0 \ \therefore\ X_0 = V_1/I_\mu \text{ and } R_0 = V_1/I_w$$

Or since the current is practically all-exciting current when a transformer is on no-load (*i.e.* $I_0 \cong I_\mu$) and as the voltage drop in primary leakage impedance is small*, hence the exciting admittance Y_0 of the transformer is given by $I_0 = V_1 Y_0$ or $Y_0 = I_0/V_1$.

The exciting conductance G_0 is given by $W = V_1^2 G_0$ or $G_0 = W/V_1^2$.

The exciting susceptance $B_0 = \sqrt{(Y_0^2 - G_0^2)}$

Example. 32.27. *In no-load test of single-phase transformer, the following test data were obtained :*

Primary voltage : 220 V ; Secondary voltage : 110 V ;

Primary current : 0.5 A ; Power input : 30 W.

Find the following :

(i) The turns ratio (ii) the magnetising component of no-load current (iii) its working (or loss) component (iv) the iron loss.

Resistance of the primary winding = 0.6 ohm.

Draw the no-load phasor diagram to scale. **(Elect. Machine A.M.I.E. 1990)**

Solution. (*i*) Turn ratio, $N_1/N_2 = 220/110 = 2$

(*ii*) $W = V_1 I_0 \cos\phi_0$; $\cos\phi_0 = 30/220 \times 0.5 = 0.273$; $\sin\phi_0 = 0.962$

$$I_\mu = I_0 \sin\phi_0 = 0.5 \times 0.962 = 0.48 \text{ A}$$

* If it is not negligibly small, then $I_0 = E_1 Y_0$ *i.e.* instead of V_1 we will have to use E_1.

(iii) $I_w = I_0 \cos \phi_0 = 0.5 \times 0.273 = 0.1365$ A

(iv) Primary Cu loss $= I_0^2 R_1 = 0.5^2 \times 0.6 = 0.15$ W

$\therefore$ Iron loss $= 30 - 0.15 = 29.85$ W

Example 32.28. *A 5 kVA 200/1000 V, 50 Hz, single-phase transformer gave the following test results :*

O.C. Test (L.V. Side) : 2000 V, 1.2 A, 90 W

S.C. Test (H.V. Side) : 50 V, 5A, 110 W

(i) Calculate the parameters of the equivalent circuit referred to the L.V. side.

(ii) Calculate the output secondary voltage when delivering 3 kW at 0.8 p.f. lagging, the input primary voltage being 200 V. Find the percentage regulation also.

(Nagpur University, November 1998)

Solution. *(i)* Shunt branch parameters from O.C. test (L.V. side) :

$R_0 = V^2/P_i = 200^2/90 = 444$ ohms, $I_{a0} = 200/444 = 0.45$ amp

$I_\mu = (1.2^2 - 0.45^2)^{0.5} = 1.11$ amp, $X_m = 200/1.11 = 180.2$ ohms

All these are referred to L.V. side.

(ii) Series-branch Parameters from S.C test (H.V side) :

Since the S.C. test has been conducted from H.V. side, the parameters will refer to H.V. side. They should be converted to the parameters referred to L.V. side by transforming them suitably.

From S.C. Test readings, $Z = 50/5 = 10$ ohms

$R = 110/25 = 4.40$ ohms, $X = (10^2 - 4.4^2)^{0.5} = 8.9$ ohms

These are referred to H.V. side.

For referring these to L.V. side, transform these using the ratio of turns, as follows :

$$r_1 = 4.40 \times (200/1000)^2 = 0.176 \text{ ohm}$$

$$x_1 = 8.98 \times (200/1000)^2 = 0.36 \text{ ohm}$$

Equivalent circuit can be drawn with R_0 and X_m calculated above and r_1 and x_1 as above.

L.V. Current at rated load= 5000/200 = 25 A

L.V. Current at 3 kW at 0.8 lagging p.f. = (3000/0.80)/200 = 18.75 A

$$\text{Regulation at this load} = 18.75\,(r_1 \cos\phi + x_1 \sin\phi)$$

$$= 18.75\,(0.176 \times 0.80 + 0.36 \times 0.6)$$

$$= +6.69 \text{ Volts} = +(6.69/200) \times 100\% = +3.345\%$$

This is referred to L.V. side, and positive sign means voltage drop.

Regulation in volts ref. to H.V. side = 6.69 × 1000/200 = 33.45 V

With 200 V across primary (*i.e.* L.V. side), the secondary (*i.e.* H.V. side)

terminal voltage $= 1000 - 33.45 = 966.55$ V

Note : Since approximate formula for voltage regulation has been used, the procedure is simpler, and faster.

32.21. Separation of Core Losses

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts *(i)* hysteresis loss $W_h = PB_{max}^{1.6} f$ as given by Steinmetz's empirical relation and *(ii)* eddy current loss $W_e = QB_{max}^2 f^2$ where Q is a constant. The total core-loss is given by

$$W_i = W_h + W_e = PB_{max}^{1.6} f^2 + QB_{max}^2 f^2$$

If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants P and Q and hence calculate hysteresis and eddy current losses separately.

Example 32.29. *In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. Since the flux density is the same in both cases, we can use the relation

Total core loss $W_i = Af + Bf^2$ or $W_i/f = A + Bf$

$\therefore$ $52/40 = A + 40B$ and $90/60 = A + 60B$; $\therefore$ $A = 0.9$ and $B = 0.01$

At 50 Hz, the two losses are

$W_h = A_f = 0.9 \times 50 = 45$ W ; $W_e = Bf^2 = 0.01 \times 50^2 = 25$ W

Example 32.30. *In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:*

Frequency (Hz)	*25*	*40*	*50*	*60*	*80*
Total loss (watt)	*18.5*	*36*	*50*	*66*	*104*

Calculate the eddy current loss per kg at a frequency of 50 Hz.

(Elect. Measur. A.M.I.E. Sec B, 1991)

Solution. When flux density and wave form factor remain constant, the expression for iron loss can be written as

$$W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

The values of W_i/f for different frequencies are as under :

f	25	40	50	60	80
W_i/f	0.74	0.9	1.0	1.1	1.3

The graph between f and W_i/f has been plotted in Fig. 32.44. As seen from it, $A = 0.5$ and $B = 0.01$

$\therefore$ Eddy current loss at 50 Hz $= Bf^2 = 0.01 \times 50^2 = 25$ W

$\therefore$ Eddy current loss/kg $= 25/10 = 2.5$ W

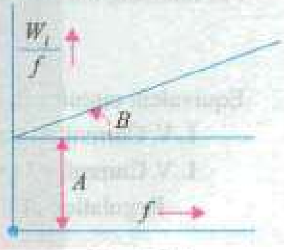


Fig. 32.44

Example 32.31. *In a test for the determination of the losses of a 440-V, 50-Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850 W. Calculate the eddy-current loss at normal voltage and frequency.*

(Elect. Inst. and Meas. Punjab Univ. 1991)

Solution. The flux density in both cases is the same because in second case voltage as well as frequency are halved. Flux density remaining the same, the eddy current loss is proportional to f^2 and hysteresis loss $\propto f$.

Hysteresis loss $\propto f = Af$ and eddy current loss $\propto f^2 = Bf^2$

where A and B are constants.

Total iron loss $\quad W_i = Af + Bf^2 \quad \therefore \quad \frac{W_i}{f} = A + Bf \quad$ ...(*i*)

Now, when $\quad f = 50$ Hz : $W_i = 2500$ W

and when $\quad f = 25$ Hz ; $W_i = 850$ W

Using these values in (*i*) above, we get, from Fig. 32.44

$$2{,}500/50 = A + 50\,B \text{ and } 850/25 = A + 25\,B \quad \therefore \quad B = 16/25 = 0.64$$

Hence, at normal p.d. and frequency

eddy current loss $= Bf^2 = 0.64 \times 50^2 =$ **1600 W**

Hystersis loss $= 2500 - 1600 = 900$ W

Example 32.32. *When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.*

Solution. Hysteresis loss $W_h \propto B_{max}^{1.6} f = PB_{max}^{1.6} f$

Eddy current loss $W_e \propto B^2_{max} f^2 = QB^2_{max} f^2$

From the relation $E = 4.44\, fNB_{max}$ A volt, we get $B_{max} \propto E/f$

Putting this value of B_{max} in the above equations, we have

$$W_h = P\left(\frac{E}{f}\right)^{1.6} f = PE^{1.6} f^{-0.6} \text{ and } W_e = Q\left(\frac{E}{f}\right)^2 f^2 = QE^2$$

In the first case, $E = 1000$ V, $f = 50$ Hz, $W_h = 650$ W, $W_e = 350$ W

$\therefore \quad 650 = P \times 1000^{1.6} \times 50^{-0.6} \qquad \therefore \quad P = 650 \times 1000^{-1.6} \times 50^{0.6}$

Similarly, $350 = Q \times 1000^2 \qquad \therefore \quad Q = 350 \times 1000^{-2}$

Hence, constants P and Q are known.

Using them in the second case, we get

$$W_h = (650 \times 1000^{-1.6} \times 50^{0.6}) \times 2000^{1.6} \times 100^{-0.6} = 650 \times 2 = 1{,}300 \text{ W}$$

$$W_e = (350 \times 1000^{-2}) \times 2{,}000^2 = 350 \times 4 = 1{,}400 \text{ W}$$

$\therefore$ Core loss under new condition is $= 1{,}300 + 1{,}400 =$ **2700 W**

Alternative Solution

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

With 1000 V at 50 Hz

$$W_h = Af \text{ or } 650 = 50\,A\,;\ A = 13$$

$$W_e = Bf^2 \text{ or } 350 = B \times 50^2\,;\ B = 7/50$$

With 2000 V at 100 Hz

$$W_h = Af = 13 \times 100 = 1300 \text{ W and}$$

$$W_e = Bf^2 = (7/50) \times 100^2 = 1400 \text{ W}$$

$\therefore$ New core loss $= 1300 + 1400 =$ **2700 W**

Example 32.33. *A transformer with normal voltage impressed has a flux density of 1.4 Wb/m² and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions ?*

(*a*) *increasing the applied voltage by 10% at rated frequency.*

(*b*) *reducing the frequency by 10% with normal voltage impressed.*

(*c*) *increasing both impressed voltage and frequency by 10 per cent.*

(Electrical Machinery-I, Madras Univ. 1985)

Solution. As seen from Ex. 32.32

$$W_h = PE^{1.6} f^{-0.6} \quad \text{and} \quad W_e = QE^2$$

From the given data, we have $3000 = PE^{1.6} f^{-0.6}$ ...(*i*)

and $1000 = QE^2$ ...(*ii*)

where E and f are the normal values of primary voltage and frequency.

(*a*) Here voltage becomes $= E + 10\%\, E = 1.1\, E$

The new hysteresis loss is $W_h = P(1.1\, E)^{1.6} f^{-0.6}$ ...(*iii*)

Dividing Eq. (*iii*) by (*i*), we get $\dfrac{W_h}{3000} = 1.1^{1.6}$; $W_h = 3000 \times 1.165 =$ **3495 W**

The new eddy-current loss is

$$W_e = Q\,(1.1.\,E)^2 \quad \therefore \quad \frac{W_e}{1000} = 1.1^2$$

$$\therefore \qquad W_e = 1000 \times 1.21 = \textbf{1210 W}$$

(*b*) As seen from Eq. (*i*) above eddy-current loss would not be effected. The new hysteresis loss is

$$W_h = \quad PE^{1.6}\,(0.9\,f)^{-0.6} \quad ...(iv)$$

From (*i*) and (*iv*), we get $\dfrac{W_h}{3000} = 0.9^{-0.6}$, $W_h = 3000 \times 1.065 = \textbf{3,196 W}$

(*c*) In this case, both *E* and *f* are increased by 10%. The new losses are as under :

$$W_h = P\,(1.1\,E)^{1.6}\,(1.1\,f)^{-0.6}$$

$$\therefore \qquad \frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6} = 1.165 \times 0.944$$

$$\therefore \qquad W_h = 3000 \times 1.165 \times 0.944 = \textbf{3,299 W}$$

As W_e is unaffected by changes in *f*, its value is the same as found in (*a*) above *i.e.* **1210 W**

Example 32.34. *A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core-loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.*

(Bharathiar Univ. Nov. 1997)

Solution. For constant flux density (*i.e.* constant V/f ratio), which is fulfilled by 2200/40 or 3300/60 figures in two cases,

$$\text{Core-loss} = Af + Bf^2$$

First term on the right-hand side represents hysteresis-loss and the second term represents the eddy-current loss.

At 40 Hz, 800 = 600 + eddy current loss.

Thus, $\quad Af = 600, \quad \text{or} \quad A = 15$

$$Bf^2 = 200, \quad \text{or} \quad B = 200/1600 = 0.125$$

At 60 Hz, $\quad$ core-loss $= 15 \times 60 + 0.125 \times 60^2$

$$= 900 + 450$$

$$= 1350 \text{ watts}$$

32.22 Short-Circuit or Impedance Test

This is an economical method for determining the following :

(*i*) Equivalent impedance ($\mathbf{Z}_{01}$ or $\mathbf{Z}_{02}$), leakage reactance (X_{01} or X_{02}) and total resistance (R_{01} or R_{02}) of the transformer as referred to the winding in which the measuring instruments are placed.

(*ii*) Cu loss at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer.

(*iii*) Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

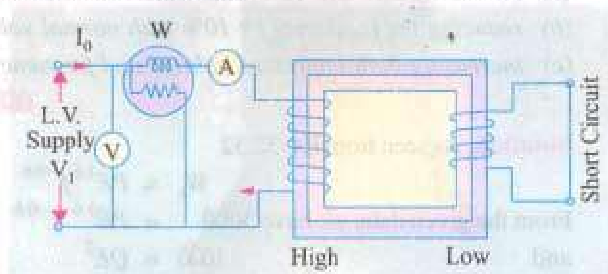


Fig. 32.45

In this test, one winding, usually the low-voltage winding, is solidly short-circuited by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current) as shown in Fig. 32.45.

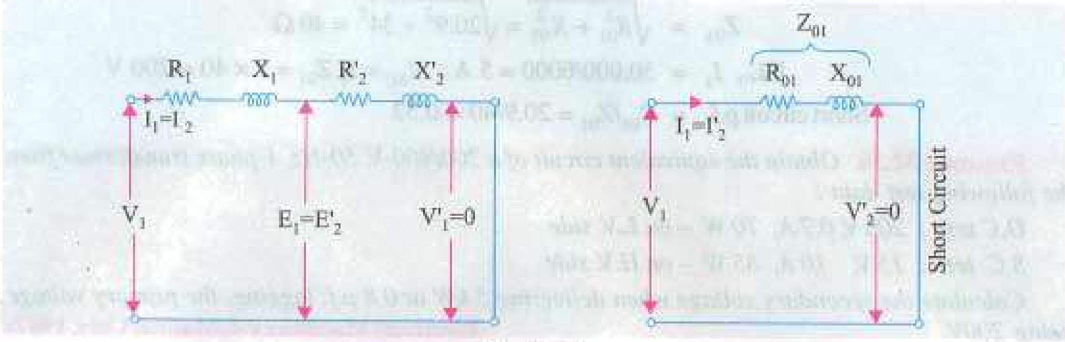


Fig. 32.46

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency (though for Cu losses it is not essential) is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary (as indicated by the respective ammeters).

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced is also a small percentage of its normal value (Art. 32.6). Hence, core losses are very small with the result that the wattmeter reading represent the full-load Cu loss or I^2R loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss. The equivalent circuit of the transformer under short-circuit condition is shown in Fig. 32.46. If V_{sc} is the voltage required to circulate rated load currents, then $Z_{01} = V_{sc}/I_1$

Also $\quad W = I_1^2 R_{01}$

$\therefore \quad R_{01} = W/I_1$

$\therefore \quad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$

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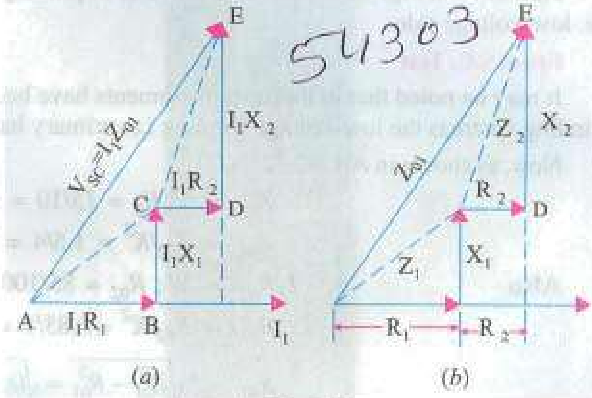


Fig. 32.47

In Fig. 32.47 (*a*) the equivalent circuit vector diagram for the short-circuit test is shown. This diagram is the same as shown in Fig. 32.34 except that all the quantities are referred to the primary side. It is obvious that the entire voltage V_{SC} is consumed in the impedance drop of the two windings.

If R_1 can be measured, then knowing R_{01}, we can find $R_2' = R_{01} - R_1$. The impedance triangle can then be divided into the appropriate equivalent triangles for primary and secondary as shown in Fig. 32.47 (*b*).

32.23. Why Transformer Rating in kVA ?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current *i.e.* it is independent of load power factor. That is why rating of transformers is in kVA and not in kW.

Example 32.35. *The primary and secondary windings of a 30 kVA 76000/230, V, 1-phase transformer have resistance of 10 ohm and 0.016 ohm respectively. The reactance of the transformer referred to the primary is 34 ohm. Calculate the primary voltage required to circulate full-load current when the secondary is short-circuited. What is the power factor on short circuit ?*

(Elect. Machines AMIE Sec. B 1991)

Solution. $\quad K = 230/6000 = 23/600, \; X_{01} = 34\,\Omega,$

$$R_{01} = R_1 + R_2/K^2 = 10 + 0.016\,(600/23)^2 = 20.9\,\Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{20.9^2 + 34^2} = 40\ \Omega$$

$$\text{F.L., } I_1 = 30{,}000/6000 = 5\text{ A} ;\ V_{SC} = I_1 Z_{01} = 5 \times 40 = 200\text{ V}$$

$$\text{Short circuit p.f.} = R_{01}/Z_{01} = 20.9/40 = 0.52$$

Example 32.36. *Obtain the equivalent circuit of a 200/400-V, 50-Hz, 1-phase transformer from the following test data :*

O.C test : 200 V, 0.7 A, 70 W – on L.V. side

S.C. test : 15 V, 10 A, 85 W – on H.V. side

Calculate the secondary voltage when delivering 5 kW at 0.8 p.f. lagging, the primary voltage being 200V. **(Electrical Machinery-I, Madras Univ. 1987)**

Solution. From O.C. Test

$$V_1 I_0 \cos\phi_0 = W_0$$

$$\therefore \quad 200 \times 0.7 \times \cos\phi_0 = 70$$

$$\cos\phi_0 = 0.5 \text{ and } \sin\phi_0 = 0.866$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35\text{ A}$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.866 = 0.606\text{ A}$$

$$R_0 = V_1/I_w = 200/0.35 = 571.4\ \Omega$$

$$X_0 = V_1/I_\mu = 200/0.606 = 330\ \Omega$$

As shown in Fig. 32.48, these values refer to primary *i.e.* low-voltage side.

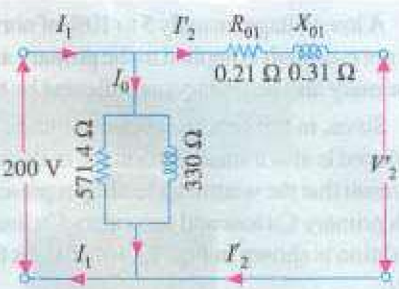


Fig. 32.48

From S.C. Test

It may be noted that in this test, instruments have been placed in the secondary *i.e.* high-voltage winding whereas the low-voltage winding *i.e.* primary has been short-circuited.

Now, as shown in Art. 32.32

$$Z_{02} = V_{sc}/I_2 = 15/10 = 1.5\ \Omega ;\ K = 400/200 = 2$$

$$Z_{01} = Z_{02}/K^2 = 1.5/4 = 0.375\ \Omega$$

Also $\quad I_2^2 R_{02} = W ;\ R_{02} = 85/100 = 0.85\ \Omega$

$$R_{01} = R_{02}/K^2 = 0.85/4 = 0.21\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\ \Omega$$

$$\text{Output kVA} = 5/0.8 ;\ \text{Output current } I_2 = 5000/0.8 \times 400 = 15.6\text{ A}$$

This value of I_2 is approximate because V_2 (which is to be calculated as yet) has been taken equal to 400 V (which, in fact, is equal to E_2 or ${}_0V_2$).

Now, $\quad Z_{02} = 1.5\ \Omega,\ R_{02} = 0.85\ \Omega \quad \therefore \quad X_{02} = \sqrt{1.5^2 - 0.85^2} = 1.24\ \Omega$

Total transformer drop as referred to secondary

$$= I_2(R_{02}\cos\phi_2 + X_{02}\sin\phi_2) = 15.6\,(0.85 \times 0.8 + 1.24 \times 0.6) = 22.2\text{ V}$$

$$\therefore \quad V_2 = 400 - 22.2 = \mathbf{377.8\ V}$$

Example 32.37. *Starting from the ideal transformer, obtain the approximate equivalent circuit of a commercial transformer in which all the constants are lumped and represented on one side.*

A 1-phase transformer has a turn ratio of 6. The resistance and reactance of primary winding are 0.9 Ω and 5 Ω respecitvely and those of the secondary are 0.03 Ω and 0.13 Ω respectively. If 330 -V at 50-Hz be applied to the high voltage winding with the low-voltage winding short-circuited, find the current in the low-voltage winding and its power factor. Neglect magnetising current.

Solution. Here $K = 1/6$; $R_{01} = R_1 + R'_2 = 0.9 + (0.03 \times 36) = 1.98\ \Omega$

$$X_{01} = X_1 + X_2' = 5 + (0.13 \times 36) = 9.68\ \Omega$$

$$\therefore \quad Z_{01} = \sqrt{(9.68^2 + 1.98^2)} = 9.9\ \Omega;\ V_{SC} = 330\ \text{V}$$

$\therefore$ Full-load primary current $I_1 = V_{sc}/Z_{01} = 330/0.9 = 100/3$ A

As I_0 is negligible, hence $I_1 = I'_2 = 100/3$ A. Now, $I'_2 = KI_2$

F.L. secondary current $I_2 = I'_2K = (100/3) \times 6 = 200$ A

Now, Power input on short-circuit $= V_{SC}I_1 \cos\phi_{SC} = \text{Cu loss} = I_1^2 R_{01}$

$$\therefore \quad (100/3)^2 \times 1.98 = 330 \times (100/3) \times \cos\phi_{SC};\ \cos\phi_{SC} = \mathbf{0.2}$$

Example 32.38. *A 1-phase, 10-kVA, 500/250-V, 50-Hz transformer has the following constants:*

Reactance : primary 0.2 Ω; secondary 0.5 Ω

Resistance : primary 0.4 Ω; secondary 0.1 Ω

Resistance of equivalent exciting circuit referred to primary, $R_0 = 1500\ \Omega$

Reactance of equivalent exciting circuit referred to primary, $X_0 = 750\ \Omega$

What would be the reading of the instruments when the transformer is connected for the open-circuit and short-circuit tests ?

Solution. While solving this question, reference may please be made to Art. 30.20 and 30.22.

O.C. Test

$$I_\mu = V_1/X_0 = 500/750 = 2/3\ \text{A};\ I_w = V_1/R_0 = 500/1500 = 1/3\ \text{A}$$

$$\therefore \quad I_0 = \sqrt{\left[1/3)^2 + (2/3)^2\right]} = 0.745\ \text{A}$$

No-load primary input $= V_1I_w = 500 \times 1/3 = 167$ W

Instruments used in primary circuit are : voltmeter, ammeter and wattmeter, their readings being 500 V, 0.745 A and 167 W respectively.

S.C. Test

Suppose S.C. test is performed by short-circuiting the *l.v.* winding *i.e.* the secondary so that all instruments are in primary.

$R_{01} = R_1 + R'_2 = R_1 + R_2/K^2$; Here $K = 1/2$ $\therefore$ $R_{01} = 0.2 + (4 \times 0.5) = 2.2\ \Omega$

Similarly, $X_{01} = X_1 + X'_2 = 0.4 + (4 \times 0.1) = 0.8\ \Omega$

$$Z_{01} = \sqrt{(2.2^2 + 0.8^2)} = 2.341\ \Omega$$

Full-load primary current

$I_1 = 10{,}000/500 = 20$ A $\therefore V_{SC} = I_1Z_{01} = 20 \times 2.341 = 46.8$ V

Power absorbed $= I_1^2 R_{01} = 20^2 \times 2.2 = 880$ W

Primary instruments will read : **46.8 V, 20 A, 880 W.**

Example 32.39. *The efficiency of a 1000-kVA, 110/220 V, 50-Hz, single-phase transformer, is 98.5 % at half full-load at 0.8 p.f. leading and 98.8 % at full-load unity p.f. Determine (i) iron loss (ii) full-load copper loss and (iii) maximum efficiency at unity p.f.*

(Elect. Engg. AMIETE Sec. A Dec. 1991)

Solution. Output at F.L. unity p.f. $= 1000 \times 1 = 1000$ kW

F.L. input $= 1000/0.988 = 1012.146$ kW

F.L. lossses $= 1012.146 - 1000 = 12.146$ kW

If F.L. Cu and iron losses are x and y respectively then

$$x + y = 12.146\ \text{kW} \qquad ...(i)$$

Input at half F.L. 0.8 p.f. = 500 × 0.8/0.985 = 406.091 kW

Total losses at half F.L. = 406.091 − 400 = 6.091 kW

Cu loss at half-load = $x\,(1/2)^2 = x/4$

$$\therefore \qquad x/4 + y = 6.091 \qquad \qquad ...(ii)$$

From Eqn. (*i*) and (*ii*), we get (*i*) x = 8.073 kW and (*ii*) y = 4.073 kW

(*iii*) kVA for $\eta_{max} = 1000 \times \sqrt{4.073/8.073} = 710.3$ kVA

Output at u.p.f. = 710.3 × 1 = 710.3 kW

Cu loss = iron loss = 4.037 kW ; Total loss = 2 × 4.037 = 8.074 kW

$\therefore \quad \eta_{max} = 710.3/(710.3 + 8.074) = 0.989$ or 98.9 %

Example 32.40. *The equivalent circuit for a 200/400-V step-up transformer has the following parameters referred to the low-voltage side.*

Equivalent resistance = 0.15 Ω ; Equivalent reactance = 0.37 Ω

Core-loss component resistance = 600 Ω ; Magnetising reactance = 300 Ω

When the transformer is supplying a load at 10 A at a power factor of 0.8 lag, calculate (i) the primary current (ii) secondary terminal voltage. **(Electrical Machinery-I, Bangalore Univ. 1989)**

Solution. We are given the following :

$R_{01} = 0.15\,\Omega, X_{01} = 0.37\,\Omega\,;\, R_0 = 600\,\Omega, X_0 = 300\,\Omega$

Using the approximate equivalent circuit of Fig. 32.41, we have,

$$I_\mu = V_1/X_0 = 200/300 = (2/3)\text{ A}$$

$$I_w = V_1/R_0 = 200/600 = (1/3)\text{ A}$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(2/3)^2 + (1/3)^2} = 0.745\text{ A}$$

As seen from Fig. 32.49

$$\tan\theta = \frac{I_w}{I_\mu} = \frac{1/3}{2/3} = \frac{1}{2}\,;\, \theta = 26.6^\circ$$

$\therefore \quad \phi_0 = 90° − 26.6° = 63.4°$; Angle between I_0 and

$I_2' = 63.4° − 36.9° = 26.5°$; $K = 400/200 = 2$

$I_2' = KI_2 = 2 \times 10 = 20$ A

(*i*) $I_1 = (0.745^2 + 20^2 + 2 \times 0.745 \times 20 \times \cos 26.5°)^{1/2}$

$= \mathbf{20.67\ A}$

(*ii*) $R_{02} = K^2 R_{01} = 2^2 \times 0.15 = 0.6\,\Omega$

$X_{02} = 22 \times 0.37 = 1.48\,\Omega$

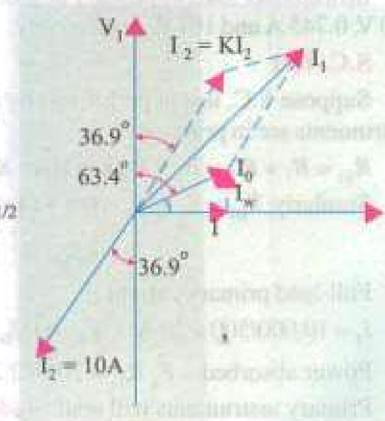


Fig. 32.49

Approximate voltage drop

$= I_2(R_{02}\cos\phi + X_{02}\sin\phi)$

$= 10\,(0.6 \times 0.8 + 1.48 \times 0.6) = 13.7$ V

$\therefore$ Secondary terminal voltage = 400 − 13.7

$= \mathbf{386.3\ V}$

Example 32.41. *The low voltage winding of a 300-kVA, 11,000/2500-V, 50-Hz transformer has 190 turns and a resistance of 0.06. The high-voltage winding has 910 turns and a resistance of 1.6 Ω. When the l.v. winding is short-circuited, the full-load current is obtained with 550-V applied to the h.v. winding. Calculate (i) the equivalent resistance and leakage reactance as referred to h.v. side and (ii) the leakage reactance of each winding.*

Solution. Assuming a full-load efficiency of 0.985, the full-load primary current is

$= E_2 = EK_1 = KV_1$ because at no-load the impedance drop is negligible.

V_2 = secondary terminal voltage on *full-load*.

The change in secondary terminal voltage from no-load to full-load is $= {}_0V_2 - V_2$. This change divided by ${}_0V_2$ is known as regulation 'down'. If this change is divided by V_2, *i.e.*, full-load secondary terminal voltage, then it is called regulation 'up'.

$$\therefore \quad \text{\% regn 'down'} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100 \text{ and \% regn 'up'} = \frac{{}_0V_2 - V_2}{V_2} \times 100$$

In further treatment, unless stated otherwise, regulation is to be taken as regulation 'down'.

We have already seen in Art. 32.16 (Fig. 32.35) that the change in secondary terminal voltage from no-load to full-load, expressed as a percentage of no-load secondary voltage is,

$$= v_r \cos\phi \pm v_x \sin\phi \qquad \text{(approximately)}$$

Or more accurately

$$= (v_r \cos\phi \pm v_x \sin\phi) + \frac{1}{200}(v_x \cos\phi \mp v_r \sin\phi)^2$$

$$\therefore \quad \text{\% regn} = v_r \cos\phi \pm v_x \sin\phi \qquad \text{...approximately.}$$

The lesser this value, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load.

(2) The regulation may also be explained in terms of primary values.

In Fig. 32.51 (*a*) the approximate equivalent circuit of a transformer is shown and in Fig. 32.51 (*b*), (*c*) and (*d*) the vector diagrams corresponding to different power factors are shown.

The secondary ***no-load*** terminal voltage as referred to primary is $E_2' = E_2/K = E_1 = V_1$ and if the secondary full-load voltage as referred to primary is $V_2' (= V_2/K)$ then

$$\text{\% regn} = \frac{V_1 - V_2'}{V_1} \times 100$$

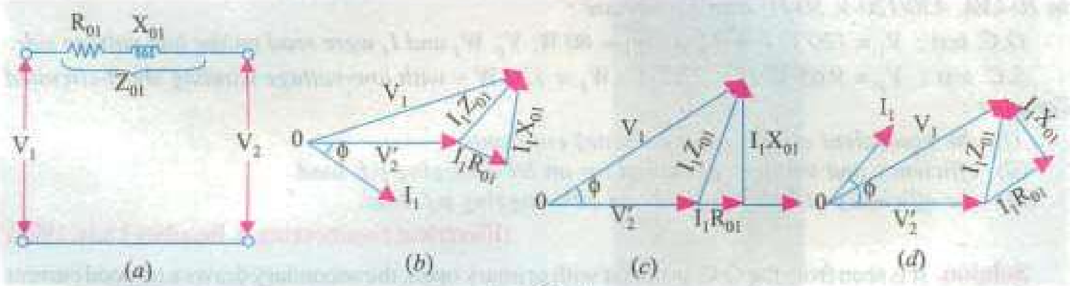


Fig. 32.51

From the vector diagram, it is clear that if angle between V_1 and V_2' is neglected, then the value of numerical difference $V_1 - V_2'$ is given by $(I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi)$ for lagging p.f.

$$\therefore \quad \text{\% regn} = \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1} \times 100 = v_r \cos\phi + v_x \sin\phi$$

$$\text{where} \quad \frac{I_1 R_{01} \times 100}{V_1} = v_r \text{ and } \frac{I_1 X_{01} \times 100}{V_1} = v_x \qquad \text{...Art. 32.16}$$

As before, if angle between V_1 and V_2' is not negligible, then

$$\text{\% regn} = (v_r \cos\phi \pm v_x \sin\phi) + \frac{1}{200}(v_x \cos\phi \mp v_r \sin\phi)^2$$

(3) In the above definitions of regulation, ***primary voltage was supposed to be kept constant*** and the changes in secondary terminal voltage were considered.

As the transformer is loaded, the secondary terminal voltage falls (for a lagging p.f.). Hence, to keep the output voltage constant, the primary voltage must be increased. The rise in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer.

Suppose primary voltage has to be raised from its rated value V_1 to V_1', then

$$\%\text{ regn.} = \frac{V_1' - V_1}{V_1} \times 100$$

Example 32.43. *A-100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively and the corresponding leakage reactances are 1.1 and 0.035 Ω respectively. The supply voltage is 2200 V. Calculate (i) equivalent impedance referred to primary and (ii) the voltage regulation and the secondary terminal voltage for full load having a power factor of 0.8 leading.*

(Elect. Machines, A.M.I.E. Sec. B, 1989)

Solution. $K = 80/400 = 1/5$, $R_1 = 0.3\ \Omega$, $R_{01} = R_1 + R_2/K^2 = 0.3 + 0.01/(1/5)^2 = 0.55\ \Omega$

$$X_{01} = X_1 + X_2/K^2 = 1.1 + 0.035/(1/5)^2 = 1.975\ \Omega$$

(i) $$Z_{01} = 0.55 + j\,1.975 = 2.05\ \angle 74.44°$$

(ii) $$Z_{02} = K^2 Z_{01} = (1/5)^2\,(0.55 + j\,1.975) = (0.022 + j\,0.079)$$

No-load secondary voltage $= KV_1 = (1/5) \times 2200 = 440$ V, $I_2 = 10 \times 10^3/440 = 227.3$ A

Full-load voltage drop as referred to secondary

$$= I_2\,(R_{02} \cos\phi - X_{02} \sin\phi)$$
$$= 227.3\,(0.022 \times 0.8 - 0.079 \times 0.6) = -6.77\text{ V}$$
$$\%\text{ regn.} = -6.77 \times 100/440 = -1.54$$

Secondary terminal voltage on load $= 440 - (-6.77) =$ **446.77 V**

Example 32.44. *The corrected instrument readings obtained from open and short-circuit tests on 10-kVA, 450/120-V, 50-Hz transformer are :*

O.C. test : $V_1 = 120$ V; $I_1 = 4.2$ A; $W_1 = 80$ W; V_1, W_1 and I_1 were read on the low-voltage side.

S.C. test : $V_1 = 9.65$ V; $I_1 = 22.2$ A ; $W_1 = 120$ W – with low-voltage winding short-circuited

Compute :

(i) the equivalent circuit (approximate) constants,
(ii) efficiency and voltage regulation for an 80% lagging p.f. load,
(iii) the efficiency at half full-load and 80% lagging p.f. load.

(Electrical Engineering-I, Bombay Univ. 1988)

Soluion. It is seen from the O.C. test, that with primary open, the secondary draws a no-load current of 4.2 A. Since $K = 120/450 = 4/15$, the corresponding no-load primary current $I_0 = 4.2 \times 4/15 = 1.12$ A.

(i) Now, $V_1 I_0 \cos\phi_0 = 80 \quad \therefore \quad \cos\phi_0 = 80/450 \times 1.12 = 0.159$ *

$\therefore \quad \phi_0 = \cos^{-1}(0.159) = 80.9°;\ \sin\phi_0 = 0.987$

$I_w = I_0 \cos\phi_0 = 1.12 \times 0.159 = 0.178$A and $I_\mu = 1.12 \times 0.987 =$ **1.1 A**

$\therefore \quad R_0 = 450/0.178 =$ **2530 Ω** and $X_0 = 450/1.1 =$ **409 Ω**

During S.C. test, instruments have been placed in primary.

$$\therefore \quad Z_{01} = 9.65/22.2 = 0.435\ \Omega$$
$$R_{01} = 120/22.2^2 = 0.243\ \Omega$$
$$X_{01} = \sqrt{0.435^2 - 0.243^2} = 0.361\ \Omega$$

The equivalent circuit is shown in Fig. 32.52.

(ii) Total approximate voltage drop as referred to primary is $I_1\,(R_{01} \cos\phi + X_{01} \sin\phi)$.

* *Assuming $\phi_1 = \phi_2 = \cos^{-1}(0.8)$.*

$$= 300{,}000/0.985 \times 11{,}000 = 27.7\text{A}$$

$$\therefore \quad Z_{01} = 550/27.7 = 19.8\ \Omega\ ;\ R'_2 = R^2/K^2 = 0.06\ (910/190)^2 = 1.38\ \Omega$$

$$\therefore \quad R_{01} = R_1 + R'_2 = 1.6 + 1.38 = 2.98\ \Omega$$

$$X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)} = \sqrt{(19.8^2 - 2.98^2)} = 19.5\ \Omega$$

Let us make another assumption that for each winding the ratio (reactance/resistance) is the same, then

$$X_1 = 19.5 \times 1.6/2.98 = 10.5\ \Omega$$

$$X'_2 = 19.5 \times 1.38/2.98 = 9.0\ \Omega\ ;\ X_2 = 9(190/910)^2 = 0.39\ \Omega$$

(a) $R_{01} = \mathbf{2.98\ \Omega}\ ;\ X_{01} = \mathbf{19.5\ \Omega}$ *(b)* $X_1 = \mathbf{10.5\ \Omega}\ ;\ X_2 = \mathbf{0.39\ \Omega}$

Example 32.42. *A 230/115 volts, single phase transformer is supplying a load of 5 Amps, at power factor 0.866 lagging. The no-load current is 0.2 Amps at power factor 0.208 lagging. Calculate the primary current and primary power factor.*

(Nagpur University Summer 2000)

Solution. L.V. current of 5 amp is referred to as a 2.5 amp current on the primary (= H.V.) side, at 0.866 lagging p.f. To this, the no load current should be added, as per the phasor diagram in Fig. 32.50. The phase angle of the load-current is 30° lagging. The no load current has a phase angle of 80° lagging. Resultant of these two currents has to be worked out. Along the reference, active components are added.

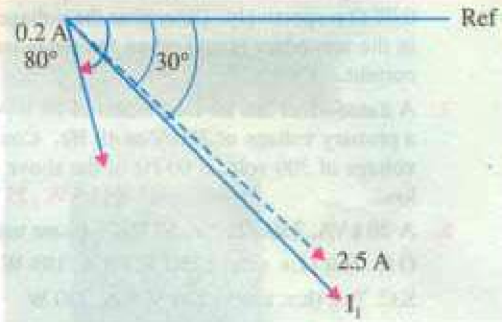


Fig. 32.50. Phasor diagram for Currents

$$\text{Active components of currents} = 2.5 \times 0.866 + 0.2 \times 0.208$$

$$= 2.165 + 0.0416$$

$$= 2.2066 \text{ amp}$$

Along the perpendicular direction, the reactive components get added up.

$$\text{Reactive component} = 2.5 \times 0.5 + 0.2 \times 0.9848$$

$$= 1.25 + 0.197 = 1.447 \text{ amp.}$$

$$I_1 = 2.2066 - j\,1.447$$

$$\phi = \tan^{-1}\frac{1.447}{2.2066} = \mathbf{33.25°\ ;\ as\ shown}$$

Tutorial Problems 32.3

1. The S.C. test on a 1-phase transformer, with the primary winding short-circuited and 30 V applied to the secondary gave a wattmeter reading of 60 W and secondary current of 10 A. If the normal applied primary voltage is 200, the transformation ratio 1 :2 and the full-load secondary current 10 A, calculate the secondary terminal p.d. at full-load current for *(a)* unity power factor *(b)* power factor 0.8 lagging. If any approximations are made, they must be explained. **[394 V, 377.6 V]**
2. A single-phase transformer has a turn ratio of 6, the resistances of the primary and secondary windings are 0.9 Ω and 0.025 Ω respectively and the leakage reactances of these windings are 5.4 Ω and 0.15 Ω respectively. Determine the voltage to be applied to the low-voltage winding to obtain a current of 100 A in the short-circuited high voltage winding. Ignore the magnetising current. **[82 V]**
3. Draw the equivalent circuit for a 3000/400-V, 1-phase transformer on which the following test results were obtained. Input to high voltage winding when l.v. winding is open-circuited : 3000 V, 0.5 A, 500

W. Input to l.v. winding when h.v. winding is short-circuited : 11 V, 100 A, 500 W. Insert the appropriate values of resistance and reactance.

[R_0 = 18,000 Ω, X_0 = 6,360 Ω, R_{01} = 2.81 Ω, X_{01} = 5.51 Ω] *(I.E.E. London)*

4. The iron loss in a transformer core at normal flux density was measured at frequencies of 30 and 50 Hz, the results being 30 W and 54 W respectively. Calculate (*a*) the hysteresis loss and (*b*) the eddy current loss at 50 Hz. **[44 W, 10 W]**
5. An iron core was magnetised by passing an alternating current through a winding on it. The power required for a certain value of maximum flux density was measured at a number of different frequencies. Neglecting the effect of resistance of the winding, the power required per kg of iron was 0.8 W at 25 Hz and 2.04 W at 60 Hz. Estimate the power needed per kg when the iron is subject to the same maximum flux density but the frequency is 100 Hz. **[3.63 W]**
6. The ratio of turns of a 1-phase transformer is 8, the resistances of the primary and secondary windings are 0.85 Ω and 0.012 Ω respectively and leakage reactances of these windings are 4.8 Ω and 0.07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary circuit when the secondary terminals are short-circuited. Ignore the magnetising current. **[176.4 W]**
7. A transformer has no-load losses of 55 W with a primary voltage of 250 V at 50 Hz and 41 W with a primary voltage of 200 V at 40 Hz. Compute the hysteresis and eddy current losses at a primary voltage of 300 volts at 60 Hz of the above transformer. Neglect small amount of copper loss at no-load. **[43.5 W ; 27 W]** *(Elect. Machines AMIE Sec. B. (E-3) Summer 1992)*
8. A 20 kVA, 2500/250 V, 50 Hz, 1-phase transformer has the following test results :

   O.C. Test (l.v. side) : 250 V, 1.4 A, 105 W

   S.C. Test (h.v. side) : 104 V, 8 A, 320 W

   Compute the parameters of the approximate equivalent circuit referred to the low voltage side and draw the circuit. **(R_0 = 592.5 Ω ; X_0 = 187.2 Ω ; R_{02} = 1.25 Ω ; X_{12} = 3 Ω)** *(Elect. Machines A.M.I.E. Sec. B Summer 1990)*
9. A 10-kVA, 2000/400-V, single-phase transformer has resistances and leakage reactances as follows :

   $R_1 = 5.2\ \Omega$, $X_1 = 12.5\ \Omega$, $R_2 = 0.2\ \Omega$, $X_2 = 0.5\ \Omega$

   Determine the value of secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with power factor 0.8 lag. The no-load current can be neglected. Draw the phasor diagram. **[376.8 V]** *(Elect. Machines, A.M.I.E. Sec B, 1989)*
10. A 1000-V, 50-Hz supply to a transformer results in 650 W hysteresis loss and 400 W eddy current loss. If both the applied voltage and frequency are doubled, find the new core losses.

    [W_h = 1300 W ; W_e = 1600 W] *(Elect. Machine, A.M.I.E. Sec. B, 1993)*
11. A 50 kVA, 2200/110 V transformer when tested gave the following results :

    O.C. test (L.V. side) : 400 W, 10 A, 110 V.

    S.C. test (H.V. side) : 808 W, 20.5 A, 90 V.

    Compute all the parameters of the equivalent ckt. referred to the H.V. side and draw the resultant ckt.

    (Rajiv Gandhi Technical University, Bhopal 2000)

[Shunt branch : R_0 = 12.1 k-ohms, X_m = 4.724 k-ohms Series branch : r = 1.923 ohms, x = 4.39 ohms]

32.24. Regulation of a Transformer

1. When a transformer is loaded with a *constant primary voltage*, the secondary voltage decreases* because of its internal resistance and leakage reactance.

Let $\quad {}_0V_2$ = secondary terminal voltage at *no-load*.

---

* Assuming lagging power factor. It will increase if power factor is leading.

Now, full-load $I_1 = 10{,}000/450 = 22.2$ A

$\therefore$ Drop $= 22.2\,(0.243 \times 0.8 + 0.361 \times 0.6) = 9.2$ V

Regulation $= 9.2 \times 100/450 = 2.04\%$

F.L. losses $= 80 + 120 = 200$ W;

F.L. output $= 10{,}000 \times 0.8 = 8000$ W

$\eta = 8000/8200 = 0.9757$ or **97.57%**

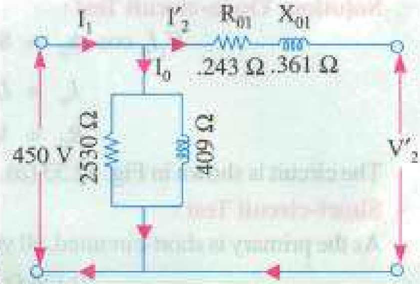


Fig. 32.52

(iii) Half-load

Iron loss $= 80$ W; Cu loss $= (1/2)^2 \times 120 = 30$ W

Total losses $= 110$ W; Output $= 5000 \times 0.8 = 4000$ W

$\therefore \quad \eta = 4000/4110 = 0.9734$ or **97.34%**

Example 32.45. *Consider a 20 kVA, 2200/220 V, 50 Hz transformer. The O.C./S.C. test results are as follows :*

O.C. test : 220 V, 4.2 A, 148 W (l.v. side)

S.C. test : 86 V, 10.5 A, 360 W (h.v. side)

Determine the regulation at 0.8 p.f. lagging and at full load. What is the p.f. on short-circuit ?

(Elect. Machines Nagpur Univ. 1993)

Solution. It may be noted that O.C. data is not required in this question for finding the regulation. Since during S.C. test instruments have been placed on the h.v. side *i.e.* primary side.

$$\therefore \quad Z_{01} = 86/10.5 = 8.19\ \Omega;\ R_{01} = 360/10.5^2 = 3.26\ \Omega$$

$$X_{01} = \sqrt{8.19^2 - 3.26^2} = 7.5\ \Omega$$

F.L. primary current, $I_1 = 20{,}000/2200 = 9.09$ A

Total voltage drop as referred to primary $= I_1\,(R_{01} \cos\phi + X_{01} \sin\phi)$

Drop $= 9.09\,(3.26 \times 0.8 + 7.5 \times 0.6) = 64.6$ V

% age regn. $= 64.6 \times 100/2200 =$ **2.9%**, p.f. on short-circuit $= R_{01}/Z_{01} = 3.26/8.19 =$ **0.4 lag**

Example 32.46. *A short-circuit test when performed on the h.v. side of a 10 kVA, 2000/400 V single phase transformer, gave the following data ; 60 V, 4 A, 100 W.*

If the l.v. side is delivering full load current at 0.8 p.f. lag and at 400 V, find the voltage applied to h.v. side. **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Here, the test has been performed on the h.v. side *i.e.* primary side.

$Z_{01} = 60/4 = 15\ \Omega$; $R_{01} = 100/4^2 = 6.25\ \Omega$; $X_{01} = \sqrt{15^2 - 6.25^2} = 13.63\ \Omega$

F.L. $I_1 = 10{,}000/2000 = 5$A

Total transformer voltage drop as referred to primary is

$$I_1\,(R_{01} \cos\phi + X_{01} \sin\phi) = 5\,(6.25 \times 0.8 + 13.63 \times 0.6) = 67\text{ V}$$

Hence, primary voltage has to be raised from 2000 V to 2067 V in order to compensate for the total voltage drop in the transformer. In that case secondary voltage on load would remain the same as on no-load.

Example 32.47. *A 250/500-V transformer gave the following test results :*

Short-circuit test : with low-voltage winding short-circuited :

20 V; 12 A, 100 W

Open-circuit test : 250 V, 1 A, 80 W on low-voltage side.

Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Open-circuit Test :

$$V_1 I_0 \cos \phi_0 = 80 \quad \therefore \quad \cos \phi_0 = 80/250 \times I = 0.32$$

$$I_w = I_0 \cos \phi_0 = I \times 0.32 = 0.32\ A,\ I_\mu = \sqrt{(1^2 - 0.32^2)} = 0.95\ A$$

$$R_0 = V_1/I_w = 250/0.32 = 781.3\ \Omega,\ X_0 = V_1/I_\mu = 250/0.95 = 263.8\ \Omega$$

The circuit is shown in Fig. 32.53 (*a*).

Short-circuit Test :

As the primary is short-circuited, all values refer to secondary winding.

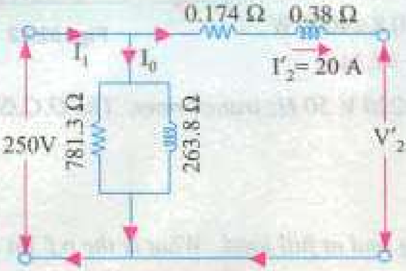


Fig. 32.53 (a)

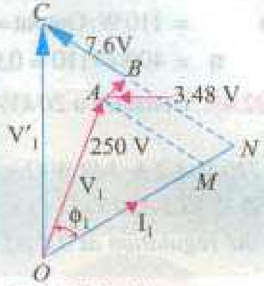


Fig. 32.53 (b)

$$\therefore \quad R_{02} = \frac{\text{short-circuit power}}{\text{F.L. secondary current}} = \frac{100}{12^2} = 0.694\ \Omega$$

$$Z_{02} = 20/12 = 1.667\ \Omega\ ;\ X_{02} = \sqrt{(1.667^2 - 0.694^2)} = 1.518\ \Omega$$

As R_0 and X_0 refer to primary, hence we will transfer these values to primary with the help of transformation ratio.

$$K = 500/250 = 2 \quad \therefore \quad R_{01} = R_{02}/K^2 = 0.694/4 = 0.174\ \Omega$$

$$X_{01} = X_{02}/K^2 = 1.518/4 = 0.38\ \Omega\ ;\ Z_{01} = Z_{02}/K^2 = 1.667/4 = 0.417\ \Omega$$

The equivalent circuit is shown in Fig. 32.53 (*a*).

Efficiency

Total Cu loss $= I_2^2 R_{02} = 100 \times 0.694 = 69.4$ W ; Iron loss = 80 W

Total loss = 69.4 + 80 = 149.4 W $\quad \therefore \quad \eta = \dfrac{5000 \times 0.8 \times 100}{4000 + 149.4} = $ **96.42%**

The applied voltage V_1' is the vector sum of V_1 and $I_1 Z_{01}$ as shown in Fig. 32.53 (*b*).

$$I_1 = 20\ \text{A}\ ;\ I_1 R_{01} = 20 \times 0.174 = 3.84\ \text{V}\ ;\ I_1 X_{01} = 20 \times 0.38 = 7.6\ \text{V}$$

Neglecting the angle between V_1 and V_1', we have

$$V_1'^2 = OC^2 = ON^2 + NC^2 = (OM + MN)^2 + (NB + BC)^2$$

$$= (250 \times 0.8 + 3.48)^2 + (250 \times 0.6 + 7.6)^2$$

$$V_1'^2 = 203.5^2 + 157.6^2 \quad \therefore \quad V_1' = \mathbf{257.4\ V}$$

Example 32.48. *A 230/230 V, 3 kVA transformer gave the following results :*

O.C. Test : 230 V, 2 amp, 100 W

S.C. Test : 15 V, 13 amp, 120 W

Determine the regulation and efficiency at full load 0.80 p.f. lagging.

(Sambalpur University, 1998)

Solution. This is the case of a transformer with turns ratio as 1 : 1. Such a transformer is mainly required for isolation.

$$\text{Rated Current} = \frac{3000}{230} = 13 \text{ amp}$$

Cu-losses at rated load = 120 watts, from S.C. test

Core losses = 100 watts, from O.C. test

At full load, VA output = **3000**

At 0.8 lag p.f., Power output = $3000 \times 0.8 = 2400$ watts

$$\text{Required efficiency} = \frac{2400}{2400 + 220} \times 100\% = 91.6\%$$

From S.C. test,
$$Z = \frac{15}{13} = 1.154 \text{ ohms}$$

$$R = \frac{120}{15 \times 15} = 0.53 \text{ ohm}, \quad X = \sqrt{1.154^2 - 0.53^2} = 1.0251 \text{ ohm}$$

Approximate voltage regulation

$$= IR\cos\phi + IX\sin\phi = 13[0.53 \times 0.8 + 1.0251 \times 0.6]$$

$$= 13[0.424 + 0.615] = 13.51 \text{ volts}$$

In terms of %, the voltage regulation $= \frac{13.51}{230} \times 100\% =$ **5.874%**

Example 32.49. *A 10 kVA, 500/250 V, single-phase transformer has its maximum efficiency of 94% when delivering 90% of its rated output at unity p.f. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging.* **(Nagpur University, November 1998)**

Solution. Rated output at unity p.f. = 10000 W. Hence, 90% of rated output = 9,000 W

Input with 94% efficiency = 9000/0.94 W

Losses = $9000((1/0.94) - 1) = 574$ W

At maximum efficiency, variable copper-loss = constant = Core loss = 574/2 = 287 W

At rated current, Let the copper-loss = P_c watts

At 90% load with unity p.f., the copper-loss is expressed as $0.90^2 \times P_c$

Hence, $P_c = 287/0.81 = 354$ W

(*b*) Output at full-load, 0.8 lag p.f. = $10{,}000 \times 0.80 = 8000$ W

At the corresponding load, Full Load copper-loss = 354 W

Hence, efficiency = 8000/(8000 + 354 + 287) = 0.926 = **92.6%**

Example 32.50. *Resistances and Leakage reactance of 10 kVA, 50 Hz, 2300/230 V single phase distribution transformer are r_1 = 3.96 ohms, r_2 = 0.0396 ohms, x_1 = 15.8 ohms, x_2 = 0.158 ohm. Subscript 1 refers to HV and 2 to LV winding (a) transformer delivers rated kVA at 0.8 p.f. Lagging to a load on the L.V. side. Find the H.V. side voltage necessary to maintain 230 V across Load-terminals. Also find percentage voltage regulation. (b) Find the power-factor of the rated load-current at which the voltage regulation will be zero, hence find the H.V. side voltage.*

(Nagpur University, November 1997)

Solution. (*a*) Rated current on L.V. side = 10,000/230 = 43.5 A. Let the total resistance and total leakage reactance be referred to L.V. side. Finally, the required H.V. side voltage can be worked out after transformation.

Total resistance,
$$r = r_1' + r_2 = 3.96 \times (230/2300)^2 + 0.0396$$
$$= 0.0792 \text{ ohms}$$

Total leakage-reactance,
$$x = x_1' + x_2 = 15.8 \times (230/2300)^2 + 0.158$$
$$= 0.316 \text{ ohm}$$

For purpose of calculation of voltage-magnitudes, approximate formula for voltage regulation can be used. For the present case of 0.8 lagging p.f.

$$V_1' = V_2 + I\,[r\cos\phi + x\sin\phi]$$
$$= 230 + 43.5\,[(0.0792 \times 0.8) + (0.316 \times 0.6)]$$
$$= 230 + 43.5\,[0.0634 + 0.1896] = 230 + 11 = 241 \text{ volts}$$

Hence, $\quad V_1 = 241 \times (2300/230) = 2410$ volts.

It means that H.V. side terminal voltage must be 2410 for keeping 230 V at the specified load.

(b) Approximate formula for voltage regulation is : $V_1' - V_2 = I\,[r\cos\phi \pm x\sin\phi]$

With Lagging p.f., +ve sign is retained. With leading power-factor, the −ve sign is applicable. For the voltage-regulation to be zero, only leading P.f. condition can prevail.

Thus, $\quad r\cos\phi - x\sin\phi = 0$

or $\quad \tan\phi = r/x = 0.0792/0.316 = 0.25$

or $\quad \phi = 14°, \quad \cos\phi = 0.97$ leading

Corresponding $\sin\phi = \sin 14° = $ **0.243**

H.V. terminal voltage required is 2300 V to maintain 230 V at Load, since Zero regulation condition is under discussion.

Example 32.51. *A 5 kVA, 2200/220 V, single-phase transformer has the following parameters.*

H.V. side : $r_1 = 3.4$ ohms, $x_1 = 7.2$ ohms

L.V. side : $r_2 = 0.028$ ohms, $x_2 = 0.060$ ohms

Transformer is made to deliver rated current at 0.8 lagging P.f. to a load connected on the L.V. side. If the load voltage is 220 V, calculate the terminal voltage on H.V. side (Neglect the exciting current). **(Rajiv Gandhi Technical University, Bhopal, Summer 2001)**

Solution. Calculations may be done referring all the parameters the L.V. side first. Finally, the voltage required on H.V. side can be obtained after transformation.

Rated current ref. to L.V. side = 5000/220 = 22.73 A

Total winding resistance ref. to L.V. side = $r_1' + r_2 = (220/2200)^2 \times 3.4 + 0.028$

Total winding-leakage-reactance ref. to L.V. side = $x_1' + x_2$

$$= (220/2200)^2 \times 7.2 + 0.060 = 0.132 \text{ ohm}$$

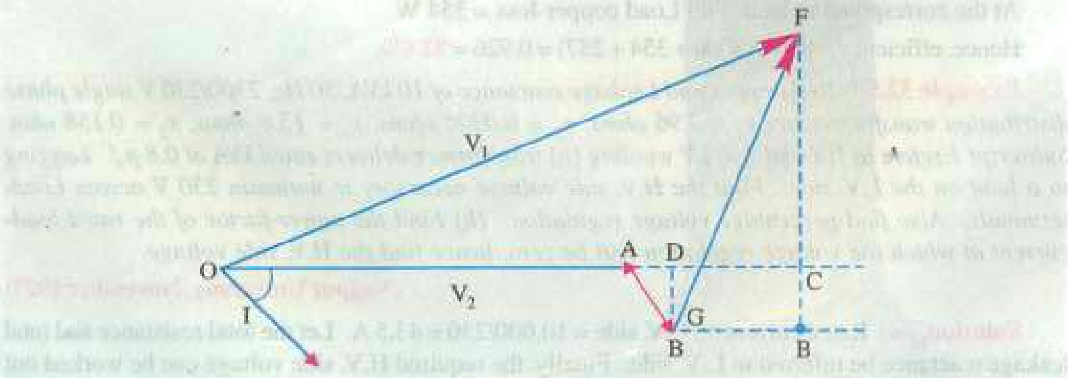


Fig. 32.53(c)

In the phasor diagram of Fig. 32.53 (*c*).

$QA = V_2 = 220$ volts, $I = 22.73$ A at lagging phase angle of 36.87°

$AB = Ir$, $AD = Ir\cos\phi = 22.73 \times 0.062 \times 0.80 = 1.127$ V

$DC = Ix\sin\phi = 22.73 \times 0.132 \times 0.60 = 1.80$ V

$OC = 220 + 1.127 + 1.80 = 222.93$ volts

$BD = Ir \sin\phi = 0.85$ V

$B'F = x \cos\phi = 2.40$ V

$CF = 2.40 - 0.85 = 1.55$ V

$V_1' = OF = (222.93^2 + 1.55^2)^{0.50} = 222.935$ volts

Required terminal voltage of H.V. side $= V_1 = 222.935 \times (2200/220) = 2229.35$ volts

[**Note.** In approximate and fast calculations, CF is often ignored for calculation of magnitude of V_1'. The concerned expression is : $V_1' = V_2 + Ir\cos\phi + Ix\sin\phi$, for lagging P.f.]

Example 32.52. *A 4-kVA, 200/400 V, single-phase transformer takes 0.7 amp and 65 W on Open-circuit. When the low-voltage winding is short-circuited and 15 V is applied to the high-voltage terminals, the current and power are 10 A and 75 W respectively. Calculate the full-load efficiency at unity power factor and full-load regulation at 0.80 power-factor lagging.*

(Nagpur University April 1999)

Solution. At a load of 4 kVA, the rated currents are :

L.V. side : 4000/200 = 20 amp

And H.V. side : 4000/400 = 10 amp

From the test data, full-load copper-loss = 75 W

And constant core-loss = 65 W

From S.C. test, $Z = 15/10 = 1.5$ ohms

$R = 75/100 = 0.75$ ohm

Hence $x = \sqrt{1.5^2 - 0.75^2} = 1.30$ ohms

All these series-parameters are referred to the H.V. side, since the S.C. test has been conducted from H.V. side.

Full-load efficiency at unity p.f. $= 4000 / (4000 + 65 + 75)$

$= 0.966 = 96.6\%$

Full load voltage regulation at 0.80 lagging p.f.

$= Ir\cos\phi + Ix\sin\phi$

$= 10\,(0.75 \times 0.80 + 1.30 \times 0.60) = 16.14$ Volts

Thus, due to loading, H.V. side voltage will drop by 16.14 volts (*i.e.* terminal voltage for the load will be 383.86 volts), when L.V. side is energized by 200-V source.

32.25. Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

(*i*) Percentage resistance at full-load

$$\% R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \% \text{ Cu loss at full-load}$$

$$\% R = \% \text{ Cu loss} = v_r \qquad \text{...Art.32.16}$$

(*ii*) Percentage reactance at full-load

$$\% X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{I_2 X_{02}}{V_2} \times 100 = v_x$$

(*iii*) Percentage impedance at full-load

$$\%Z = \frac{I_1 Z_{01}}{V_1} \times 100 = \frac{I_2 Z_{02}}{V_2} \times 100$$

(*iv*)

$$\%Z = \sqrt{(\%R^2 + \%X^2)}$$

It should be noted from above that the reactances and resistances in ohm can be obtained thus :

$$R_{01} = \frac{\%R \times V_1}{100 \times I_1} = \frac{\%\text{ Cu loss} \times V_1}{100 \times I_1}\text{ ; Similarly } R_{02} = \frac{\%R \times V_2}{100 \times I_2} = \frac{\%\text{ Cu loss} \times V_2}{100 \times I_2}$$

$$X_{01} = \frac{\%X \times V_1}{100 \times I_1} = \frac{v_x \times V_1}{100 \times I_1}\text{ ; Similarly } X_{02} = \frac{\%X \times V_2}{100 \times I_2} = \frac{v_x \times V_2}{100 \times I_2}$$

It may be noted that percentage resistance, reactance and impedance have the same value whether referred to primary or secondary.

Example 32.53. *A 3300/230 V, 50-kVA, transformer is found to have impedance of 4% and a Cu loss of 1.8% at full-load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant ?*

Solution. $\%X = \sqrt{(\%Z^2 - \%R^2)} = \sqrt{(4^2 - 1.8^2)} = 3.57\%$ (Cu loss = % R)

Full load $I_1 = 50,000/3300 = 15.2$ A (assuming 100% efficiency). Considering primary winding, we have

$$\%R = \frac{R_{01} I_1 \times 100}{V_L} = 1.8 \quad \therefore R_{01} = \frac{1.8 \times 3300}{100 \times 15.2} = \mathbf{3.91\ \Omega}$$

Similarly $$\%X = \frac{X_{01} I_1 \times 100}{V_1} = 3.57 \quad \therefore X_{01} = \frac{3.57 \times 3300}{100 \times 15.2} = \mathbf{7.76\ \Omega}$$

Similarly $$Z_{01} = \frac{4 \times 3300}{100 \times 15.2} = \mathbf{8.7\ \Omega}$$

Now $$\frac{\text{Short-circuit current*}}{\text{Full load current}} = \frac{100}{4} \quad \therefore \text{S.C. current} = 15.2 \times 25 = \mathbf{380\ A}$$

Example 32.54. *A 20-kVA, 2200/220-V, 50-Hz distribution transformer is tested for efficiency and regulation as follows :*

O.C. test : 220 V	*4.2 A,*	*148 W*	*– l.v side*
S.C. test : 86 V	*10.5 A,*	*360 W*	*– l.v. side*

Determine **(a)** *core loss* **(b)** *equivalent resistance referred to primary* **(c)** *equivalent resistance referred to secondary* **(d)** *equivalent reactance referred to primary* **(e)** *equivalent reactance referred to secondary* **(f)** *regulation of transformer at 0.8 p.f. lagging current* **(g)** *efficiency at full-load and half the full-load at 0.8 p.f. lagging current.*

Solution. (*a*) As shown in Art 32.9, no-load primary input is practically equal to the core loss. Hence, core loss as found from no-load test, is **148 W.**

(*b*) From S.C. test, $R_{01} = 360/10.5^2 = \mathbf{3.26\ \Omega}$

(*c*) $R_{02} = K^2 R_{01} = (220/2200)^2 \times 3.26 = \mathbf{0.0326\ \Omega}$

---

* Short circuit $I_{SC} = \frac{V_1}{Z_{01}}$ Now, $Z_{01} = \frac{V_1 \times \%Z}{100 \times I_1}$

$$I_{SC} = \frac{V_1 \times 100 \times I_1}{V_1 \times \%Z} = \frac{100 \times I_1}{\%Z} \quad \therefore \frac{I_{SC}}{I_1} = \frac{100}{\%Z}$$

(d) $$Z_{10} = \frac{V_{SC}}{I_{SC}} = \frac{86}{10.5} = 8.19\ \Omega$$

$$X_{01} = \sqrt{(8.19^2 - 3.26^2)} = \mathbf{7.51\ \Omega}$$

(e) $$X_{02} = K^2 X_{01} = (220/2200)^2 \times 7.51 = \mathbf{0.0751\ \Omega}$$

(f) We will use the definition of regulation as given in Art. 32.24 (3).

We will find the rise in primary voltage necessary to maintain the output terminal voltage constant from no-load to full-load.

$$\text{Rated primary current} = 20{,}000/2200 = 9.1\ \text{A}$$

$$V_1' = \sqrt{[2200 \times 0.8 + 9.1 \times 3.26)^2 + (2200 \times 0.6 + 9.1 \times 7.51)^2]} = \mathbf{2265\ V}$$

$$\therefore \quad \%\ \text{regn} = \frac{2265 - 2200}{2200} \times 100 = \mathbf{2.95\%}$$

We would get the same result by working in the secondary. Rated secondary current = **91 A.**

$${}_0V_2 = \sqrt{(220 \times 0.8 + 91 \times 0.0326)^2 + (220 \times 0.6 + 91 \times 0.0751)^2]} = 226.5\ V$$

$$\therefore \quad \%\ \text{regns.} = \frac{226.5 - 220}{220} \times 100 = \mathbf{2.95\%}$$

(g) Core loss $= 1.48$ W. It will be the same for all loads.

Cu loss at full load $= I_1^2 R_{01} = 9.1^2 \times 3.26 = 270$ W

Cu loss at half full-load $= 4.55^2 \times 3.26 = 67.5$ W (or F.L. Cu loss/4)

$$\therefore \quad \eta \text{ at full-load} = \frac{22{,}000 \times 0.8 \times 100}{20{,}000 \times 0.8 + 148 + 270} = \mathbf{97.4\%}$$

$$\therefore \quad \eta \text{ at half-load} = \frac{10{,}000 \times 0.8 \times 100}{10{,}000 \times 0.8 + 148 + 67.5} = \mathbf{97.3\%}$$

Example 32.55. *Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop is 5% of the voltage, when the power factor is (i) 0.80 Lag (ii) unity (iii) 0.80 Leading.* **(Madras University, 1997)**

Solution. When 1% of output is the ohmic loss, p.u. resistance of the transformer, $\varepsilon_r = 0.01$

When 5% is the reactance drop, p.u. reactance of the transformer $\varepsilon_x = 0.05$

(i) Per Unit regulation of the transformer at full-load, 0.8 Lagging p.f.
$= 0.01 \times \cos\phi + 0.05 \times \sin\phi = 0.01 \times 0.8 + 0.05 \times 0.06 = 0.038$ or 3.8%

(ii) Per Unit regulation at unity p.f. $= 0.01 \times 1 = 0.01$ or 1%

(iii) Per Unit regulation at 0.08 Leading p.f. $= 0.01 \times 0.8 - 0.05 \times 0.6 = -0.022$ or -2.2%

Example 32.56. *The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, single phase transformer is 97% and occurs at 3/4th full-load u.p.f. If the impedance is 10% calculate the regulation at fullload, 0.8 p.f. Lag.* **(Madurai Kamraj University, November 1997)**

Solution. At unity p.f. with 3/4th full load, the output of the transformer

$$= 500 \times 0.75 \times 1\ \text{kW} = 375\ \text{kW}$$

$$0.97 = \frac{375}{375 + 2\,P_i}$$

where P_i = core loss in kW, at rated voltage.

At maximum efficiency, $x^2 P_c = P_i$

$$(0.75)^2 P_c = P_i$$

where $x = 0.75$, *i.e.* 3/4th which is the fractional loading of the transformer

P_c = copper losses in kW, at rated current

$$P_i = \frac{1}{2}\left\{(375)\times\left(\frac{1}{0.97}-1\right)\right\} = ½\times 375\times\frac{3}{97} = 5.8 \text{ kW}$$

$$P_c = 5.8/(0.75)^2 = 10.3 \text{ kW}$$

Full load current in primary (H.V.) winding $= \frac{500\times 1000}{3300} =$ **151.5 amp**

Total winding resistance ref. to primary

$$= \frac{10.3\times 1000}{(151.5)^2} = 0.44876 \text{ ohm}$$

$$\varepsilon_r = \% \text{ resistance} = \frac{151.5\times 0.44876}{3300}\times 100\ \% = \mathbf{2.06\%}$$

$$\varepsilon_z = \% \text{ Impedance} = 10\%$$

$$\varepsilon_x = \% \text{ reactance} = \sqrt{100-4.244} = \mathbf{9.7855\%}$$

By Approximate formula at 0.8 p.f. lag

$$\% \text{ regulation} = \varepsilon_r\cos\phi + \varepsilon_x\sin\phi$$
$$= 2.06\times 0.8 + 9.7855\times 0.6$$
$$= 1.648 + 5.87 = \mathbf{7.52\ \%}$$

Example 32.57. *A transformer has copper-loss of 1.5% and reactance-drop of 3.5% when tested at full-load. Calculate its full-load regulation at (i) u.p.f. (ii) 0.8 p.f. Lagging and (iii) 0.8 p.f. Leading.* **(Bharathithasan Univ. April 1997)**

Solution. The test-data at full-load gives following parameters :

p.u. resistance = 0.015, p.u. reactance = 0.035

(*i*) Approximate Voltage – Regulation at unity p.f. full load

$$= 0.015\cos\phi + 0.035\sin\phi$$
$$= 0.015 \text{ per unit} = \mathbf{1.5\%}$$

(*ii*) Approximate Voltage – Regulation at 0.80 Lagging p.f.

$$= (0.015\times 0.8) + (0.035\times 0.6) = 0.033 \text{ per unit} = \mathbf{3.3\%}$$

(*iii*) Approximate Voltage Regulation at 0.8 leading p.f.

$$= I_r\cos\phi - I_x\sin\phi$$
$$= (0.015\times 0.8) - (0.035\times 0.6) = -0.009 \text{ per unit} = \mathbf{-0.9\%}$$

32.26. Kapp Regulation Diagram

It has been shown that secondary terminal voltage falls as the load on the transformer is increased when p.f. is lagging and it increases when the power factor is leading. In other words, secondary terminal voltage not only depends on the load but on power factor also (Art. 32.16). For finding the voltage drop (or rise) which is further used in determining the regulation of the transformer, a graphical construction is employed which was proposed by late Dr. Kapp.

For drawing Kapp regulation diagram, it is necessary to know the equivalent resistance and reactance as referred to secondary *i.e.* R_{02} and X_{02}. If I_2 is the secondary load current, then secondary terminal voltage on load V_2, is obtained by subtracting $I_2 R_{02}$ and $I_2 X_{02}$ voltage drops vectorially from secondary no-load voltage ${}_0V_2$.

Now, ${}_0V_2$ is constant, hence it can be represented by a circle of constant radius OA as in Fig. 32.54. This circle is known as no-load or open-circuit e.m.f. circle. For a given load, OI_2 represents the load current and is taken as the reference vector, CB represents $I_2 R_{02}$ and is parallel to OI_2, AB represents $I_2 X_{02}$ and is drawn at right angles to CB. Vector OC obviously represents $I_2 X_{02}$ and is drawn at right angles to CB. Vector OC obviously represents secondary terminal voltage V_2. Since I_2 is constant, the drop triangle ABC remains constant in size. It is seen that end point C of V_2 lies on another circle whose centre is O'. This point O' lies at a distance of $I_2 X_{02}$ vertically below the point O and a distance of $I_2 R_{02}$ to its left as shown in Fig. 32.54.

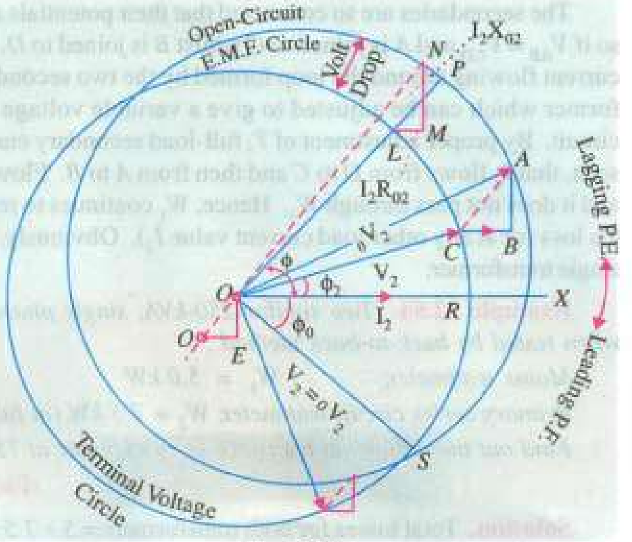


Fig. 32.54

Suppose it is required to find the voltage drop on full-load at a lagging power factor of cos φ, then a radius OLP is drawn inclined at an angle of φ with OX. $LM = I_2 R_{02}$ and is drawn horizontal $MN = I_2 X_{02}$ and is drawn perpendicular to LM. Obviously, ON is no-load voltage $_0V_2$. Now, $ON = OP = {_0V_2}$. Similarly, OL is V_2. The voltage drop $= OP - OL = LP$.

Hence, percentage regulation 'down' is $= \dfrac{OP - OL}{OP} \times 100 = \dfrac{LP}{OP} \times 100$

It is seen that for finding voltage drop, triangle LMN need not be drawn, but simply the radius OLP.

The diagram shows clearly how the secondary terminal voltage falls as the angle of lag increases. Conversely, for a leading power factor, the fall in secondary terminal voltage decreases till for an angle of ϕ_0 leading, the fall becomes zero; hence $V_2 = {_0V_2}$. For angles greater than ϕ_0, the secondary terminal voltage V_2 becomes greater than $_0V_2$.

The Kapp diagram is very helpful in determining the variation of regulation with power factor but it has the disadvantage that since the lengths of the sides of the impedance triangle are very small as compared to the radii of the circles, the diagram has to be drawn on a very large scale, if sufficiently accurate results are desired.

32.27. Sumpner of Back-to-Back Test

This test provides data for finding the regulation, efficiency and heating under load conditions and is employed only when two similar transformers are available. One transformer is loaded on the other and both are connected to supply. The power taken from the supply is that necessary for supplying the losses of both transformers and the negligibly small loss in the control circuit.

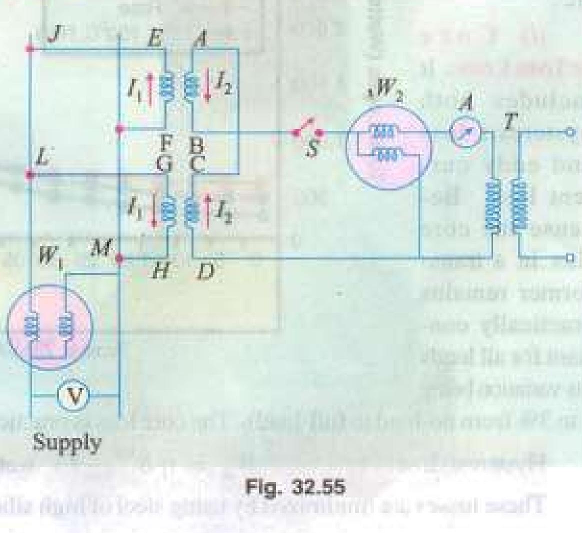


Fig. 32.55

As shown in Fig. 32.55, primaries of the two transformers are connected in parallel across the same a.c. supply. With switch S open, the wattmeter W_1 reads the core loss for the two transformers.

The secondaries are so connected that their potentials are in opposition to each other. This would so if $V_{AB} = V_{CD}$ and A is joined to C whilst B is joined to D. In that case, there would be no secondary current flowing around the loop formed by the two secondaries. T is an auxiliary low-voltage transformer which can be adjusted to give a variable voltage and hence current in the secondary loop circuit. By proper adjustment of T, full-load secondary current I_2 can be made to flow as shown. It is seen, that I_2 flows from D to C and then from A to B. Flow of I_1 is confined to the loop $FEJLGHMF$ and it does not pass through W_1. Hence, W_1 continues to read the core loss and W_2 measures full-load Cu loss (or at any other load current value I_2). Obviously, the power taken in is twice the losses of a single transformer.

Example 32.58. *Two similar 250-kVA, single-phase transformers gave the following results when tested by back-to-back method :*

Mains wattmeter, $W_1 = 5.0$ kW

Primary series circuit wattmeter, $W_2 = 7.5$ kW *(at full-load current).*

Find out the individual transformer efficiencies at 75% full-load and 0.8 p.f. lead.

(Electrical Machines-III, Gujarat Univ. 1986)

Solution. Total losses for both transformers = 5 + 7.5 = 12.5 kW

F.L. loss for each transformer = 12.5/2 = 6.25 kW

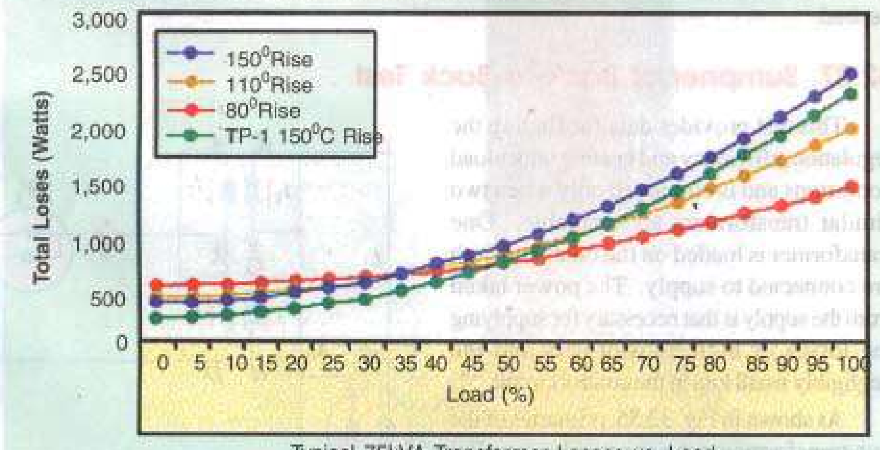
$$\text{Copper-loss at 75\% load} = \left(\frac{3}{4}\right)^2 \times \frac{7.5}{2}\text{ kW} = 2.11\text{ kW}$$

Output of each transformer at 75% F.L. and 0.8 p.f. = $(250 \times 0.75) \times 0.8 = 150$ kW

$$\eta = \frac{150}{150 + 2.5 + 2.11} = 97\%$$

32.28. Losses in a Transformer

In a static transformer, there are no friction or windage losses. Hence, the only losses occuring are :



Typical 75kVA Transformer Losses vs. Load

(i) Core or Iron Loss : It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load). The core loss is practically the same at all loads.

Hysteresis loss $W_h = \eta B^{1.6}_{max} f V$ watt; eddy current loss $W_e = PB^2_{max} f^2 t^2$ watt

These losses are minimized by using steel of high silicon content for the core and by using very thin laminations. Iron or core loss is found from the *O.C.* test. ***The input of the transformer when on no-load measures the core loss.***

(ii) **Copper loss.** This loss is due to the ohmic resistance of the transformer windings. Total Cu loss $= I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} + I_2^2R_{02}$. It is clear that Cu loss is proportional to (current)^2 or kVA^2. In other words, Cu loss at half the full-load is one-fourth of that at full-load.

The value of Cu loss is found from the short-circuit test (Art. 32.22).

32.29. Efficiency of a Transformer

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input–the two being measured in the same units (either watts or kilowatts).

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

But a transformer being a highly efficient piece of equipment, has very small loss, hence it is impractical to try to measure transformer, efficiency by measuring input and output. These quantities are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from ;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output + losses}} = \frac{\text{Output}}{\text{Output + Cu loss + iron loss}}$$

or

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$$

It may be noted here that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to VA. Hence, at any volt-ampere load, the efficiency depends on power factor, being maximum at a power factor of unity.

Efficiency can be computed by determining core loss from no-load or open-circuit test and Cu loss from the short-circuit test.

32.30. Condition for Maximum Efficiency

$$\text{Cu loss} = I_1^2R_{01} \text{ or } I_2^2R_{02} = W_{cu}$$

$$\text{Iron loss} = \text{Hysteresis loss} + \text{Eddy current loss} = W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1I_1 \cos\phi_1$$

$$\eta = \frac{V_1I_1\cos\phi_1 - \text{losses}}{V_1I_1\cos\phi_1} = \frac{V_1I_1\cos\phi_1 - I_1^2R_{01} - W_i}{V_1I_1\cos\phi_1}$$

$$= 1 - \frac{I_1R_{01}}{V_1\cos\phi_1} - \frac{W_i}{V_1I_1\cos\phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1\cos\phi_1} + \frac{W_i}{V_1I_1^2\cos\phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1\cos\phi_1} = \frac{W_i}{V_1I_1^2\cos\phi_1} \quad \text{or} \quad W_i = I_1^2R_{01} \quad \text{or} \quad I_2^2R_{02}$$

or **Cu loss = Iron loss**

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i/R_{02})}$.

It is this value of the output current which will make the Cu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

Note. *(i)* If we are given iron loss and full-load Cu loss, then the load at which two losses would be equal (*i.e.* corresponding to maximum efficiency) is given by

$$= \text{Full load} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L. Cu loss}}\right)}$$

In Fig. 32.56, Cu losses are plotted as a percentage of power input and the efficiency curve as deduced from these is also shown. It is obvious that the point of intersection of the Cu and iron loss curves gives the point of maximum efficiency. It would be seen that the efficiency is high and is practically constant from 15% full-load to 25% overload.

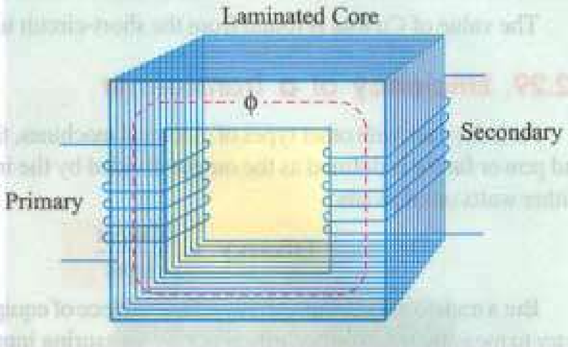


Fig. 32.56

(ii) The efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} + W_i} \times 100$$

where x = ratio of actual to full-load kVA

W_i = iron loss in kW ; W_{cu} = Cu loss in kW.

Example 32.59. *In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on (i) full load (ii) half full-load.* **(Elect. Engg. & Electronic, Bangalore Univ. 1990 and Similar example in U.P. Technical University 2001)**

Solution. *(i)* **Full-load Unity p.f.**

$$\text{Total loss} = 350 + 400 = 750\text{ W}$$

$$\text{F.L. output at u.p.f.} = 25 \times 1 = 25\text{ kW} ; \eta = 25/25.75 = 0.97 \text{ or } \mathbf{97\%}$$

(ii) **Half F.L. Unity p.f.**

Cu loss = 400 × $(1/2)^2$ = 100 W. Iron loss remains constant at 350 W, Total loss = 100 + 350 = 450 W.

Half-load output at u.p.f. = 12.5 kW

$$\therefore \quad \eta = 12.5/(12.5 + 0.45) = \mathbf{96.52\%}$$

Example 32.60. *If P_1 and P_2 be the iron and copper losses of a transformer on full-load, find the ratio of P_1 and P_2 such that maximum efficiency occurs at 75% full-load.* **(Elect. Machines AMIE Sec. B, Summer 1992)**

Solution. If P_2 is the Cu loss at full-load, its value at 75% of full-load is $= P_2 \times (0.75)^2 = 9P_2/16$. At maximum efficiency, it equals the iron loss P_1 which remains constant throughout. Hence, at maximum efficiency.

$$P_1 = 9P_2/16 \quad \text{or} \quad P_1/P_2 = 9/16.$$

Example 32.61. *A 11000/230 V, 150-kVA, 1-phase, 50-Hz transformer has core loss of 1.4 kW and F.L. Cu loss of 1.6 kW. Determine*

(i) the kVA load for max. efficiency and value of max. efficiency at unity p.f.

(ii) the efficiency at half F.L. 0.8 p.f. leading **(Basic Elect. Machine, Nagpur Univ. 1993)**

Solution. (*i*) Load kVA corresponding to maximum efficiency is

$$= \text{F.L. kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}} = 250 \times \sqrt{\frac{1.6}{1.4}} = 160 \text{ kVA}$$

Since Cu loss equals iron loss at maximum efficiency, total loss = 1.4 + 1.4 = 2.8 kW ; output = 160 × 1 = 160 kW

$$\eta_{max} = 160/162.8 = 0.982 \text{ or } \mathbf{98.2\%}$$

(*ii*) Cu loss at half full-load = $1.6 \times (1/2)^2 = 0.4$ kW ; Total loss = 1.4 + 0.4 = 1.8 kW

Half F.L. output at 0.8 p.f. = (150/2) × 0.8 = 60 kW

$\therefore$ Efficiency = 60/(60 + 1.8) = 0.97 or **97%**

Example 32.62. *A 5-kVA, 2,300/230-V, 50-Hz transformer was tested for the iron losses with normal excitation and Cu losses at full-load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs :*

1.25 2.5 3.75 5.0 6.25 7.5

Plot efficiency vs kVA output curve. **(Elect. Engg.-I, Bombay Univ. 1987)**

Solution. F.L. Cu loss = 112 W ; Iron loss = 40 W

(*i*) Cu loss at 1.25 kVA = $112 \times (1.25/5)^2 = 7$ W

Total loss = 40 + 7 = 47 W Output = 1.25 × 0.8 = 1 kW = 1,000 W

η = 100 × 1,000/1,047 = **95.51 %**

(*ii*) Cu loss at 2.5 kVA = $112 \times (2.5/5)^2 = 28$ W

Total loss = 40 + 28 = 68 W

Output = 2.5 × 0.8 = 2 kW

η = 2,000 × 100/2,068 = **96.71%**

(*iii*) Cu loss at 3.75 kVA

$= 112 \times (3.75/5)^2 = 63$ W

Total loss = 40 + 63 = 103 W

η = 3,000 × 100/3,103 = **96.68 %**

(*iv*) Cu loss at 5 kVA

= 112 W

Total loss = 152 W = 0.152 kW

Output = 5 × 0.8 = 4 kW

η = 4 × 100/4.142 = **96.34 %**

(*v*) Cu loss at 6.25 kVA

$= 112 \times (6.25/5)^2 = 175$ W

Total loss = 125 W = 0.125 kW ; Output = 6.25 × 0.8 = 5 kW

η = 5 × 100/5.215 = **95.88 %**

(*vi*) Cu loss at 7.5 kVA = $112 \times (7.5/5)^2 = 252$ W

Total loss = 292 W = 0.292 kW ; Output = 7.5 × 0.8 = 6 kW

η = 6 × 100/6.292 = **95.36 %**

The curve is shown in Fig. 32.57.

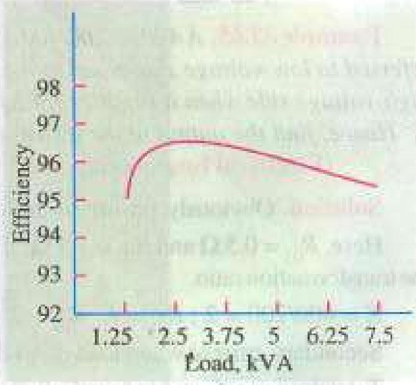


Fig. 32.57

Example 32.63. *A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume negligible magnetizing current and p.f. 0.8 at all loads.* **(Elect. Technology Punjab Univ. Jan. 1991)**

Solution. As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

$$\text{F.L. output} = 200 \times 0.8 = 160 \text{ kW} ; \text{ F.L. input} = 160/0.98 = 163.265 \text{ kW}$$

$$\text{F.L. losses} = 163.265 - 160 = 3.265 \text{ kW}$$

This loss consists of F.L. Cu loss x and iron loss y.

$$\therefore \quad x + y = 3.265 \text{ kW} \quad ...(i)$$

It is also given that η_{max} occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

$$\therefore \quad \text{Cu loss at 75 \% of F.L.} = x\,(3/4)^2 = 9\,x/16$$

Since y remains constant, hence $9\,x/16 = y$ ...(ii)

Substituting the value of y in Eqn. (i), we get $x + 9\,x/16 = 3265$ or $x = 2090$ W; $y = 1175$ W

Half-load Unity p.f.

$$\text{Cu loss} = 2090 \times (1/2)^2 = 522 \text{ W} ; \text{ total loss} = 522 + 1175 = 1697 \text{ W}$$

$$\text{Output} = 100 \times 0.8 = 80 \text{ kW} ; \eta = 80/81.697 = 0.979 \text{ or } \mathbf{97.9\ \%}$$

Example 32.64. *A 25-kVA, 1-phase transformer, 2,200 volts to 220 volts, has a primary resistance of 1.0 Ω and a secondary resistance of 0.01 Ω. Find the equivalent secondary resistance and the full-load efficiency at 0.8 p.f. if the iron loss of the transformer is 80% of the full-load Cu loss.*

(Elect. Technology, Utkal Univ. 1998)

Solution. $K = 220/2,200 = 1/10$; $R_{02} = R_2 + K_2 R_1 = 0.01 + 1/100 = \mathbf{0.02\ \Omega}$

$$\text{Full-load } I_2 = 25,000/220 = 113.6 \text{ A} ; \text{ F.L. Cu loss} = I_2^2 R_{02} = 113.6^2 \times 0.02 = 258 \text{ W.}$$

$$\text{Iron loss} = 80\% \text{ of } 258 = 206.4 \text{ W} ; \text{ Total loss} = 258 + 206.4 = 464.4 \text{ W}$$

$$\text{F.L. output} = 25 \times 0.8 = 20 \text{ kW} = 20,000 \text{ W}$$

$$\text{Full-load } \eta = 20,000 \times 100/(20,000 + 464.4) = \mathbf{97.7\ \%}$$

Example 32.65. *A 4-kVA, 200/400-V, 1-phase transformer has equivalent resistance and reactance referred to low-voltage side equal to 0.5 Ω and 1.5 Ω respectively. Find the terminal voltage on the high-voltage side when it supplies 3/4th full-load at power factor of 0.8, the supply voltage being 220 V. Hence, find the output of the transformer and its efficiency if the core losses are 100 W.*

(Electrical Engineering ; Bombay Univ. 1985)

Solution. Obviously, primary is the low-voltage side and the secondary, the high voltage side.

Here, $R_{01} = 0.5\ \Omega$ and $X_{01} = 1.5\ \Omega$. These can be transferred to the secondary side with the help of the transformation ratio.

$K = 400/200 = 2$; $R_{02} = K^2 R_{01} = 2^2 \times 0.5 = 2\ \Omega$; $X_{02} = K^2 X_{01} = 4 \times 1.5 = 6\ \Omega$

Secondary current when load is 3/4 the, full-load is $= (1,000 \times 4 \times 3/4)/400 = 7.5$ A

Total drop as referred to transformer secondary is

$= I_2\,(R_{02} \cos \phi + X_{02} \sin \phi)$* $= 7.5\,(2 \times 0.8 + 6 \times 0.6) = 39$ V

$\therefore$ Terminal voltage on high-voltage side under given load condition is

$$= 400 - 39 = \mathbf{361\ V}$$

$$\text{Cu loss} = I_2^2 R_{02} = 7.5^2 \times 2 = 112.5 \text{ W} \qquad \text{Iron loss} = 100 \text{ W}$$

$$\text{Total loss} = 212.5 \text{ W} \qquad \text{output} = (4 \times 3/4) \times 0.8 = \mathbf{2.4\ kW}$$

$$\text{Input} = 2,400 + 212.5 = 2,612.5 \text{ W} \qquad \eta = 2,400 \times 100/2,612.5 = \mathbf{91.87\ \%}$$

Example 32.66. *A 20-kVA, 440/220 V, 1-φ, 50 Hz transformer has iron loss of 324 W. The Cu loss is found to be 100 W when delivering half full-load current. Determine (i) efficiency when*

* Assuming a lagging power factor

delivering full-load current at 0.8 lagging p.f. and (ii) the percent of full-load when the efficiency will be maximum. **(Electrotechnique-II, M.S. Univ., Baroda 1987)**

Solution. F.L. Cu loss $= 2^2 \times 100 = 400$ W ; Iron loss = 324 W

(*i*) F.L. efficiency at 0.8 p.f. $= \dfrac{20 \times 0.8}{(20 \times 0.8) + 0.724} \times 100 =$ **95.67 %**

(*ii*) $\dfrac{\text{kVA for maximum}}{\text{F.L. kVA}} = \sqrt{\dfrac{\text{Iron loss}}{\text{F.L. Cu loss}}} = \sqrt{\dfrac{324}{400}} = 0.9$

Hence, efficiency would be maximum at **90** % of F.L.

Example 32.67. *Consider a 4-kVA, 200/400 V single-phase transformer supplying full-load current at 0.8 lagging power factor. The O.C./S.C. test results are as follows :*

O.C. test	:	*200 V,*	*0.8 A,*	*70 W*	*(L.V. side)*
S.C. test	:	*20 V,*	*10 A,*	*60*	*(H.V. side)*

Calculate efficiency, secondary voltage and current into primary at the above load.

Calculate the load at unity power factor corresponding to maximum efficiency.

(Elect. Machines Nagpur Univ. 1993)

Solution. Full-load, $I_2 = 4000/400 = 10$ A

It means that S.C. test has been carried out with full secondary flowing. Hence, 60 W represents full-load Cu loss of the transformer.

Total F.L. losses = 60 + 70 = 130 W ; F.L. output = $4 \times 0.8 = 3.2$ kW

F.L. $\eta = 3.2/3.33 = 0.96$ or **96 %**

S.C. Test

$Z_{02} = 20/10 = 2\,\Omega$; $I_2^2 R_{02} = 60$ or $R_{02} = 60/10^2 = 0.6\,\Omega$; $X_{02} = \sqrt{2^2 - 0.6^2} = 1.9\,\Omega$

Transformer voltage drop as referred to secondary

$$= I_2(R_{02}\cos\phi + X_{02}\sin\phi) = 10\,(0.6 \times 0.8 + 1.9 \times 0.6) = 16.2 \text{ V}$$

$$\therefore \quad V_2 = 400 - 16.2 = 383.8 \text{ V}$$

Primary current = 4000/200 = 20 A

kVA corresponding to $\eta_{max} = 4 \times \sqrt{70/60} = 4.32$ kVA

$\therefore$ Load at u.p.f. corresponding to $\eta_{max} = 4.32 \times 1 =$ **4.32 kW**

Example 32.68. *A 600 kVA, 1-phase transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full-load at 0.8 power factor lag.*

(Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution.

$$\eta = \frac{x \times kVA \times \cos\phi}{(x \times kVA) \times \cos\phi + W_i + x^2 W_{Cu}} \times 100$$

where x represents percentage of full-load

W_i is iron loss and W_{Cu} is full-load Cu loss.

At F.L., u.p.f. Here $x = 1$

$$\therefore \quad 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_1 + I^2 W_{Cu}} \times 100,\; W_i + W_{Cu} = 52.174 \text{ kW} \quad ...(i)$$

At half F.L. UPF. Here $x = 1/2$

$$92 = \frac{1/2 \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{Cu}} \times 100 ;$$

$\therefore \quad W_i + 0.25 \quad W_{Cu} = 26.087$ kW ...(ii)

From (i) and (ii), we get, $W_i = 17.39$ kW, $W_{Cu} = 34.78$ kW

60 % F.L. 0.8 p.f. (lag) Here, $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8 \times 100}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2\ 34.78} = 85.9\ \%$$

Example 32.69. *A 600-kVA, 1-ph transformer when working at u.p.f. has an efficiency of 92 % at full-load and also at half-load. Determine its efficiency when it operates at unity p.f. and 60 % of full-load.* **(Electric. Machines, Kerala Univ. 1987)**

Solution. The fact that efficiency is the same *i.e.* 92 % at both full-load and half-load will help us to find the iron and copper losses.

At full-load

Output = 600 kW ; Input = 600/0.92 = 652.2 kW ; Total loss = 652.2 − 600 = 52.2 kW

Let $\quad x$ = Iron loss — It remains constant at all loads.

$\quad y$ = F.L. Cu loss — It is ∝ (kVA)^2. $\quad \therefore \quad x + y = 52.2$ ...(i)

At half-load

Output = 300 kW ; Input = 300/0.92 $\quad \therefore \quad$ Losses = (300/0.92 − 300) = 26.1 kW

Since Cu loss becomes one-fourth of its F.L. value, hence

$$x + y/4 = 26.1$$

Solving for x and y, we get $\quad x$ = **17.4 kW** ; $y = 34.8$ kW ...(ii)

At 60 % full-load

Cu loss = $0.6^2 \times 34.8 = 12.53$ kW ; Total loss = 17.4 + 12.53 = 29.93 kW

Output = $600 \times 0.6 = 360$ kW $\quad \therefore \quad \eta = 360/389.93 = 0.965$ or **96.5 %**

Example 32.70. *The maximum efficiency of a 100-kVA, single phase transformer is 98% and occurs at 80% of full load at 8 p.f. If the leakage impedance of the transformer is 5 %, find the voltage regulation at rated load of 0.8 power factor lagging.*

(Elect. Machines-I, Nagpur Univ. 1993)

Solution. Since maximum efficiency occurs at 80 percent of full-load at 0.8 p.f.,

Output at $\eta_{max} = (100 \times 0.8) \times 0.8 = 64$ kW ; Input = 64/0.98 = 65.3 kW

$\therefore \quad$ Total loss = 65.3 − 64 = 1.3 kW. This loss is divided equally between Cu and iron.

$\therefore \quad$ Cu loss at 80 % of full-load = 1.3/2 = 0.65 kW

Cu loss at full-load = $0.65/0.8^2 = 1$ kW

$$\%\ R = \frac{\text{Cu loss}}{V_2 I_2} \times 100 = 1 \times \frac{100}{100} = 1\ \% = v_r\ ;\ v_x = 5\ \%$$

$$\therefore \quad \%\text{ age regn.} = (1 \times 0.8 + 5 \times 0.6) + \frac{1}{200}(5 \times 0.8 - 1 \times 0.6)^2 = 0.166\ \%$$

Example 32.71. *A 10 kVA, 5000/440-V, 25-Hz single phase transformer has copper, eddy current and hysteresis losses of 1.5, 0.5 and 0.6 per cent of output on full load. What will be the percentage losses if the transformer is used on a 10-kV, 50-Hz system keeping the full-load current constant ? Assume unity power factor operation. Compare the full load efficiencies for the two cases.*

(Elect. Machines, A.M.I.E., Sec. B, 1991)

Solution. We know that $E_1 = 4.44 f N_1 B_1 A$.. When both excitation voltage and frequency are doubled, flux remains unchanged.

F.L. output at upf = 10 kVA × 1 = 10 kW

F.L. Cu loss = 1.5 × 10/100 = 0.15 kW ; Eddy current loss

= 0.5 × 10/100 = 0.05 kW ; Hysteresis loss = 0.6 × 10/100 = 0.06 kW

Now, full-load current is kept constant but voltage is increased from 5000 V to 10,000 V. Hence, output will be doubled to 20 kW. Due to constant current, Cu loss would also remain constant.

New Cu loss = 0.15 kW, % Cu loss = (0.15/20) × 100 = 0.75 %

Now, eddy current loss $\propto f^2$ and hysteresis loss $\propto f$.

New eddy current loss = $0.05\,(50/25)^2 = 0.2$ kW, % eddy current loss = (0.2/20) × 100 = 1 %

Now, $W_h = 0.06 \times (50/25) = 0.12$ kW, % $W_h = (0.12/20) \times 100 = 0.6$ %

$$\eta_1 = \frac{10}{10+0.15+0.05+0.06} \times 100 = \mathbf{87.4\ \%}$$

$$\eta_2 = \frac{20}{20+0.15+0.2+0.12} \times 100 = \mathbf{97.7\ \%}$$

Example 32.72. *A 300-kVA, single-phase transformer is designed to have a resistance of 1.5 % and maximum efficiency occurs at a load of 173.2 kVA. Find its efficiency when supplying full-load at 0.8 p.f. lagging at normal voltage and frequency.* **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. $$\% R = \frac{\text{F.L. Cu loss}}{\text{Full-load } V_2 I_2} \times 100 \; ; \; 1.5 = \frac{\text{F.L. Cu loss}}{300 \times 1000} \times 100$$

∴ $$\text{F.L. Cu loss} = 1.5 \times 300 \times 1000/100 = 4500 \text{ W}$$

Also, $$173.2 = 300\sqrt{\frac{\text{Iron Loss}}{4500}} \; ; \text{ Iron loss} = 1500 \text{ W}$$

$$\text{Total F.L. loss} = 4500 + 1500 = 6 \text{ kW}$$

$$\text{F.L. } \eta \text{ at } 0.8 \text{ p.f.} = \frac{300 \times 0.8}{(300 \times 0.8) + 6} \times 100 = \mathbf{97.6\ \%}$$

Example 32.73. *A single phase transformer is rated at 100-kVA, 2300/230-V, 50 Hz. The maximum flux density in the core is 1.2 Wb/m² and the net cross-sectional area of the core is 0.04 m². Determine*

(a) The number of primary and secondary turns needed.

(b) If the mean length of the magnetic circuit is 2.5 m and the relative permeability is 1200, determine the magnetising current. Neglect the current drawn for the core loss.

(c) On short-circuit with full-load current flowing, the power input is 1200 W and an open-circuit with rated voltage, the power input was 400 W. Determine the efficiency of the transformer at 75 % of full-load with 0.8 p.f. lag.

(d) If the same transformer is connected to a supply of similar voltage but double the frequency (i.e., 100 Hz). What is the effect on its efficiency ? **(Elect. Engg., Bombay Univ. 1988)**

Solution. (*a*) Applying e.m.f. equation of the transformer to the primary, we have

$$2300 = 4.44 \times 50 \times N_1 \times (1.2 \times 0.0.4) \quad \therefore \quad N_1 = \mathbf{216}$$

$$K = 230/2300 = 1/10 \quad N_2 = KN_1 = 216/10 = 21.6 \text{ or } \mathbf{22}$$

(*b*) $$AT = H \times l = \frac{B}{\mu_0 \mu_r} \times l = \frac{1.2 \times 2.5}{4\pi \times 10^{-7} \times 1200} = 1989 \quad \therefore I = \frac{1989}{216} = \mathbf{9.21\ A}$$

(*c*) $$\text{F.L. Cu loss} = 1200 \text{ W} - \text{S.C. test} \; ; \text{ Iron loss} = 400 \text{ W} - \text{O.C. test}$$

$$\text{Cu loss at 75 \% of F.L.} = (0.75)^2 \times 1200 = 675 \text{ W}$$

$$\text{Total loss} = 400 + 675 = 1075 \text{ kW}$$

Output = 100 × (3/4) × 0.8 = 60 kW ; η = (60/61.075) × 100 = **98.26 %**

(*d*) When frequency is doubled, iron loss is increased because

(*i*) hysteresis loss is doubled – $W_h \propto f$

(*ii*) eddy current loss is quadrupled – $W_e \propto f^2$

Hence, efficiency will be decreased.

Example 32.74. *A transformer has a resistance of 1.8 % and a reactance of 5.4 %. (a) At full load, what is the power-factor at which the regulation will be : (i) Zero, (ii) positive-maximum ? (b) If its maximum efficiency occurs at full-load (at unity p.f.), what will be the efficiency under these conditions ?*

Solution : Approximate percentage regulation is given, in this case, by the relationship $1.8 \cos\phi \pm 5.4 \sin\phi$.

(*a*) **Regulation :**

(*i*) If regulation is zero, negative sign must be applicable. This happens at leadings p.f.

$$\text{Corresponding p.f.} = \tan\phi = 1.8/5.4 = 0.333 \text{ leading}$$

$$\phi = \mathbf{18.44^0 \text{ leading}}$$

(*ii*) For maximum positive regulation, lagging p.f. is a must. From phasor diagram, the result can be obtained.

$$\text{Corresponding } \tan\phi = 5.4/1.8 = 3, \phi = 71.56 \text{ lagging}$$

$$\% \text{ Voltage regulation} = 1.8 \cos\phi + 5.4 \sin\phi = \mathbf{5.7\ \%}$$

(*b*) **Efficiency :** Maximum efficiency occurs at such a load when

$$\text{Iron losses} = \text{Copper losses}$$

This means Iron-losses are 1.8 %.

$$\text{Efficiency} = 100/(100 + 1.8 + 1.8) = \mathbf{96.52\ \%}$$

Example 32.75. *A 10 kVA, 1 phase, 50 Hz, 500/250 V transformer gave following test results :*

OC test (LV) side : 250 V, 3.0 A, 200 W

SC test (LV) side : 15 V, 30 A, 300 W.

Calculate efficiency and regulation at full load, 0.8 p.f. lagging.

(Nagpur University, Summer 2000)

Solution. For efficiency calculations, full load current should be calculated, on the L.V. side in this case,

$$\text{F.L. Current} = \frac{10{,}000}{250} = 40 \text{ amp}$$

Short-circuit test data have been given at 30 A current on the L.V. side.

$$i^2 r \text{ losses at 40 A L.V. side} = \left(\frac{40}{30}\right)^2 \times 300 \text{ Watts} = 533.3 \text{ Watts}$$

At rated voltage, iron losses (from O.C. test) = 200 Watts

F.L. Output at 0.8 P.F. = $10{,}000 \times 0.8 = 8000$ Watts

$$\text{Hence, } \eta = \frac{8000}{8000 + 4733.3} \times 100\ \% = \mathbf{91.6\ \%}$$

For regulation, series resistance and reactance parameters of the equivalent circuit have to be evaluated, from the S.C. test.

$$\text{Series Impedance, } Z = \frac{15}{30} = 0.5 \text{ ohm}$$

$$\text{Series resistance, } r = \frac{300}{30 \times 30} = 0.333 \text{ ohm}$$

Series reactance, $x = \sqrt{0.5^2 - 0.333^2} = $ **0.373 ohm**

By Approximate formula,

p.u. regulation at full load, 0.8 p.f. lagging

$$= \frac{40}{250}[0.333 \times 0.8 - 0.373 \times 0.6] = 6.82 \times 10^{-3} \text{ p.u.}$$

When converted into volts, this is $6.82 \times 10^{-3} \times 250 =$ **1.70 volt**

Example 32.76. *A 40 kVA, 1-ph, transformer has an iron loss of 400 W, and full copper loss of 800 W. Find the load at which maximum efficiency is achieved at unity power factor.*

(Amravati University, Winter 1999)

Solution. If x = fraction of rated load at which the efficiency is maximum.

$$P_i = \text{Iron–loss} = 400 \text{ W}$$

$$P_c = \text{F.L. copper–loss} = 800 \text{ W}$$

Then $\quad x^2 P_c = P_i$

On substitution of numerical values of P_i and P_c, we get

$$x = 0.707$$

Hence, the efficiency is maximum, at unity p.f. and at 70.7 % of the rated Load. At this load, copper–loss = Iron–loss = 0.40 kW

$$\text{Corresponding output} = 40 \times 0.707 \times 1$$

$$= 28.28 \text{ kW}$$

$$\text{Corresponding efficiency} = \frac{28.28}{28.28 + 0.4 + 0.4} = \textbf{97.25 \%}$$

Extension to Question : (a) *At what load (s) at unity p.f. the efficiency will be 96.8 % ?*

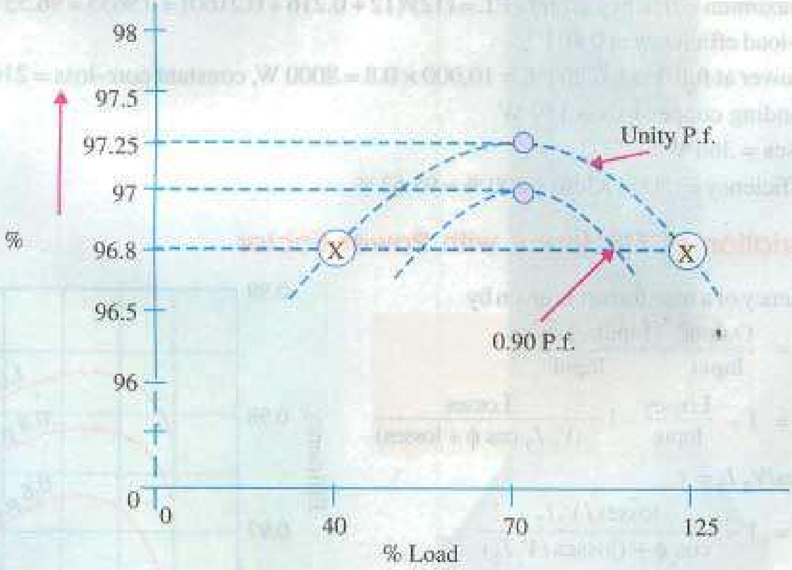


Fig. 32.58. Efficiency variation with load

Solution. Let x = Fractional load at which the concerned efficiency occurs, at unity p.f.

$$\frac{40\,x}{40\,x + 0.8\,x^2 + 0.40} = 0.968$$

This gives the following values of x : $\qquad x_1 = 1.25 \qquad x_2 = 0.40$

Thus, at 40 % and at 125 % of the rated load, the efficiency will be **96.8 %** as marked on the graph, in Fig. 32.58.

(b) How will maximum-efficiency condition be affected if the power factor is 0.90 lagging ?

Solution.

The condition for efficiency-variation-statement is that the power factor remains constant. Thus, for 0.90 lagging p.f., another curve (Lower curve in Fig. 32.58) will be drawn for which the maximum efficiency will occur at the same value of x (= 0.707), but

$$\text{Maximum efficiency} = \frac{40\, x \cos\phi}{40\cos\phi + 0.80\, x^2 + 0.40}$$

$$= \frac{28.28 \times 0.90}{(28.28 \times 0.90) + 0.80} = 97\ \%$$

Example 32.77. *A 10 kVA, 500/250 V, single-phase transformer gave the following test results: S.C. Test (H.V. side) : 60 V, 20 A, 150 W*

The maximum efficiency occurs at unity power factor and at 1.20 times full-load current. Determine full-load efficiency at 0.80 p.f. Also calculate the maximum efficiency.

(Rajiv Gandhi Technical University, Bhopal, Summer 2001)

Solution. Full-load current on H.V. side = 10,000/500 = 20 Amp

S.C. test has been conducted from H.V. side only. Hence, full-load copper-loss, at unity p.f. = 150 watts

(a) Maximum efficiency occurs at 1.2 times full-load current, at unity p.f. corresponding copper-loss = $(1.2)^2 \times 150 = 216$ watts

At maximum efficiency, copper-loss = core-loss = 216 watts

Corresponding Power-output = $1.2 \times 10{,}000 \times 1.0 = 12$ kW

Hence, maximum efficiency at unity P.f. = (12)/(12 + 0.216 + 0.2160) = 0.9653 = 96.53 %

(b) Full-load efficiency at 0.80 P.f.

Output Power at full-load, 0.80 P.f. = $10{,}000 \times 0.8 = 8000$ W, constant core-loss = 216 W

Corresponding copper-loss = 150 W

Total losses = 366 W

Hence, efficiency = $(8000/8366) \times 100\ \% = 95.63\ \%$.

32.31. Variation of Efficiency with Power Factor

The efficiency of a transformer is given by

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input - Losses}}{\text{Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{(V_2 I_2 \cos\phi + \text{losses})}$$

Let, losses/$V_2 I_2 = x$

$$\therefore \quad \eta = 1 - \frac{\text{losses}/V_2 I_2}{\cos\phi + (\text{losses}/V_2 I_2)}$$

$$= 1 - \frac{x}{(\cos\phi + x)} = 1 - \frac{x/\cos\phi}{1 + (x/\cos\phi)}$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Fig. 32.59.

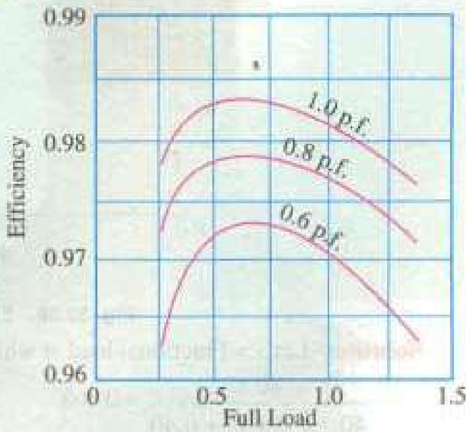


Fig. 32.59

Tutorial Problems 32.4

1. A 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate (*a*) iron loss at F.L. (*b*) Cu loss at F.L. (*c*) efficiency at half-load. Ignore magnetising current and assume a p.f. of 0.8 at all loads.
   [(*a*) 1.777 kW (*b*) 2.09 kW (*c*) 97.92%]
2. A 600 kVA, 1-ph transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full load at 0.8 power factor lag.
   [90.59 %] (*Elect. Machines, A.M.I.E. Sec. B, 1992*)
3. Find the efficiency of a 150 kVA transformer at 25 % full load at 0.8 p.f. lag if the copper loss at full load is 1600 W and the iron loss is 1400 W. Ignore the effects of temperature rise and magnetising current. **[96.15 %] (*Elect. Machines, A.M.I.E. Sec. B, 1991*)**
4. The F.L. Cu loss and iron loss of a transformer are 920 W and 430 W respectively. (*i*) Calculate the loading of the transformer at which efficiency is maximum (*ii*) what would be the losses for giving maximum efficiency at 0.85 of full-load if total full-load losses are to remain unchanged ?
   [(*a*) 68.4% of F.L. (*ii*) W_i = 565 W ; W_{cu} = 785 W]
5. At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 2.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8. **[93.58%] (City & Guilds London)**
6. A transformer, when tested on full-load, is found to have Cu loss 1.8% and reactance drop 3.8%. Calculate its full-load regulation (*i*) at unity p.f. (*ii*) 0.8 p.f. lagging (*iii*) 0.8 p.f. leading.
   [(*i*) 1.80% (*ii*) 3.7 % (*iii*) −0.88%]
7. With the help of a vector diagram, explain the significance of the following quantities in the open-circuit and short-circuit tests of a transformer (*a*) power consumed (*b*) input voltage (*c*) input current.
   When a 100-kVA single-phase transformer was tested in this way, the following data were obtained : On open circuit, the power consumed was 1300 W and on short-circuit the power consumed was 1200 W. Calculate the efficiency of the transformer on (*a*) full-load (*b*) half-load when working at unity power factor. **[(*a*) 97.6% (*b*) 96.9%] (*London Univ.*)**
8. An 11,000/230-V, 150-kVA, 50-Hz, 1-phase transformer has a core loss of 1.4 kW and full-load Cu loss of 1.6 kW. Determine (*a*) the kVA load for maximum efficiency and the minmum efficiency (*b*) the efficiency at half full-load at 0.8 power factor lagging. **[140.33 kVA, 97.6% ; 97%]**
9. A single-phase transformer, working at unity power factor has an efficiency of 90 % at both half-load and a full-load of 500 kW. Determine the efficiency at 75 % of full-load. **[90.5%] (*I.E.E. London*)**
10. A 10-kVA, 500/250-V, single-phase transformer has its maximum efficiency of 94 % when delivering 90 % of its rated output at unity power factor. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging. **[92.6%] (*Elect. Machinery, Mysore Univ, 1979*)**
11. A single-phase transformer has a voltage ratio on open-circuit of 3300/660-V. The primary and secondary resistances are 0.8 Ω and 0.03 Ω respectively, the corresponding leakage reactance being 4 Ω and 0.12 Ω. The load is equivalent to a coil of resistance 4.8 Ω and inductive reactance 3.6 Ω. Determine the terminal voltage of the transformer and the output in kW. **[636 V, 54 kW]**
12. A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (*a*) 100 kVA output at 0.8 p.f. lagging (*b*) 50 kVA output at unity power factor. **[(*a*) 97.44% (*b*) 98.09%]**
13. A 10-kVA, 440/3300-V, 1-phase transformer, when tested on open circuit, gave the following figures on the primary side : 440V ; 1.3 A ; 115 W.
   When tested on short-circuit with full-load current flowing, the power input was 140 W. Calculate the efficiency of the transformer at (*a*) full-load unity p.f. (*b*) one quarter full-load 0.8 p.f.
   [(*a*) 97.51% (*b*) 94.18%] (*Elect. Engg-I, Sd. Patel Univ. June 1977*)
14. A 150-kVA single-phase transformer has a core loss of 1.5 kW and a full-load Cu loss of 2 kW. Calculate the efficiency of the transformer (*a*) at full-load, 0.8 p.f. lagging (*b*) at one-half full-load

unity p.f. Determine also the secondary current at which the efficiency is maximum if the secondary voltage is maintained at its rated value of 240 V. **[(a) 97.17% (b) 97.4% ; 541 A]**

15. A 200-kVA, 1-phase, 3300/400-V transformer gave the following results in the short-circuit test. With 200 V applied to the primary and the secondary short-circuited, the primary current was the full-load value and the input power 1650 W. Calculate the secondary p.d. and percentage regulation when the secondary load is passing 300 A at 0.707 p.f. lagging with normal primary voltage.

[380 V ; 480%]

16. The primary and secondary windings of a 40-kVA, 6600/250-V, single-phase transformer have resistances of 10 Ω and 0.02 Ω respectively. The leakage reactance of the transformer referred to the primary is 35 Ω. Calculate
(a) the primary voltage required to circulate full-load current when the secondary is short-circuited.
(b) the full-load regulations at (i) unity (ii) 0.8 lagging p.f. Neglect the no-load current.

[(a) 256 V (b) (i) 2.2% (ii) 3.7%] *(Elect. Technology, Kerala Univ. 1979)*

17. Calculate :
(a) F.L. efficiency at unity p.f.
(b) The secondary terminal voltage when supplying full-load secondary current at p.f. (i) 0.8 lag (ii) 0.8 lead for the 4-kVA, 200/400 V, 50 Hz, 1-phase transformer of which the following are the test figures :

Open circuit with 200 V supplied to the primary winding-power 60 W. Short-circuit with 16 V applied to the h.v. winding-current 8 A, power 40 W. **[0.97 ; 383 V ; 406 V]**

18. A 100-kVA, 6600/250-V, 50-Hz transformer gave the following results :
O.C. test : 900 W, normal voltage.
S.C. test (data on h.v. side) : 12 A, 290 V, 860 W
Calculate
(a) the efficiency and percentage regulation at full-load at 0.8 p.f. lagging.
(b) the load at which maximum efficiency occurs and the value of this efficiency at p.f. of unity, 0.8 lag and 0.8 lead. **[(a) 97.3%, 4.32% (b) 81 kVA, 97.8%, 97.3% ; 97.3%]**

19. The primary resistance of a 440/110-V transformer is 0.5 Ω and the secondary resistance is 0.04 Ω. When 440 V is applied to the primary and secondary is left open-circuited, 200 W is drawn from the supply. Find the secondary current which will give maximum efficiency and calculate this efficiency for a load having unity power factor.

[53 A ; 93.58%] *(Basic Electricity & Electronics, Bombay Univ. 1981)*

20. Two tests were performed on a 40-kVA transformer to predetermine its efficiency. The results were:
Open circuit : 250 V at 500 W
Short circuit : 40 V at F.L. current, 750 W both tests from primary side.
Calculate the efficiency at rated kVA and 1/2 rated kVA at (i) unity p.f. (ii) 0.8 p.f.

[96.97% ; 96.68% ; 96.24% ; 95.87%]

21. The following figures were obtained from tests on a 30-kVA, 3000/110-V transformer :
O.C. test : 3000 V 0.5 A 350 W ; S.C. test : 150 V 10 A 500 W
Calculate the efficiency of the transformer at
(a) full-load, 0.8 p.f. (b) half-load, unity p.f.
Also, calculate the kVA output at which the efficiency is maximum. **[96.56% ; 97% ; 25.1 kVA]**

22. The efficiency of a 400 kVA, 1-phase transformer is 98.77 % when delivering full load at 0.8 power factor, and 99.13 % at half load and unity power factor. Calculate (a) the iron loss, (b) the full load copper loss. **[(a) 1012 W (b) 2973 W]** *(Rajiv Gandhi Technical University, 2000)*

32.32. All-day Efficiency

The world first 5,000 KVA amorphous transformer commissioned in August 2001 in Japan

The ordinary or commercial efficiency of a transformer is given by the ratio

$$\frac{\text{Output in watts}}{\text{Input in watts}}$$

But there are certain types of transformers whose performance cannot be judged by this efficiency. Transformers used for supplying lighting and general network *i.e.*, distribution transformers have their primaries energised all the twenty-four hours, although their secondaries supply little or no-load much of the time during the day except during the house lighting period. It means that whereas core loss occurs throughout the day, the Cu loss occurs only when the transformers are loaded. Hence, it is considered a good practice to design such transformers so that core losses are very low. The Cu losses are relatively less important, because they depend on the load. The performance of such is compared on the basis of *energy* consumed during a certain time period, usually a day of 24 hours.

$$\therefore \qquad \eta_{\text{all-day}} = \frac{\text{Output in kWh}}{\text{Input in kWh}} \text{ (For 24 hours)}$$

This efficiency is always less than the commercial efficiency of a transformer.

To find this all-day efficiency or (as it is also called) energy efficiency, we have to know the load cycle on the transformer *i.e.*, how much and how long the transformer is loaded during 24 hours. Practical calculations are facilitated by making use of a ***load factor***.

Example 32.78. *Find the all-day efficiency of 500-kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW respectively. During a day of 24 hours, it is loaded as under :*

No. of hours	*Loading in kW*	*Power factor*
6	400	0.8
10	300	0.75
4	100	0.8
4	0	–

(Elect. Machines, Nagpur Univ. 1993)

Solution. It should be noted that a load of 400 kW at 0.8 p.f. is equal to 400/0.8 = 500 kVA. Similarly, 300 kW at 0.75 p.f. means 300/0.75 = 400 kVA and 100 kW at 0.8 p.f. means 100/0.8 = 125 kVA *i.e.*, one-fourth of the full-load.

$$\begin{aligned}
\text{Cu loss at F.L. of 500 kVA} &= 4.5\text{ kW}\\
\text{Cu loss at 400 kVA} &= 4.5\times(400/500)^2 = 2.88\text{ kW}\\
\text{Cu loss at 125 kVA} &= 4.5\times(125/500)^2 = 0.281\text{ kW}\\
\text{Total Cu loss in 24 hrs} &= (6\times4.5)+(10\times2.88)+(4\times0.281)+(4\times0)\\
&= 56.924\text{ kWh}
\end{aligned}$$

The iron loss takes place throughout the day irrespective of the load on the transformer because its primary is energized all the 24 hours.

$\therefore$ Iron loss in 24 hours $= 24 \times 3.5 = 84$ kWh

Total transformer loss $= 56.924 + 84 = 140.924$ kWh

Transformer output is 24 hrs $= (6 \times 400) + (10 \times 300) + (4 \times 100) = 5800$ kWh

$$\therefore \quad \eta_{all\text{-}day} = \frac{\text{output}}{\text{output + losses}} = \frac{5800}{5800 + 140.924} = 0.976 \text{ or } \mathbf{97.6\ \%}$$

Example 32.79. *A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency.* **(Elect. Engg. Punjab Univ. 1990)**

Solution. It should be noted that lighting transformers are taken to have a load p.f. of unity.

Iron loss for 24 hour $= 1.5 \times 24 = 36$ kWh ; F.L. Cu loss $= 1.5$ kW

$\therefore$ Cu loss for 3 hours on F.L. $= 1.5 \times 3 = 4.5$ kWh

Cu loss at half full-load $= 1.5/4$ kW

Cu loss for 4 hours at half the load $= (1.5/4) \times 4 = 1.5$ kWh

Total losses $= 36 + 4.5 + 1.5 = 42$ kWh

Total output $= (100 \times 3) + (50 \times 4) = 500$ kWh

$\therefore$ $\eta_{all\text{-}day} = 500 \times 100/542 = \mathbf{92.26\ \%}$

Incidentally, ordinary or commercial efficiency of the transformer is

$= 100/(100 + 3) = 0.971$ or **97.1 %**

Example 32.80. *Two 100-kW transformers each has a maximum efficiency of 98 % but in one the maximum efficiency occurs at full-load while in the other, it occurs at half-load. Each transformer is on full-load for 4 hours, on half-load for 6 hours and on one-tenth load for 14 hours per day. Determine the all-day efficiency of each transformer.* **(Elect. Machines-I, Vikram Univ. 1988)**

Solution. Let x be the iron loss and y the full-load Cu loss. If the ordinary efficiency is a maximum at $1/m$ of full-load, then $x = y/m^2$.

Now, output $= 100$ kW ; Input $= 100/0.98$

$\therefore$ Total losses $= 100/0.98 - 100 = 2.04$ kW

$\therefore$ $y + x/m^2 = 2.04$

Ist Transformer

Here $m = 1$; $y + y = 2.04$; $y = 1.02$ kW and $x = 1.02$ kW

Iron loss for 24 hours $= 1.02 \times 24 = 24.48$ kWh

Cu loss for 24 hours $= 4 \times 1.02 + 6 \times (1.02/4) + 14\,(1.02/10^2) = 5.73$ kWh

Total loss $= 24.48 + 5.73 = 30.21$ kWh

$= 4 \times 100 + 6 \times 50 + 14 \times 10 = 840$ kWh

$\therefore$ $\eta_{all\text{-}day} = 840/870.21 = 0.965$ or **96.5 %**

2nd Transformer

Here $1/m = 1/2$ or $m = 2 \therefore y + y/4 = 2.04$

or $y = 1.63$ kW ; $x = 0.14$ kW

Output $= 840$ kWh ...as above

Iron loss for 24 hours $= 0.41 \times 24 = 9.84$ kWh

Cu loss for 24 hours $= 4 \times 1.63 + 6\,(1.63/4) + 14\,(1.63/10^2) = 9.19$ kWh

Total loss $= 9.84 + 9.19 = 19.03$ kWh

$\therefore$ $\eta_{all\text{-}day} = 840/859.03 = 0.978$ or **97.8 %**

Example 32.81. *A 5-kVA distribution transformer has a full-load efficiency at unity p.f. of 95 %, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows :*

No load for	*10 hours*	*Quarter load for*	*7 hours*
Half load for	*5 hours*	*Full load for*	*2 hours*

Assume load p.f. of unity. **(Power Apparatus-I, Delhi Univ. 1987)**

Solution. Let us first find out the losses from the given commercial efficiency of the transformer.

Output = 5 × 1 = 5 kW ; Input = 5/0.95 = 5.264 kW

Losses = (5.264 − 5.000) = 0.264 kW = 264W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$$\therefore \quad \text{Cu loss at F.L. of 5 kVA} = 264/2 = 132 \text{ W} ; \text{Iron loss} = 132 \text{ W}$$

$$\text{Cu loss at one-fourth F.L.} = (1/4)^2 \times 132 = 8.2 \text{ W}$$

$$\text{Cu loss at one-half F.L.} = (1/2)^2 \times 132 = 33 \text{ W}$$

$$\text{Quarter load Cu loss for 7 hours} = 7 \times 8.2 = 57.4 \text{ Wh}$$

$$\text{Half-load Cu loss for 5 hours} = 5 \times 33 = 165 \text{ Wh}$$

$$\text{F.L. Cu loss for 2 hours} = 2 \times 132 = 264 \text{ Wh}$$

$$\text{Total Cu loss during one day} = 57.4 + 165 + 264 = 486.4 \text{ Wh} = 0.486 \text{ kWh}$$

$$\text{Iron loss in 24 hours} = 24 \times 132 = 3168 \text{ Wh} = 3.168 \text{ kWh}$$

$$\text{Total losses in 24 hours} = 3.168 + 0.486 = 3.654 \text{ kWh}$$

Since load p.f. is to be assumed as unity.

F.L. output = 5 × 1 = 5 kW ; Half F.L. output = (5/2) × 1 = 2.5 kW

Quarter load output = (5/4) × 1 = 1.25 kW

Transformer output in a day of 24 hours = (7 × 1.25) + (5 × 2.5) + (2 × 5) = 31.25 kWh

$$\eta_{\text{all-day}} = \frac{31.25}{(31.25 + 3.654)} \times 100 = \mathbf{89.53\ \%}$$

Example 32.82. *Find "all day" efficiency of a transformer having maximum efficiency of 98 % at 15 kVA at unity power factor and loaded as follows :*

12 hours – 2 kW at 0.5 p.f. lag

6 hours – 12 kW at 0.8 p.f. lag

6 hours – at no load. **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution.
$$\text{Output} = 15 \times 1 = 15 \text{ kW, input} = 15/0.98$$

$$\text{Losses} = (15/0.98 - 15) = 0.306 \text{ kW} = 306 \text{ W}$$

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$$\therefore \quad \text{Cu loss at 15 kVA} = 306/2 = 153 \text{ W, Iron loss} = 153 \text{ W}$$

$$2 \text{ kW at 0.5 p.f.} = 2/0.5 = 4 \text{ kVA}, 12 \text{ kW at 0.8 p.f.} = 12/0.8 = 15 \text{ kVA}$$

$$\text{Cu loss at 4 kVA} = 153\,(4/15)^2 = 10.9 \text{ W} ; \text{Cu loss at 15 kVA} = 153 \text{ W.}$$

$$\text{Cu loss in 12 hrs} = 12 \times 10.9 = 131 \text{ Wh} ; \text{Cu loss in 6 hr} = 6 \times 153 = 918 \text{ Wh.}$$

$$\text{Total Cu loss for 24 hr} = 131 + 918 = 1050 \text{ Wh} = 1.05 \text{ kWh}$$

$$\text{Iron loss for 24 hrs} = 24 \times 153 = 3{,}672 \text{ Wh} = 3.672 \text{ kWh}$$

$$\text{Output in 24 hrs} = (2 \times 12) + (6 \times 12) = 96 \text{ kWh}$$

$$\text{Input in 24 hrs} = 96 + 1.05 + 3.672 = 100.72 \text{ kWh}$$

$$\therefore \quad \eta_{\text{all-day}} = 96 \times 100/100.72 = \mathbf{95.3\ \%}$$

Example 32.83. *A 150-kVA transformer is loaded as follows :*

Load increases from zero to 100 kVA in 3 hours from 7 a.m. to 10.00 a.m., stays at 100 kVA from 10 a.m. to 6 p.m. and then the transformer is disconnected till next day. Assuming the load to be resistive and core-loss equal to full-load copper loss of 1 kW, determine the all-day efficiency and the ordinary efficiency of the transformer. **(Electrical Machines-II, Indore Univ. 1990)**

Solution. Since load is resistive, its p.f. is unity.

Average load from 7 a.m. to 10 a.m. $= (0+100)/2 = 50$ kVA *i.e.*, one-third F.L.

Load from 10 a.m. to 6 p.m. $= 100$ kVA *i.e.*, 2/3 of F.L.

Ordinary Efficiency

In this case, load variations are not relevant.

Output $= 150 \times 1 = 150$ kW ; Iron loss = Cu loss = 1 kW ; Total loss = 2 kW.

$\therefore$ Ordinary $\eta = 150/(150+2) = 0.9868$ or **98.68 %**

All-day Efficiency

$$\begin{aligned}
\text{Cu loss from 7-10 a.m.} &= 3 \times (1/3)^2 \times 1 = 0.333 \text{ kWh} \\
\text{Cu loss from 10 a.m. to 6.p.m.} &= 8 \times (2/3)^2 \times 1 = 3.555 \text{ kWh} \\
\text{Total Cu loss for 24 hrs} &= 0.333 + 3.555 = 3.888 \text{ kWh} \\
\text{Total iron loss for 24 hrs} &= 24 \times 1 = 24 \text{ kWh} \\
\text{Losses for a day of 24 hrs} &= 27.888 \text{ kWh} \\
\text{Output for 24 hrs} &= 3 \times (50 \times 1) + 8\,(100 \times 1) = 950 \text{ kWh}
\end{aligned}$$

$$\therefore \quad \eta_{\text{all-day}} = \frac{950 \times 100}{(950 + 27.888)} = \mathbf{97.15\ \%}$$

Example 32.84. *Find the all-day efficiency of a 50 kVA distribution transformer having full load efficiency of 94 % and full-load copper losses are equal to the constant iron losses. The loading of the transformer is as follows, the power factor being 1.0.*

(i) No load for 10 hours *(ii) Half load for 5 hours*

(iii) 25 % load for 6 hours *(iv) Full load for 3 hours.* **(Sambalpur University, 1998)**

Solution. At full load unity p.f.

$$\text{efficiency} = 94\,\% = \frac{50,000}{50,000 + 2P_i}$$

$$2P_i = \left[\frac{50,000}{0.94} - 50,000\right], \quad \text{or} \quad P_i = \frac{1}{2} \times 50,000 \left[\frac{1-0.94}{0.94}\right],$$

$$P_i = 25,000 \times \frac{0.06}{0.94} = 1596 \text{ Watts}$$

Hence, full load Cu-losses = 1596 Watts

(*a*) Energy required in overcoming Cu-losses, during 24 hours

(*i*) No load for 10 hours : zero

(*ii*) At half load, Cu-losses $= (0.5)^2 \times 1596$ Watts $= 399$

$$\text{Energy in 5 hours} = \frac{399 \times 5}{1000} \text{ kWh} = 1.995 \text{ kWh}$$

(*iii*) At 25 % load, Cu-loss $= (0.25)^2 \times 1596 = 99.75$ Watts

$$\text{Energy in 6 hours} = \frac{6 \times 99.75}{1000} = 0.5985 \text{ kWh}$$

(*iv*) Energy lost during 3 hours of full load $= \dfrac{1596 \times 3}{1000} = 4.788$ kWh

(*b*) Energy lost in constant core-losses for 2 hours $= \dfrac{1596}{1000} \times 24 = 38.304$ kWh

(c) Energy required by the load = $25 \times 5 + 12.5 \times 6 + 50 \times 3 = 125 + 75 + 150 = 350$ kWh

$$\text{All-day efficiency} = \frac{350}{350 + 38.304 + 7.3815} \times 100 = \mathbf{88.454\ \%}$$

Example 32.85. *A 10 kVA, 1-ph transformer has a core-loss of 40 W and full load ohmic loss of 100 W. The daily variation of load on the transformer is as follows :*

6 a.m. to	*1 p.m.*	*3 kW at 0.60 p.f.*
1 p.m. to	*5 p.m.*	*– 8kW at 0.8 p.f.*
5 p.m. to	*1 a.m.*	*full load at u.p.f.*
1 a.m. to	*6 a.m.*	*no load*

Determine all day efficiency of the transformer **(Amravati University, 1999)**

Solution. Fractional loading (= x) and the output kWh corresponding to load variations can be worked out in tabular form, as below :

S.N.	*Number of hours*	$x = \frac{\text{load kVA}}{\text{Xmer Rating}}$	$x^2 P_c$ *in kW*	*Output in kWh*	*Copper Loss in kWh*
1	7	$\frac{3/0.6}{10} = 0.5$	$0.50^2 \times 0.10 = 0.025$	$3 \times 7 = 21$	$0.025 \times 7 = 0.175$
2	4	$\frac{8/0.8}{10} = 1.0$	0.10	$8 \times 4 = 32$	$0.1 \times 4 = 0.40$
3	8	$\frac{10/1}{10} = 1.0$	0.10	$10 \times 8 = 80$	$0.1 \times 8 = 0.8$
4	5	Zero	Zero	Zero	Zero
			Output in kWh	$21 + 32 + 80 = 133$	
			Ohmic Loss, in kWh		$0.175 + 0.40 + 0.80 = 1.375$
		Core loss during 24 Hrs = $\frac{400}{1000} \times 24 = 0.96$ kWh			

$$\text{Hence, Energy efficiency (= All day Efficiency)} = \frac{133}{133 + 1.375 + 0.96} \times 100 = \mathbf{98.3\ \%}$$

Example 32.86. *A transformer has its maximum efficiency of 0.98 at 15 kVA at unity p.f. During a day, it is loaded as follows :*

12 hours	:	*2 kW*	*at*	*0.8 p.f.*
6 hours	:	*12 kW*	*at*	*0.8 p.f.*
6 hours	:	*18 kW*	*at*	*0.9 p.f.*

Find the all day efficiency. **(Manonmaniam Sundaranar Univ. April 1998)**

Solution. Let 15 kVA be treated as full load.

Output at maximum efficiency = 15000×1 watts

Input = 15000/0.98 watts

Losses = Input – Output = $15000\ (1/0.98 - 1) = 15000 \times 2/98 = 306$ Watts

At maximum efficiency, since the variable copper-loss and constant core-loss are equal.

Full load copper-loss = Constant core-loss = 306/2 = 153 Watts

Let the term x represents the ratio of required Load/Full load.

Output = $15 \times \cos \phi$

Following tabular entries simplify the calculations for all-day efficiency.

S.N.	x	Hrs	x^2P_c in kW	Energy in Copper -loss in kWh	Output during the period in kWh
1	$\frac{2/0.5}{15} = \frac{4}{15}$	12	0.01088	0.131	24
2	$\frac{12/0.8}{15} = 1.0$	6	0.153	0.918	72
3	$\frac{18/0.9}{15} = 4/3$	6	0.272	1.632	108

Total output during the day = 204 kWh

Total copper-loss during the day = 2.681 kWh

Total core-loss during the day = $0.153 \times 24 = 3.672$

All day efficiency = $(204/210.353) \times 100 = 96.98$ %

Tutorial Problems 32.5

1. A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.

   The transformer undergoes a daily load cycle as follows :

Load	Power factor	Load duration
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

   Determine its all-day efficiency. *(Electrical Engineering, MS Univ. Baroda 1979)*

2. What is meant by energy efficiency of a transformer ?

   A 20-kVA transformer has a maximum efficiency of 98 percent when delivering three-fourth full-load at u.p.f. If during the day, the transformer is loaded as follows :

12 hours	No load
6 hours	12 kWh, 0.8 p.f.
6 hours	20 kW, u.p.f.

   Calculate the energy efficiency of the transformer.

   (Electrical Technology-III, Gwalior Univ., 1980)

32.33. Auto-transformer

It is a transformer with one winding only, part of this being common to both primary and secondary. Obviously, in this transformer the primary and secondary are not electrically *isolated* from each other as is the case with a 2-winding transformer. But its theory and operation are similar to those of a two-winding transformer. Because of one winding, it uses less copper and hence is cheaper. It is used where transformation ratio differs little from unity. Fig. 32.60 shows both step down and step-up auto-transformers.

As shown in Fig. 32.60 (*a*), *AB*, is primary winding having N_1 turns and *BC* is secondary winding having N_2 turns. Neglecting iron losses and no-load current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

The current in section CB is vector difference* of I_2 and I_1. But as the two currents are practically in phase opposition, the resultant current is $(I_2 - I_1)$ where I_2 is greater than I_1.

As compared to an ordinary 2-winding transformer of same output, an auto-transformer has higher efficiency but smaller size. Moreover, its voltage regulation is also superior.

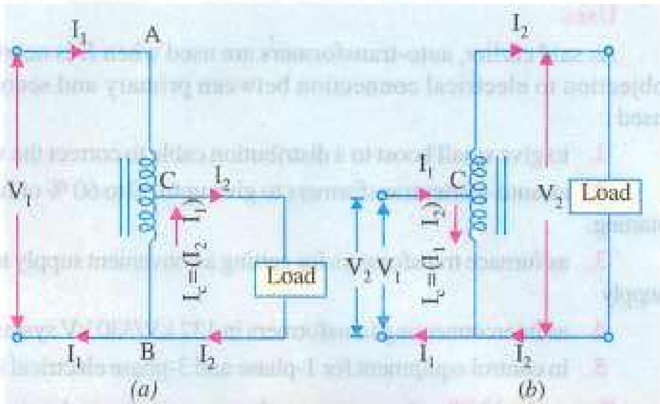


Fig. 32.60

Saving of Cu

Volume and hence weight of Cu, is proportional to the length and area of the cross-section of the conductors. Now, length of conductors is proportional to the number of turns and cross-section depends on current. Hence, weight is proportional to the product of the current and number of turns.

With reference to Fig. 32.60,

Wt. of Cu in section AC is $\propto (N_1 - N_2)\, I_1$; Wt. of Cu in section BC is $\propto N_2\,(I_2 - I_1)$.

$\therefore$ Total Wt. of Cu in auto-transformer $\propto (N_1 - N_2)\, I_1 + N_2\,(I_2 - I_1)$

If a two-winding transformer were to perform the same duty, then

Wt. of Cu on its primary $\propto N_1 I_1$; Wt. of Cu on secondary $\propto N_2 I_2$

Total Wt. of Cu $\propto N_1 I_1 + N_2 I_2$

$$\therefore \quad \frac{\text{Wt. of Cu in auto-transformer}}{\text{Wt. of Cu in ordinary transformer}} = \frac{(N_1 - N_2)\, I_1 + N_2\,(I_2 - I_1)}{N_1 I_1 + N_2 I_2}$$

$$= 1 - \frac{2\dfrac{N_2}{N_1}}{1 + \dfrac{N_2}{N_1} \times \dfrac{I_2}{I_1}} = 1 - \frac{2K}{2} = 1 - K \quad \left(\because \frac{N_2}{N_1} = K \,;\, \frac{I_2}{I_1} = \frac{1}{K}\right)$$

Wt. of Cu in auto-transformer $(W_a) = (1 - K) \times$ (Wt. of Cu in ordinary transformer W_0)

$\therefore$ Saving $= W_0 - W_a$

$= W_0 - (1 - K)\, W_0 = KW_0$

$\therefore$ Saving $= K \times$ (Wt. of Cu in ordinary transformer)

Hence, saving will increase as K approaches unity.

It can be proved that power transformed *inductively* is input $(1 - K)$.

The rest of the power $= (K \times \text{input})$ is *conducted* directly from the source to the load *i.e.*, it is transferred **conductively** to the load.


Step up auto-transformer

* In fact, current flowing in the common winding of the auto-transformer is always equal to the difference between the primary and secondary currents of an ordinary transformer.

Uses

As said earlier, auto-transformers are used when K is nearly equal to unity and where there is no objection to electrical connection between primary and secondary. Hence, such transformers are used :

1. to give small boost to a distribution cable to correct the voltage drop.
2. as auto-starter transformers to give upto 50 to 60 % of full voltage to an induction motor during starting.
3. as furnace transformers for getting a convenient supply to suit the furnace winding from a 230-V supply
4. as interconnecting transformers in 132 kV/330 kV system.
5. in control equipment for 1-phase and 3-phase electrical locomotives.

Example 32.87. *An auto-transformer supplies a load of 3 kW at 115 volts at a unity power factor. If the applied primary voltage is 230 volts, calculate the power transferred to the load (a) inductively and (b) conductively.* **(Basic Elect. Machines, Nagpur Univ, 1991)**

Solution. As seen from Art 32.33

Power transferred inductively = Input $(1 - K)$

Power transferred conductively = Input $\times K$

Now, $K = 115/230 = 1/2$, input $\cong$ output = 3 kW

$\therefore$ Inductively transferred power = $3(1 - 1/2)$ = **1.5 kW**

Conductivley transferred power = $(1/2) \times 3$ = **1.5 kW**

Example 32.88. *The primary and secondary voltages of an auto-transformer are 500 V and 400 V respectively. Show with the aid of diagram, the current distribution in the winding when the secondary current is 100 A and calculate the economy of Cu in this particular case.*

Solution. The circuit is shown in Fig, 30.61.

$K = V_2/V_1 = 400/500 = 0.8$

$\therefore \quad I_1 = KI_2 = 0.8 \times 100 = 80$ A

The current distribution is shown in Fig. 32.61.

Saving = $KW_0 = 0.8\ W_0$ - Art 32.33

$\therefore$ Percentage saving = 0.8×100 = **80**

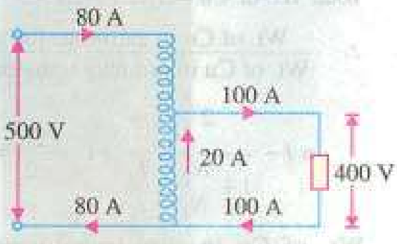


Fig. 32.61

Example 32.89. *Determine the core area, the number of turns and the position of the tapping point for a 500-kVA, 50-Hz, single-phase, 6,600/5,000-V auto-transformer, assuming the following approximate values : e.m.f. per turn 8 V. Maximum flux density 1.3 Wb/m².*

Solution.
$$E = 4.44 f \Phi_m N \text{ volt}$$
$$\Phi_m = \frac{E/N}{4.44 f} = \frac{8}{4.44 \times 50} = 0.03604 \text{ Wb}$$

Core area = $0.03604/1.3 = 0.0277\ \text{m}^2$ = **277 cm²**

Turns of h.v. side = 6600/8 = **825** ; Turns of *L.V.* side = 5000/8 = **625**

Hence, tapping should be 200 turns from high voltage end or 625 turns from the common end.

32.34. Conversion of 2-Winding Transformer into Auto-transformer

Any two-winding transformer can be converted into an auto-transformer either step-down or step-up. Fig. 32.62 (*a*) shows such a transformer with its polarity markings. Suppose it is a 20-kVA, 2400/240 V transformer. If we employ *additive* polarity between the high-voltage and low-voltage sides, we get a step-up auto-transformer. If, however, we use the *subtractive* polarity, we get a step-down auto-transformer.

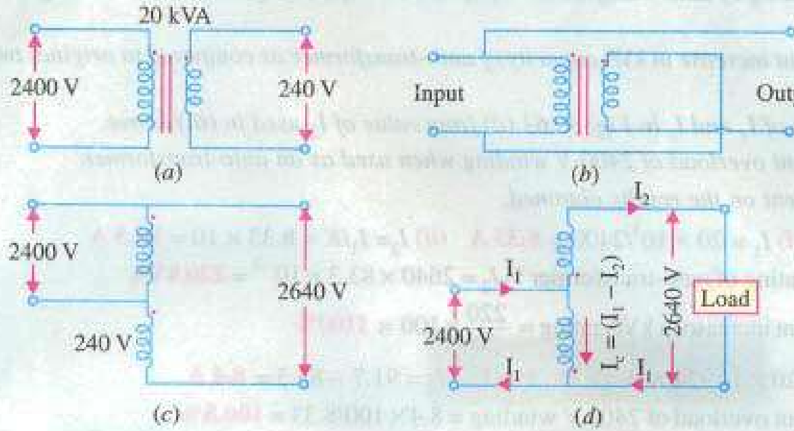


Fig. 32.62

(a) Additive Polarity

Connections for such a polarity are shown in Fig. 32.62 (*b*). The circuit is re-drawn in Fig. 32.62 (*c*) showing common terminal of the transformer at the top whereas Fig. 32.62 (*d*) shows the same circuit with common terminal at the bottom. Because of additive polarity, $V_2 = 2400 + 240 = 2640$ V and V_1 is 2400 V. There is a marked increase in the kVA of the auto-transformer (Ex. 32.90). As shown in Fig. 32.62 (*d*), common current flows *towards* the common terminal. The transformer acts as a step-up transformer.

(b) Subtractive Polarity

Such a connection is shown in Fig. 32.63 (*a*). The circuit has been re-drawn with common polarity at top in Fig. 32.63 (*b*) and at bottom in Fig. 32.63 (*c*). In this case, the transformer acts as a step-down auto-transformer.

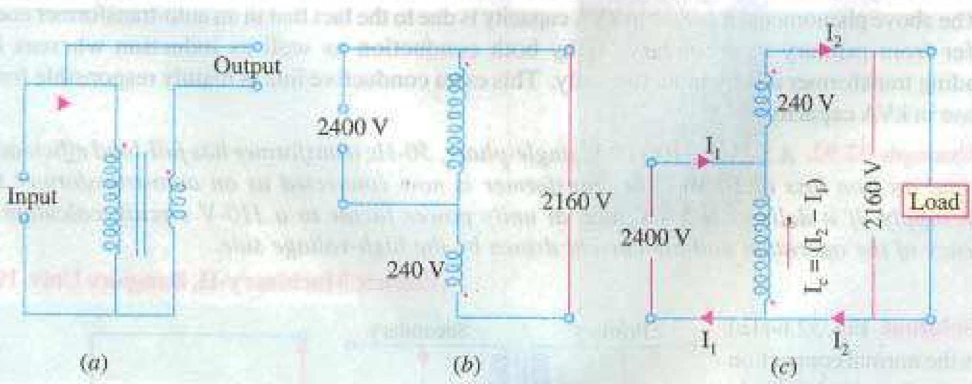


Fig. 32.63

The common current flows *away* from the common terminal. As will be shown in Example 32.91, in this case also, there is a very large increase in kVA rating of the auto-transformer though not as marked as in the previous case. Here, $V_2 = 2400 - 240 = 2160$ V.

Example 32.90. *For the 20-kVA, 2400/240-V two-winding step-down transformer shown in Fig. 32.63 (a) connected as an auto-transformer with additive polarity as shown in Fig. 30.61 (d), compute* (*i*) *original current capacity of HV-windings.*

(*ii*) *original current capacity of LV-windings.*

(*iii*) *kVA rating of auto-transformer using current capacity of current LV winding as calculated in (ii) above.*

(*iv*) *per cent increase in kVA capacity of auto-transformer as compared to original two-winding transformer.*

(*v*) *values of I_1 and I_c in Fig. 30.61 (d) from value of I_2 used in (iii) above.*

(*vi*) *per cent overload of 2400-V winding when used as an auto-transformer.*

(*vii*) *comment on the results obtained.*

Solution. (*i*) $I_1 = 20 \times 10^3/2400 =$ **8.33 A** (*ii*) $I_2 = I_1/K = 8.33 \times 10 =$ **83.3 A**

(*iii*) kVA rating of auto-transformer $V_2I_2 = 2640 \times 83.3 \times 10^{-3} =$ **220 kVA**

(*iv*) Per cent increase in kVA rating $= \dfrac{220}{20} \times 100 =$ **1100%**

(*v*) $I_1 = 220 \times 10^3/2400 =$ **91.7A**, $I_c = I_1 - I_2 = 91.7 - 83.3 =$ **8.4 A**

(*vi*) Per cent overload of 2400 V winding = 8.4 × 100/8.33 = **100.8%**

(*vii*) As an auto-transformer, the kVA has increased tremendously to 1100% of its original value with *LV* coil at its rated current capacity and *HV* coil at negligible overload *i.e.* 1.008 × rated load.

Example 32.91. *Repeat Example 30.64 for subtractive polarity as shown in Fig. 32.62 (c).*

Solution. (*i*) $I_1 =$ **8.33 A** (*ii*) $I_2 =$ **83.3 A**

(*iii*) New kVA rating of auto-transformer is $2160 \times 83.3 \times 10^{-3} =$ **180 kVA**

(*iv*) Per cent increase in kVA rating $= \dfrac{180}{20} \times 100 =$ **900%**

(*v*) $I_1 = 180 \times 10^3/2400 =$ **75 A**, $I_c = I_2 - I_1 = 83.3 - 75 =$ **8.3 A**

(*vi*) Per cent overload of 2400V winding = 8.3 × 100/8.33 = **100%**

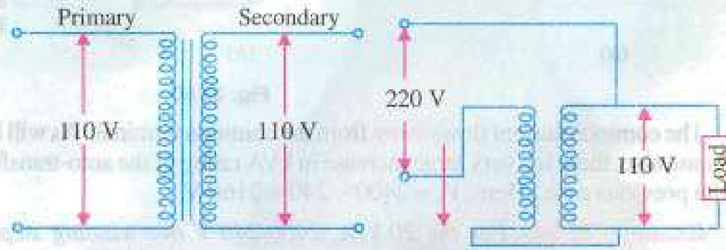
(*vii*) In this case, kVA has increased to 900% of its original value as a two-winding transformer with both low-voltage and high-voltage windings carrying their rated currents.

The above phenomenal increase in kVA capacity is due to the fact that in an auto-transformer energy transfer from primary to secondary is by both conduction as well as induction whereas in a 2-winding transformer it is by induction only. This extra conductive link is mainly responsible for the increase in kVA capacity.

Example 32.92. *A 5-kVA, 110/110-V, single-phase, 50-Hz transformer has full-load efficiency of 95% and an iron loss of 50 W. The transformer is now connected as an auto-transformer to a 220-V supply. If it delivers a 5-kW load at unity power factor to a 110-V circuit, calculate the efficiency of the operation and the current drawn by the high-voltage side.*

(Electric Machinery-II, Banglore Univ. 1991)

Solution. Fig. 32.64 (*a*) shows the normal connection for a 2-winding transformer. In Fig. 32.64 (*b*) the same unit has been connected as an auto transformer. Since the two windings are connected in series, voltage across each is 110 V.



(*a*) 2-Winding Transformer (*b*) Auto-Transformer

Fig. 32.64

The iron loss would

remain the same in both connections. Since the auto-transformer windings will each carry but half the current as compared to the conventional two-winding transformer, the copper loss will be one-fourth of the previous value.

Two-winding Transformer

$$\eta = 0.95 \qquad \therefore\ 0.95 = \frac{\text{output}}{\text{output + losses}} = \frac{5{,}000}{5{,}000 + 50 + \text{Cu loss}}$$

$\therefore$ Cu loss = 212 W

Auto-transformer

Cu loss = 212/4 = 53 W; Iron loss = 50 W $\therefore\ \eta = \dfrac{5{,}000}{5{,}000 + 53 + 50} = 0.9797$ or **97.97%**

Current of the h.v. side = 5103/220 = **23.2 A**

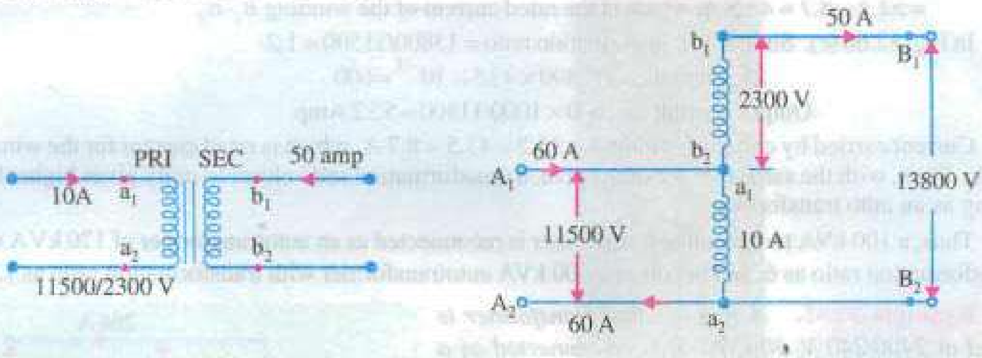
Example 32.93. *A transformer has a primary voltage rating of 11500 volts and secondary voltage rating of 2300 volts. Two windings are connected in series and the primary is connected to a supply of 11500 volts, to act as a step-up auto transformer. Determine the voltage output of the transformer.*

Question extended : If the two winding transformer is rated at 115 kVA, what will be the kVA raitng of the auto-transformer ? **(Madras University, 1997)**

Solution. As in Fig. 32.65 (*a*), 115 kVA, 11500/2300 V, transformer has the current ratings of 10 A and 50 A.

Referring to Fig. 32.65 (*b*), the step-up connections have been shown. Winding currents have to be at the same rated values. As in Fig. 32.65 (*b*), the voltage obtainable at $B_1 - B_2$ is 13800 *V*, and from b_1 a load-current of 50 A can be supplied.

kVA rating = $13800 \times 50 \times 10^{-3} = 690$



(*b*) 2 winding transformer Rating : 115kVA (*b*) Auto transformer

Fig. 32.65

Example 32.94. *An 11500/2300 V transformer is rated at 100 kVA as a 2 winding transformer. If the two windings are connected in series to form an auto-transformer, what will be the possible voltage ratios ?* **(Manonmaniam Sundaranar Univ. April 1998)**

Solution. Fig. 32.66 (*a*) shows this 2-winding transformer with rated winding currents marked .

Rated current of 11.5 kV winding = 100 × 100/11500 = 8.7 Amp

Rated current of 2300 V winding = 43.5 Amp

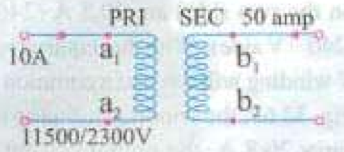


Fig. 32.66 (*a*). 2 winding transformer

Fig. 32.66 (*b*) and Fig. 32.66 (*c*) show autotransformer connections. On H.V. side, they have a rating of 13.8 kV. On L.V. side, with connection as in Fig. 32.66 (*b*), the rating is 2300 V. On L.V. side of Fig. 32.66 (*c*), the output is at 11.5 kV.

Thus, possible voltage ratios are : 13800/2300 V and 13800/11500 V.

With both the connections, step-up or step-down versions are possible.

Extension of Question : Calculate kVA ratings in the two cases.

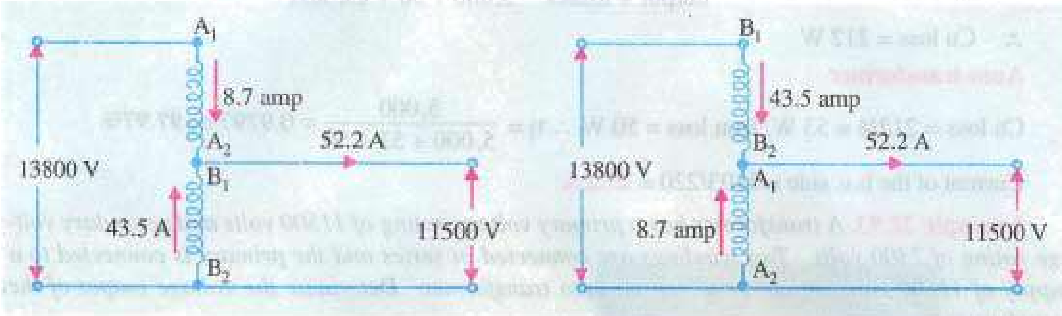


Fig. 32.66

Windings will carry the rated currents, while working out kVA outputs.

In Fig. 32.66 (*b*), Input current (into terminal A_1 of windings A_1-A_2) can be 8.7 Amp with H.V.-side-voltage ratings as 13.8 kV. Transformation ratio = 13800/2300 = 6

Hence, kVA rating $= 13.8 \times 8.7 = 120$

Output current $= 120 \times 1000/2300 = 52.2$ Amp

Current in the winding B_1-B_2

= Difference of Output current and Input current

$= 52.2 - 8.7 = 43.5$ A, which is the rated current of the winding B_1-B_2.

In Fig. 32.66 (*c*). Similarly, transformation ratio = 13800/11500 = 1.2

kVA rating $= 13800 \times 43.5 \times 10^{-3} = 600$

Output current $= 600 \times 1000/11500 = 52.2$ Amp

Current carried by common winding $= 52.2 - 43.5 = 8.7$ A, which is rated current for the winding A_1-A_2. Thus, with the same two windings give, a transformation ratio closer to unity gives higher kVA rating as an auto transformer.

Thus, a 100 kVA two winding transformer is reconnected as an autotransformer of 120 kVA with transformation ratio as 6, and becomes a 600 kVA autotransformer with transformation ratio as 1.2.

Example 32.95. *A two-winding transformer is rated at 2400/240 V, 50-kVA. It is re-connected as a step-up auto-transformer, with 2400 V input. Calculate the rating of the auto-transformer and the inductively and conductively transferred powers while delivering the rated output at unity power-factor.*

(Nagpur University, Winter 1999)

Solution. With 50 kVA as the rating, the rated currents on the two sides are 20.8 A (2400-V side) and 208 A (240 – V side). With the required re-connection, the 2400-V winding will work as a common winding. As shown in Fig. 32.67, the winding common to input and output can carry 20.8 A, the output current can be 208 A with a voltage of 2640 V, which means that the output of this auto

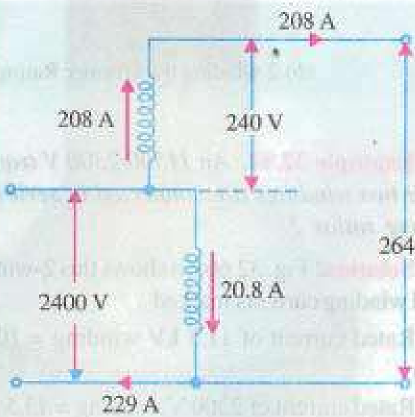


Fig. 32.67

transformer is (2640 × 208) = 550 kVA.

The corresponding input-current is

$(208 \times 2640/2400) = 229$ A.

The ratio of turns in this case is given by

$k = 2640/2400 = 1.1$

With the step-up job, and $k = 1.1$

$$\frac{\text{Rating of Auto-transformer}}{\text{Rating as two-winding transformer}} = \frac{k}{k-1} = \frac{1.1}{0.1}$$

This gives the rating as auto-transformer of 550 kVA.

At unity power-factor, the rated load = 550 kW.

Out of this, the "*inductively*" transferred power

$$\begin{aligned} &= \text{Power handled by the common winding} \\ &= (2400 \text{ V}) \times (20.8 \text{ A}) \times 10^{-3} = 50 \text{ kW.} \\ &= \text{Rated output as a two-winding transformer.} \\ \text{Remaining Power} &= 550 \text{ kW} - 50 \text{ kW} = 500 \text{ kW.} \end{aligned}$$

This power of 500 kW is "*conductively*" transferred as is clear from the division of currents at the input node. Out of the total current of 229 A from the source, 208 A goes straight to the output. The remaining current of 20.8 A is through the "*common*" and "*inductive*" path, as marked in the Fig. 32.67.

32.35. Parallel Operation of Single-phase Transformers

For supplying a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it as shown in Fig. 32.68. It is seen that primary windings are connected to the supply bus bars and secondary windings are connected to the load bus-bars. In connecting two or more than two transformers in parallel, it is essential that their terminals of similar polarities are joined to the same bus-bars as in Fig. 32.68. If this is not done, the two e.m.fs. induced in the secondaries which are paralleled with incorrect polarities, will act together in the local secondary circuit even when supplying no load and will hence produce the equivalent of a dead short-circuit as shown in Fig. 32.69.

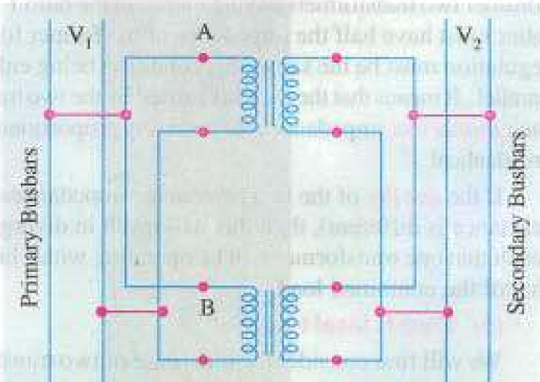


Fig. 32.68

There are certain definite conditions which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their kVA ratings. The conditions are :

1. Primary windings of the transformers should be suitable for the supply system voltage and frequency.
2. The transformers should be properly connected with regard to polarity.
3. The voltage ratings of both primaries and secondaries should be identical. In other words, the transformers should have the same turn ratio *i.e.* transformation ratio.
4. The percentage impedances should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factors.

5. With transformers having different kVA ratings, the equivalent impedances should be inversely proportional to the individual kVA rating if circulating currents are to be avoided.

Of these conditions, (1) is easily comprehended ; condition (2) is absolutely essential (otherwise paralleling with incorrect polarities will result in dead short-circuit). There is some lattitude possible with conditions (3) and (4). If condition (3) is not exactly satisfied *i.e.* the two transformers have slightly different transformation or voltage ratios, even then parallel operation is possible. But due to inequality of induced e.m.fs. in secondaries, there will be even on no-load, some circulating current between them (and therefore between the primary windings also) when secondary terminals are connected in parallel. When secondaries are loaded, this localized circulating current will tend to produce unequal loading condition. Hence, it may be impossible to take full kVA output from the parallel connected group without one of the transformers becoming over-heated.

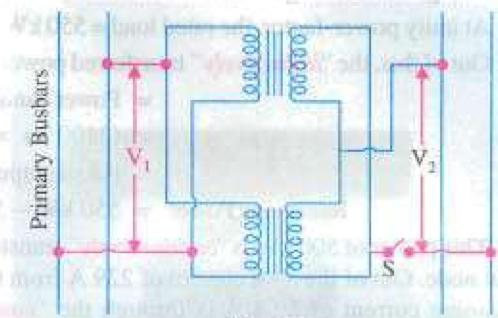


Fig. 32.69

If condition (4) is not exactly satisfied *i.e.* impedance triangles are not identical in shape and size, parallel operation will still be possible, but the power factors at which the two transformers operate will be different from the power factor of the common load. Therefore, in this case, the two transformers will not share the load in proportion to their kVA ratings.

It should be noted that the impedances of two transformers may differ in magnitude and in quality (*i.e.* ratio of equivalent resistance to reactance). It is worthwhile to distinguish between the *percentage* and *numerical* value of an impedance. For example, consider two transformers having ratings in the ratio 1 : 2. It is obvious that to carry double the current, the latter must have half the impedance of the former for the same regulation. For parallel operation, the regulation must be the same, this condition being enforced by the very fact of their being connected in parallel. It means that the currents carried by the two transformers are proportional to their ratings provided their *numerical* impedances are inversely proportional to these ratings and their *percentage* impedances are identical.

If the *quality* of the two percentage impedances is different (*i.e.* ratio of percentage resistance to reactance is different), then this will result in divergence of phase angle of the two currents, with the result that one transformer will be operating with a higher and the other with a lower power factor than that of the combined load.

(a) Case 1. Ideal Case

We will first consider the ideal case of two transformers having the same voltage ratio and having impedance voltage triangles identical in size and shape.

Let E be the no-load secondary voltage of each transformer and V_2 the terminal voltage ; I_A and I_B the currents supplied by them and I-the total current, lagging behind V_2 by an angle ϕ (Fig. 32.70(*a*)

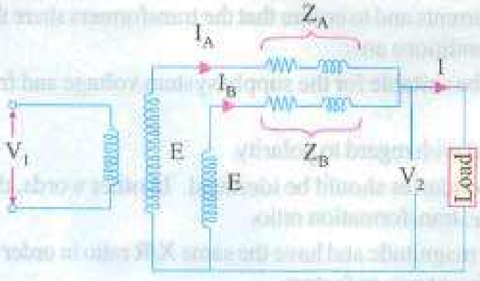


Fig. 32.70 (a)

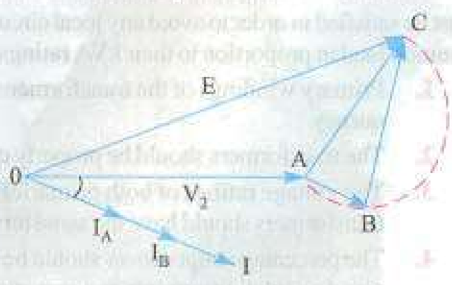


Fig. 32.70 (b)

In Fig. 32.70 (*b*) a single triangle *ABC* represents the identical impedance voltage triangles of both the transformers. The currents I_A and I_B of the individual transformers are in phase with the load current *I* and are inversely proportional to the respective impedances. Following relations are obvious.

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B;\ \mathbf{V}_2 = \mathbf{E} - \mathbf{I}_A\mathbf{Z}_A = \mathbf{E} - \mathbf{I}_B\mathbf{Z}_B = \mathbf{E} - \mathbf{I}\mathbf{Z}_{AB}$$

Also $\quad \mathbf{I}_A\mathbf{Z}_A = \mathbf{I}_B\mathbf{Z}_B \quad \text{or} \quad \mathbf{I}_A/\mathbf{I}_B = \mathbf{Z}_B/\mathbf{Z}_A$

$\therefore \quad \mathbf{I}_A = \mathbf{I}\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B) \quad \text{and} \quad \mathbf{I}_B = \mathbf{I}\mathbf{Z}_A/(\mathbf{Z}_A + \mathbf{Z}_B)$

(*b*) Case 2. Equal Voltage Ratios

Let us assume that no-load voltages of both secondaries is the same *i.e.* $E_A = E_B = E$, and that the two voltages are coincident *i.e.* there is no phase difference between E_A *and* E_B, which would be true if the magnetising currents of the two transformers are not much different from each other. Under these conditions, both primaries and secondaries of the two transformers can be connected in parallel and there will circulate no current between them on ***on-load***.

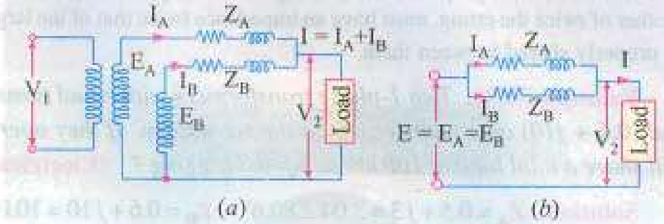


Fig. 32.71

However, if we neglect the magnetising admittances, the two transformers can be connected as shown by their equivalent circuits in Fig. 32.71. The vector diagram is shown in Fig. 32.72.

From Fig. 32.71 (*a*) or (*b*) it is seen that it represents two impedances in parallel. Considering all values ***consistently with reference to secondaries***, let

$\mathbf{Z}_A, \mathbf{Z}_B$ = impedances of the transformers

$\mathbf{I}_A, \mathbf{I}_B$ = their respective currents

$\mathbf{V}_2$ = common terminal voltage

$\mathbf{I}$ = combined current

It is seen that $\mathbf{I}_A\mathbf{Z}_A = \mathbf{I}_B\mathbf{Z}_B = \mathbf{I}\mathbf{Z}_{AB}$ ...(*i*)

where $\mathbf{Z}_{AB}$ is the combined impedance of $\mathbf{Z}_A$ and $\mathbf{Z}_B$ in parallel.

$$1/\mathbf{Z}_{AB} = 1/\mathbf{Z}_A + 1/\mathbf{Z}_B$$

Hence $\quad \mathbf{Z}_{AB} = \mathbf{Z}_A\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B)$ ...(*ii*)

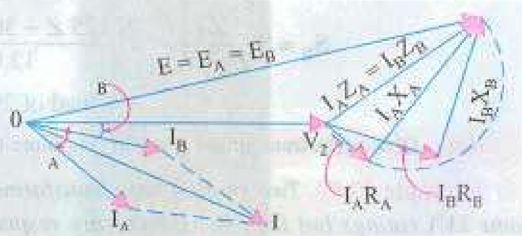


Fig. 32.72

From equation (*i*), we get

$$\mathbf{Z}_A = \mathbf{I}\mathbf{Z}_{AB}\mathbf{Z}_A = \mathbf{I}\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B) \text{ and } \mathbf{I}_B = \mathbf{I}\mathbf{Z}_{AB}/\mathbf{Z}_B = \mathbf{I}\mathbf{Z}_A/(\mathbf{Z}_A + \mathbf{Z}_B)$$

Multiplying both sides by common terminal voltage $\mathbf{V}_2$, we have

$$\mathbf{V}_2\mathbf{I}_A = \mathbf{V}_2\mathbf{I}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B};\ \text{Similarly}\ \mathbf{V}_2\mathbf{I}_B = \mathbf{V}_2\mathbf{I}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}$$

Let $\mathbf{V}_2\mathbf{I} \times 10^{-3} = \mathbf{S}$ the combined load kVA. Then, the kVA carried by each transformer is

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{S}\frac{1}{1 + \mathbf{Z}_A/\mathbf{Z}_B} \quad \text{and} \quad \mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{S}\frac{1}{1 + \mathbf{Z}_B/\mathbf{Z}_A} \quad ...(iii)$$

Hence, $\mathbf{S}_A$ and $\mathbf{S}_B$ are obtained in magnitude as well as in phase from the above vectorial expressions.

The above problem may be solved graphically, although somewhat more laboriously. As shown in Fig. 32.72, drawn I_A and I_B with an angular difference of $(\phi_A - \phi_B)$ and magnitude (according to some suitable scale) inversely proportional to the respective impedances. Vector sum of I_A and I_B gives total combined current *I*. The phase angle and magnitude of *I* will be known from the conditions of loading, so that angle ϕ

between V_2 and I will be known. Inserting this, the transformer currents I_A and I_B become known in magnitude and phase with respect to V_2.

Note. (*a*) In equation (*iii*) above, it is not necessary to use the ohmic values of resistances and reactances, because only impedance ratios are required.

(*b*) The two percentage impedances must be adjusted to the same kVA in the case of transformers of different rating as in Ex. 32.101.

(*c*) From equation (*iii*) above, it is seen that if two transformers having the same rating and the same transformation ratio are to share the load equally, then their impedances should be equal *i.e.* equal resistances and reactances and not numerical equality of impedances. In general, for transformers of different ratings but same transformation ratio, their equivalent impedances must be inversely proportional to their ratings if each transformer is to assume a load in proportion to its rating. For example, as said earlier, a transformer operating in parallel with another of twice the rating, must have an impedance twice that of the large transformer in order that the load may be properly shared between them.

Example 32.96. *Two 1-phase transformers with equal turns have impedances of (0.5 + j3) ohm and (0.6 + j10) ohm with respect to the secondary. If they operate in parallel, determine how they will share a total load of 100 kW at p.f. 0.8 lagging ?* **(Electrical Technology, Madras Univ. 1987)**

Solution. $\mathbf{Z}_A = 0.5 + j3 = 3.04\angle 80.6°$ $\mathbf{Z}_B = 0.6 + j10 = 10.02\angle 86.6°$

$$\mathbf{Z}_A + \mathbf{Z}_B = 1.1 + j13 = 13.05\angle 85.2°$$

Now, a load of 100 kW at 0.8 p.f. means a kVA of 100/0.8 = 125. Hence,

$$\mathbf{S} = 125\angle -36.9°$$

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{125\angle -36.9° \times 10.02\angle 86.6°}{13.05\angle 85.2°} = 96\angle -35.5°$$

$$= \text{a load of } 96 \times \cos 35.5° = 78.2 \text{ kW}$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{125\angle -36.9° \times 3.04\angle 80.6°}{13.05\angle 85.2°} = 29.1\angle -41.5°$$

$$= \text{a load of } 29.1 \times \cos 41.5° = \mathbf{21.8\ kW}$$

Note. Obviously, transformer *A* is carrying more than its due share of the common load.

Example 32.97. *Two single-phase transformers A and B are connected in parallel. They have same kVA ratings but their resistances are respectively 0.005 and 0.01 per unit and their leakage reactances 0.05 and 0.04 per unit. If A is operated on full-load at a p.f. of 0.8 lagging, what will be the load and p.f. of B ?* **(A.C. Machines-I, Jadavpur Univ. 1985)**

Solution. In general, $\mathbf{S}_A = \mathbf{S}\dfrac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}$ and $\mathbf{S}_B = \mathbf{S}\dfrac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}$

where S is the total kVA supplied and $\mathbf{Z}_A$ and $\mathbf{Z}_B$ are the percentage impedances of these transformers.

$$\frac{\mathbf{S}_B}{\mathbf{S}_A} = \frac{\mathbf{Z}_A}{\mathbf{Z}_B}$$

Now, $\mathbf{Z}_A = 0.005 + j\,0.05$ per unit ; % $\mathbf{Z}_A = 0.5 + j5$ and %$\mathbf{Z}_B = 1 + j4$

Let $\mathbf{S}_A = S_A\angle -36.87°$

where S_A represents the rating of transformer *A* (and also of *B*).

$$\mathbf{S}_B = \mathbf{S}_A\angle -36.87° \times \frac{(0.5 + j5)}{(1 + j4)}$$

$$= S_A\angle -36.87° \times \frac{20.7}{17}\angle 8.3° = 1.22\ S_A\angle -28.57°$$

It is obvious that transformer B is working **22%** over-load and its power factor is

$$\cos 28.57° = \mathbf{0.878\ (lag)}$$

Example 32.98. *Two 1-phase transformers A and B rated at 250 kVA each are operated in parallel on both sides. Percentage impedances for A and B are (1 + j 6) and (1.2 + j 4.8) respectively. Compute the load shared by each when the total load is 500 kVA at 0.8 p.f. lagging.*

(Electrical Machines-II, Indore Univ. 1989)

Solution.

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_A+\mathbf{Z}_B} = \frac{1+j\,6}{2.2+j\,10.8} = 0.55\angle 2.1° \; ; \; \frac{\mathbf{Z}_B}{\mathbf{Z}_A+\mathbf{Z}_B} = \frac{1.2+j\,4.8}{2.2+j\,10.8} = 0.45\angle -2.5°$$

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A+\mathbf{Z}_B} = 500\angle -36.9° \times 0.45\angle -2.5° = \mathbf{225\angle -39.4°}$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A+\mathbf{Z}_B} = 500\angle -36.9° \times 0.55\angle 2.1° = \mathbf{275\angle -34.8°}$$

Obviously, transformer B is overloaded to the extent of $(275 - 250) \times 100/250 = 10\%$. It carries $(275/500) \times 100 = 55\%$ of the total load.

Example 32.99. *Two 100-kW, single-phase transformers are connected in parallel both on the primary and secondary. One transformer has an ohmic drop of 0.5% at full-load and an inductive drop of 8% at full-load current. The other has an ohmic drop of 0.75% and inductive drop of 2%. Show how will they share a load of 180 kW at 0.9 power factor.*

(Elect. Machines-I, Calcutta Univ. 1988)

Solution. A load of 180 kW at 0.9 p.f. means a kVA of $180/0.9 = 200$

$\therefore$ Load $\quad \mathbf{S} = 200 - 25.8°$

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(0.5+j\,8)}{(1.25+j\,12)} = \frac{(0.5+j\,8)(1.25-j\,12)}{1.25^2+12^2}$$

$$= \frac{96.63+j\,4}{145.6} = \frac{96.65\angle 2.4°}{145.6} = 0.664\angle 2.4°$$

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(0.75+j\,4)(1.25-j\,12)}{145.6}$$

$$= \frac{48.94-j\,4}{145.6} = \frac{49.1\angle -5°}{145.6}$$

$$= 0.337\angle -5°$$

$$\mathbf{S}_1 = \mathbf{S}\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = 200\angle -25.8° \times 0.337\angle -5° = 67.4\angle -30.8°$$

$$\therefore \quad kW_1 = 67.4 \times \cos 30.8° = 67.4 \times 0.859 = 57.9\ kW$$

$$\mathbf{S}_2 = 200\angle -25.8° \times 0.664\angle 2.4° = 132.8\angle -23.4$$

$$kW_2 = 132.8 \times \cos 23.4° = 132.8 \times 0.915 = 121.5\ kW$$

Note. Second transformer is working 21.5% over-load. Also, it shares 65.7% of the total load.

Example 32.100. *A load of 200 kW at 0.85 power factor lagging is to be shared by two transformers A and B having the same ratings and the same transformation ratio. For transformer A, the full-load resistive drop is 1% and reactance drop 5% of the normal terminal voltage. For transformer B the corresponding values are : 2% and 6%. Calculate the load kVA supplied by each transformer.*

Solution. $\quad \mathbf{Z}_A = 1 + j5;\ Z_B = Z + j\,6$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_B}} = \frac{1+j5}{2+j6} = 0.8 + j\,0.1\,; \quad \frac{\mathbf{Z_B}}{\mathbf{Z_A}} = \frac{2+j6}{1+j5} = 1.23 - j\,0.514$$

Load kVA = kW/p.f. = 200/0.85 = 235

$$\textbf{kVA} = \mathbf{S} = 235(0.85 - j\,0.527)$$

$$= 200 - j\,123.8 \qquad (\cos\phi = 0.85;\ \sin\phi = 0.527)$$

$$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A}+\mathbf{Z_B}} = \mathbf{S}\frac{1}{1+(\mathbf{Z_A}/\mathbf{Z_B})}$$

$$= \frac{200 - j\,123.8}{1+(0.8+j0.1)} = \frac{200 - j\,123.8}{1.8 + j0.1} = 107.3 - j\,74.9$$

$$S_A = \sqrt{(107.3^2 + 74.9^2)} = 131;\ \cos\phi_A = 107.3/131 = 0.82 \text{ (lag)}$$

Similarly
$$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A}+\mathbf{Z_B}} = \mathbf{S}\frac{1}{1+(\mathbf{Z_B}/\mathbf{Z_A})}$$

$$= \frac{200 - j\,123.8}{1+(1.23 - j0.514)} = \frac{200 - j\,123.8}{2.23 - j\,0.154} = 93 - j\,49$$

$\therefore$
$$S_B = \sqrt{(93^2 + 49^2)} = 105;\ \cos\phi_B = 90/105 = 0.888 \text{ (lag)}$$

(As a check, $S = S_A + S_B = 131 + 105 = 236$. The small error is due to approximations made in calculations.)

Example 32.101. *Two 2,200/110-V, transformers are operated in parallel to share a load of 125 kVA at 0.8 power factor lagging. Transformers are rated as below :*

A : 100 kVA ; 0.9% resistance and 10% reactance

B : 50 kVA ; 1.0% resistance and 5% reactance

Find the load carried by each transformer. **(Elect. Technology, Utkal Univ. 1989)**

Solution. It should be noted that the percentages given above refer to different ratings. As pointed out in Art. 32.35, these should be adjusted to the same basic kVA, say, 100 kVA.

$$\%\mathbf{Z_A} = 0.9 + j\,10\,; \qquad \%\mathbf{Z_B} = (100/50)(1 + j5) = (2 + j\,10)$$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_A}+\mathbf{Z_B}} = \frac{0.9 + j\,10}{(2.9 + j20)} = \frac{(0.9 + j\,10)\,(2.9 - j\,20)}{2.9^2 + 20^2}$$

$$= \frac{(202.6 + j\,11)}{408.4} = \frac{202.9\angle 3.1°}{408.4} = 0.4968\angle 3.1°$$

$$\frac{\mathbf{Z_B}}{\mathbf{Z_A}+\mathbf{Z_B}} = \frac{(2 + j\,10)(2.9 - j\,20)}{408.4} = \frac{206 - j\,11}{408.4}$$

$$= \frac{206.1\angle -3.1°}{408.4} = 0.504\angle -3.1°$$

Also $\cos\phi = 0.8,\ \phi = \cos^{-1}(0.8) = 36.9°$

$$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A}+\mathbf{Z_B}} = 125\angle -36.9° \times 0.504\angle -3.1° = 63\angle -40°$$

$$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A}+\mathbf{Z_B}} = 125\angle -36.9° \times 0.4968\angle 3.1° = 62.1\angle -33.8°$$

Example 32.102. *A 500-kVA transformer with 1% resistance and 5% reactance is connected in parallel with a 250-kVA transformer with 1.5% resistance and 4% reactance. The secondary voltage of each transformer is 400 V on no-load. Find how they share a load of 750-kVA at a p.f. of 0.8 lagging.* **(Electrical Machinery-I, Madras Univ. 1987)**

Solution. It may be noted that percentage drops given above refer to different ratings. These should be adjusted to the same basic kVA *i.e.* 500 kVA.

$$\% \mathbf{Z}_A = 1 + j5 = 5.1 \angle 78.7° \; ; \% \mathbf{Z}_B = \left(\frac{500}{250}\right)(1.5 + j4) = 3 + j8 = 8.55 \angle 69.4°$$

$$\%(\mathbf{Z}_A + \mathbf{Z}_B) = 4 + j13 = 13.6 \angle 72.9° \; ; \mathbf{S} = 750 \angle -36.9°$$

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{750 \angle -36.9° \times 8.55 \angle 69.4°}{13.6 \angle 72.9°} = 470 \angle -40.4°$$

$$= 470 \text{ kVA at p.f. of } 0.762 \text{ lagging}$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{750 \angle -36.9° \times 5.1 \angle 78.7°}{13.6 \angle 72.9°} = 280 \angle -31.1°$$

$$= 280 \text{ kVA at p.f. } 0.856 \text{ lagging}$$

Note. The above solution has been attempted vectorially, but in practice, the angle between I_A and I_B is so small that if instead of using vectorial expressions, arithmetic expressions were used, the answer would not be much different. In most cases, calculations by both vectorial and arithmetical methods generally yield results that do not differ sufficiently to warrant the more involved procedure by the vector solution. The above example will now be attempted arithmetically :

$$Z_A = 5.1\,\Omega, Z_B = 8.55\,\Omega \; ; I_A/I_B = Z_B/Z_A = 8.55/5.1 = 1.677 \therefore I_A = 1.677\, I_B$$

$$\text{Total current} = 750{,}000/400 = 1875 \text{ A}$$

$$I = I_A + I_B \; ; 1875 = 1.677 I_B + I_B = 2.677\, I_B \therefore I_B = 1875/2.677$$

$$\therefore \quad S_B = 400 \times 1875/2.677 \times 100 = 280 \text{ kVA}$$

$$I_A = 1.677 \times 1875/2.677$$

$$S_A = 400 \times 1.677 \times 1875/2.677 \times 1000 = 470 \text{ kVA}$$

Example 32.103. *Two single-phase transformers A and B of equal voltage ratio are running in parallel and supply a load of 1000 A at 0.8 p.f. lag. The equivalent impedances of the two transformers are (2 + j3) and (2.5 + j5) ohms respectively. Calculate the current supplied by each transformer and the ratio of the kW output of the two transformers.*

(Electrical Machines-I, Bombay Univ. 1986)

Solution. $\mathbf{Z}_A = (2 + j3), \mathbf{Z}_B = (2.5 + j5)$

Now, $$\frac{\mathbf{I}_A}{\mathbf{I}_B} = \frac{\mathbf{Z}_B}{\mathbf{Z}_A} = \frac{2.5 + j5}{2 + j3} = (1.54 + j0.2) \; ; \mathbf{I}_A = \mathbf{I}_B\,(1.54 + j0.2)$$

Taking secondary terminal voltage as reference vector, we get

$$\mathbf{I} = 1000(0.8 - j0.6) = 800 - j600 = 200\,(4 - j3)$$

Also, $$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = \mathbf{I}_B\,(1.54 + j0.2) + \mathbf{I}_B = \mathbf{I}_B\,(2.54 + j0.2)$$

$$\therefore \quad 200(4 - j3) = \mathbf{I}_B\,(2.54 + j0.2) \; ; \mathbf{I}_B = 294.6 - j259.5 = \mathbf{392.6 \angle -41.37°}$$

$$\mathbf{I}_A = \mathbf{I}_B\,(1.54 + j0.2) = (294.6 - j259.5)(1.54 + j0.2) = 505.6 - j340.7$$

$$= \mathbf{609.7 \angle -33.95°}$$

The ratio of the kW output is given by the ratio of the in-phase components of the two currents.

$$\frac{\text{output of } A}{\text{output of } B} = \frac{505.6}{294.6} = \frac{1.7}{1}$$

Note. Arithmetic solution mentioned above could also be attempted.

Example 32.104. *Two transformers A and B, both of no-load ratio 1,000/500-V are connected in parallel and supplied at 1,000 V. A is rated at 100 kVA, its total resistance and reactance being 1% and 5% respectively, B is rated at 250 kVA, with 2% resistance and 2% reactance. Determine the load on each transformer and the secondary voltage when a total load of 300 kVA at 0.8 power factor lagging is supplied.*

Solution. Let the percentage impedances be adjusted to the common basic kVA of 100.

Then $$\%\mathbf{Z}_A = (1 + j\,5)\;;\;\%\mathbf{Z}_B = (100/250)(2 + j\,2) = (0.8 + j\,0.8)$$

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{1 + j5}{(1.8 + j5.8)} = 0.839\angle 5.9$$

$$\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{0.8 + j0.8}{(1.8 + j5.8)} = 0.1865\angle -27.6°$$

Now, $$\mathbf{S} = 300\angle -36.9° = 240 - j\,180$$

$$\mathbf{S}_A = \mathbf{S}.\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = 300\angle -36.9° \times 0.1865\angle -27.6° = 55.95\angle -64.5°$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = 300\angle -36.9° \times 0.839\angle 5.9° = 251.7\angle -31°$$

Since $\mathbf{Z}_A$ and $\mathbf{Z}_B$ are in parallel, their combined impedance on 100 kVA basis is

$$\mathbf{Z}_{AB} = \frac{\mathbf{Z}_A\,\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{(1 + j5)(0.8 + j0.8)}{(1.8 + j5.8)} = 0.6 + j0.738$$

Percentage drop over $\mathbf{Z}_{AB}$ is $= (0.6 \times 240/100) + (0.738 \times 180/100) = 1.44 + 1.328 = 2.768\%$

$$\therefore \quad V_2 = \left(500 - \frac{500 \times 2.768}{100}\right) = \mathbf{486.12\ V}$$

Example 32.105. *Two 1-φ transformers are connected in parallel at no-load. One has a turn ratio of 5,000/440 and a rating of 200 kVA, the other has a ratio of 5,000/480 and a rating of 350 kVA. The leakage reactance of each is 3.5%.*

What is the no-load circulation current expressed as a percentage of the nominal current of the 200 kVA transformer.

Solution. The normal currents are

$$200 \times 10^3/440 = 455\ \text{A and } 350 \times 10^3/480 = 730\ \text{A}$$

Reactances seen from the secondary side are

$$\frac{3.5}{100} \times \frac{440}{455} = 0.034\,\Omega,\quad \frac{3.5}{100} \times \frac{480}{730} = 0.023\,\Omega$$

The difference of induced voltage is 40 V. The circulating current is

$I_C = 40/0.057 = 704\ \text{A} =$ **1.55 times** the normal current of 200 kVA unit.

Tutorial Problems 32.6

1. Two single-phase transformers *A* and *B* of equal voltage ratio are running in parallel and supplying a load requiring 500 A at 0.8 power factor lagging at a terminal voltage of 400 V. The equivalent impedances of the transformers, as referred to secondary windings, are $(2 + j\,3)$ and $(2.5 + j\,5)$ ohm. Calculate the current supplied by each transformer.

   (**Note.** The student is advised to try by arithmetic method also). **[I_A = 304 A ; I_B = 197 A]**

2. Two single-phase transformers *A* and *B* are operating in parallel and supplying a common load of 1000 kVA at 0.8 p.f. lagging. The data regarding the transformers is as follows :

Transformer	Rating	%Resistance	% Reactance
A	750 kVA	3	5
B	500 kVA	2	4

Determine the loading of each transformer. **[$S_A = 535 \angle -34.7°$; $S_B = 465 \angle -39.3°$]**

3. Two transformers *A* and *B* give the following test results. With the low-tension side short-circuited, *A* takes a current of 10 A at 200 V, the power input being 1000 W. Similarly, *B* takes 30 A at 200 V ; the power input being 1,500 W. On open circuit, both transformers give a secondary voltage of 2200 when 11,000 volts are applied to the primary terminals. These transformers are connected in parallel on both high tension and low tension sides Calculate the current and power in each transformer when supplying a load of 200 A at 0.8 power factor lagging. The no-load currents may be neglected.

(**Hint** : Calculate $\mathbf{Z}_A$ and $\mathbf{Z}_B$ from S.C. test data)

[I_A = 50.5 A, P_A = 100 kW ; I_B = 151 A, P_B = 252 kW] *(London University)*

4. Two 6600/250-V transformers have the following short-circuit characteristics : Applied voltage 200 V, current 30 A, power input 1,200 W for one of the transformers ; the corresponding data for the other transformer being 120 V, 20 A and 1,500 W. All values are measured on the H.V. side with the L.V. terminals short circuited. Find the approximate current and the power factor of each transformer when working in parallel with each other on the high and low voltage sides and taking a total load of 150 kW at a p.f. of 0.8 lagging from the high voltage bus-bars.

[I_A = 13.8 A, cos ϕA = 0.63 ; I_B = 15.35 A , cos ϕB = 0.91] *(Electrical Engg-IV, Baroda Univ. 1978)*

5. Two 11,000/2,200-V, 1-phase transformers are connected in parallel to supply a total load of 200 at 0.8 p.f. lagging at 2,200 V. One transformer has an equivalent resistance of 0.4 Ω and equivalent reactance of 0.8 Ω referred to the low-voltage side. The other has equivalent resistance of 0.1 Ω and a reactance of 0.3 Ω. Determine the current and power supplied by each transformer.

[52 A ; 148 A ; 99 A ; 252 kW]

6. A 2,000-kVA transformer (*A*) is connected in parallel with a 4,000-kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage volt drops in the windings at the rated loads are as follows :

Transformer *A* : resistance 2% ; reactance 8%

Transformer *B* : resistance 1.6% ; reactance 3%

[A : 860 kVA, 0.661 lag ; B : 4170 kVA, 0.824 lag] *(A.C. Machines-I, Jadavpur Univ. 1979)*

7. Two single-phase transformers work in parallel on a load of 750 A at 0.8 p.f. lagging. Determine secondary voltage and the output and power factor of each transformer. Test data are :

Open circuit : 11,00/13,300 V for each transformer

Short circuit : with h.v. winding short-circuit

Transformer *A* : secondary input 200 V, 400 A, 15 kW

Transformer *B* : secondary input 100 V, 400 A, 20 kW

[3,190 VA : 80 kVA, 0.65 lag ; B : 1,615 kVA ; 0.86 lag]

(*c*) Case 3. Unequal Voltage Ratios

In this case, the voltage ratios (or transformation ratios) of the two transformers are different. It means that their no-load secondary voltages are unequal. Such cases can be more easily handled by phasor algebra than graphically.

Let $\mathbf{E}_A$, $\mathbf{E}_B$ = no-load secondary e.m.f.s of the two transformers.

$\mathbf{Z}_L$ = load impedance across the secondary.

The equivalent circuit and vector diagram are also shown in Fig. 32.73 and 32.74.

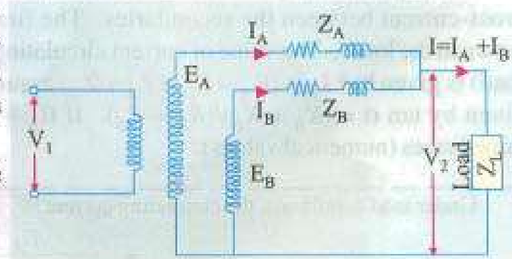


Fig. 32.73

It is seen that even when secondaries are on no-load, there will be some cross-current in them because of inequality in their induced e.m.fs. This circulating current I_C is given by

$$\mathbf{I_C} = (\mathbf{E_A} - \mathbf{E_B})/(\mathbf{Z_A} + \mathbf{Z_B}) \qquad ...(i)$$

As the induced e.m.fs. of the two transformers are equal to the total drops in their respective circuits.

$\therefore$ $\quad \mathbf{Z_A} = \mathbf{I_A}\,\mathbf{Z_A} + \mathbf{V_2}\ ;\ \mathbf{E_B} = \mathbf{I_B Z_B} + \mathbf{V_2}$

Now, $\quad \mathbf{V_2} = \mathbf{IZ_L} = (\mathbf{I_A} + \mathbf{I_B})\,\mathbf{Z_L}$

where $\quad \mathbf{Z_L}$ = load impedance

$$\mathbf{E_A} = \mathbf{I_A}\,\mathbf{Z_A} + (\mathbf{I_A} + \mathbf{I_B})\,\mathbf{Z_L} \qquad ...(ii)$$

$$\mathbf{E_B} = \mathbf{I_B Z_B} + (\mathbf{I_A} + \mathbf{I_B})\,\mathbf{Z_L} \qquad ...(iii)$$

$$\therefore \quad \mathbf{E_A} - \mathbf{E_B} = \mathbf{I_A}\,\mathbf{Z_A} - \mathbf{I_B}\,\mathbf{Z_B} \qquad ...(iv)$$

$$\mathbf{I_A} = [(\mathbf{E_A} - \mathbf{E_B}) + \mathbf{I_B}\,\mathbf{Z_B}]/\mathbf{Z_A}$$

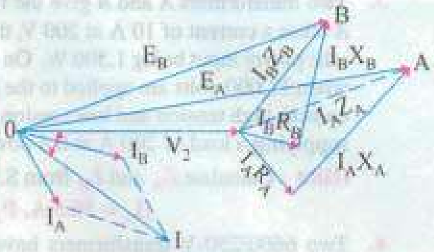


Fig. 32.74

Substituting this value of $\mathbf{I_A}$ in equation (*iii*), we get

$$\mathbf{E_B} = \mathbf{I_B Z_B} + [\{(\mathbf{E_A} - \mathbf{E_B}) + \mathbf{I_B Z_B}\}/\mathbf{Z_A} + \mathbf{I_B}]/\mathbf{Z_L}$$

$$\mathbf{I_B} = [\mathbf{E_B Z_A} - (\mathbf{E_A} - \mathbf{E_B})\mathbf{Z_L}\ ]/[\mathbf{Z_A Z_B} + \mathbf{Z_L}.(\mathbf{Z_A} + \mathbf{Z_B})] \qquad ...(v)$$

From the symmetry of the expression, we get

$$\mathbf{I_A} = [\mathbf{E_A Z_B} + (\mathbf{E_A} - \mathbf{E_B})\mathbf{Z_L}[\mathbf{Z_A Z_B} + \mathbf{Z_L}(\mathbf{Z_A} + \mathbf{Z_B})] \qquad ...(vi)$$

Also, $\quad \mathbf{I} = \mathbf{I_A} + \mathbf{I_B} = \dfrac{\mathbf{E_A Z_B} + \mathbf{E_B Z_A}}{\mathbf{Z_A Z_B} + \mathbf{Z_L}\,(\mathbf{Z_A} + \mathbf{Z_B})}$

By multiplying the numerator and denominator of this equation by $1/\mathbf{Z_A Z_B}$ and the result by $\mathbf{Z_L}$ we get

$$\mathbf{V_2} = \mathbf{IZ_L} = \frac{\mathbf{E_A}/\mathbf{Z_A} + \mathbf{E_B Z_B}}{1/\mathbf{Z_A} + 1/\mathbf{Z_B} + 1/\mathbf{Z_L}}$$

The two equations (*v*) and (*vi*) then give the values of secondary currents. The primary currents may be obtained by the division of transformation ratio *i.e.* K and by addition (if not negligible) of the no-load current. Usually, $\mathbf{E_A}$ and $\mathbf{E_B}$ have the same phase (as assumed above) but there may be some phase difference between the two due to some difference of internal connection in parallel of a star/star and a star/delta 3-phase transformers.

If $\mathbf{Z_A}$ and $\mathbf{Z_B}$ are small as compared to $\mathbf{Z_L}$ *i.e.* when the transformers are not operated near short-circuit conditions, then equations for $\mathbf{I_A}$ and $\mathbf{I_B}$ can be put in a simpler and more easily under-standable form. Neglecting $\mathbf{Z_A Z_B}$ in comparison with the expression $\mathbf{Z_L}(\mathbf{Z_A} + \mathbf{Z_B})$, we have

$$\mathbf{I_A} = \frac{\mathbf{E_A Z_B}}{\mathbf{Z_L}(\mathbf{Z_A} + \mathbf{Z_B})} + \frac{\mathbf{E_A} - \mathbf{E_B}}{\mathbf{Z_A} + \mathbf{Z_B}} \qquad ...(vii)$$

$$\mathbf{I_B} = \frac{\mathbf{E_B Z_A}}{\mathbf{Z_L}(\mathbf{Z_A} + \mathbf{Z_B})} - \frac{\mathbf{E_A} - \mathbf{E_B}}{\mathbf{Z_A} + \mathbf{Z_B}} \qquad ...(viii)$$

The physical interpretation of the second term in equations (*vii*) and (*viii*) is that it represents the cross-current between the secondaries. The first term shows how the actual load current divides between the loads. The value of current circulating in transformer secondaries (even when there is no-load) is given by* $\mathbf{I_C} = (\mathbf{E_A} - \mathbf{E_B})/(\mathbf{Z_A} + \mathbf{Z_B})$ assuming that $\mathbf{E_A} > \mathbf{E_B}$. It lags behind $\mathbf{E_A}$ by an angle α given by $\tan\alpha = (X_A + X_B)/(R_A + R_B)$. If $E_A + E_B$ the ratios of the currents are inversely as the impedances (numerical values).

* Under load conditions, the circulating current is

$$I_C = \frac{E_A - E_B}{Z_A + Z_B + Z_A Z_B/Z_L}$$

If $\quad Z_L = \infty$ *i.e.* on open-circuit, the expression reduces to that given above.

If in Eq. (*iv*) we substitute $\mathbf{I_B} = \mathbf{I} - \mathbf{I_A}$ and simplify, we get

$$\mathbf{I_A} = \frac{\mathbf{IZ_B}}{\mathbf{Z_A + Z_B}} + \frac{\mathbf{E_A - E_B}}{\mathbf{Z_A + Z_B}} \quad ...(ix)$$

Similarly, if we substitute $\mathbf{I_A} = \mathbf{I} - \mathbf{I_B}$ and simplify, then

$$\mathbf{I_B} = \frac{\mathbf{IZ_A}}{\mathbf{Z_A + Z_B}} + \frac{\mathbf{E_A - E_B}}{\mathbf{Z_A + Z_B}} \quad ...(x)$$

In a similar manner, value of terminal voltage $\mathbf{V_2}$ is given by

$$\mathbf{V_2 = IZ_L} = \frac{\mathbf{E_A Z_B + E_B Z_A - IZ_A Z_B}}{\mathbf{Z_A + Z_B}} \quad ...(xi)$$

These expressions give the values of transformer currents and terminal voltage in terms of the load current. The value of V_2 may also be found as under :

As seen from Fig. 30.70.

$$\mathbf{I_A} = (\mathbf{E_A} - \mathbf{V_2})/\mathbf{Z_A} = (\mathbf{E_A} - \mathbf{V_2})\mathbf{Y_A} ; \mathbf{I_B} = (\mathbf{E_B} - \mathbf{V_2})\mathbf{Y_B}$$

$$\mathbf{I} = \mathbf{V_2 Y_L} = \mathbf{I_A} + \mathbf{I_B} \text{ or } \mathbf{V_2 Y_L} = (\mathbf{E_A} - \mathbf{V_2})\mathbf{Y_A} + (\mathbf{E_B} - \mathbf{V_2})\mathbf{Y_B}$$

$$\therefore \quad \mathbf{V_2(Y_L + Y_A + Y_B)} = \mathbf{E_A Y_A + E_B Y_B} \text{ or } \mathbf{V_2} = \frac{\mathbf{E_A Y_A + E_B Y_B}}{\mathbf{Y_L + Y_A + Y_B}} \quad ...(xii)$$

Eq. (*xi*) gives $\mathbf{V_2}$ in terms of load current. But if only load kVA is given, the problem becomes more complicated and involves the solution of a quadratic equation in V_2.

Now, $\mathbf{S} = \mathbf{V_2 I}$. When we substitute this value of **I** in Eq. (*xi*), we get

$$\mathbf{V_2} = \frac{\mathbf{E_A Z_B + E_B Z_A - SZ_A Z_B / V_2}}{\mathbf{(Z_A + Z_B)}}$$

or $$\mathbf{V_2^2(Z_A + Z_B)} - \mathbf{V(E_A Z_B E_B Z_A)} + \mathbf{SZ_A Z_B} = 0$$

When $\mathbf{V_2}$ becomes known, then $\mathbf{I_A}$ and $\mathbf{I_B}$ may be directly found from

$$\mathbf{V_2} = \mathbf{E_A - I_A Z_A} \text{ and } \mathbf{V_2} = \mathbf{E_B - I_B Z_B}.$$

Note. In the case considered above, it is found more convenient to work with *numerical* values of impedances instead of % values.

Example 32.106. *Two transformers A and B are joined in parallel to the same load. Determine the current delivered by each transformer having given : open-circuit e.m.f. 6600 V for A and 6,400 V for B. Equivalent leakage impedance in terms of the secondary = 0.3 + j3 for A and 0.2 + j1 for B. The load impedance is 8 + j6.* **(Elect. Machines-I, Indore Univ. 1987)**

Solution.
$$\mathbf{I_A} = \frac{\mathbf{E_A Z_B + (E_A - E_B) Z_L}}{\mathbf{Z_A Z_B + Z_L (Z_A + Z_B)}}$$

Here $\mathbf{E_A} = 6{,}600$ V ; $\mathbf{E_B} = 6{,}400$ V ; $\mathbf{Z_L} = 8 + j\,6$; $\mathbf{Z_A} = 0.3 + j\,3$; $\mathbf{Z_B} = 0.2 + j\,1$

$$\mathbf{I_A} = \frac{6600(0.2 + j1) + (6600 - 6400)(8 + j6)}{(0.3 + j3)(0.2 + j1) + (8 + j6)(0.3 + j3 + 0.2 + j1)}$$

$$117 - j\,156 = 195\text{A in magnitude}$$

Similarly, $$\mathbf{I_B} = \frac{\mathbf{E_B Z_A - (E_A - E_B) Z_L}}{\mathbf{Z_A Z_B + Z_L (Z_A + Z_B)}}$$

$$= \frac{6400\,(0.3 + j3) - (6600 - 6400)\,(8 + j6)}{(0.3 + j3) + (0.2 + j1)\,(8 + j6) + (0.5 + j\,4)}$$

$$= 349 - j\,231 = 421 \text{ A (in magnitude)}$$

Example 32.107. *Two 1-φ transformers, one of 100 kVA and the other of 50 kVA are connected in parallel to the same bus-bars on the primary side, their no-load secondary voltages being 1000 V and 950 V respectively. Their resistances are 2.0 and 2.5 per cent respectively and their reactances 8 and 6 percent respectively. Calculate no-load circulating current in the secondaries.*

(Adv. Elect Machines, A.M.I.E. Sec. B, 1991)

Solution. The circuit connections are shown in Fig. 32.75.

1st transformer

Normal secondary current = 100,000/1000 = 100 A

$$R_A = \frac{1000 \times 2.0}{100 \times 100} = 0.2\ \Omega\ ;\ X_A = \frac{1000 \times 8}{100 \times 100} = 0.8\ \Omega$$

2nd Transformer

Normal secondary current = 50,000/950 = 52.63 A

$$R_B = 950 \times 2.5/100 \times 52.63 = 0.45\ \Omega$$

$$X_B = 850 \times 6/100 \times 52.63 = 1.08\ \Omega$$

Combined impedance of the two secondaries is

$$Z = \sqrt{(R_A + R_B)^2 + (X_A + X_B)^2}$$

$$= \sqrt{0.65^2 + 1.88^2} = 1.99\ \Omega$$

$\therefore\ I_c = (1000 - 950)/1.99 =$ **25.1 A** ; $\alpha = \tan^{-1}(1.88/0.65) =$ **71°**

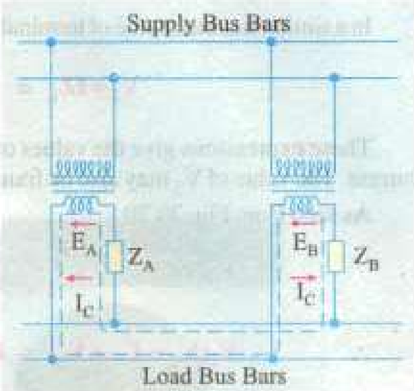


Fig. 32.75

Example 32.108. *Two single-phase transformers, one of 1000-kVA and the other of 500-kVA are connected in parallel to the same bus-bars on the primary side ; their no-load secondary voltages being 500 V and 510 V respectively. The impedance voltage of the first transformer is 3% and that of the second 5%. Assuming that ratio of resistance to reactance is the same and equal to 0.4 in each. What will be the cross current when the secondaries are connected in parallel ?*

(Electrical Machines-I, Madras Univ. 1985)

Solution. Let us first determine the ohmic value of the two impedances. Also, let the secondary voltage be 480 V*.

Full-load $I_A = 1000 \times 1000/480 = 2083$ A ; F.L. $I_B = 500 \times 1000/480 = 1042$ A

$$Z_A = \frac{\%Z_A \times E_A}{100 \times I_A} = \frac{9 \times 500}{100 \times 2083} = 0.0072\ \Omega$$

$$Z_B = \frac{5 \times 510}{100 \times 1042} = 0.0245\ \Omega$$

$$I_C = \frac{E_B - E_A}{Z_A + Z_B} = \frac{510 - 500}{(0.0072 + 0.0245)} = \textbf{315.4 A}$$

Note. Since the value of X/R is the same for the two transformers, there is no phase difference between E_A and E_B.

Example 32.109. *Two transformers A and B of ratings 500 kVA and 250 kVA are supplying a load kVA of 750 at 0.8 power factor lagging. Their open-circuit voltages are 405V and 415 V respectively. Transformer A has 1% resistance and 5% reactance and transformer B has 1.5% resistance and 4% reactance. Find (a) cross-current in the secondaries on no-load and (b) the load shared by each transformer.*

* Though it is chosen arbitrarily, its value must be less than either of the two no-load e.m.f.s.

Solution. As said earlier, it is more convenient to work with ohmic impedances and for that purpose, we will convert percentage value into numerical values by assuming 400 volt as the terminal voltage (this value is arbitrary but this assumption will not introduce appreciable error).

Now $$I_A R_A = 1\% \text{ of } 400 \quad \therefore \quad R_A = \frac{1}{100} \times \frac{400}{1250} = 0.0032\ \Omega$$

where $$I_A = 500{,}000/400 = 1250 \text{ A}$$

$$I_A X_A = 5\% \text{ of } 400\ ;\ X_A = \frac{5}{100} \times \frac{400}{1250} = 0.016\ \Omega\ (i.e.\ X_A = 5R_A)$$

In a similar way, we can find R_B and X_B ; $R_B = 0.0096\ \Omega$; $X_B = 0.0256\ \Omega$

$$\therefore \quad \mathbf{Z}_A = 0.0032 + j\,0.016 = 0.0163 \angle 78.5°\ ;\ \mathbf{Z}_B = 0.0096 + j\,0.0256 = 0.0275 \angle 69.4°$$

$$Z_A + Z_B = 0.0128 + j\,0.0416 = 0.0436 \angle 72.9°$$

Next step is to calculate load impedance. Let $\mathbf{Z}_L$ be the load impedance and V_2 the terminal voltage which has been assumed as 400 *V*.

$$\therefore \quad (V_2^{\ 2}/Z_L) = 750\angle -36.9°$$

$$\therefore \quad \mathbf{Z}_L = 400^2 \times 10^{-3}/750 \angle -36.9° = 0.214 \angle 36.9° = (0.171 + j0.128)\ \Omega$$

(*a*) $$\mathbf{I}_C = \frac{\mathbf{E}_A - \mathbf{E}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{(405 - 415)}{0.0436 \angle 72.9°} = -230 \angle -72.9°$$

(*b*) $$\mathbf{I}_A = \frac{405 \times 0.0275 \angle 69.4° + (405 - 415) \times 0.214 \angle 36.9°}{0.0163 \angle 78.5° \times 0.0275 \angle 69.4° + 0.214 \angle 36.9° \times 0.0436 \angle 72.9°}$$

$$= 970 \angle -35°$$

Similarly, $$I_B = \frac{415 \times 0.0163 \angle 78.5° - (405 - 415) \times 0.214 \angle 36.9°}{0.0163 \angle 78.5° \times 0.0275 \angle 69.4° + 214 \angle 369° \times 0.0436 \angle 72.9°}$$

$$\therefore \quad \mathbf{S}_A = 400 \times 970 \times 10^{-3} \angle -35° = 388 \angle -35° \text{ kVA}\ ;\ \cos\phi_A = \cos 35° = 0.82 \text{ (lag)}$$

$$\mathbf{S}_B = 400 \times 875 \times 10^{-3} \angle 42.6° = 350 \angle -42.6° \text{ kVA}$$

$$\cos\phi_B = \cos 42.6° = 0.736 \text{ (lag)}$$

Example 32.110. *Two transformers A and B are connected in parallel to a load of (2 + j 1.5) ohms. Their impedances in secondary terms are $Z_A = (0.15 + j\,0.5)$ ohm and $Z_B = (0.1 + j\,0.6)$ ohm. Their no-load terminal voltages are $E_A = 207 \angle 0°$ volt and $E_B = 205 \angle 0°$ volt. Find the power output and power factor of each transformer.* **(Elect. Machines-I, Punjab Univ. 1991)**

Solution. Using the equations derived in Art. 30.34 (*c*), we have

$$\mathbf{I}_A = \frac{\mathbf{E}_A \mathbf{Z}_B + (\mathbf{E}_A - \mathbf{E}_B)\mathbf{Z}_L}{\mathbf{Z}_A \mathbf{Z}_B + \mathbf{Z}_L(\mathbf{Z}_A + \mathbf{Z}_B)}$$

$$\mathbf{Z}_A = (0.15 + j\,0.5)\ \Omega\ ;\ Z_B = (0.1 + j\,0.6)\ \Omega\ ;\ \mathbf{Z}_L = (2 + j\,1.5) = 2.5 \angle 36.9°$$

$$\therefore \quad \mathbf{I}_A = \frac{207(0.1 + j\,0.6) + (207 - 205)\,(2 + j1.5)}{(0.15 + j0.5)(0.1 + j\,0.6) + (2 + j1.5)(0.25 + j1.1)}$$

$$= \frac{24.7 + j\,127.2}{-1.435 + j\,2.715} = \frac{129.7 \angle 79°}{3.07 \angle 117.9°} = 42.26 \angle -38.9° A = (32.89 - j26.55) A$$

$$\mathbf{I}_B = \frac{\mathbf{E}_B \mathbf{Z}_A - (\mathbf{E}_A - \mathbf{E}_B)\mathbf{Z}_L}{\mathbf{Z}_A \mathbf{Z}_B + \mathbf{Z}_L(\mathbf{Z}_A + \mathbf{Z}_B)} = \frac{205(0.15 + j\,0.5) - 2(2 + j\,1.5)}{-1.435 + j\,2.715} = \frac{103 \angle 75°}{3.07 \angle 117.9°}$$

$$= 33.56 \angle -42.9° = (24.58 - j\,22.84)\ A$$

Now $$\mathbf{V}_2' = \mathbf{I}\mathbf{Z}_L = (\mathbf{I}_A + \mathbf{I}_B)\,\mathbf{Z}_L$$

$$= (57.47 - j\,49.39)\,(2 + j\,1.5) = 189 - j\,12.58 = 189.4 \angle -3.9°$$

p.f. angle of transformer $A = -3.9° - (-38.9°) = 35°$

$\therefore$ p.f. of $A = \cos 35° = 0.818$ (lag) ; p.f. of $B = \cos[-3.9° - (-42.9°)] = 0.776$ (lag)

Power output of transformer A is $P_A = 189.4 \times 42.26 \times 0.818 =$ **6,548 W**

Similarly, $P_B = 189.4 \times 33.56 \times 0.776 =$ **4,900 W**

Example 32.111. *Two transformers have the following particulars :*

	Transformer A	*Transformer B*
Rated current	*200 A*	*600 A*
Per unit resistance 0.02	*0.025*	
Per unit reactance	*0.05*	*0.06*
No-load e.m.f.	*245 V*	*240 V*

Calculate the terminal voltage when they are connected in parallel and supply a load impedance of (0.25 + j 0.1) Ω. **(Elect. Machines-I, Sd. Patel Univ. 1981)**

Solution. Impedance in ohms $= \mathbf{Z}_{pu} \times$ N.L. e.m.f./full-load current

$$\mathbf{Z}_A = (245/200)(0.02 + j\,0.05) = 0.0245 + j\,0.0613\ \Omega = 0.066\ \angle 68.2°$$

$$Z_B = (240/600)(0.025 + j\,0.06) = 0.01 + j\,0.024\ \Omega = 0.026\ \angle 67.3°$$

$$\mathbf{Z} = (0.25 + j\,0.1) = 0.269\ \angle 21.8° \ ;\ Z_A + Z_B = 0.0345 + j\,0.0853 = 0.092\ \angle 68°$$

$$\mathbf{Z_L(Z_A + Z_B)} = 0.269 \times 0.092\ \angle 89.8° = 0.0247\ \angle 89.8° = (0 + j\,0.0247)$$

$$\mathbf{Z_A Z_B} = 0.066 \times 0.026\ \angle 135.5° = (-0.001225 + j\,0.001201)$$

$$\therefore\ \mathbf{Z_A Z_B + Z_L(Z_A + Z_B)} = (-0.00125 + j\,0.259) = 0.0259\ \angle 92.7°$$

Let us take $\mathbf{E_A}$ as reference quantity.

Also $\mathbf{E_b}$ is in phase with $\mathbf{E_A}$ because transformers are in parallel on both sides.

$$\mathbf{E_A Z_B} = 245(0.01 + j\,0.0245) = 2.45 + j\,5.87$$

$$\mathbf{E_B Z_A} = 240(0.0245 + j\,0.0613) = 5.88 + j\,14.7$$

$$\mathbf{E_A Z_B + E_B Z_A} = 8.33 + j\,20.57 = 22.15\ \angle 67.9°$$

Now, $$\mathbf{I} = \frac{\mathbf{E_A Z_B + E_B Z_A}}{\mathbf{Z_A Z_B + Z_L(Z_A + Z_B)}} = \frac{22.15\ \angle 67.9°}{0.0259\ \angle 92.7°} = 855\ \angle -24.8°$$

$$\therefore\ \mathbf{V_2} = \mathbf{IZ_L} = 885\ \angle -24.8° \times 0.269\ \angle 21.8° = 230 - 3°$$

Tutorial Problems 29.1

1. A 1000-kVA and a 500-kVA, 1-phase transformers are connected to the same bus-bars on the primary side. The secondary e.m.fs. at no-load are 500 and 510 V respectively. The impedance voltage of the first transformer is 3.4% and of the second 5%. What cross-current will pass between them when the secondaries are connected together in parallel ? Assuming that the ratio of resistance to reactance is the same in each, what currents will flow in the windings of the two transformers when supplying a total load of 1200 kVA.

   [(*i*) 290 A (*ii*) 1577 and 900 A] (*City & Guilds, London*)

2. Two transformers A and B are connected in parallel to supply a load having an impedance of $(2 + j\,1.5\Omega)$. The equivalent impedances referred to the secondary windings are $0.15 + j\,0.5\ \Omega$ and $0.1 + j\,0.6\ \Omega$ respectively. The open-circuit e.m.f. of A is 207 V and of B is 205 V. Calculate (*i*) the voltage at the load (*ii*) the power supplied to the load (*iii*) the power output of each transformer and (*iv*) the kVA input to each transformer.

   [(*i*) 189 ∠− 3.8° V (*ii*) 11.5 kW (*iii*) 6.5 kW, 4.95 kW (iv) 8.7 kVA, 6.87 kVA]

QUESTIONS AND ANSWERS ON TRANSFORMERS

Q.1. How is magnetic leakage reduced to a minimum in commerical transformers ?

Ans. By interleaving the primary and secondary windings.

Q.2. Mention the factors on which hysteresis loss depends ?

Ans. (*i*) Quality and amount of iron in the core (*ii*) Flux density and (*iii*) Frequency.

Q.3. How can eddy current loss be minimised ?

Ans. By laminating the core.

Q.4. In practice, what determines the thickness of the laminae or stampings ?

Ans. Frequency.

Q.5. Does the transformer draw any current when its secondary is open ?

Ans. Yes, no-load primary current.

Q.6. Why ?

Ans. For supplying no-load iron and copper losses in primary.

Q.7. Is Cu loss affected by power factor ?

Ans. Yes, Cu loss varies inversely with power factor.

Q.8. Why ?

Ans. Cu loss depends on current in the primary and secondary windings. It is well-known that current required is higher when power factor is lower.

Q.9. What effects are produced by change in voltage ?

Ans. 1. Iron loss.........varies approximately as V^2.

2. Cu loss.........it also varies as V^2 but decreases with an increase in voltage ***if constant kVA output is assumed***.

3. Efficiency.........for distribution transformers, efficiency at fractional loads decreases with increase in voltage while at full load or overload it increases with increase in voltage and *vice-versa*.

4. Regulation.........it varies as V^2 but decreases with increase in voltage ***if constant kVA output is assumed.***

5. Heating.........for constant kVA output, iron temperatures increase whereas Cu temperatures decrease with increase in voltages and ***vice-versa***.

Q. 10. How does change in frequency affect the operation of a given transformer ?

Ans. 1. Iron loss ....increases with a decrease in frequency. A 60-Hz transformer will have nearly 11% higher losses when worked on 50Hz instead of 60 Hz. However, when a 25-Hz transformer is worked on 60 Hz, iron losses are reduced by 25%.

2. Cu loss.........in distribution transformers, it is independent of frequecy.

3. Efficiency.........since Cu loss is unaffected by change in frequency, a given transformer efficiency is less at a lower frequency than at a higher one.

4. Regulation.........regulation at unity power factor is not affected because *IR* drop is independent of frequency. Since reactive drop is affected, regulation at low power factors decreases with a decrease in frequency and *vice-versa*. For example, the regulation of a 25-Hz transformer when operated at 50-Hz and low power factor is much poorer.

5. Heating.........since total loss is greater at a lower frequency, the temperature is increased with decrease in frequency.

OBJECTIVE TESTS – 32

1. A transformer transforms
   (a) frequency
   (b) voltage
   (c) current
   (d) voltage and current.
2. Which of the following is not a basic element of a transformer ?
   (a) core
   (b) primary winding
   (c) secondary winding
   (d) mutual flux.
3. In an ideal transformer,
   (a) windings have no resistance
   (b) core has no losses
   (c) core has infinite permeability
   (d) all of the above.
4. The main purpose of using core in a transformer is to
   (a) decrease iron losses
   (b) prevent eddy current loss
   (c) eliminate magnetic hysteresis
   (d) decrease reluctance of the common magnetic circuit.
5. Transformer cores are laminated in order to
   (a) simplify its construction
   (b) minimise eddy current loss
   (c) reduce cost
   (d) reduce hysteresis loss.
6. A transformer having 1000 primary turns is connected to a 250-V a.c. supply. For a secondary voltage of 400 V, the number of secondary turns should be
   (a) 1600 (b) 250
   (c) 400 (d) 1250
7. The primary and secondary induced e.m.fs. E_1 and E_2 in a two-winding transformer are **always**
   (a) equal in magnitude
   (b) antiphase with each other
   (c) in-phase with each other
   (d) determined by load on transformer secondary.
8. A step-up transformer increases
   (a) voltage (b) current
   (c) power (d) frequency.
9. The primary and secondary windings of an ordinary 2-winding transformer **always** have
   (a) different number of turns
   (b) same size of copper wire
   (c) a common magnetic circuit
   (d) separate magnetic circuits.
10. In a transformer, the leakage flux of each winding is proportional to the current in that winding because
    (a) Ohm's law applies to magnetic circuits
    (b) leakage paths do not saturate
    (c) the two windings are electrically isolated
    (d) mutual flux is confined to the core.
11. In a two-winding transformer, the e.m.f. per turn in secondary winding is **always**.......the induced e.m.f. power turn in primary.
    (a) equal to K times
    (b) equal to $1/K$ times
    (c) equal to
    (d) greater than.
12. In relation to a transformer, the ratio 20 : 1 indicates that
    (a) there are 20 turns on primary one turn on secondary
    (b) secondary voltage is 1/20th of primary voltage
    (c) primary current is 20 times greater than the secondary current.
    (d) for every 20 turns on primary, there is one turn on secondary.
13. In performing the short circuit test of a transformer
    (a) high voltage side is usually short circuited
    (b) low voltage side is usually short circuited
    (c) any side is short circuited with preference
    (d) none of the above.
    (Elect. Machines, A.M.I.E. Sec. B, 1993)
14. The equivalent resistance of the primary of a transformer having $K = 5$ and $R_1 = 0.1$ ohm when referred to secondary becomes.......ohm.
    (a) 0.5
    (b) 0.02
    (c) 0.004
    (d) 2.5
15. A transformer has negative voltage regulation when its load power factor is
    (a) zero
    (b) unity
    (c) leading
    (d) lagging.

16. The primary reason why open-circuit test is performed on the low-voltage winding of the transformer is that it
    (*a*) draws sufficiently large on-load current for convenient reading
    (*b*) requires least voltage to perform the test
    (*c*) needs minimum power input
    (*d*) involves less core loss.
17. No-load test on a transformer is carried out to determine
    (*a*) copper loss
    (*b*) magnetising current
    (*c*) magnetising current and no-load loss
    (*d*) efficiency of the transformer.
18. The main purpose of performing open-circuit test on a transformer is to measure its
    (*a*) Cu loss
    (*b*) core loss
    (*c*) total loss
    (*d*) insulation resistance.
19. During short-circuit test, the iron loss of a transformer is negligible because
    (*a*) the entire input is just sufficient to meet Cu losses only
    (*b*) flux produced is a small fraction of the normal flux
    (*c*) iron core becomes fully saturated
    (*d*) supply frequency is held constant.
20. The iron loss of a transformer at 400 Hz is 10 W. Assuming that eddy current and hysteresis losses vary as the square of flux density, the iron loss of the transformer at rated voltage but at 50 Hz would be....... watt.
    (*a*) 80 (*b*) 640
    (*c*) 1.25 (*d*) 100
21. In operating a 400 Hz transformer at 50 Hz
    (*a*) only voltage is reduced in the same proportion as the frequency
    (*b*) only kVA rating is reduced in the same proportion as the frequency
    (*c*) both voltage and kVA rating are reduced in the same proportion as the frequency
    (*d*) none of the above.
22. The voltage applied to the h.v. side of a transformer during short-circuit test is 2% of its rated voltage. The core loss will be.......percent of the rated core loss.
    (*a*) 4 (*b*) 0.4
    (*c*) 0.25 (*d*) 0.04
23. Transformers are rated in kVA instead of kW because
    (*a*) load power factor is often not known
    (*b*) kVA is fixed whereas kW depends on load p.f.
    (*c*) total transformer loss depends on volt-ampere
    (*d*) it has become customary.
24. When a 400-Hz transformer is operated at 50 Hz its kVA rating is
    (*a*) raduced to 1/8
    (*b*) increased 8 times
    (*c*) unaffected
    (*d*) increased 64 times.
25. At relatively light loads, transformer efficiency is low because
    (*a*) secondary output is low
    (*b*) transformer losses are high
    (*c*) fixed loss is high in proportion to the output
    (*d*) Cu loss is small.
26. A 200 kVA transformer has an iron loss of 1 kW and full-load Cu loss of 2kW. Its load kVA corresponding to maximum efficiency is .... kVA.
    (*a*) 100 (*b*) 141.4
    (*c*) 50 (*d*) 200
27. If Cu loss of a transformer at 7/8th full load is 4900 W, then its full-load Cu loss would be ....watt.
    (*a*) 5600 (*b*) 6400
    (*c*) 375 (*d*) 429
28. The ordinary efficiency of a given transformer is maximum when
    (*a*) it runs at half full-load
    (*b*) it runs at full-load
    (*c*) its Cu loss equals iron loss
    (*d*) it runs slightly overload.
29. The output current corresponding to maximum efficiency for a transformer having core loss of 100 W and equivalent resistance referred to secondary of 0.25 Ω is .... ampere.
    (*a*) 20
    (*b*) 25
    (*c*) 5
    (*d*) 400
30. The maximum efficiency of a 100-kVA transformer having iron loss of 900 kW and F.L. Cu loss of 1600 W occurs at .... kVA.
    (*a*) 56.3 (*b*) 133.3
    (*c*) 75 (*d*) 177.7

31. The all-day efficiency of a transformer depends primarily on
(a) its copper loss
(b) the amount of load
(c) the duration of load
(d) both (b) and (c).

32. The marked increase in kVA capacity produced by connecting a 2 winding transformer as an autotransfomer is due to
(a) increase in turn ratio
(b) increase in secondary voltage
(c) increase in transformer efficiency
(d) establishment of conductive link between primary and secondary.

33. The kVA rating of an ordinary 2-winding transformer is increased when connected as an autotransformer because
(a) transformation ratio is increased
(b) secondary voltage is increased
(c) energy is transferred both inductively and conductivity
(d) secondary current is increased.

34. The saving in Cu achieved by converting a 2-winding transformer into an autotransformer is determined by
(a) voltage transformation ratio
(b) load on the secondary
(c) magnetic quality of core material
(d) size of the transformer core.

35. An autotransformer having a transformation ratio of 0.8 supplies a load of 3 kW. The power transferred conductively from primary to secondary is.......kW.
(a) 0.6 (b) 2.4
(c) 1.5 (d) 0.27

36. The essential condition for parallel opearation of two 1-ϕ transformers is that they should have the same
(a) polarity
(b) kVA rating
(c) voltage ratio
(d) percentage impedance.

37. If the impedance triangles of two transformers operating in parallel are not identical in shape and size, the two transformers will
(a) share the load unequally
(b) get heated unequally
(c) have a circulatory secondary current even when unloaded
(d) run with different power factors.

38. Two transformers A and B having equal outputs and voltage ratios but unequal percentage impedances of 4 and 2 are operating in parallel. Transformer A will be running over-load by .... percent.
(a) 50 (b) 66
(c) 33 (d) 25

ANSWERS

1. d 2. d 3. d 4. d 5. b 6. a 7. c 8. a 9. c 10. b 11. c 12. d 13. b 14. d 15. c 16. a 17. c 18. b 19. b 20. b 21. b 22. d 23. c 24. a 25. c 26. b 27. b 28. c 29. a 30. c 31. d 32. d 33. c 34. a 35. b 36. a 37. d 38. c